The FRG approach to gauge theories &
applications to QCD

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Disclaimer

Results before ’06?

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↓

Introductory talk to gauge theories, Lefkada ’06

Litim

Outline

- **FRG in gauge theories**
  - FRG in Yang-Mills theory
  - gauge symmetry, gauge fixing & regularisation
  - dynamical hadronisation
- **Yang-Mills theory**
  - propagators
  - confinement
  - finite temperature
- **QCD**
  - confinement & chiral symmetry breaking
  - many-flavour QCD
  - phase diagram of QCD
- **Summary & outlook**

Functional RG in gauge theories

FunMethods in gauge theories

FunMethods: FRG-DSE-2PI-...

The FRG in Yang-Mills theory

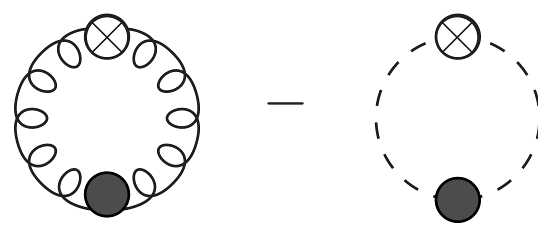
Functional RG in gauge theories

Wetterich '93

$$\textcolor{red}{k\partial_k}\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + \textcolor{red}{R_k(p)}}\,\textcolor{red}{k\partial_k R_k(p)}$$

- **Yang Mills Theory:** $\phi = (A, C, \bar{C})$ RG-scale k: $t = \ln k$

$$\partial_t \Gamma_k[\phi] = \tfrac{1}{2}$$



- **Fermions are straightforward** though 'physically' complicated
  - **no sign problem** numerics as in scalar theories
  - **chiral fermions** reminder: Ginsparg-Wilson fermions from RG arguments
  - **bound states via dynamical hadronisation** effective field theory techniques

Functional RG in gauge theories

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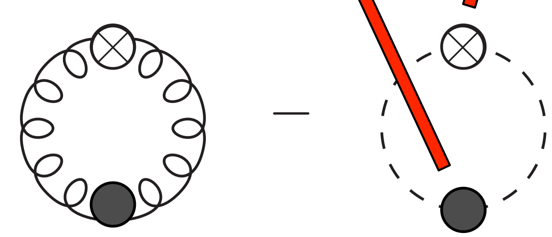

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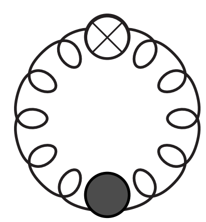
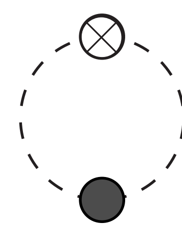
$\partial_t \Gamma_k[\phi] \;=\; \frac{1}{2}$  $-$ 

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FunMethods: FRG-DSE-2PI-...

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Complementary to lattice!

e.g. finite volume scaling: Braun, Klein, Piasecki '10

Gauge symmetry, gauge fixing & regularisation

Gauge symmetry

- **non-Abelian gauge symmetry:** $U = e^{i\omega} \in SU(N)$

$$\delta_\omega : \; gA_\mu \to U^{-1} g A_\mu U - i\, U^{-1} \partial_\mu U$$

- **classical action is invariant under gauge transformations**

$$S_{\rm YM}[A^U] = S_{\rm YM}[A] \qquad \textbf{with} \qquad S_{\rm YM}[A] = \frac{1}{2} \int_x {\rm tr}\, F_{\mu\nu}^2$$

and field strength

$$F^a_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i g f^{abc} A^b A^c$$

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- **gauge symmetry** ⟷ **redundancy in field degrees of freedom**
  - **gauge fixing (necessarily breaking of gauge invariance)**
  - **gauge invariant variables (necessarily non-local)**

Gauge fixing

- **gauge fixing and ghost term (Jacobian), e.g. covariant gauge** $\partial_\mu A_\mu = 0$

$$\frac{1}{2\xi}\int_x (\partial A)^2 + \int_x \bar{C}\cdot\partial D\cdot C$$

- **Slavnov-Taylor identities for effective action** $\Gamma[\phi]$ **with** $\phi = (A\,,\,C\,,\,\bar{C})$

$$\delta_\omega(\Gamma - S_{\rm cl}) + \text{loops} = 0$$

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- **BRST Master equation with anti-fields**

$$\int_x \frac{\delta\Gamma}{\delta\phi}\frac{\delta\Gamma}{\delta\phi^*} = 0$$

EoM for anti-fields ϕ^* **are the symmetry transformations**

Regularisation

- **Cut-off term**

$$\frac{1}{2}\int_x A\cdot R_k^A\cdot A+\int_x \bar{C}\cdot R_k^C\cdot C$$

- **Slavnov-Taylor identities for effective action** $\Gamma[\phi]$

$$\delta_\omega(\Gamma - S_{\rm cl}) + \text{loops} = \frac{1}{2}\text{Tr}\,\left(U R_k U^{-1}\right) G_k[\phi]$$

Bonini, Ellwanger, Litim, Marchesini, Morris, JMP, Reuter, Weber, Wetterich, ...

- **BRST Master equation with anti-fields**

$$\int_x \frac{\delta\Gamma}{\delta\phi}\frac{\delta\Gamma}{\delta\phi^*} = \Delta\Gamma[\phi,\phi^*]$$

STI/ME → modified STI/modified ME

Igarashi, Itoh, Itou, Kugo, Sonodo, ...

Gauge invariant flows

- **covariant cut-off**

Morris '99
Morris, Rosten'06
Arnone, Morris, Rosten'06
Rosten'10

$$\mathrm{tr} \int F_{\mu\nu}\, c^{-1}(D^2/k^2)\, F_{\mu\nu} \rightarrow$$

Polchinski flow

- **SU(N) → spontaneously broken SU(N|N)**

- **Pauli-Villars fields**

Locality?

Gauge invariant flows

- **covariant cut-off**

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→ **Polchinski flow**

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Complicated

Gauge invariant flows

- **covariant cut-off**

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$$\mathrm{tr} \int F_{\mu\nu}\, c^{-1}(D^2/k^2)\, F_{\mu\nu}$$ → **Polchinski flow**

- **SU(N) → spontaneously broken SU(N|N)**

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Remarks

- **Polchinski flow: well-suited for formal developments**

- **Wetterich flow: well-suited for numerics**

Gauge invariant flows

- **covariant cut-off**

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Morris, Rosten'06
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  - **SU(N) → spontaneously broken SU(N|N)**
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Results

- **two-loop YM beta function & one-loop QCD beta function**

Manifestly gauge invariant

- **one loop computation for thin Wilson loops**
- **speculation of infrared slavery scenario**

within ?renormalised? strong coupling expansion

Gauge invariant flows

- **geometrical approach**

Branchina, Meissner, Veneziano '03
JMP '03

$$\phi_\mu(A) = \bar{A}_\mu + a_\mu + O(a^2)$$

- **effective action only depends on gauge invariant part of** ϕ

- **Non-locality & modified Nielsen identity** JMP '03

Gauge invariant flows

- **geometrical approach**

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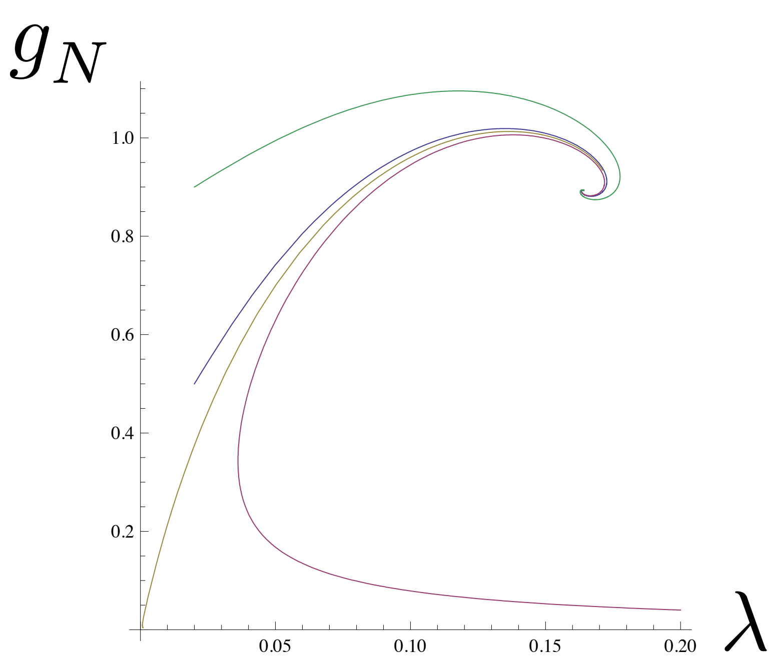

Results

UV fixed point in gravity

bi-metric expansion

Diploma thesis Donkin '08
Donkin, JMP, work in prep.

Gauge invariant flows

- **background field approach**

Reuter, Wetterich '93

$$\phi_\mu(A) = \bar{A}_\mu + a_\mu$$

  - **linear split in ϕ**

  - **background gauge invariance & modified fluctuation STIs**

Reuter, Wetterich '97
Freire, Litim, JMP '00

Gauge invariant flows

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Reuter, Wetterich '94

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Reuter, Wetterich '97
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→ **gauge-fixed setting**

see gravity talks

Gauge invariant flows

- **background field approach** — Reuter, Wetterich '94

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Reuter, Wetterich '97
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Remarks

  - **'single metric' truncation in YM**

one loop beta function non-universal! — Litim, JMP '02

  - **'Einstein-Hilbert' truncation in YM**

no confinement! — see confinement section

Gauge invariant flows

Curci-Ferrari-Delbourgo-Jarvis model

Tissier, Wschebor '08

- **deformation of Yang-Mills theory**

- **locality?**

see talk of M. Tissier

Dynamical hadronisation

see talk of C. Wetterich

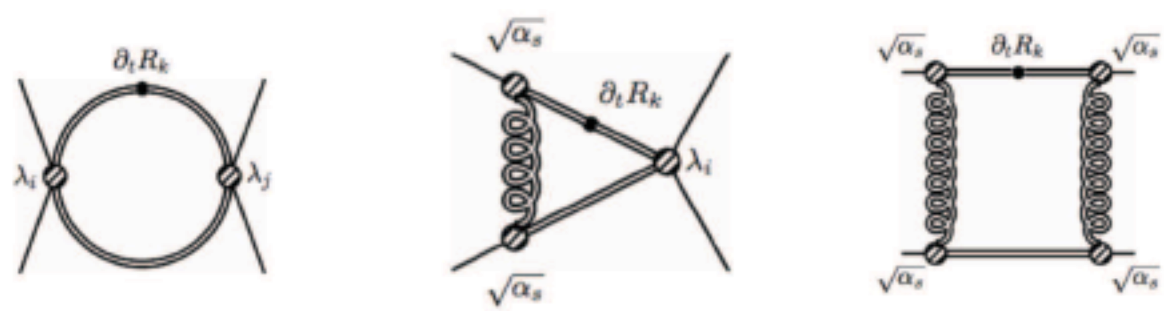
Dynamical hadronisation

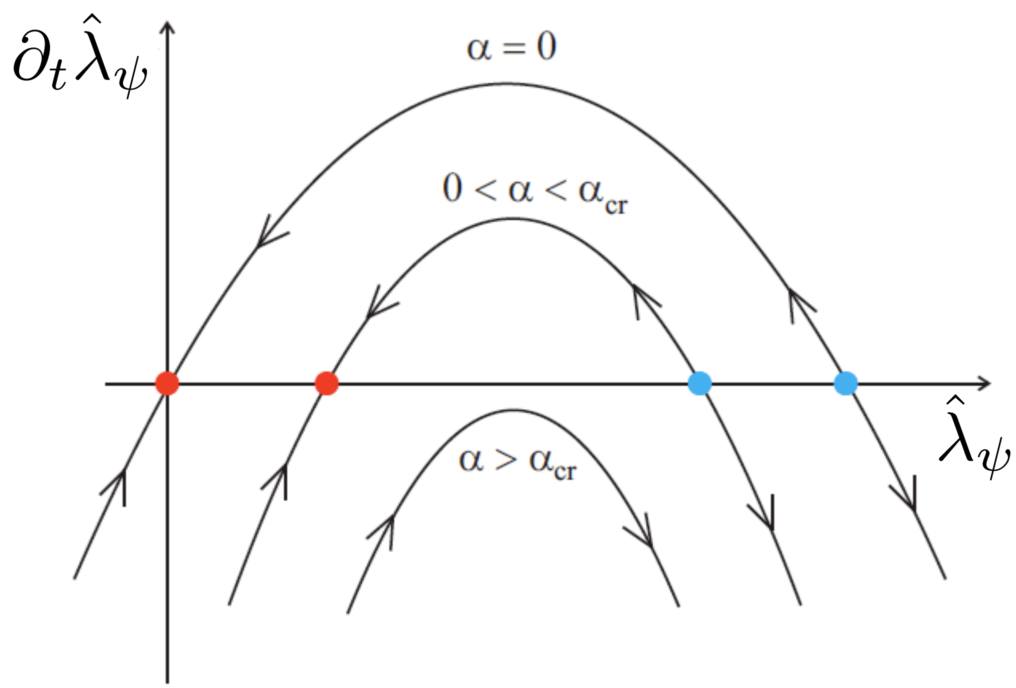
A glimpse at chiral symmetry breaking

Flow of four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale k

$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi - A(\tfrac{T}{k})\,\hat{\lambda}_\psi^2 - B(\tfrac{T}{k})\,\hat{\lambda}_\psi\,\alpha_s - C(\tfrac{T}{k})\,\alpha_s^2$$

$\Uparrow$ $\Uparrow$ $\Uparrow$





$m^2 \propto \frac{1}{\lambda}$

$\Longrightarrow$

$\alpha_s > \alpha_{s,crit}$: chiral symmetry breaking

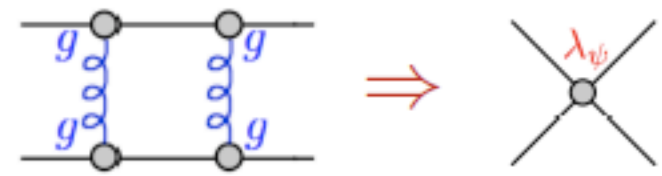
Gies, Wetterich '02

Gies, Jaeckel '05

Braun, Gies '06

Dynamical hadronisation

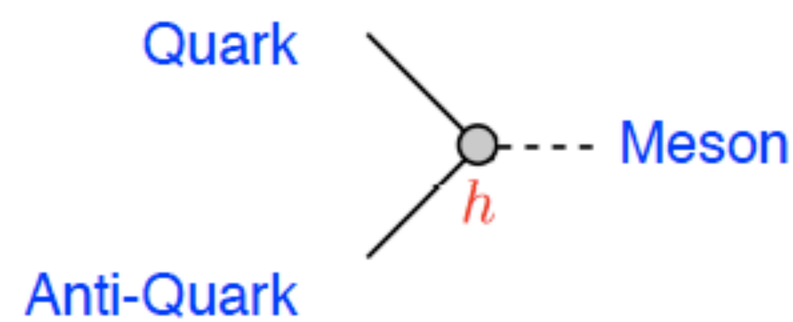
$$g\,\bar\psi \not{A}\,\psi \quad \xrightarrow{\int_p}$$



Hubbard-Stratonovich

$$\lambda_\psi\,(\bar\psi\psi)^2 = h\,\bar\psi\psi\,\sigma - \tfrac{1}{2}m^2\sigma^2$$

with $m^2 = -\dfrac{h^2}{2\lambda_\psi}$ and EoM(σ) $\Longrightarrow$



+Baryons and Glueballs

+Baryonisation

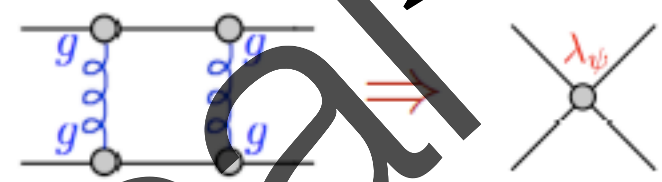
Dynamical degrees of freedom

Quarks, Gluons $\psi,\ A$ $\quad\Longrightarrow\quad$ $\psi,\ A$ + Mesons, Baryons $\phi \sim \bar\psi\psi,\ b \sim \psi^3$

Dynamical hadronisation

Dynamical hadronisation ⟸ (Functional RG-flows)

$g\,\bar\psi \not{A}\,\psi \quad \xrightarrow{\int_{k-dk}^{k}}$



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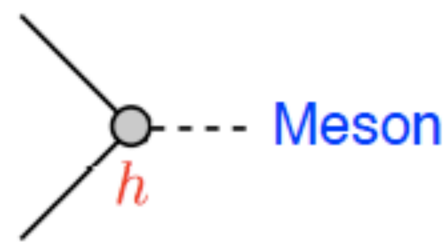

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Gies, Wetterich '01

JMP '05

also very successfully used in ultracold atoms!

Flörchinger, Wetterich '09

Yang-Mills theory

first you walk ...

Propagators

Propagators

- **Yang-Mills propagators in Landau gauge (’96 - today)**

  - **DSE, FRG, Stochastic Quantisation, Lattice** von Smekal, Hauck, Alkofer ‘96

    FRG: Litim, Nedelko, JMP, von Smekal ’03

    Aguilar, Alkofer, Binosi, Bicudo, Bloch, Boucaud, Bogolubsky, Bornyakov, Bowman, Braun, Cucchieri, De Soto, Dudal, Ellwanger, Fischer, Fister, Gies, Gracey, Haas, Hirsch, Huber, Ilgenfritz, Langfeld, Leder, Leinweber, Leroy, Litim, Llanes-Estrada, Nakamura, Natale, Nedelko, Maas, Mendes, Micheli, Müller-Preußker, Oliveira, Papavassilio, JMP, Pene, Petreczky, Quandt, Reinhardt, Rodriguez-Quintero, Schwenzer, Silva, Skullerud, Sorella, Spielmann, Stamatescu, Sternbeck, Vandersickel, Verschelde, von Smekal, Wambach, Watson, Weber, Wetterich, Williams, Zwanziger, ...

    - **Numerical solutions**

    - **Analytic IR-asymptotics** IR-scaling & Gribov ambiguity

Propagators

- **Yang-Mills propagators in Landau gauge (’96 - today)**

  - **FRG**

Braun
Fischer Fister Gies Haas Leder
Litim Nedelko
JMP Reinhardt
Weber

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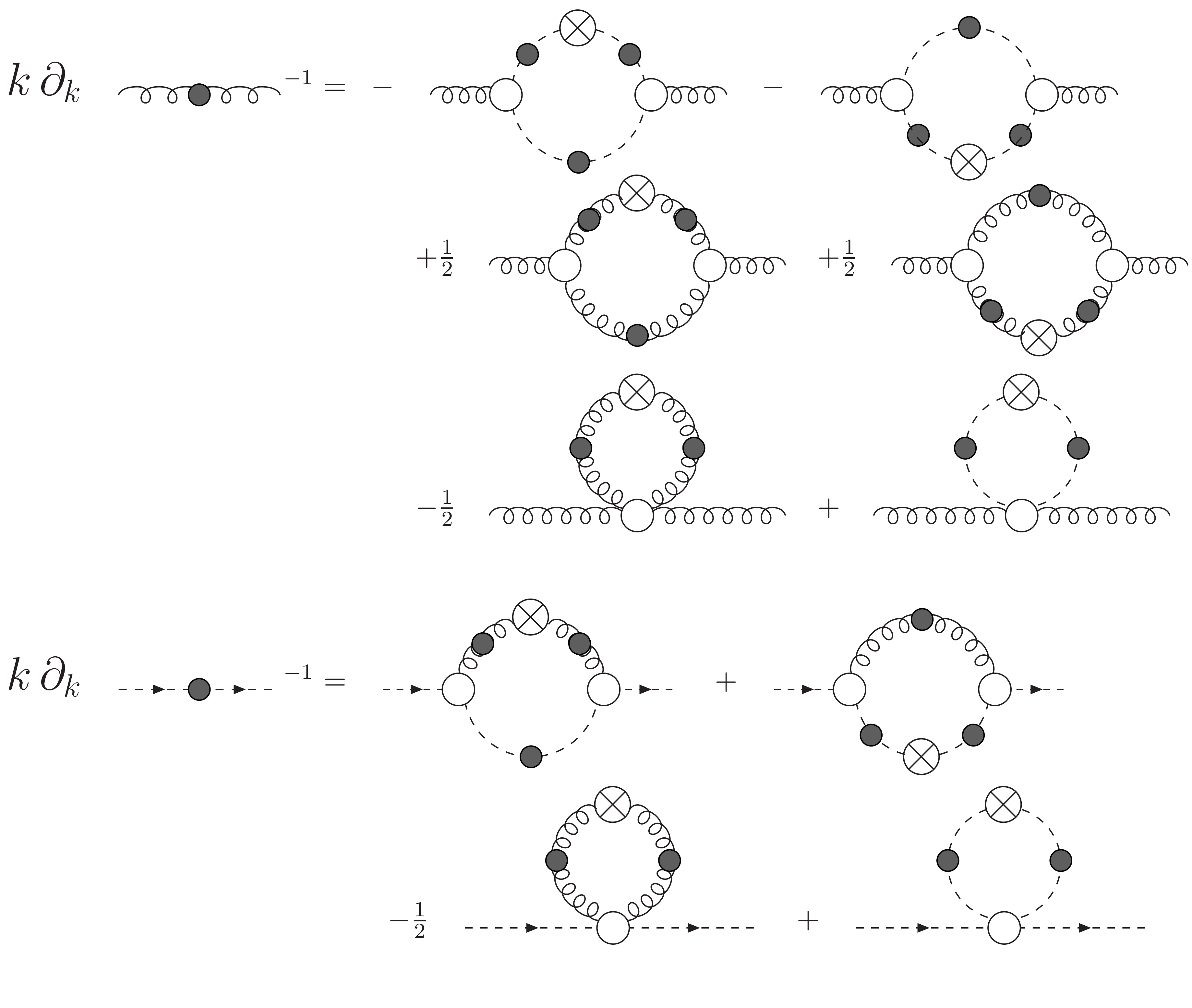
Propagators



Propagators

Truncation schemes

- derivative expansion
- vertex expansion
- BMW expansion
- resummation schemes & 'Ward identities'
- mixtures & more

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Propagators

JMP, in prep.

- **full momentum dependence of propagators**
- **vertices with momentum-dependent RG-dressing**
- **functional optimisation** JMP '05

Optimisation, Litim '00

- **functional relations between diagrams:** Flow=Flow(DSE,2PI)
- **scaling/decoupling via boundary condition at** $p^2 = 0$

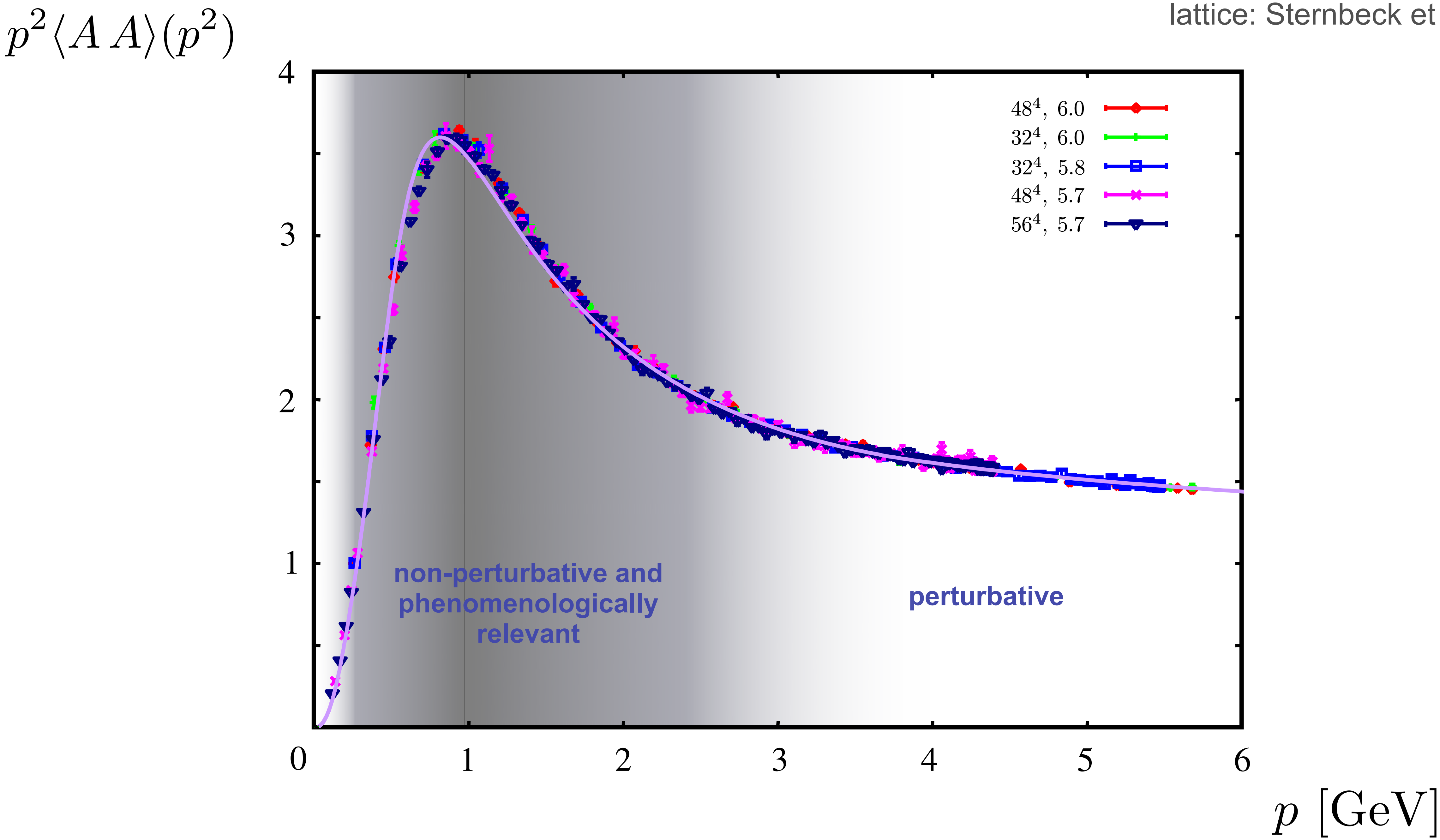
$$k\partial_k \langle A(p)A(-p)\rangle = \mathrm{Flow}_{\mathrm{A}}[\langle AA\rangle, \langle C\bar{C}\rangle]$$

$$k\partial_k \langle C(p)\bar{C}(-p)\rangle = \mathrm{Flow}_{\mathrm{C}}[\langle AA\rangle, \langle C\bar{C}\rangle]$$

Propagators

FRG: Fischer, Maas, JMP'08; JMP, in prep.

lattice: Sternbeck et al. '06

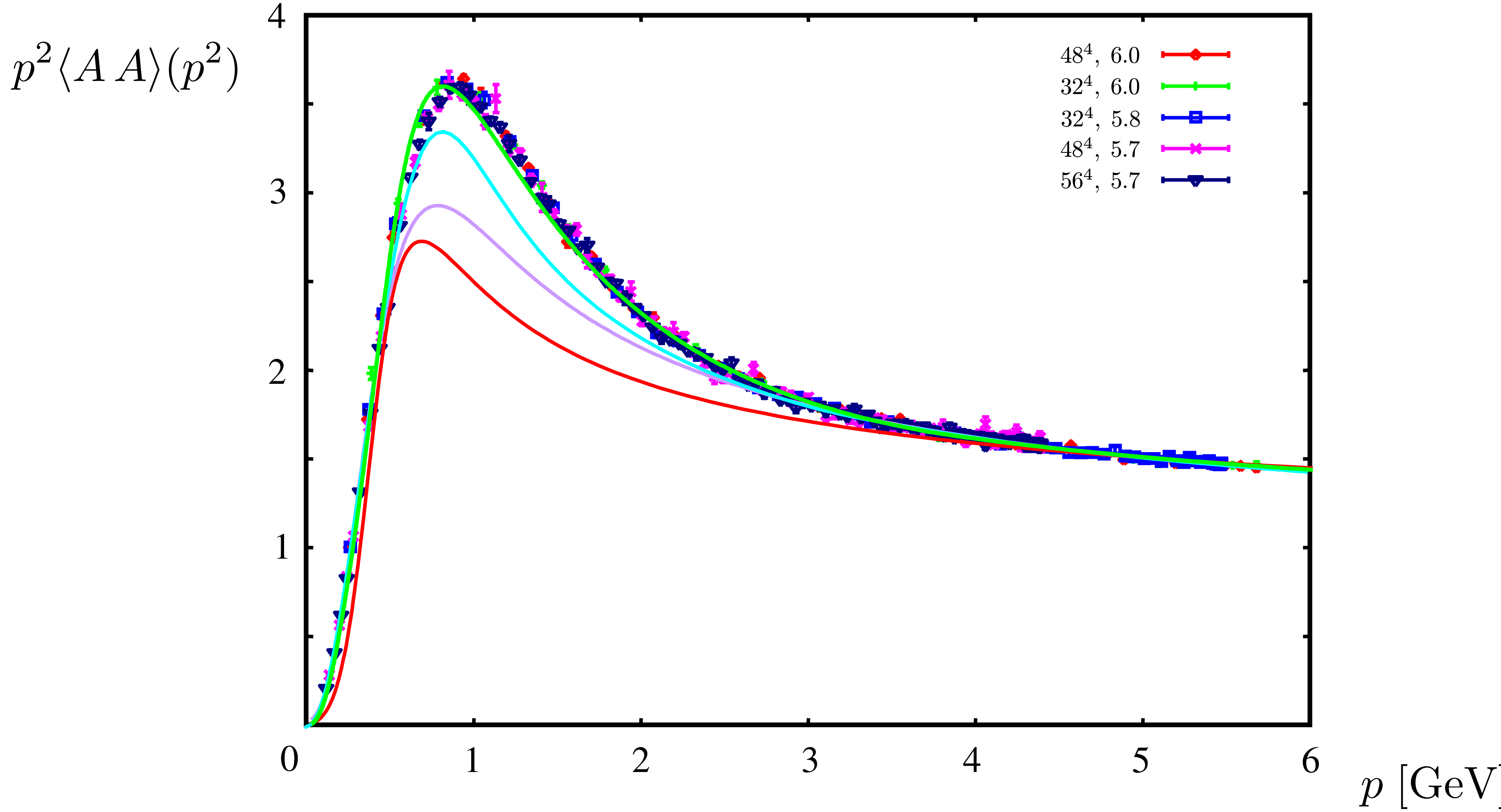


Propagators phenomenologically well described in 1/N expansion

Propagators

Pure Yang-Mills, $T = 0$

lattice: Sternbeck et al. ’06



- von Smekal, Hauck, Alkofer ’97
- Lerche, von Smekal, Phys. ’02
  Fischer, Alkofer, Phys.Rev. ’02
- JMP, Litim, Nedelko, von Smekal ‘03; JMP ’06 (unpublished)
- Fischer, Maas, JMP’08; JMP, in prep.

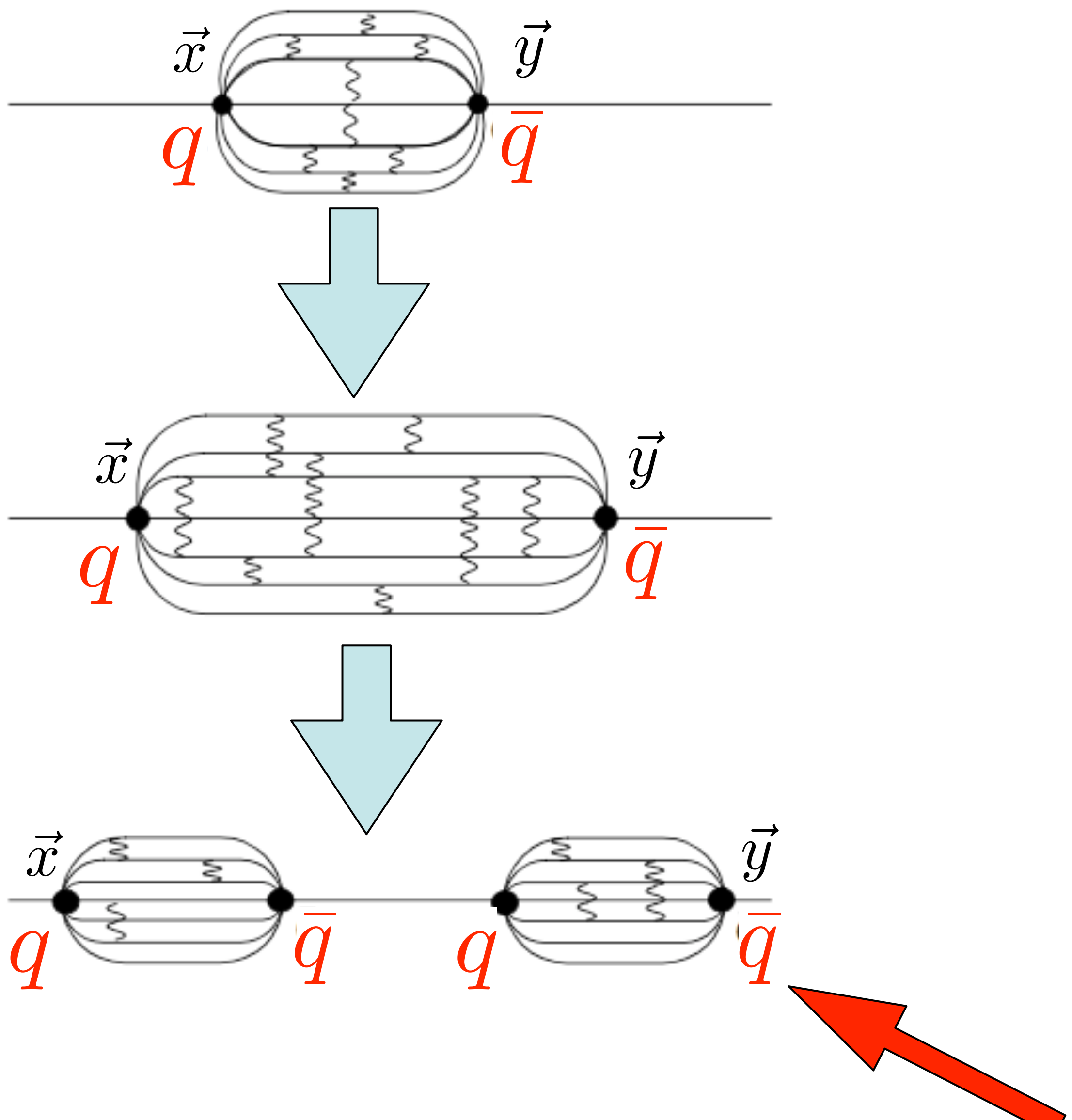
then you run ...

Confinement

Confinement

$\boldsymbol{r} = |\vec{x} - \vec{y}|$

$$\langle q(\vec{x})\bar{q}(\vec{y})\rangle \simeq e^{-\beta F_{q\bar{q}}(r)}$$



$$F_{q\bar{q}} \simeq -\frac{1}{r}$$

$$F_{q\bar{q}} \simeq \sigma r$$

$$F_{q\bar{q}} \simeq \sigma r_0$$

string breaking at $r \approx 1.1 fm$

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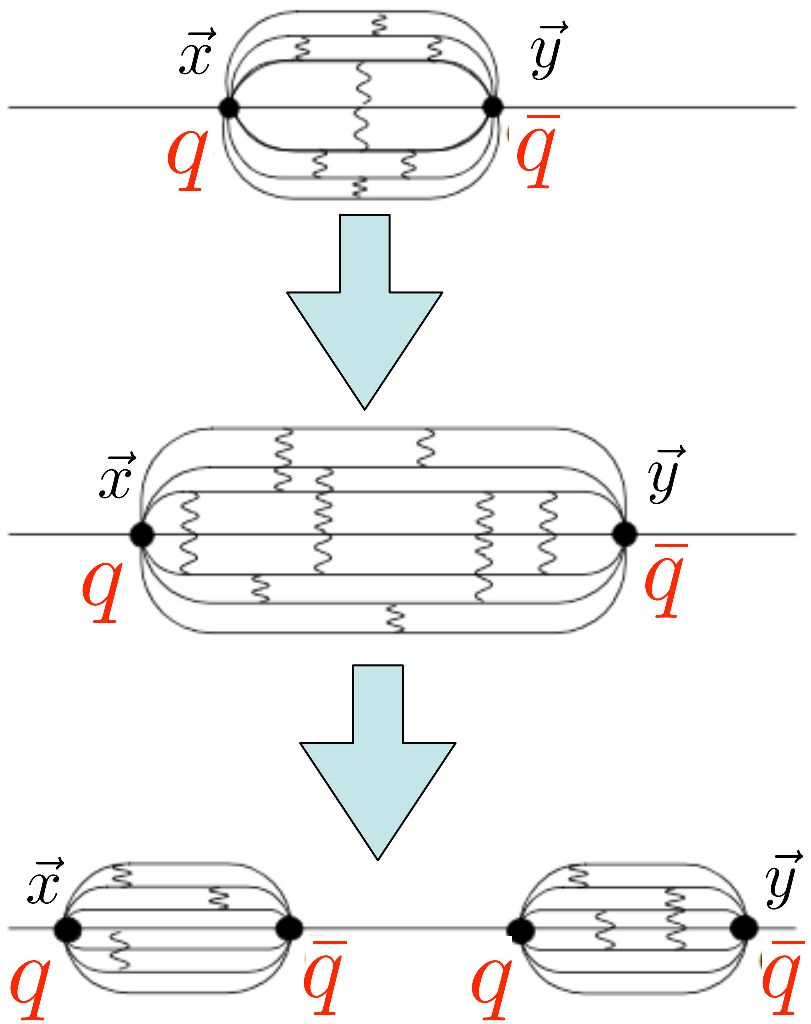

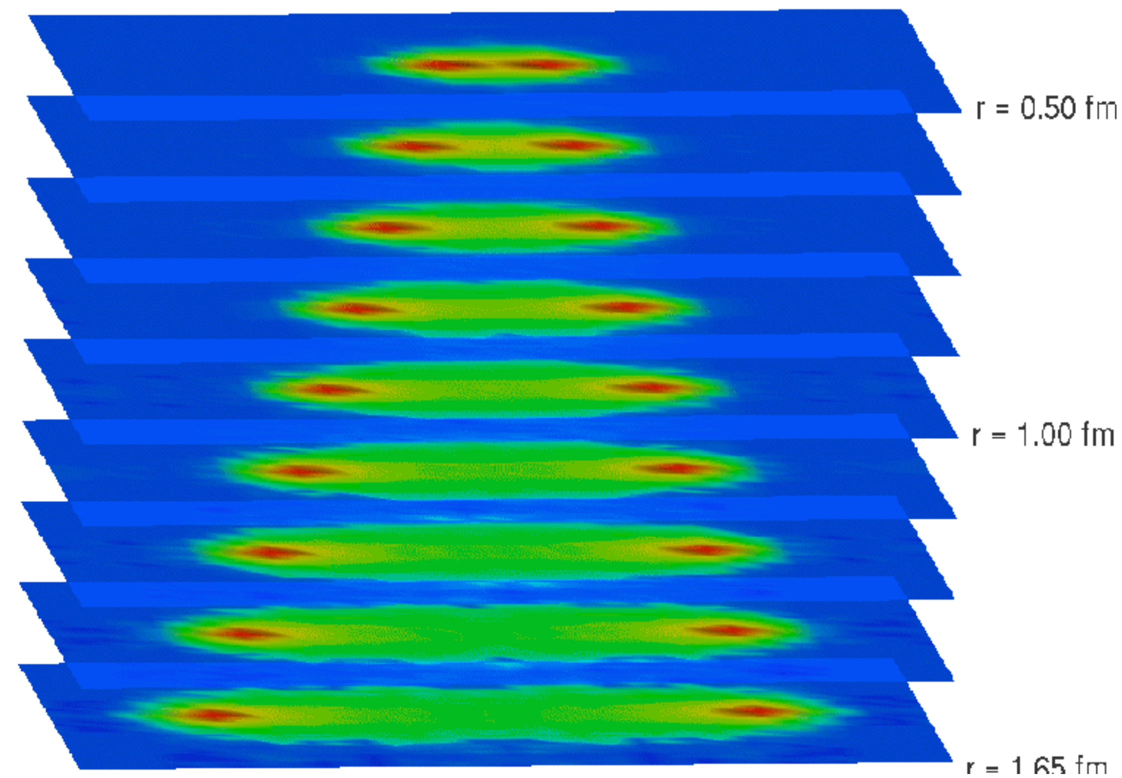

Bali et al. '94

string breaking at $r \approx 1.1 fm$

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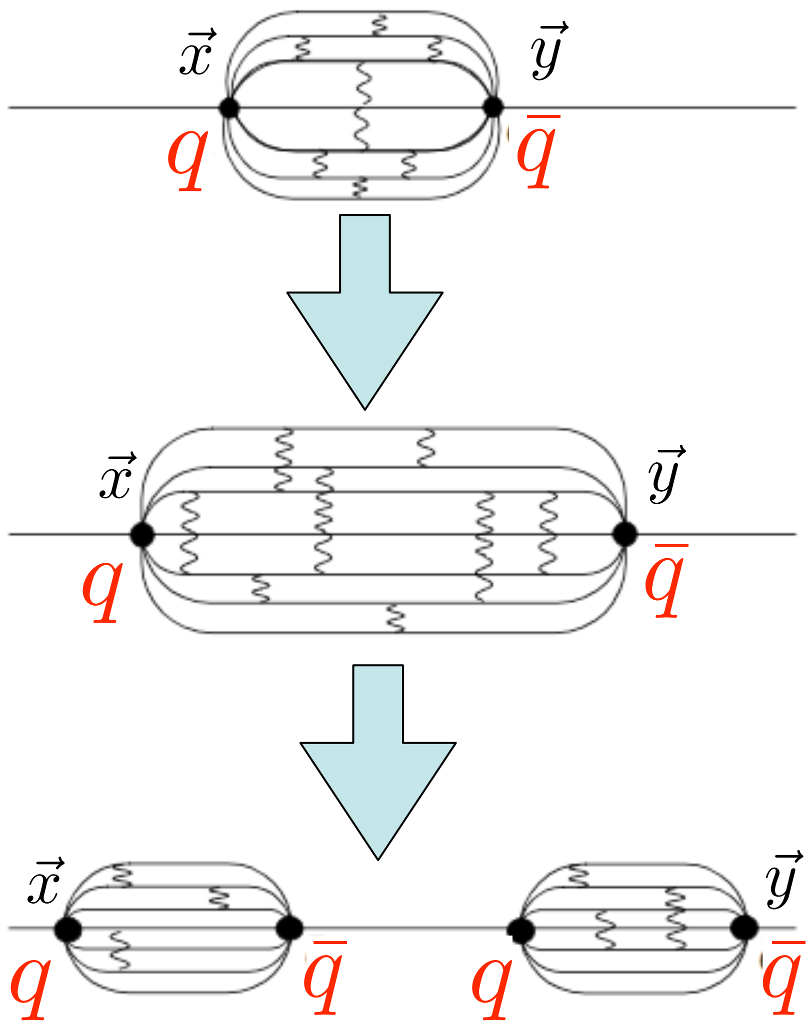
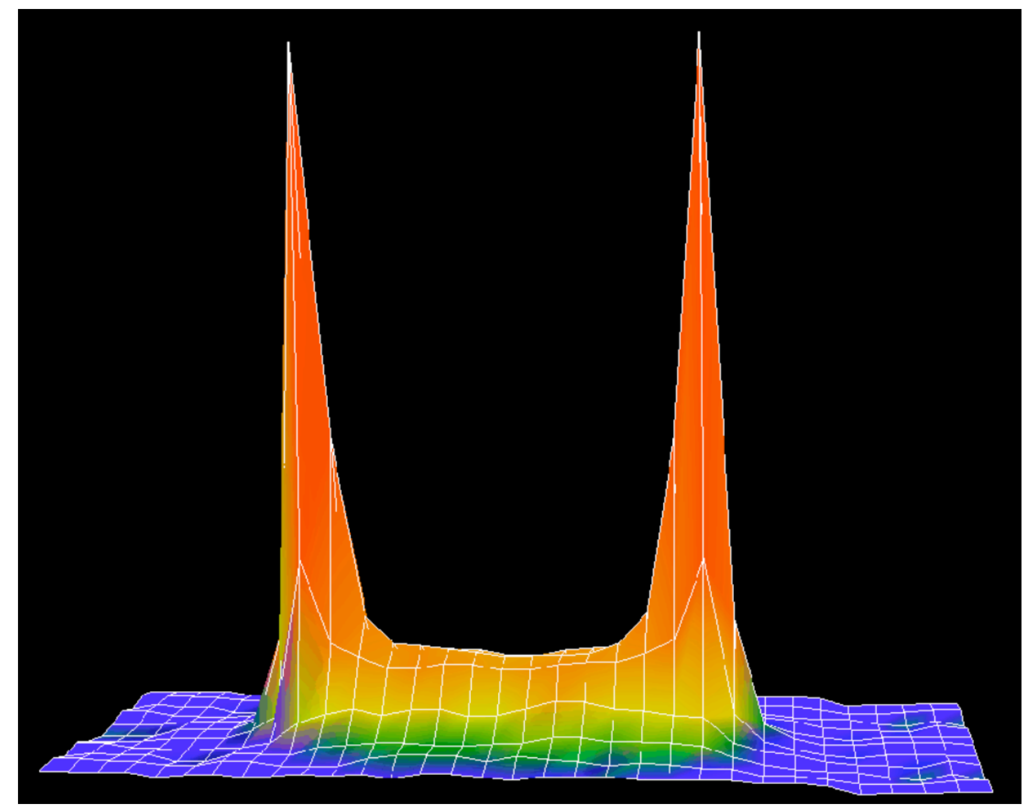


Energy density



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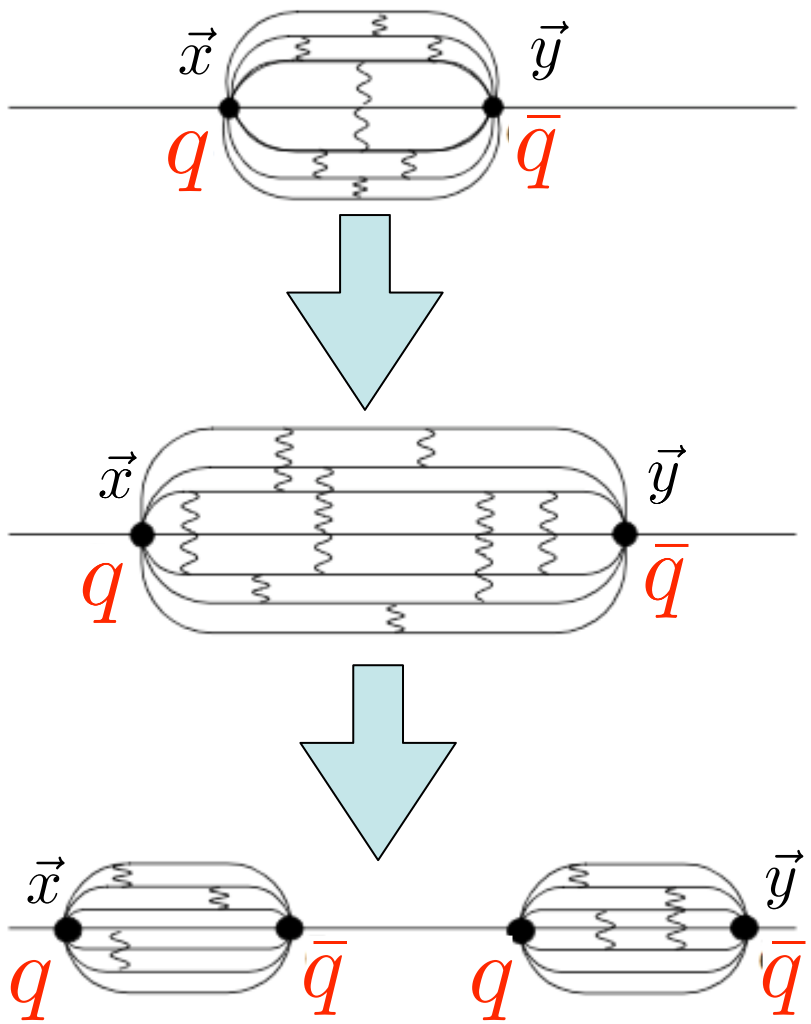


Order parameter $\sim \, '\langle q \rangle'$

$$\Phi = e^{-\frac{1}{2}\beta F_{q\bar{q}}(\infty)}$$

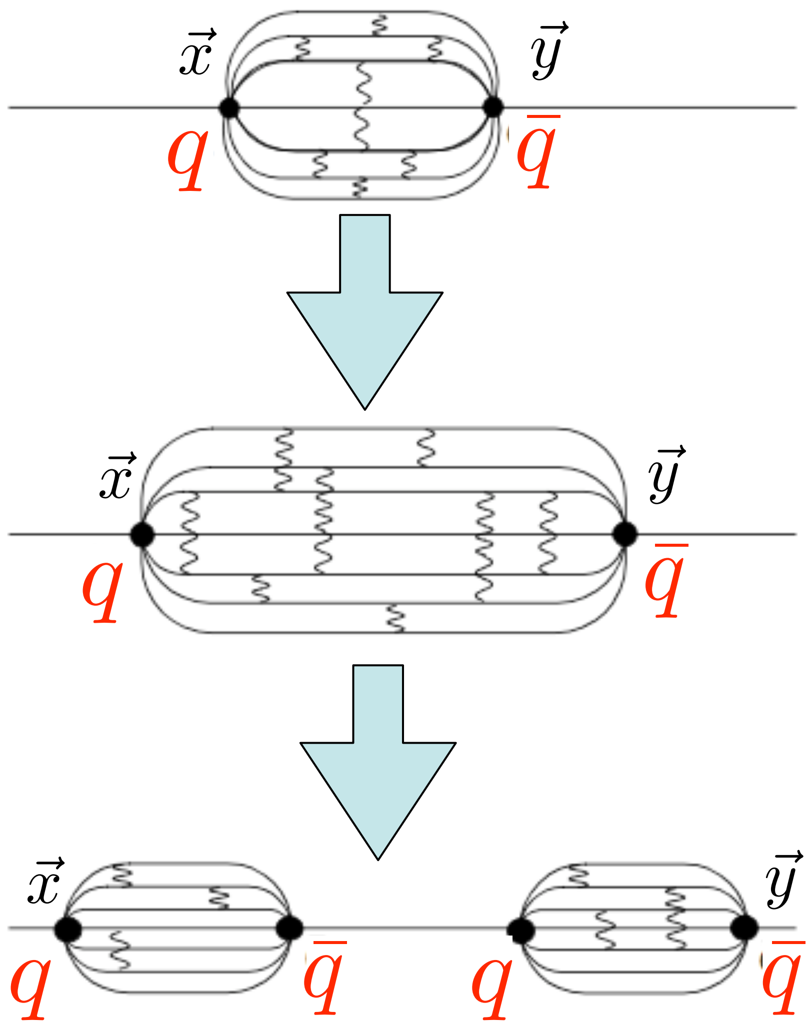
- **Confinement:** $\Phi = 0$
- **Deconfinement:** $\Phi \neq 0$

Φ **Polyakov loop**

$$\Phi = \frac{1}{3}\langle \mathrm{Tr}\, \mathcal{P} \exp\{ig \int_0^{1/T} dx_0 A_0\}\rangle$$

Confinement

$r = |\vec{x} - \vec{y}|$



string breaking at $r \approx 1.1 fm$

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Symmetry

- Z_3 **- symmetry:** $q \rightarrow zq$
- **broken by dynamical quarks**

Confinement

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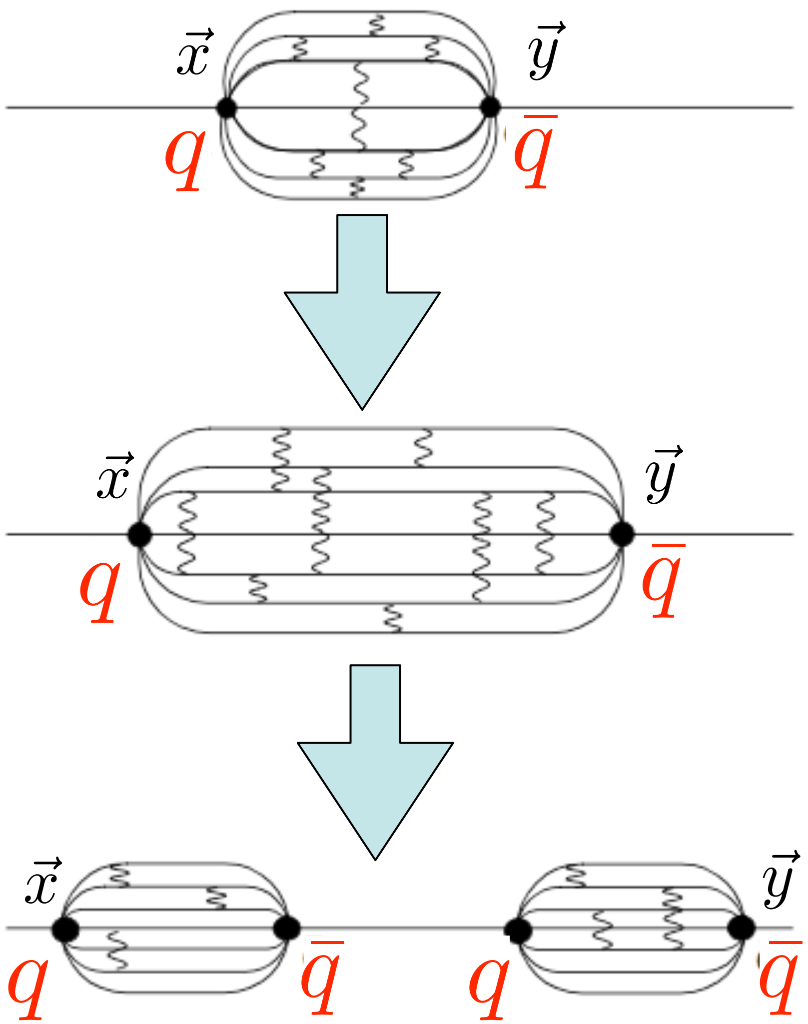

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Mechanism

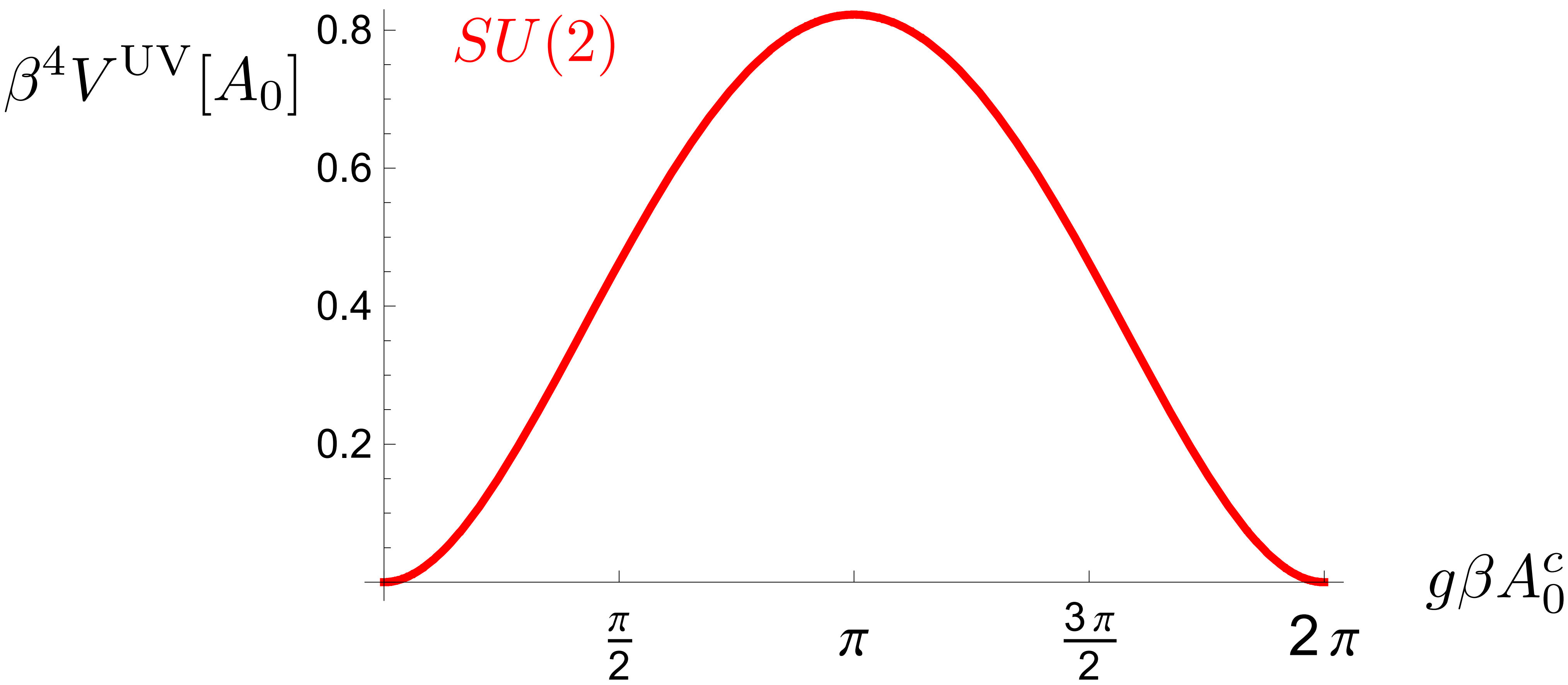
- **not fully resolved**

Confinement

Perturbation theory

Gross, Pisarski, Yaffe '81
Weiss '81

$$V^{\mathrm{UV}}[A_0] = \frac{1}{2\Omega}\,\mathrm{Tr}\log S^{(2)}_{AA}[A_0] - \frac{1}{\Omega}\mathrm{Tr}\log S^{(2)}_{C\bar{C}}[A_0]$$



$$SU(2):\ \Phi[A_0] = \cos\frac{1}{2}\beta g A_0^c \qquad \text{with} \qquad A_0 = A_0^c\,\frac{\sigma_3}{2}$$

Confinement

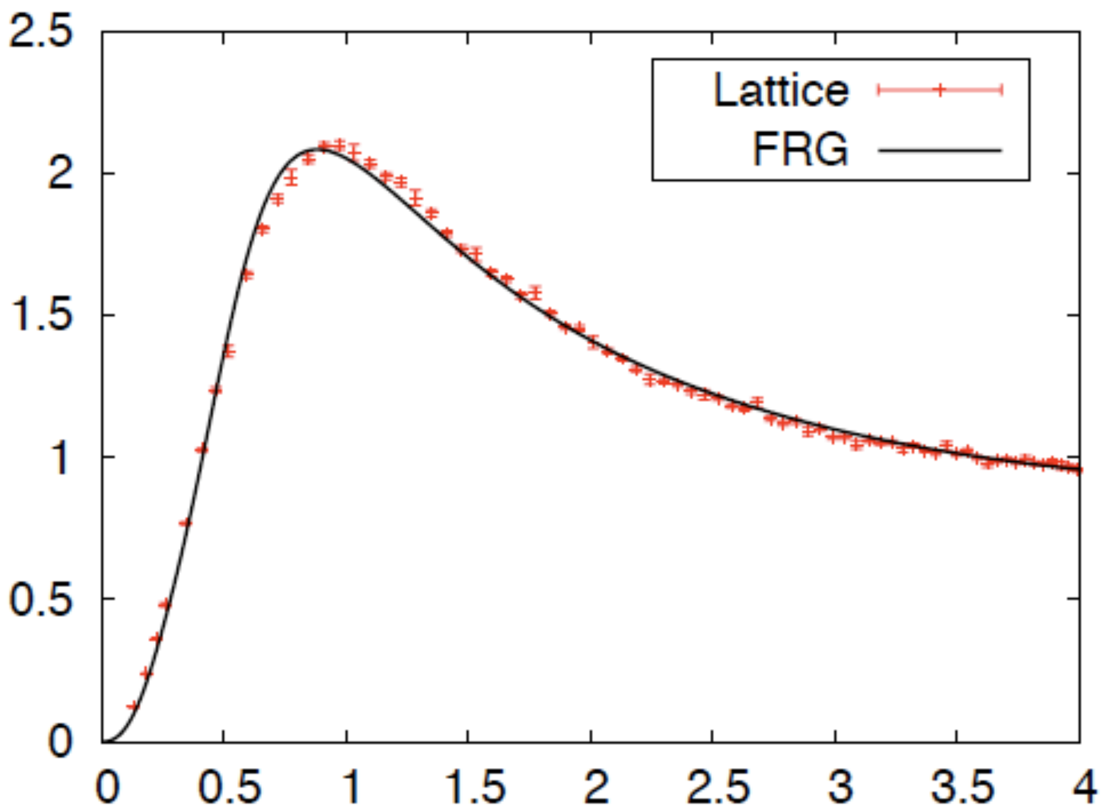
Continuum methods ← (Functional RG-flows)

Braun, Gies, JMP '07

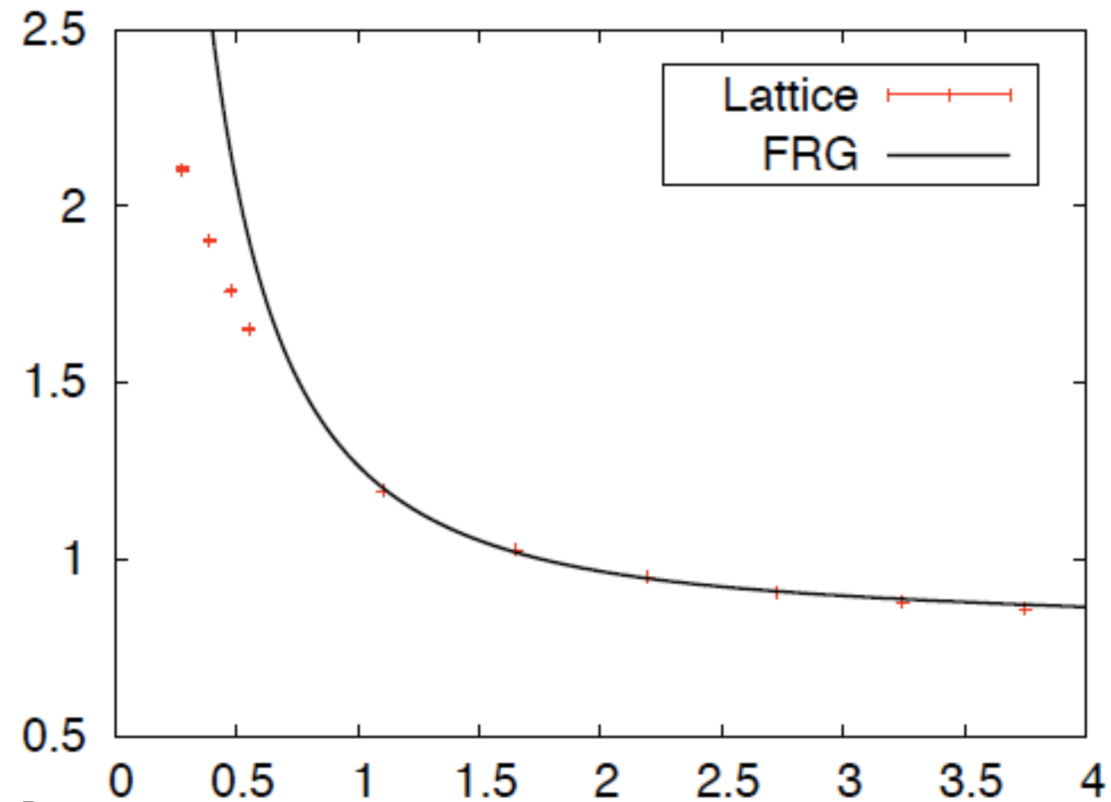
$$V[A_0] = -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) - \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

$$p_0 \to 2\pi T n - g A_0$$

$p^2\langle A\,A\rangle(p^2)$



$p^2\langle C\,\bar{C}\rangle(p^2)$



p [GeV]

Lattice data: Sternbeck et al '07

Fischer, Maas, JMP '08

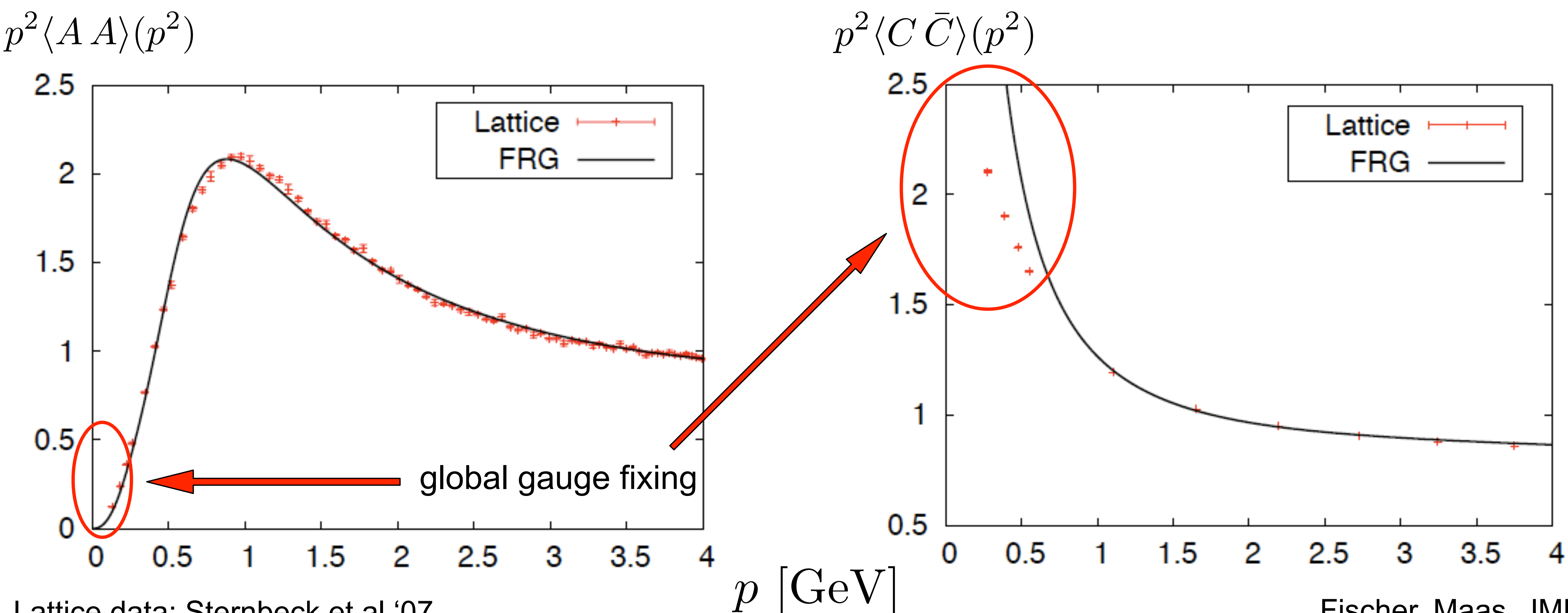
→ **Confinement criterion!**

Confinement

Continuum methods ← (Functional RG-flows)

Braun, Gies, JMP '07

$$V[A_0] = -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) - \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$



Lattice data: Sternbeck et al '07

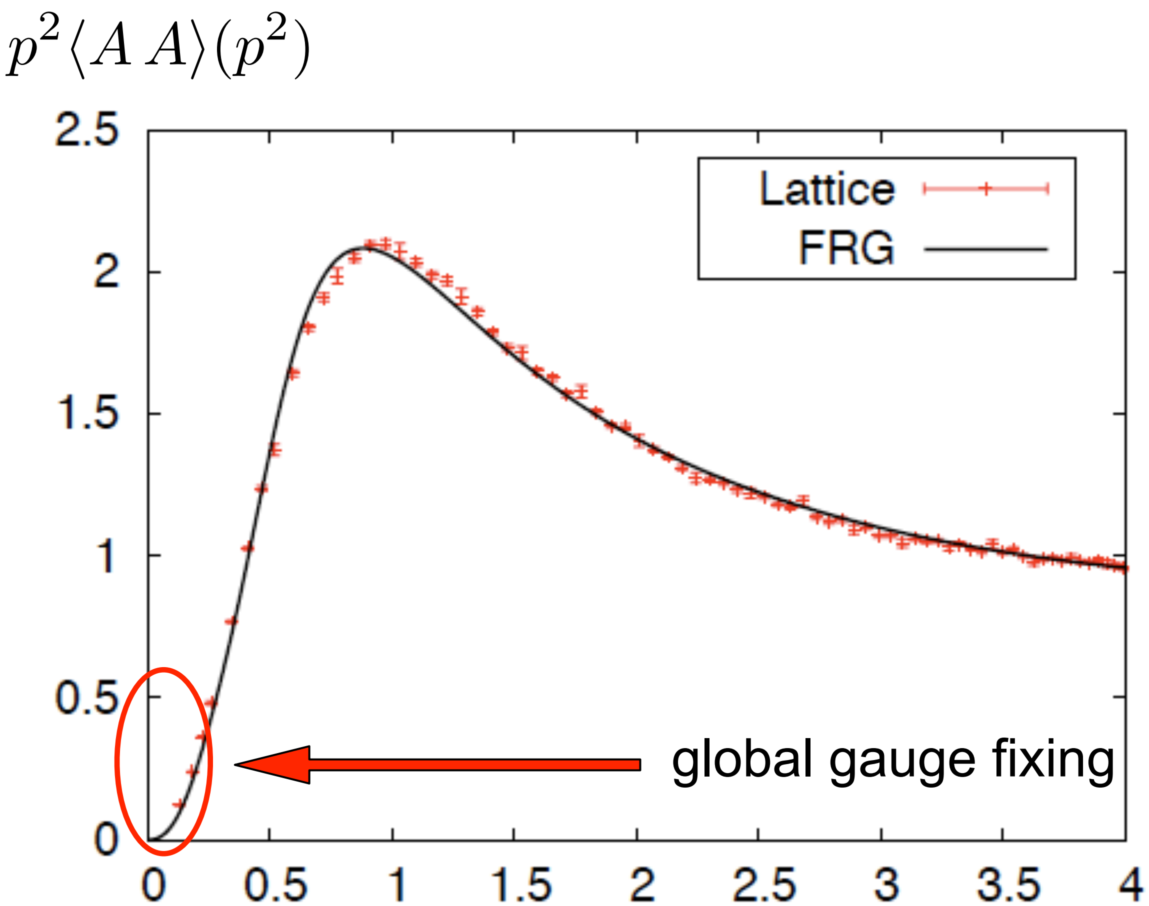
Fischer, Maas, JMP '08

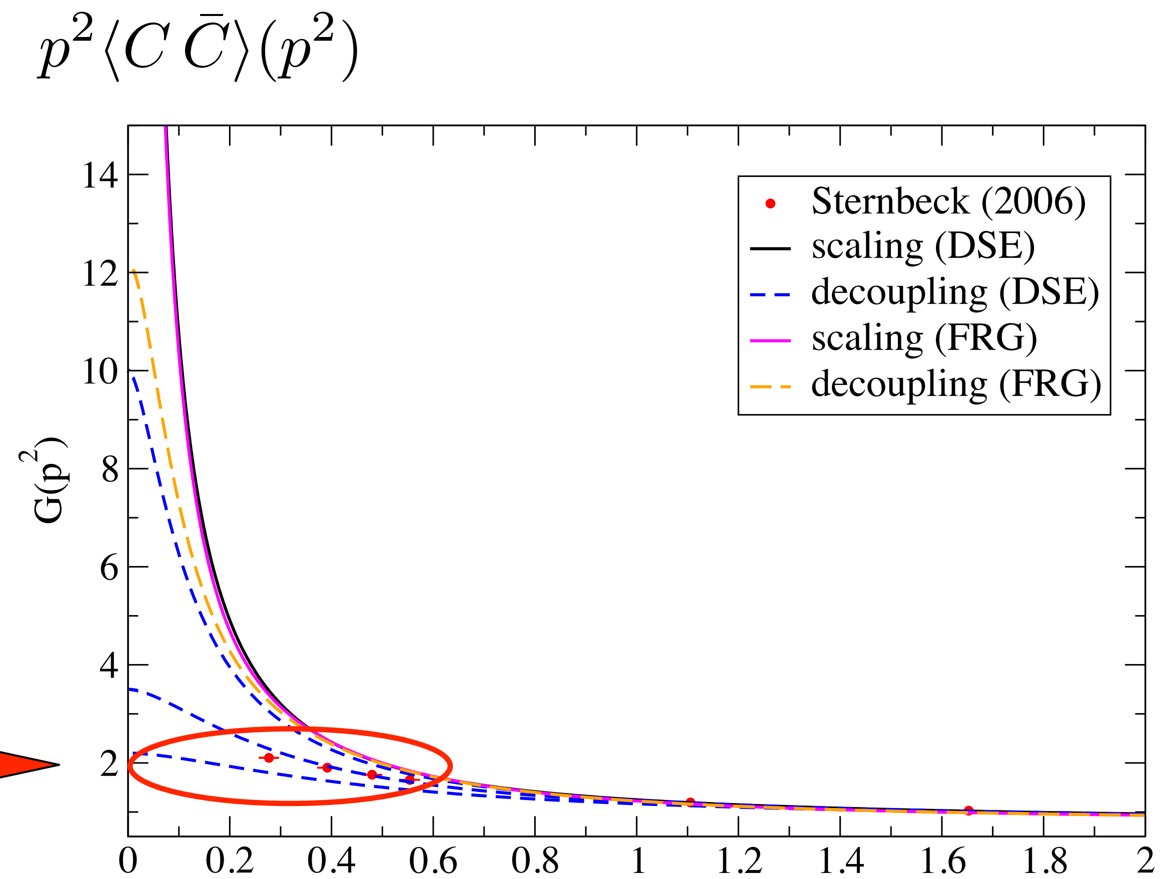
Confinement

Continuum methods ⟵ (Functional RG-flows)

Braun, Gies, JMP '07

$$V[A_0] = -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) - \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$





p [GeV]

Lattice data: Sternbeck et al '07

Fischer, Maas, JMP '08

Confinement

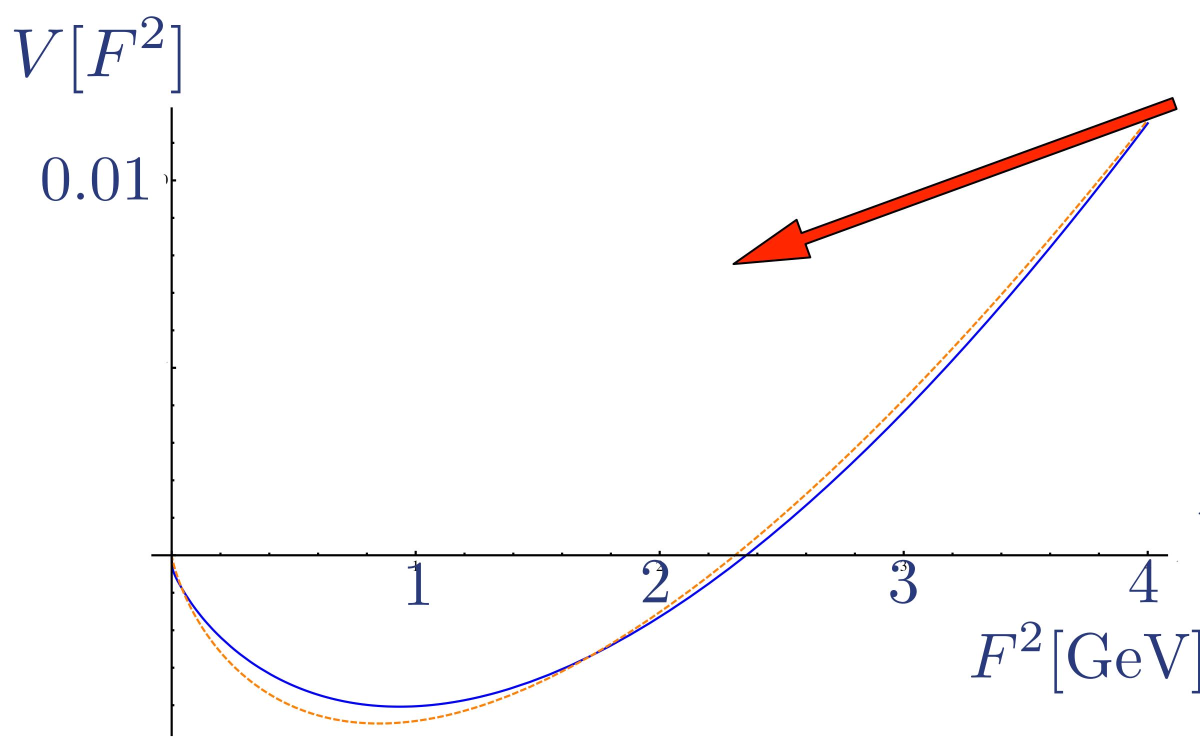
→ Confinement criterion!

---

infrared behaviour of propagators & confinement

IR gluon	**IR ghost**
$p^2\langle A\,A\rangle(p^2) \propto (p^2)^{\kappa_A}$	$p^2\langle C\,\bar{C}\rangle(p^2) \propto (p^2)^{\kappa_C}$

scaling	$\kappa_A = -2\kappa_c : \kappa_C \simeq 0.595...$	$\kappa_C < 0.605$	$\kappa_C > 1/4$
decoupling	$\kappa_A = -1 \;\&\; \kappa_c = 0$	✓	✓
		infrared stability	**confinement**
		Eichhorn, Gies, JMP, in prep.	**Braun, Gies, JMP '07**

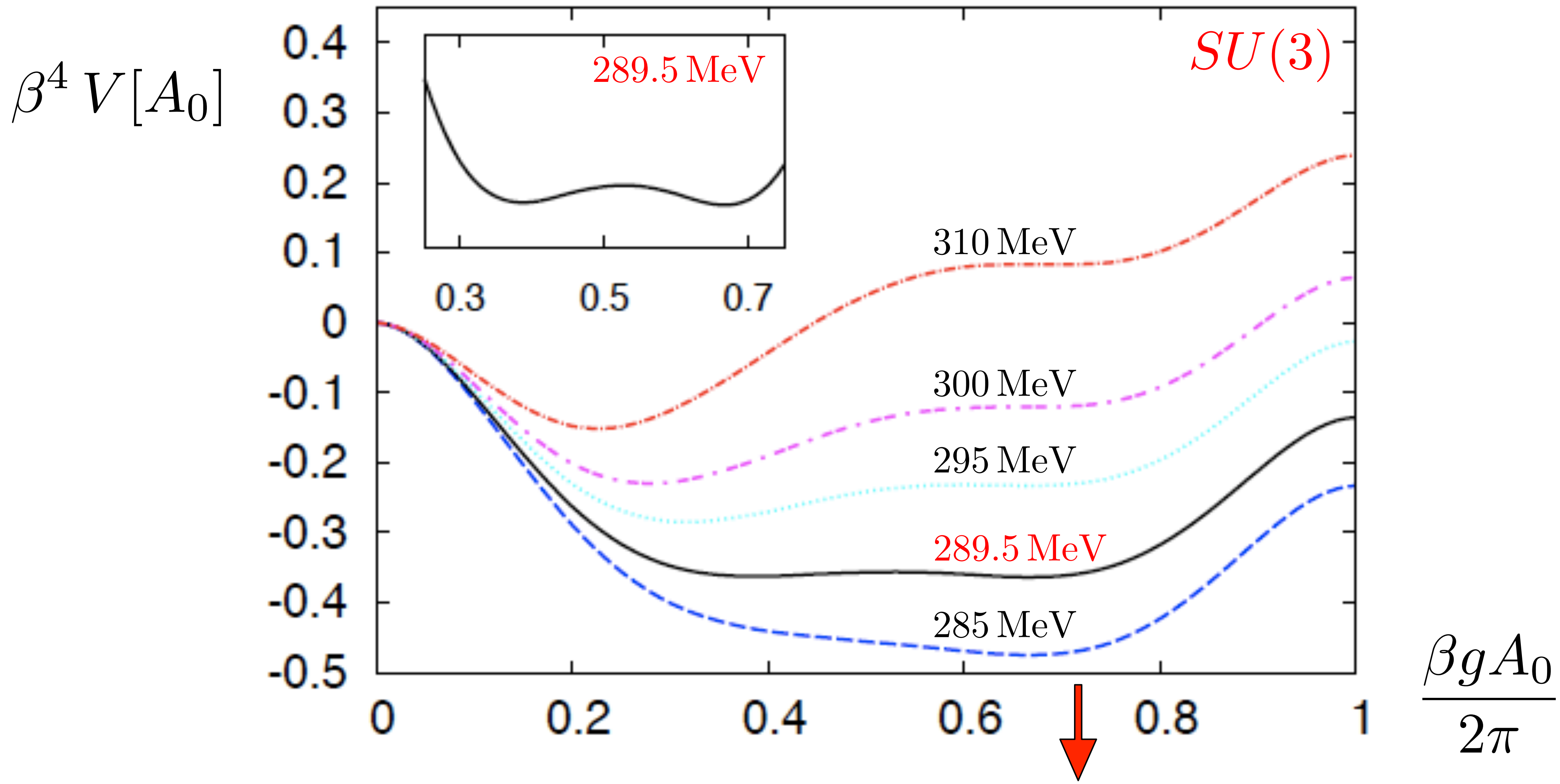


$F^2 \log F^2$ **- model** → **String tension:** $\approx 747\mathrm{MeV}$

Confinement

Order Parameter

$T_c = 289.5 \pm 10\,\mathrm{MeV}$ $\quad$ $T_c/\sqrt{\sigma} = 0.658 \pm 0.023$ $\quad$ $\mathrm{lattice}: T_c/\sqrt{\sigma} = 0.646$



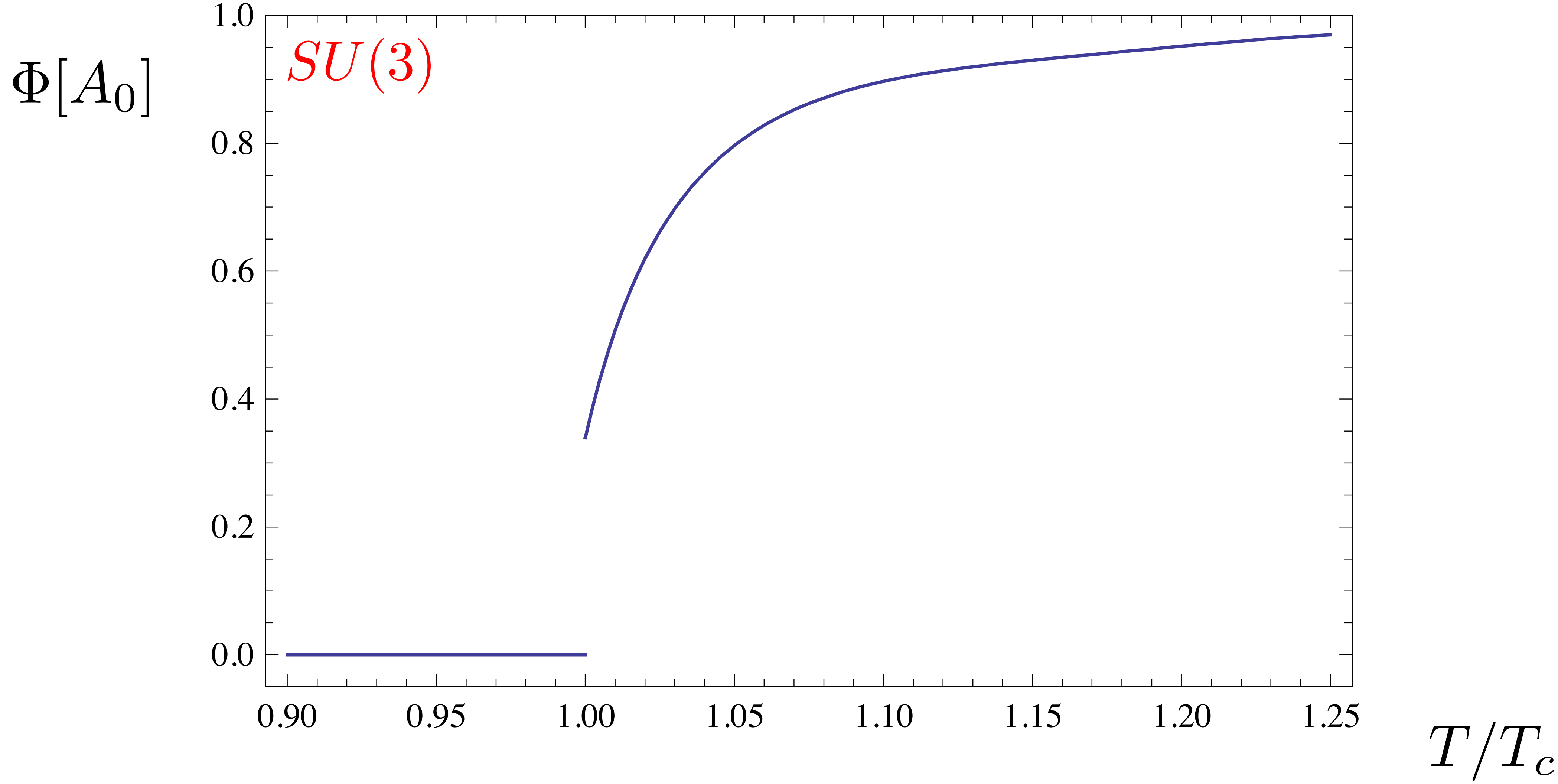
$$\Phi[A_0] = \frac{1}{3}\left(1 + 2\cos\frac{1}{2}\beta g A_0\right) \longrightarrow \Phi\left[\frac{4}{3}\pi\frac{1}{\beta g}\right] = 0$$

Braun, Gies, JMP '07

Confinement

Order Parameter

$T_c = 289.5 \pm 10\,\mathrm{MeV}$ $T_c/\sqrt{\sigma} = 0.658 \pm 0.023$ $\mathrm{lattice}: T_c/\sqrt{\sigma} = 0.646$



SU(N), Sp(2), E(7): Braun, Eichhorn, Gies, JMP '10

Braun, Gies, JMP '07

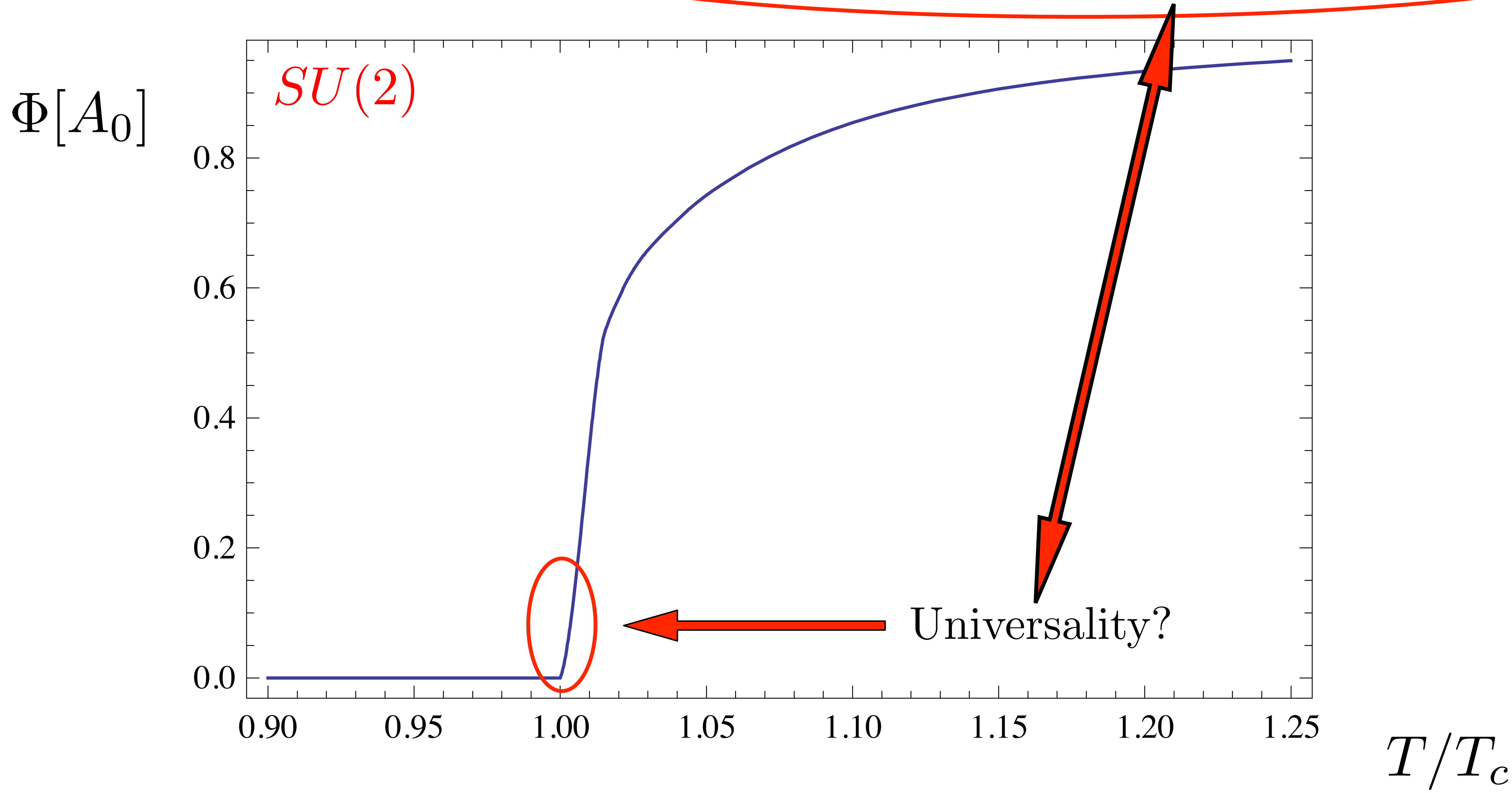
Confinement

Order Parameter

$T_c = 266 \pm 10\,\mathrm{MeV}$

$T_c/\sqrt{\sigma} = 0.605 \pm 0.023$ ⟷ lattice : $T_c/\sqrt{\sigma} = 0.709$

$V'' -$ terms



Braun, Gies, JMP '07

Confinement

Order Parameter

- Fresh results with V'' -terms in Landau gauge

$$T_c = 315 \pm 20\,\mathrm{MeV}$$

$$T_c/\sqrt{\sigma} = 0.716 \pm 0.046 \quad \longleftrightarrow \quad \mathrm{lattice}: T_c/\sqrt{\sigma} = 0.709$$

$$V'' - \mathrm{terms}$$

Braun, Gies, JMP, Spallek, in prep.

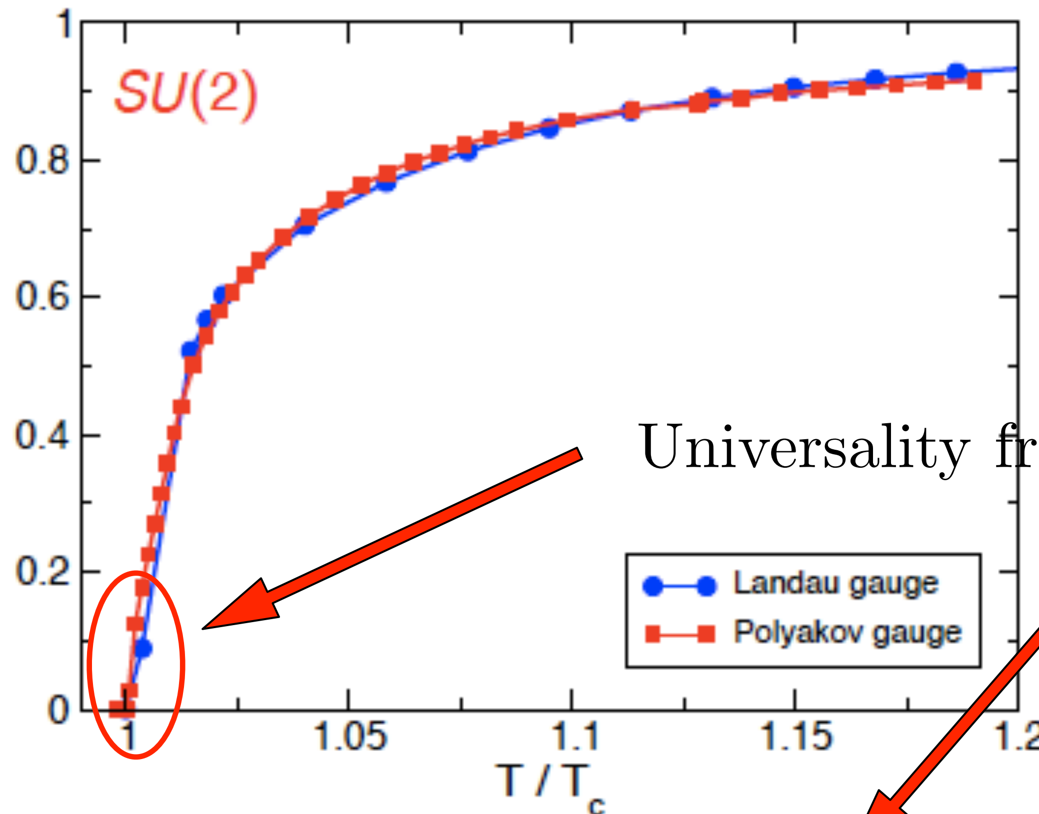
Confinement

Universality & gauge independence

Polyakov gauge: $A_0 = A_0^c(\vec{x})\sigma_3$

RG-flow : $V[A_0] = -\int dt\, \text{flow}[V''[A_0]\,,\,\alpha_s]$

$\Phi[A_0]$



$\text{Universality from } V'' - \text{terms}$

T/T_c

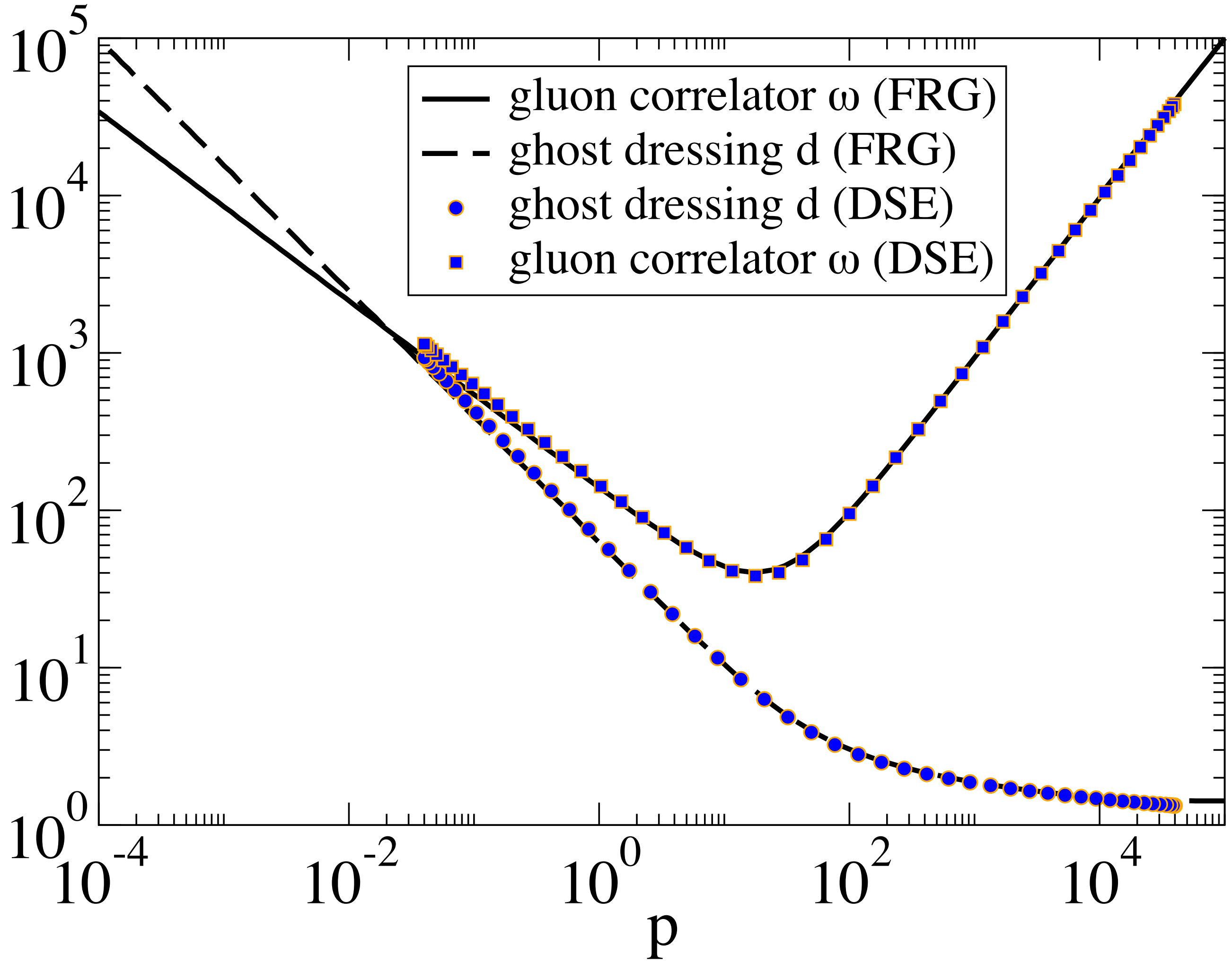
- — —: Polyakov gauge: crit. exp. $\nu = 0.65$ $\quad \nu_{\text{Ising}} = 0.63$
- — —: Landau gauge propagators

JMP, Marhauser ‘08

Confinement

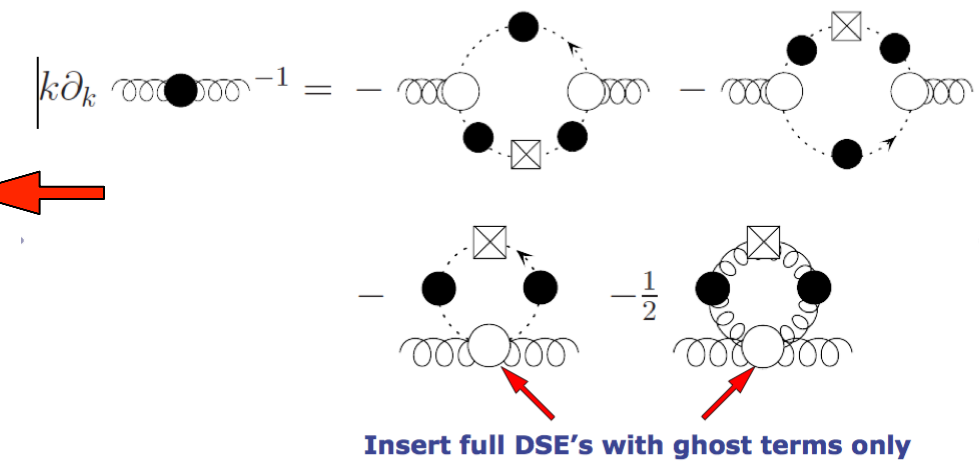
Coulomb gauge

see talk of M. Leder



Braun, Gies, JMP '07

Confinement

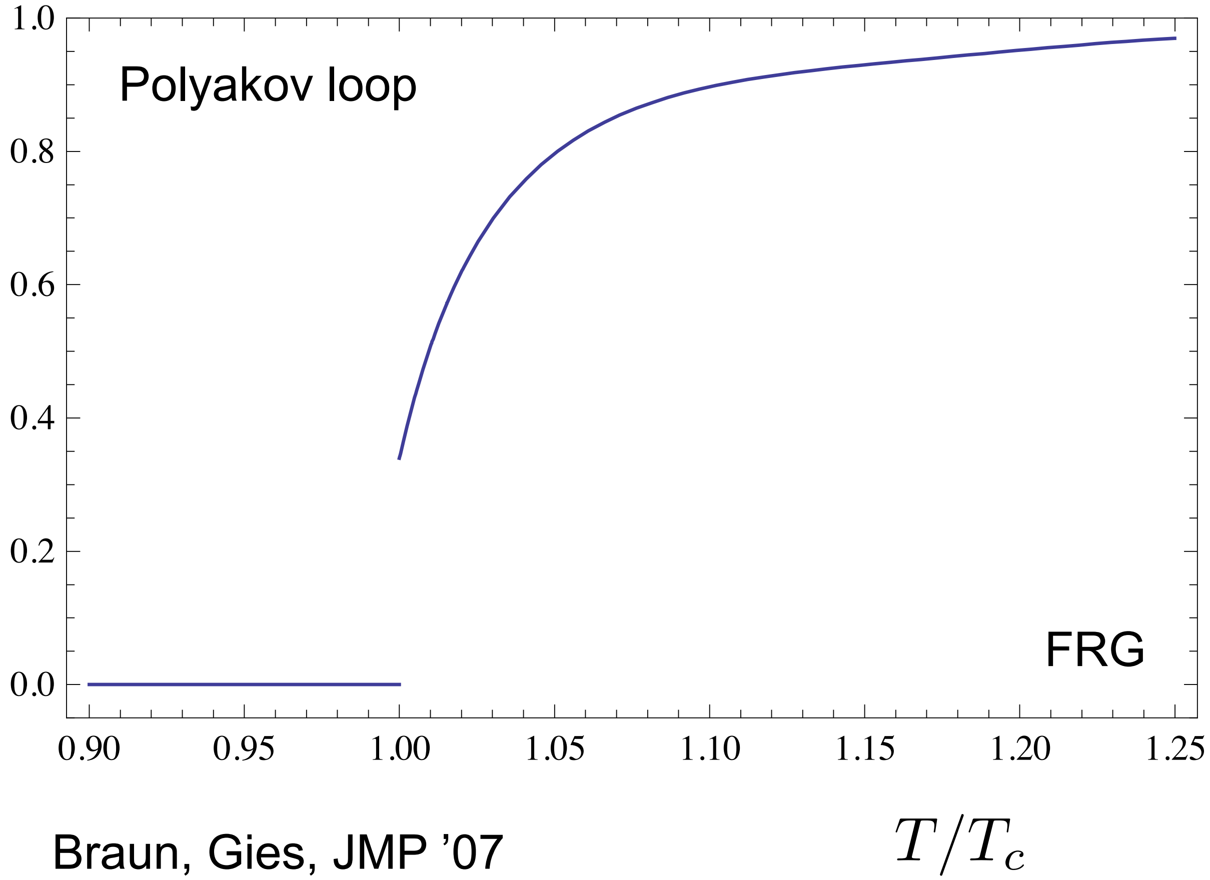


T_c, work in progress

Leder, JMP, Reinhardt, Weber '10

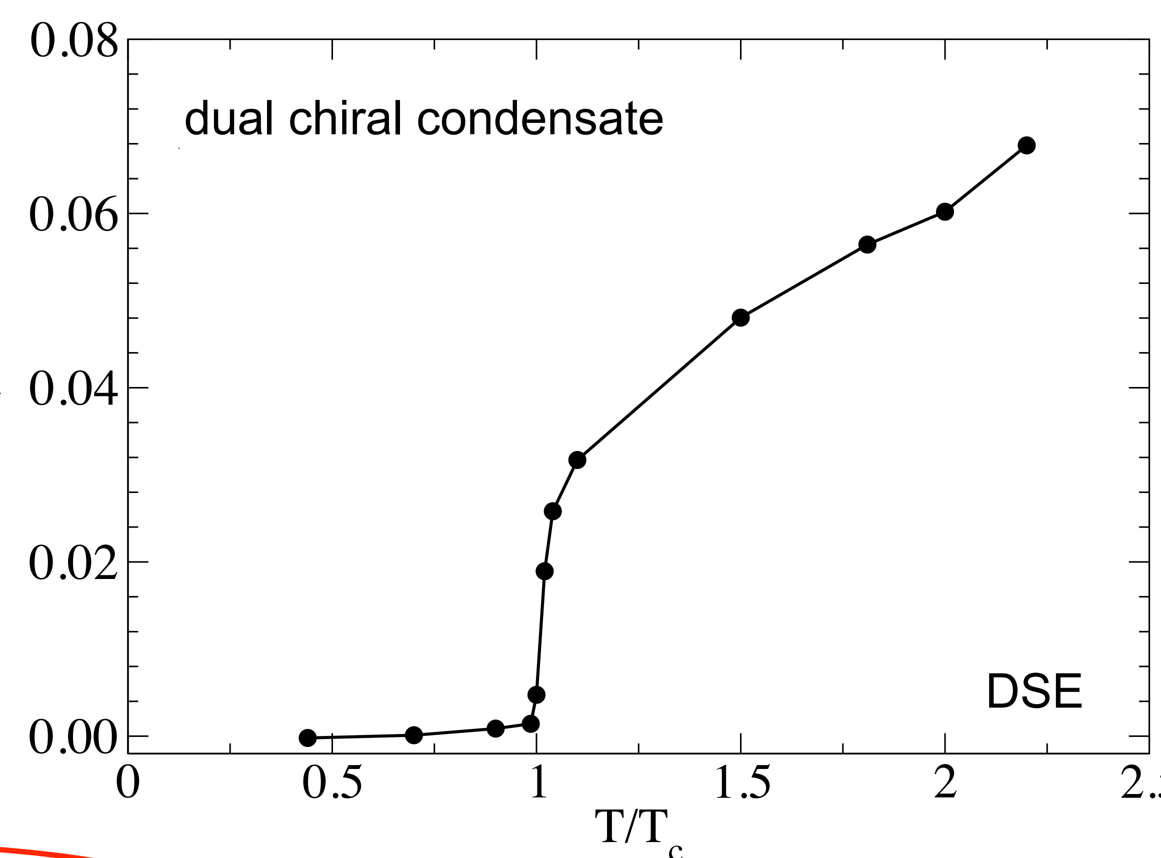
Dual chiral condensate

Order parameters from functional methods



Braun, Gies, JMP ’07

$SU(3)$



Fischer, Maas, Müller ’10

Dual order parameters

Gattringer ‘06
Synatschke, Wipf, Wozar ’07
Bruckmann, Hagen, Bilgici, Gattringer ‘08

Functional methods

Fischer, ’09; Fischer, Maas, Müller ’10
Braun, Haas, Marhauser, JMP ‘09 ← imaginary chemical potential

Finite temperature

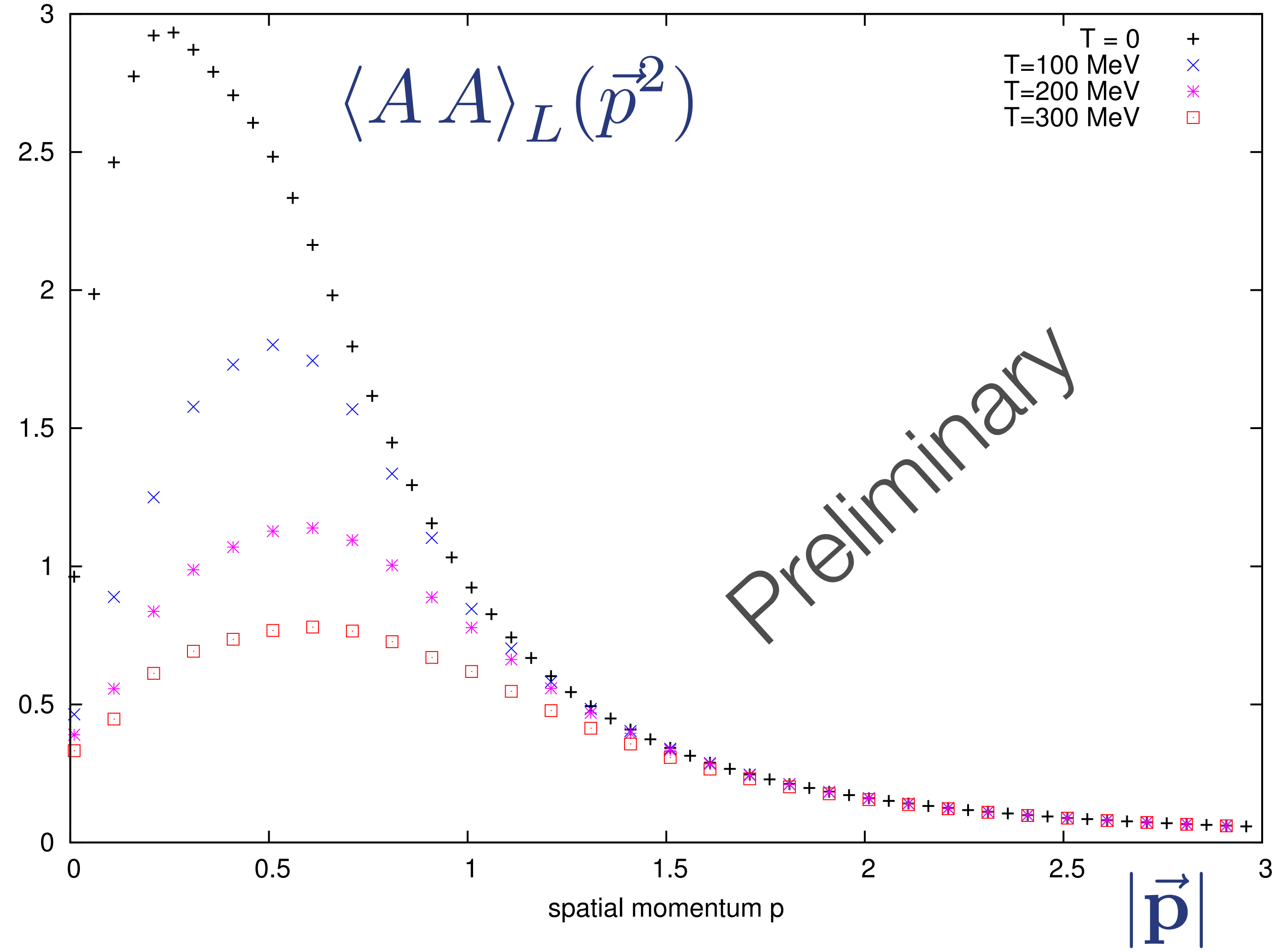
see talk of L. Fister

Finite temperature

see talk of L. Fister

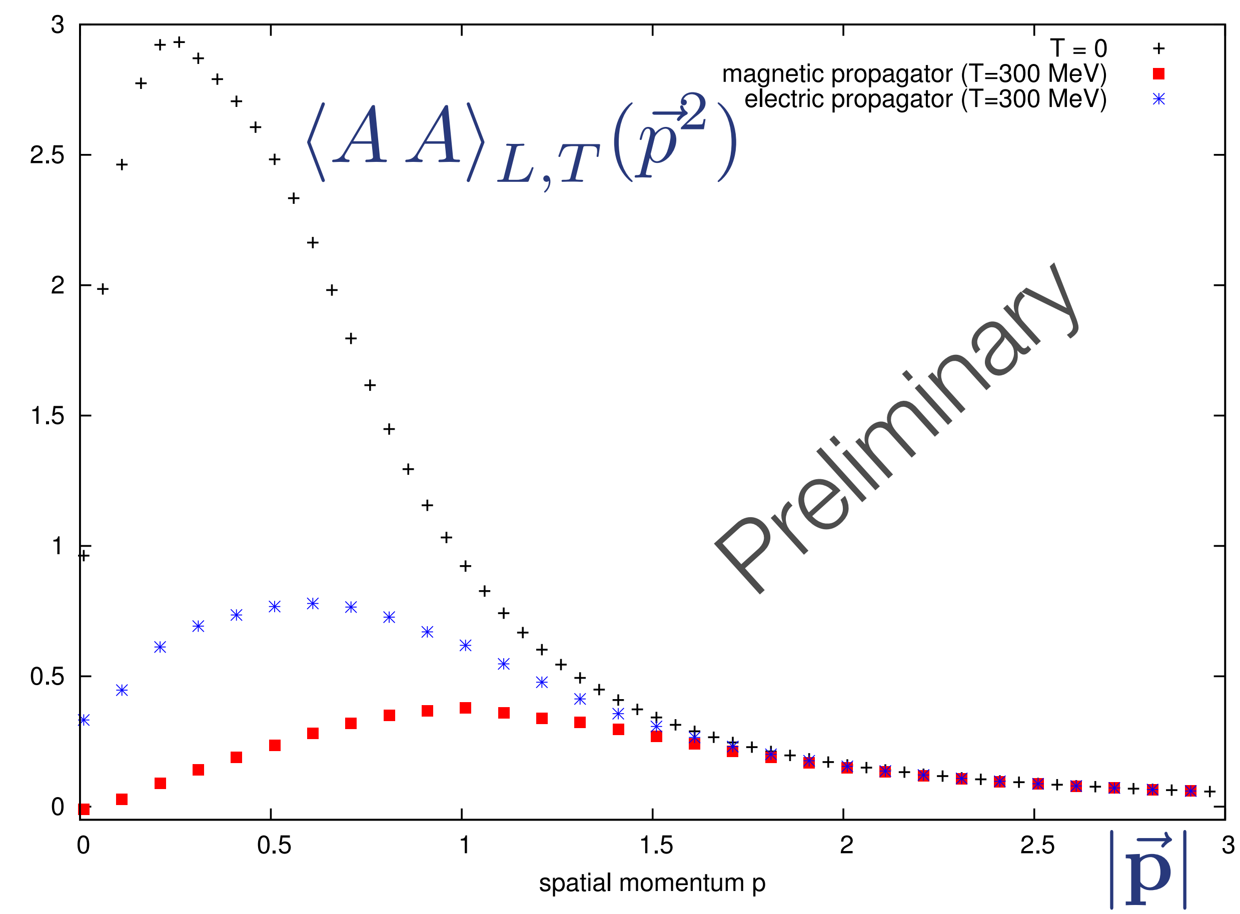
Finite temperature propagators

Fister, JMP, work in progress



longitudinal propagator

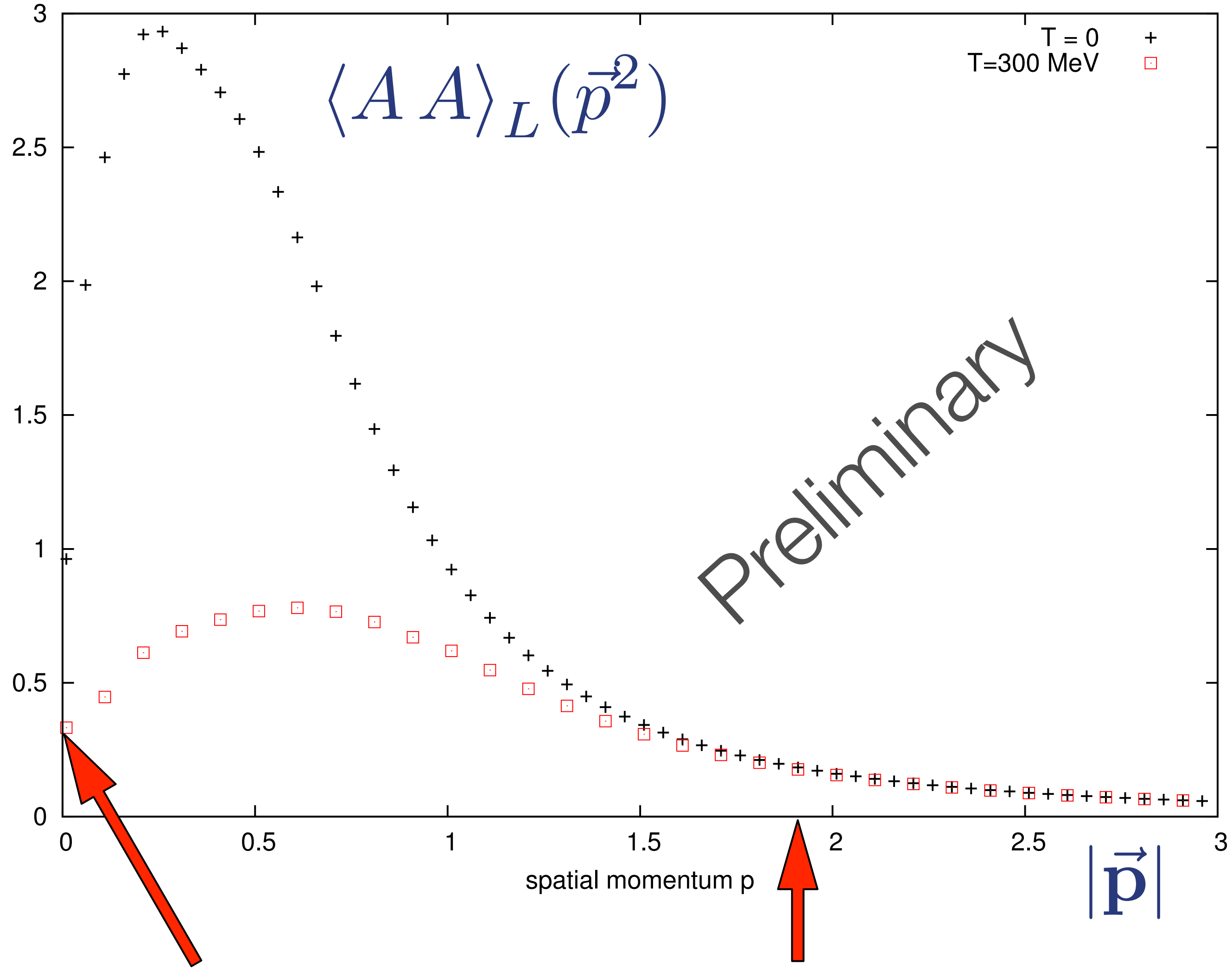
transversal propagator



Finite temperature propagators

Fister, JMP, work in progress

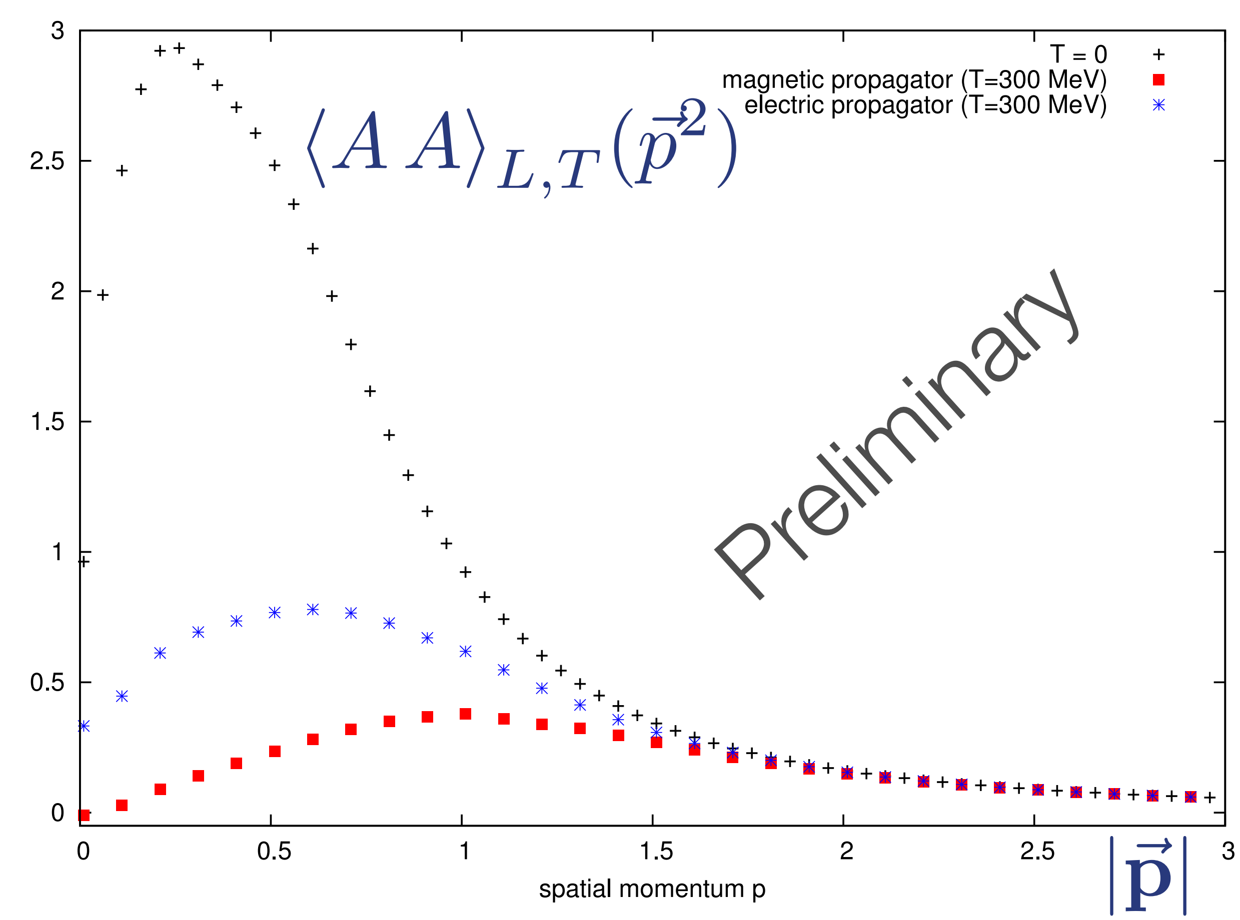
longitudinal propagator



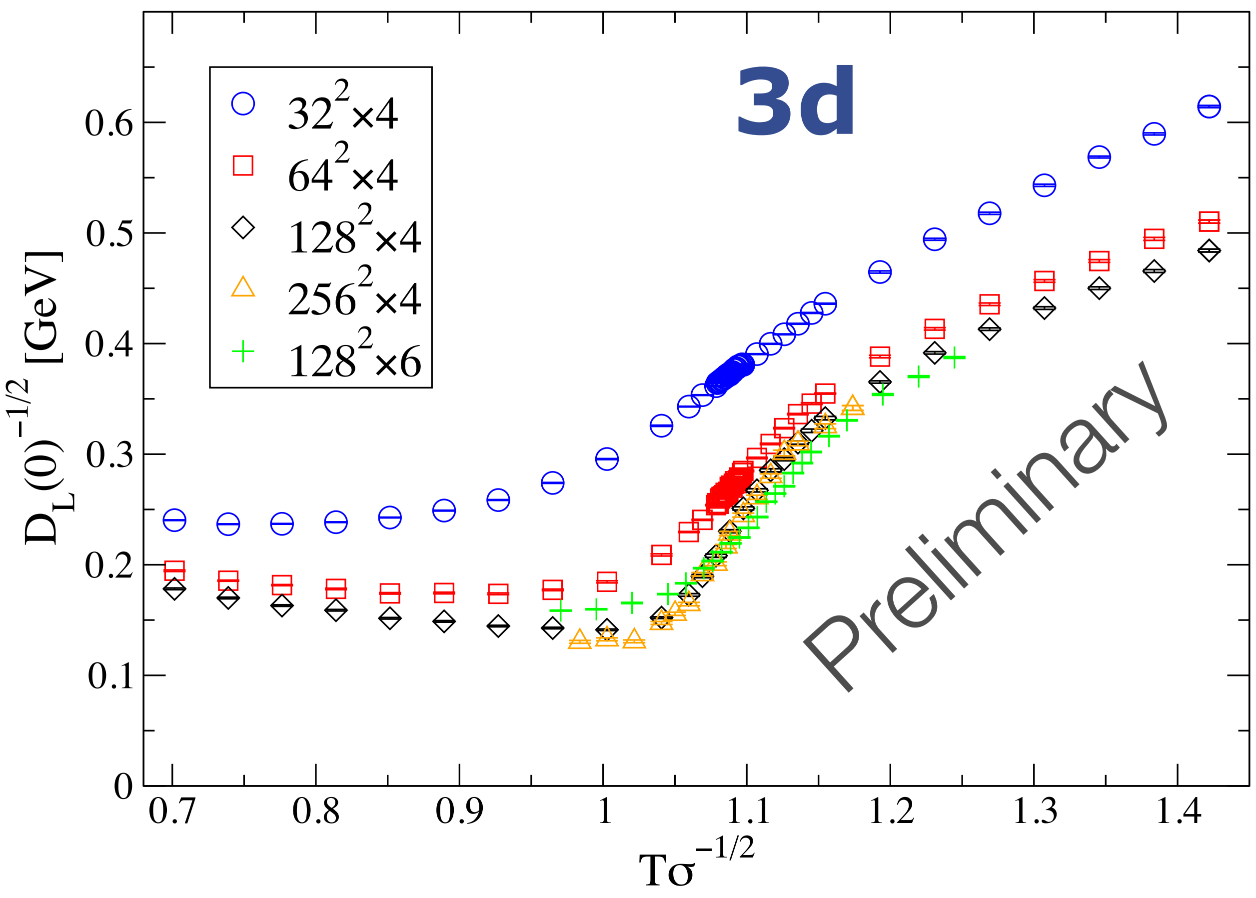
thermal mass

$2\pi\mathrm{T}$

transversal propagator



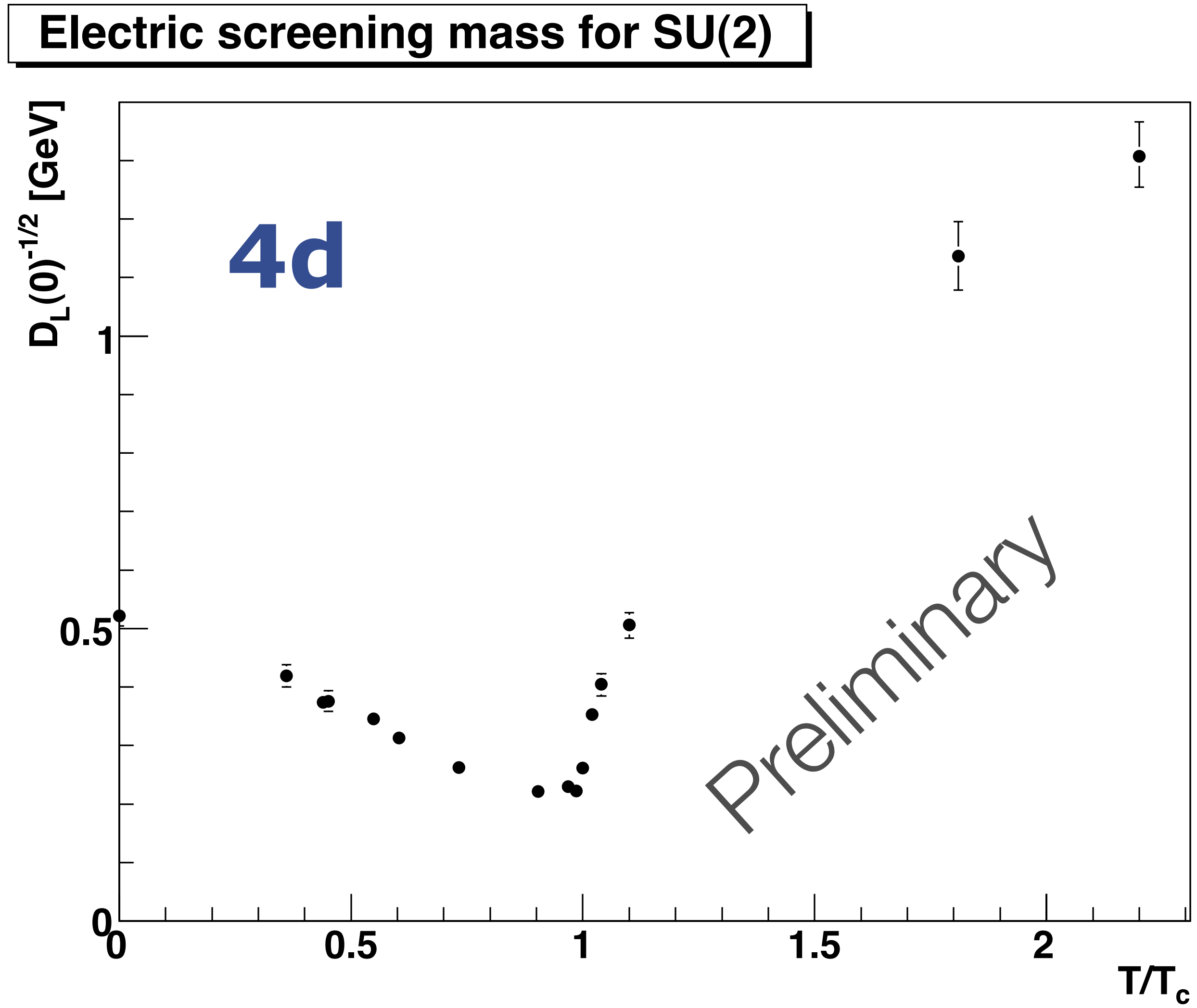
Finite temperature propagators



Maas, JMP, Spielmann, von Smekal, work in prep.

$$\mathbf{D_L(0) = \langle A\, A \rangle_T(0)}$$

$$\nu \approx \mathbf{1}$$



critical scaling in Landau gauge props?

$$\mathbf{D_L(0)^{-1/2} \propto |T - T_c|^{\nu} + \cdots}$$

$$\nu \approx \mathbf{0.68}$$

keep running ...

QCD

Confinement & chiral symmetry breaking

see talk of L. Haas

Chiral symmetry breaking

chiral symmetry

Generation	first	second	third	Charge
Mass [MeV]	1.5-4	1150-1350	170×10^3	
Quark	u	c	t	$\frac{2}{3}$
Quark	d	s	b	$-\frac{1}{3}$
Mass [MeV]	4-8	80-130	$(4.1\text{-}4.4)\times10^3$	

chiral symmetry breaking: $\Delta m \approx 400 MeV$

2 light flavours, one heavy flavour $2+1$

chiral symmetry breaking

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Mass [MeV]	1.5-4	1150-1350	170×10^3	
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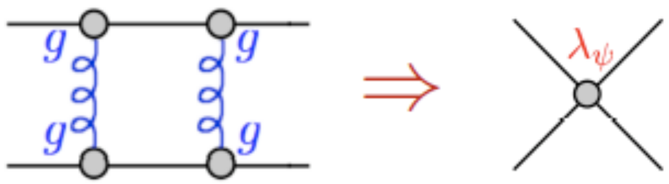
Chiral symmetry breaking



$\propto \alpha_s^2$

$$\int d^4x\, \lambda_\psi \left[(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2 \right]$$

$$\langle \bar{q}q \rangle \neq 0$$

mass term: $\langle \bar{q}q \rangle \, \bar{q}q$

Generation	first	second	third	Charge
Mass [MeV]	1.5-4	1150-1350	170×10^3	
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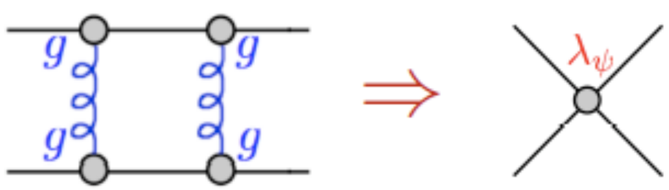
Chiral symmetry breaking



$\propto \alpha_s^2$

$$\int d^4x\, \lambda_\psi \left[(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2\right]$$

$\langle \bar{q}q \rangle \neq 0$

mass term: $\langle \bar{q}q \rangle\, \bar{q}q$

Order parameter

$$\sigma = \langle \bar{q}q \rangle$$ chiral condensate

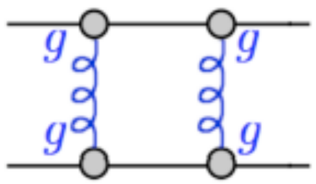
- **chiral symmetry:** $\sigma = 0$
- **symmetry breaking:** $\sigma \neq 0$

Symmetry

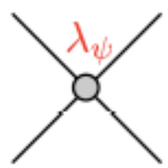
$SU_L(N_f) \times SU_R(N_f)$

- **broken to** $SU(N_f)$

Chiral symmetry breaking



$\Rightarrow$



$\propto \alpha_s^2$

$$\int d^4x\, \lambda_\psi \left[(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2 \right]$$

$$\langle \bar{q}q \rangle \neq 0$$

mass term: $\langle \bar{q}q \rangle\, \bar{q}q$

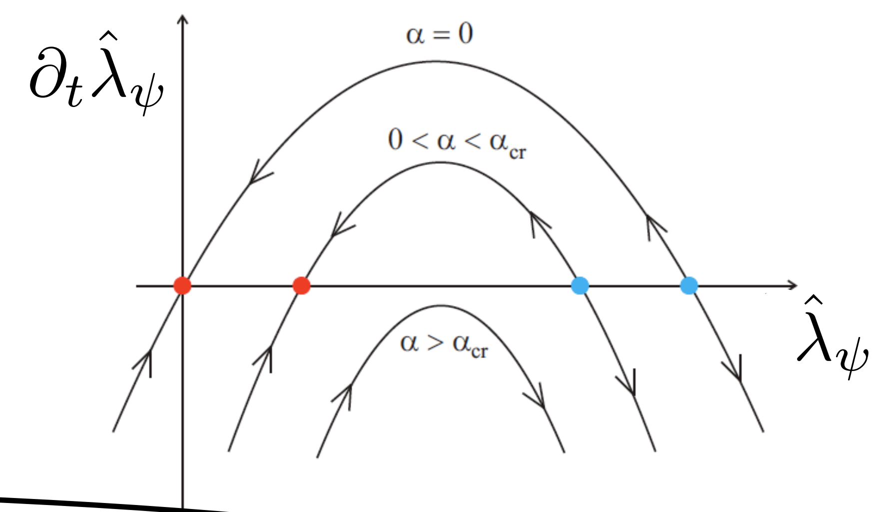
$\alpha_s > \alpha_{s,\text{crit}}$

Order parameter

$$\sigma = \langle \bar{q}q \rangle$$

chiral condensate

- **chiral symmetry:** $\sigma = 0$
- **symmetry breaking:** $\sigma \neq 0$



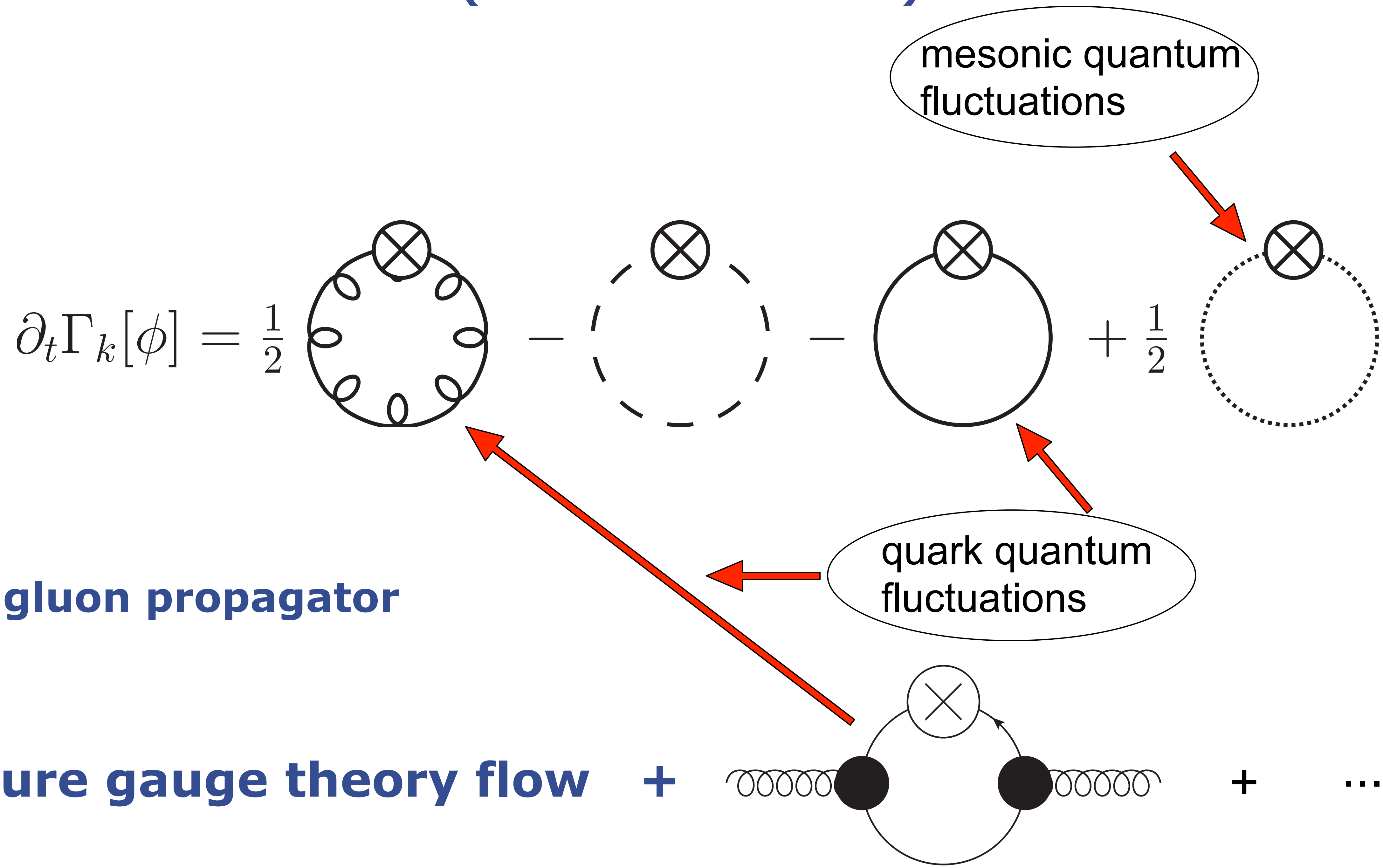
Chiral symmetry breaking directly sensitive to size of α_s

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods ⟵ (Functional RG-flows)

- **RG-flow of Effective Action (Effective Potential)**

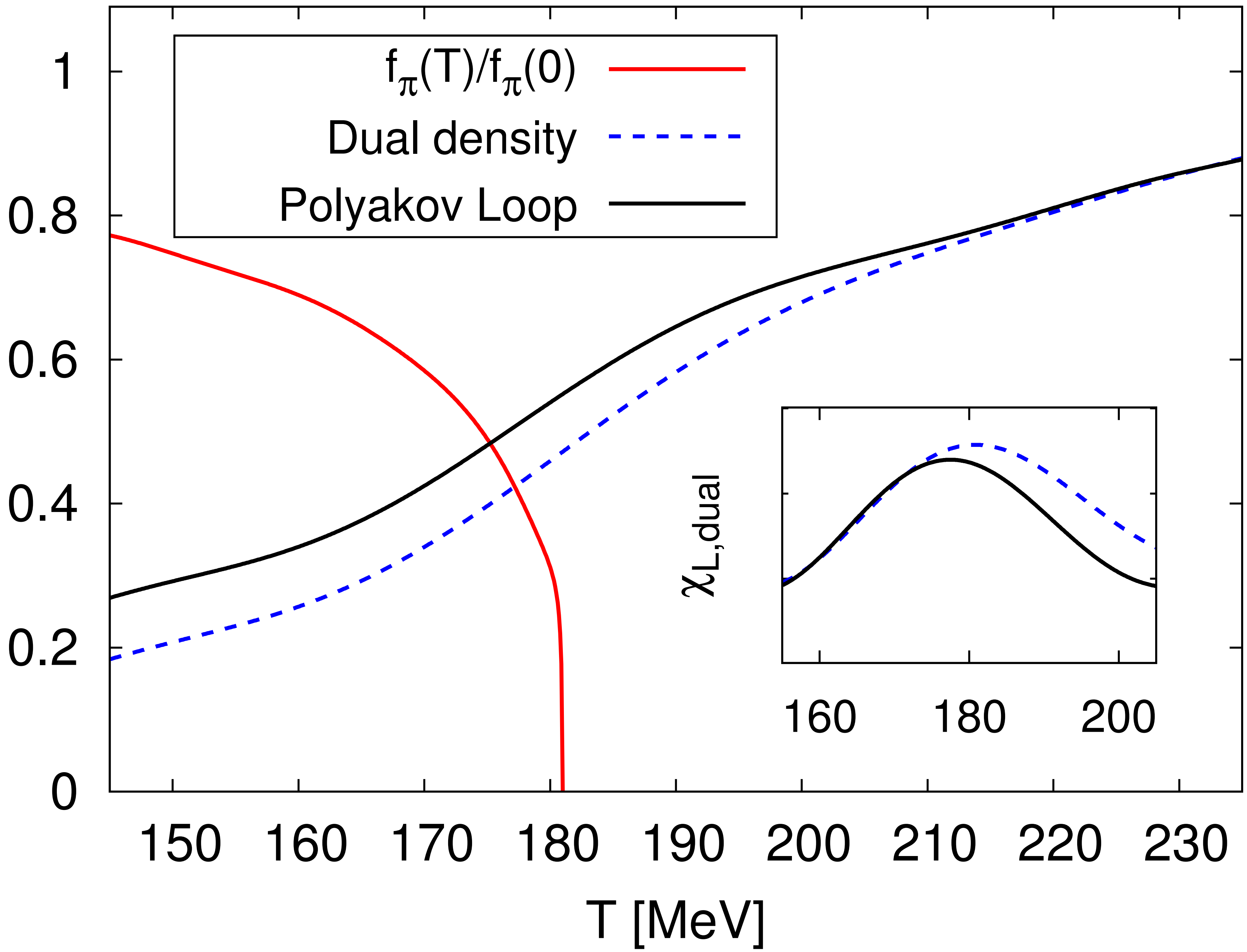
- **flow of gluon propagator**



Naturally encorporates PQM/PNJL models as specific low order trunations

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods

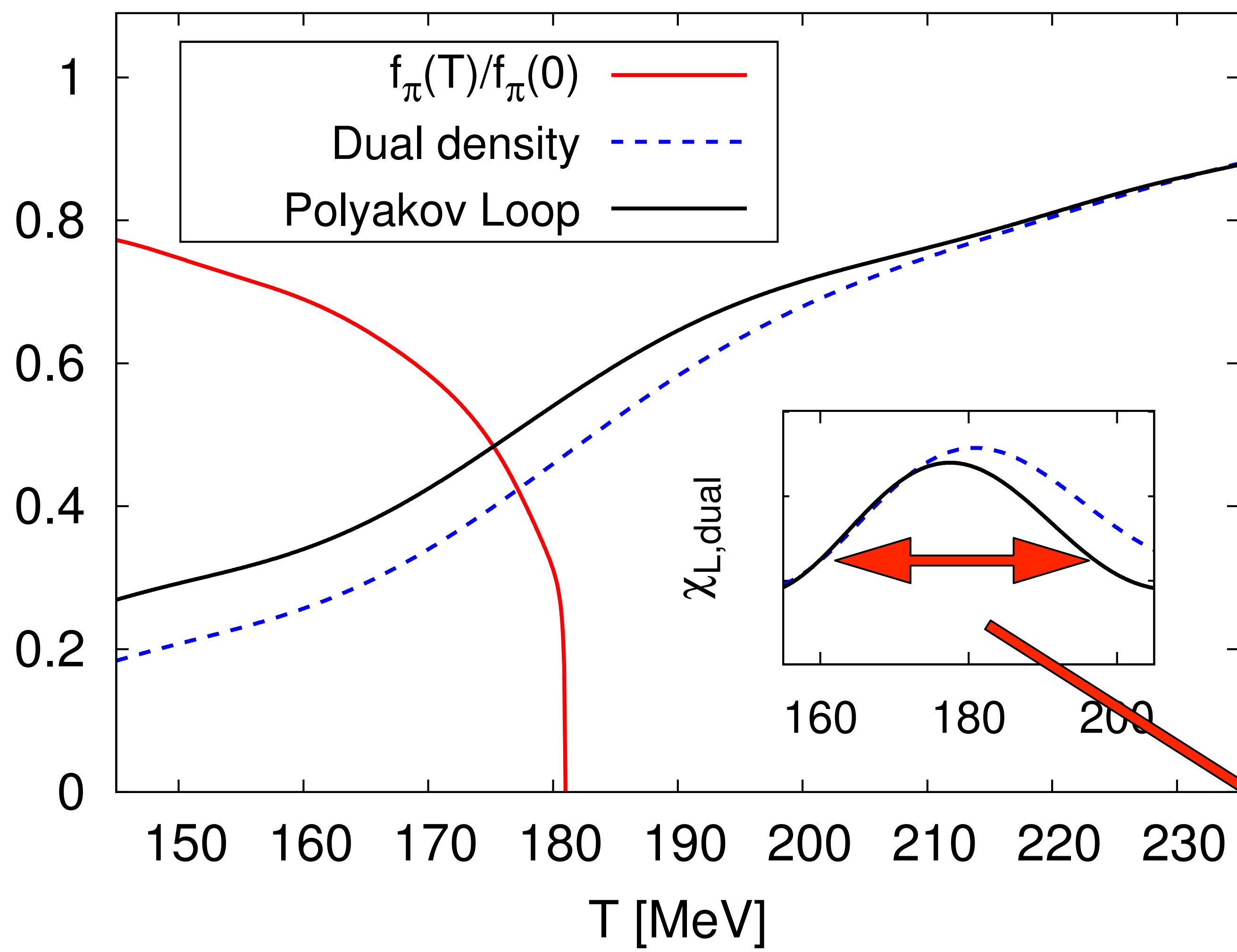


$$T_\chi \simeq T_{\rm conf} \simeq 180\mathrm{MeV}$$

Braun, Haas, Marhauser, JMP '09

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods



compatible with hotQCD '10

$T_{c,\mathrm{conf}} \simeq T_{c,\chi} \lesssim 170\mathrm{MeV}$

$N_f = 2 + 1$

$T_\chi \simeq T_{\mathrm{conf}} \simeq 180\mathrm{MeV}$

$N_f = 2$

compatible with Aoki et al '09

$171\mathrm{MeV} \simeq T_{c,\mathrm{conf}} \simeq T_{c,\chi} \simeq 150\mathrm{MeV}$

$N_f = 2 + 1$

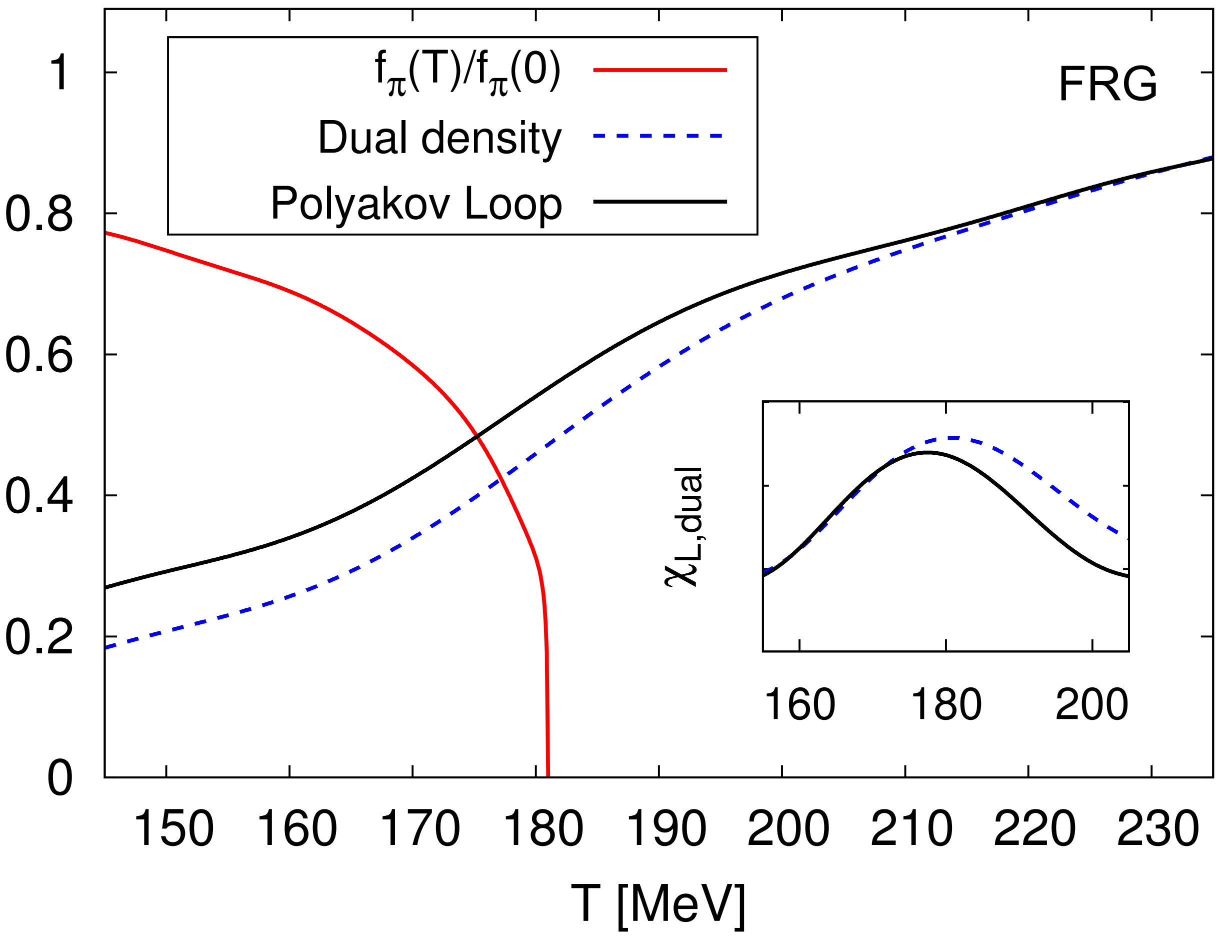
± 20 MeV

Braun, Haas, Marhauser, JMP '09

Full dynamical QCD: N_f = 2

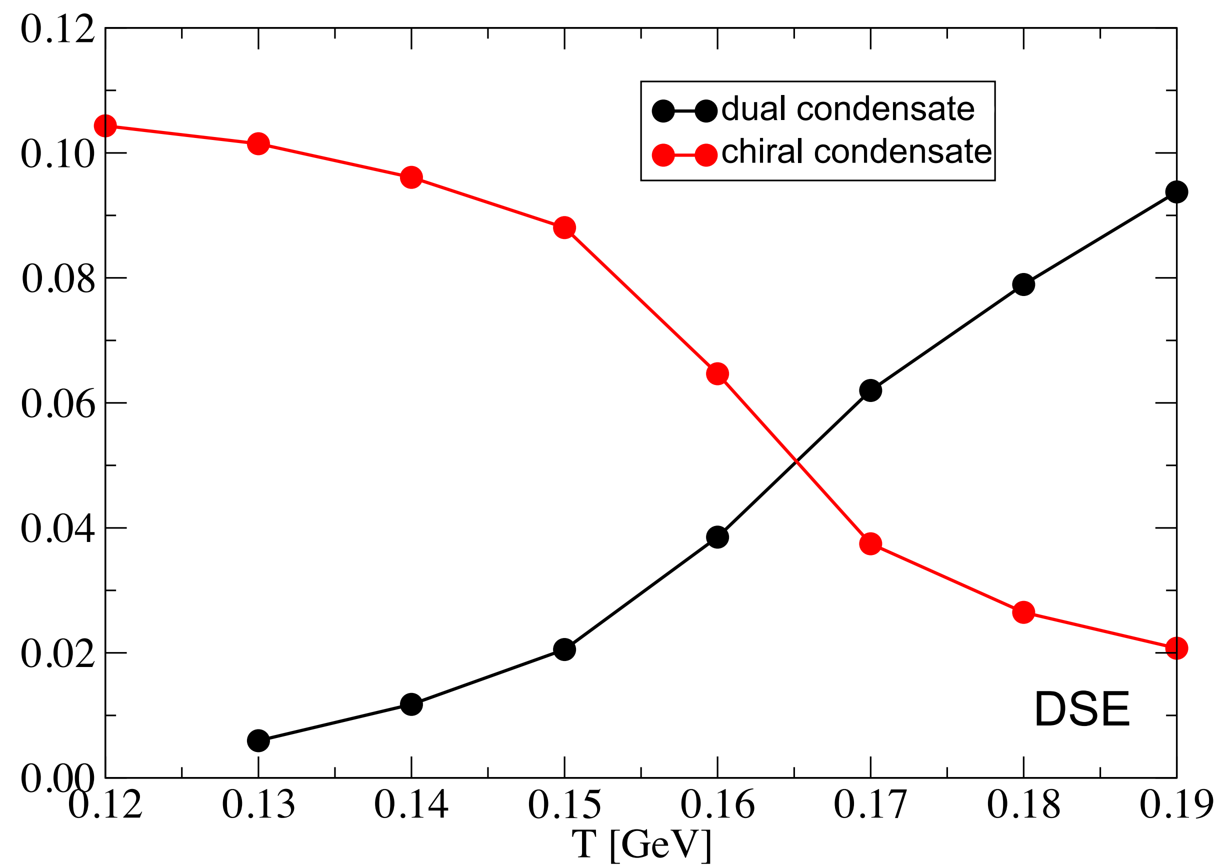
Continuum methods

chiral limit



Braun, Haas, Marhauser, JMP '09

phys. quark masses

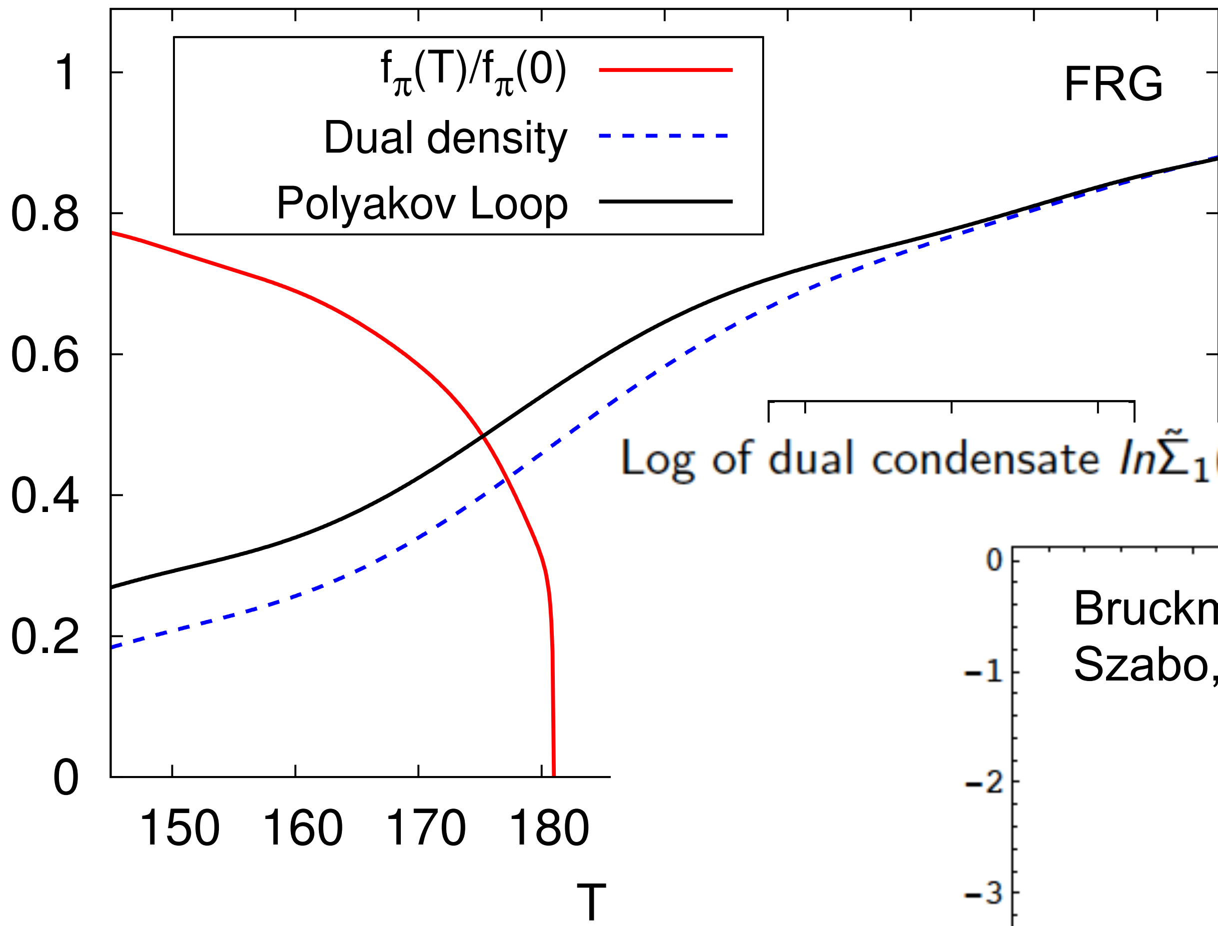


Fischer, Lücker, Müller, in prep.

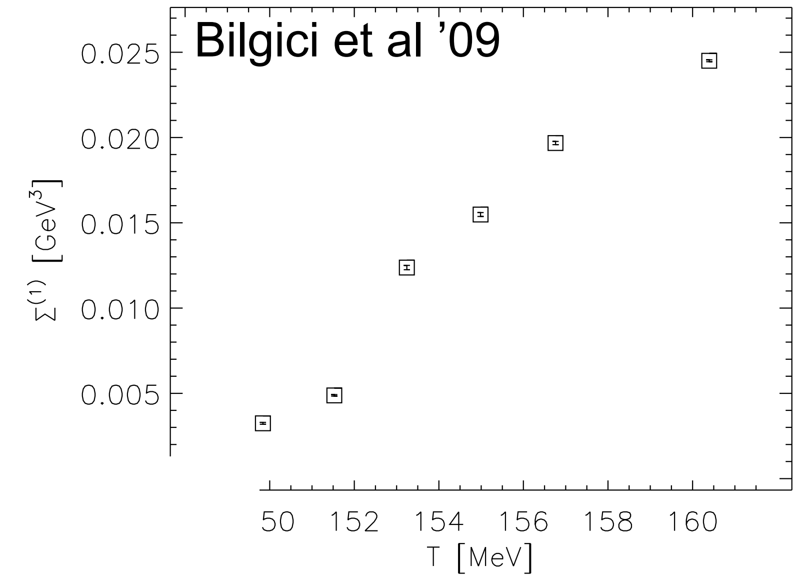
Full dynamical QCD: N_f = 2

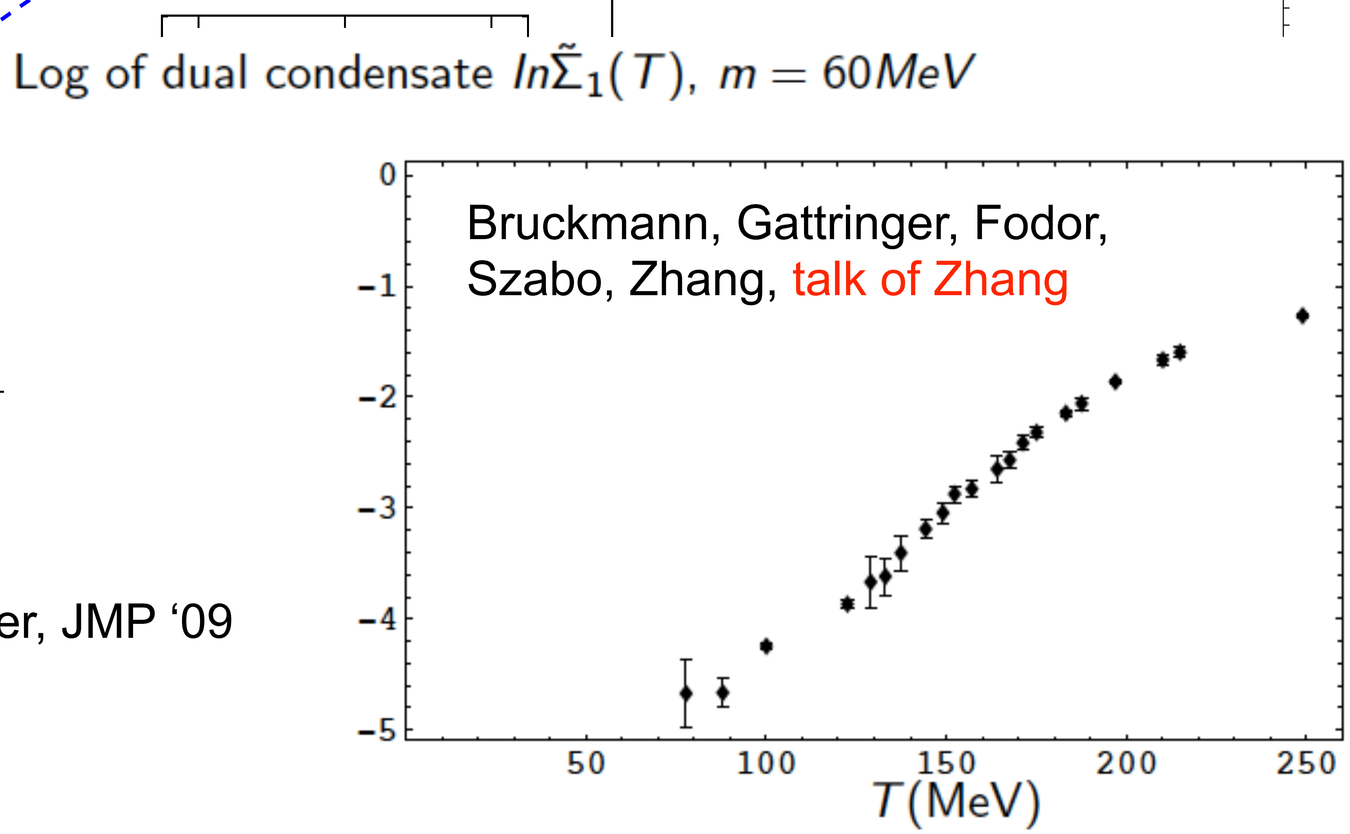
Continuum methods & lattice

chiral limit

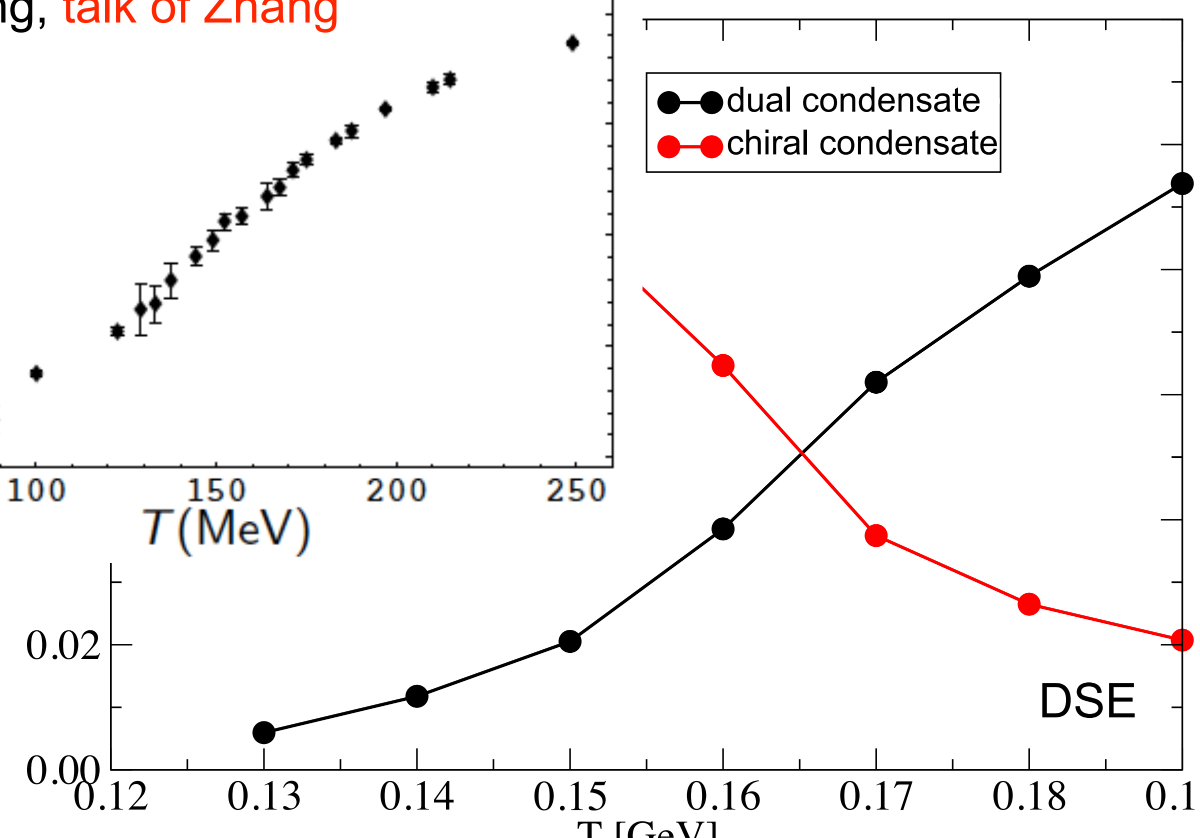


Braun, Haas, Marhauser, JMP ‘09





phys. quark masses



Fischer, Lücker, Müller, in prep.

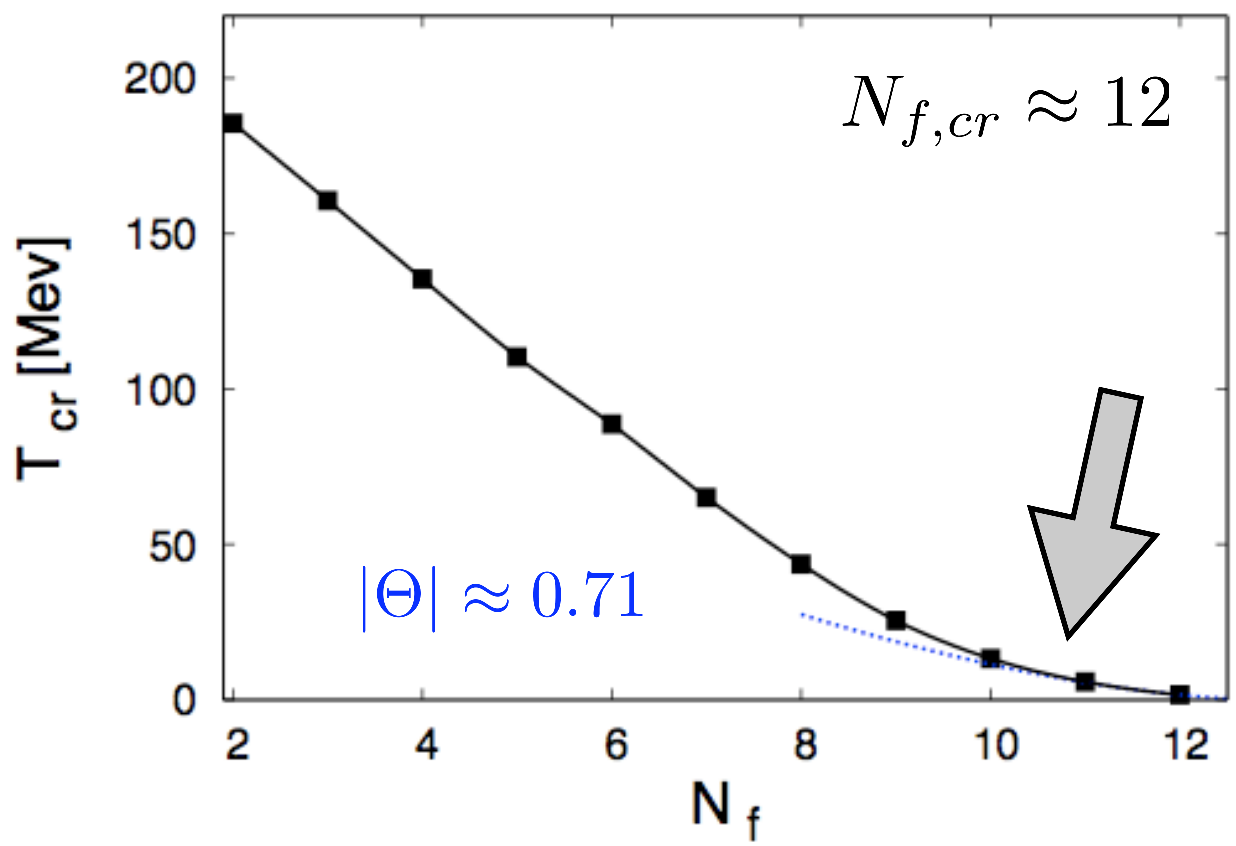
Comparison!

Many-flavour QCD

see talk of J. Braun

Many-flavour QCD

J. Braun, H. Gies '05,'06, '09



new scaling law:

$$\mathcal{O} \simeq \mu_0^{d_{\mathcal{O}}} F_{\mathcal{O}}(N_f)|N_f - N_f^{\rm cr}|^{\frac{d_{\mathcal{O}}}{|\Theta|}}$$

with $\mathcal{O} = f_\pi, \langle\bar\psi\psi\rangle, T_{\rm cr}, \ldots$

- critical number (RG error estimate): $N_{f,cr} \simeq 10\,..\,12$ (H. Gies & J. Jaeckel '05; JB & H. Gies '05)
- walking technicolor: $N_{f,cr} \simeq 12$ (Dietrich, Sannino, Tuominen '05; Dietrich, Sannino '06)
- state-of-the-art lattice studies: $9 < N_{f,cr} \lesssim 12$ (Appelquist, Fleming, Neil '08, '09; Deuzeman, Lombardo, Pallante '08; Fodor et al. '08, '09; Fodor, Holland, Kuti, Nogradi, Schroeder '09; Jin, Mawhinney '09)
- "conformal phase" for $N_{f,cr} < N_f < 16.5$: asymptotic freedom but no χSB

finish ...

Phase diagram of QCD

see talk of L.M. Haas

Polyakov loop extended chiral models: T. Herbst
B.J. Schaefer

finite volume scaling in chiral models: B. Klein

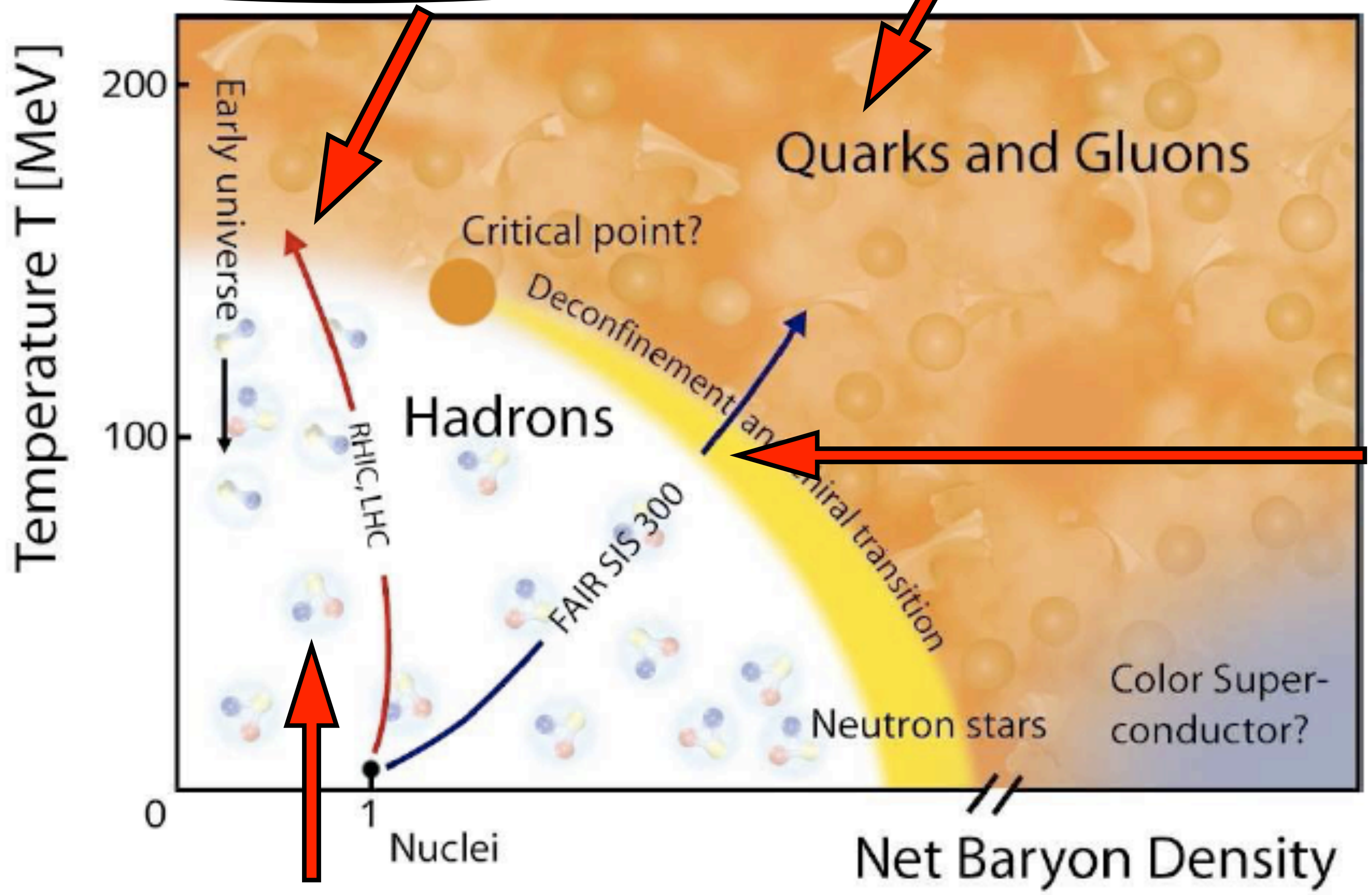
Phase diagram of QCD

Strongly correlated quark-gluon-plasma

'RHIC serves the perfect fluid'

massless quarks (chiral symmetry)

deconfinement



FAIR, www.gsi.de

quarkyonic:

confinement & chiral symmetry?

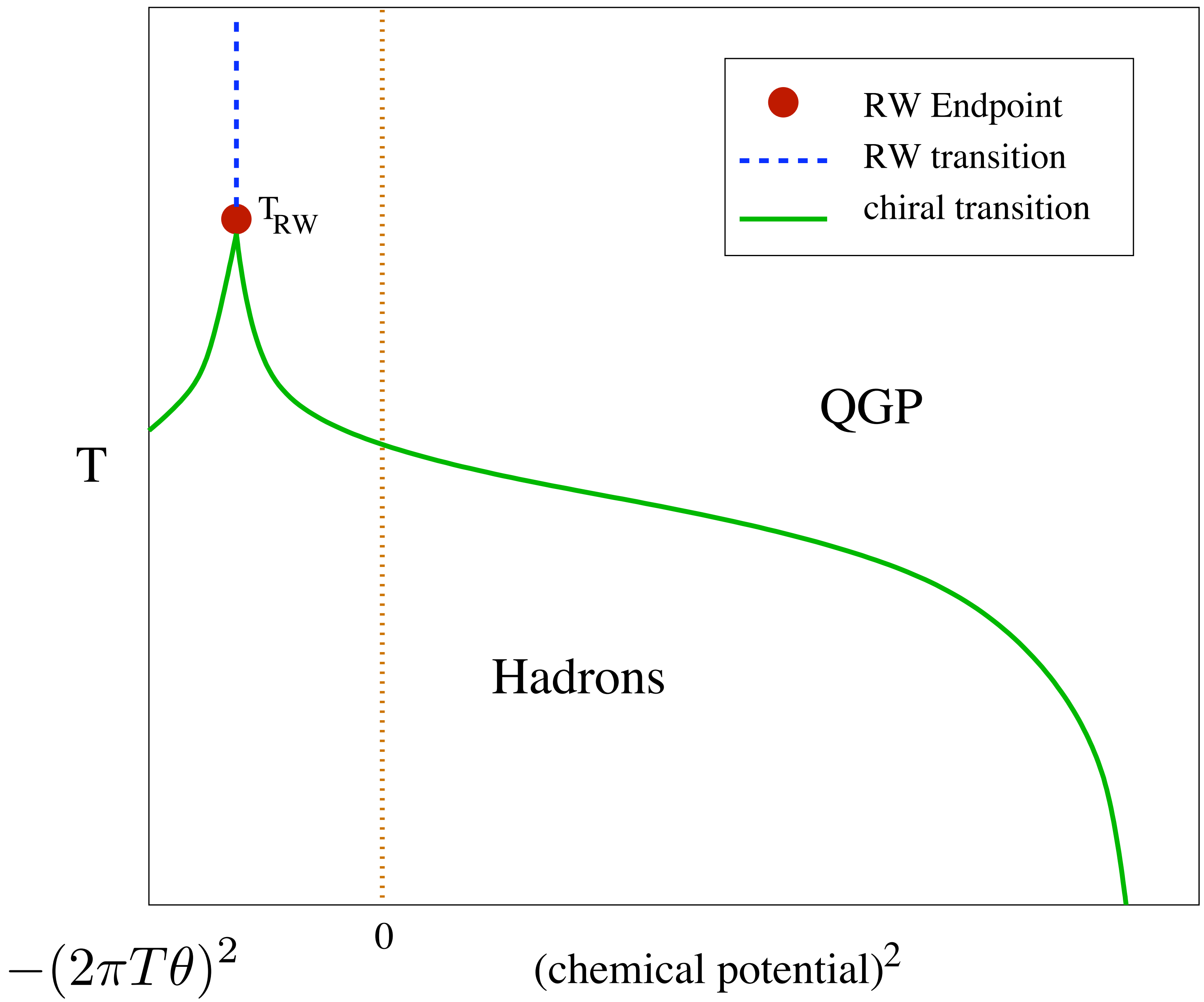
hadronic phase

confinement & chiral symmetry breaking

Imaginary chemical potential

Lattice & Continuum QCD

$$\psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x) \qquad \text{with} \qquad \mu_I = 2\pi T\theta$$

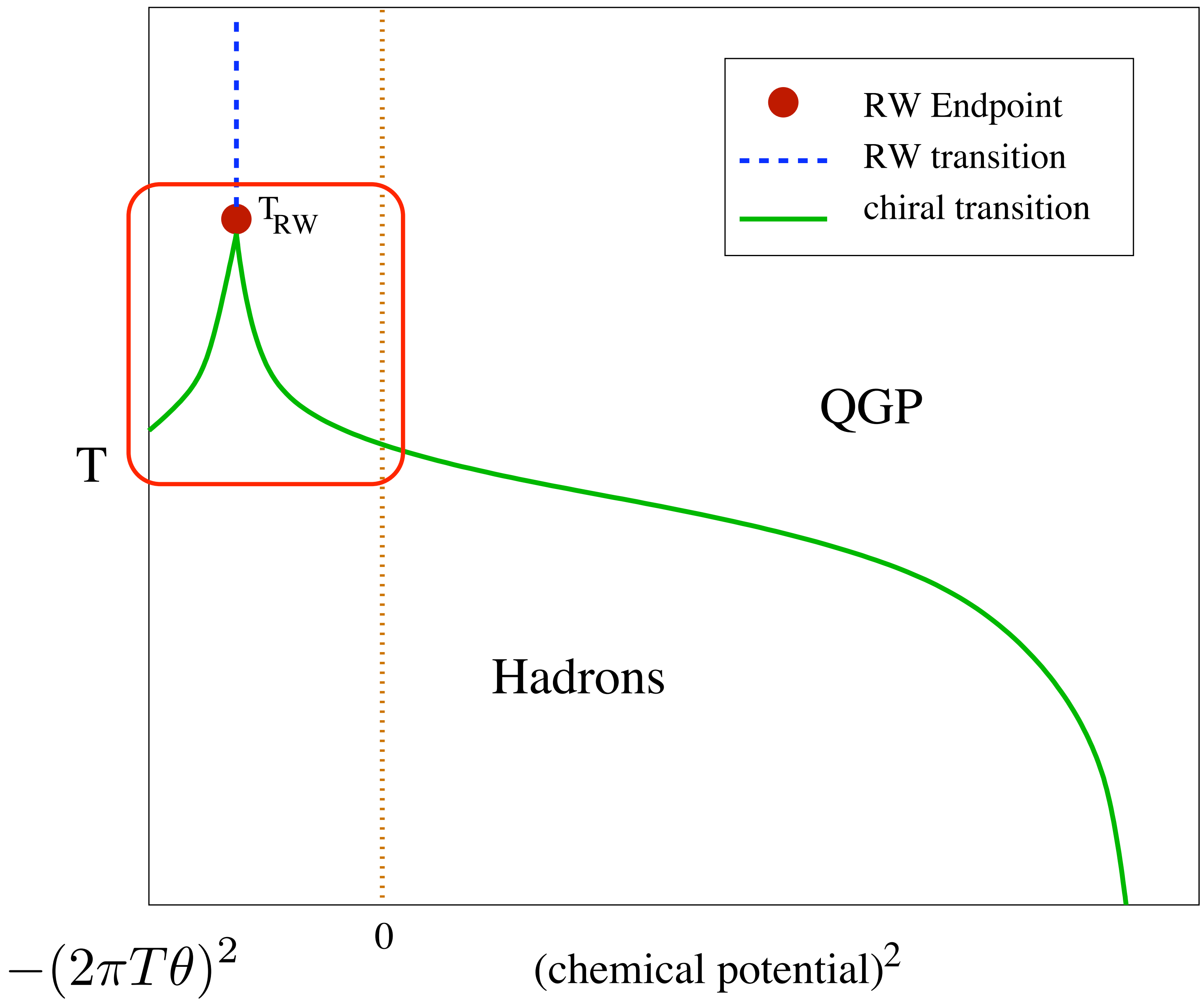


- Roberge-Weiss symmetry: $\theta \to \theta + 1/3$

Imaginary chemical potential

Lattice & Continuum QCD

$$\psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x) \qquad \text{with} \qquad \mu_I = 2\pi T\theta$$



- Roberge-Weiss symmetry: $\theta \to \theta + 1/3$

Imaginary chemical potential

Lattice & Continuum QCD

$$\mathcal{O}_\theta = \langle O[e^{2\pi i\theta t/\beta}\psi]\rangle \qquad \text{with} \qquad \psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x)$$

imaginary chemical potential $\mu = 2\pi i\theta/\beta$ for $\psi_\theta = e^{2\pi i\theta t/\beta}\psi$

$$z = e^{2\pi i\theta_z} \longrightarrow \tilde{\mathcal{O}} = \int_0^1 d\theta\, \mathcal{O}_\theta e^{-2\pi i\theta}$$

order parameter for confinement

Dual order parameter

- **Lattice**

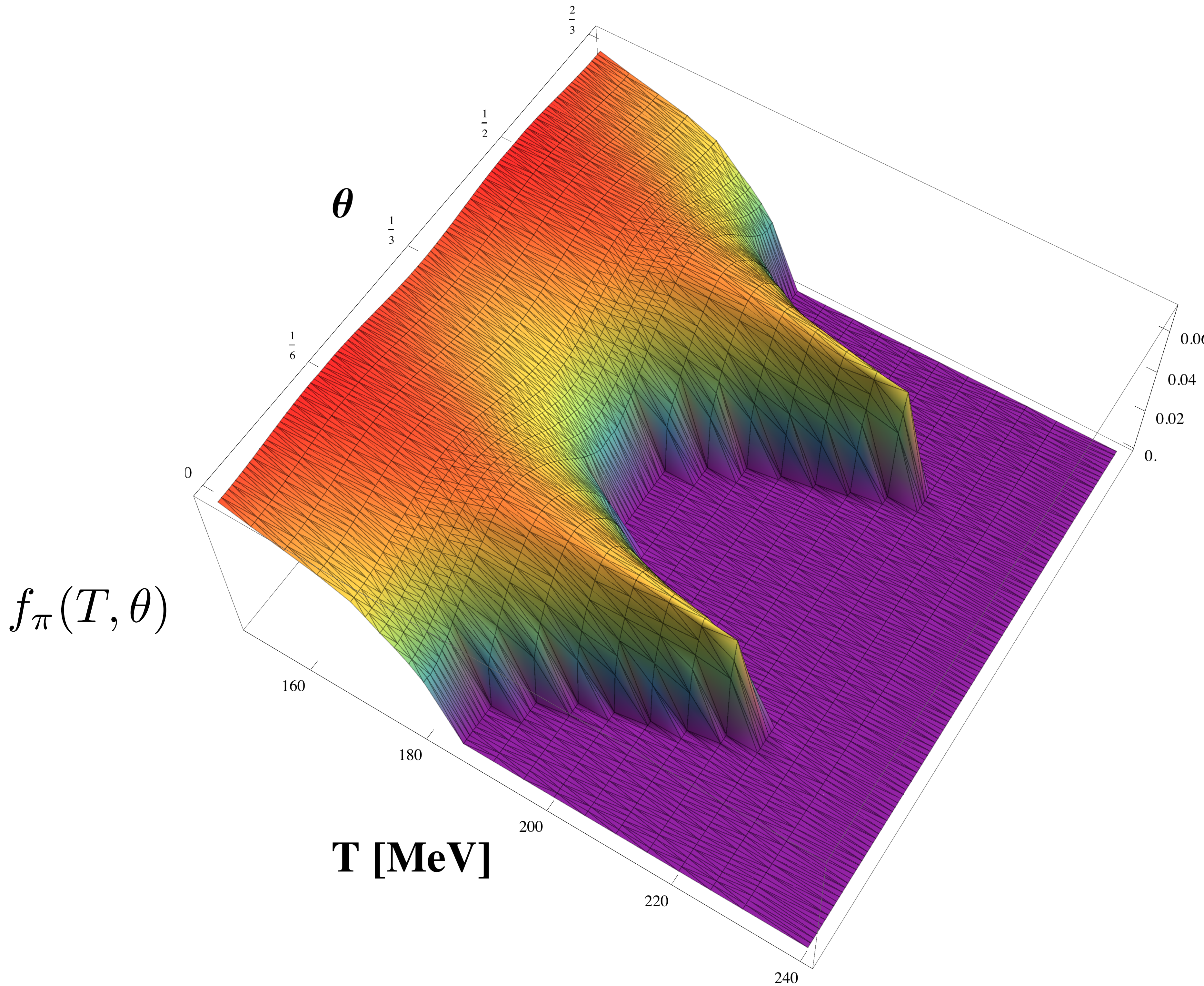
  Gattringer '06
  Synatschke, Wipf, Wozar '07
  Bruckmann, Hagen, Bilgici, Gattringer '08

- **Functional methods**

  Fischer, '09; Fischer, Maas, Müller '10
  Braun, Haas, Marhauser, JMP '09 ← imaginary chemical potential

Imaginary chemical potential

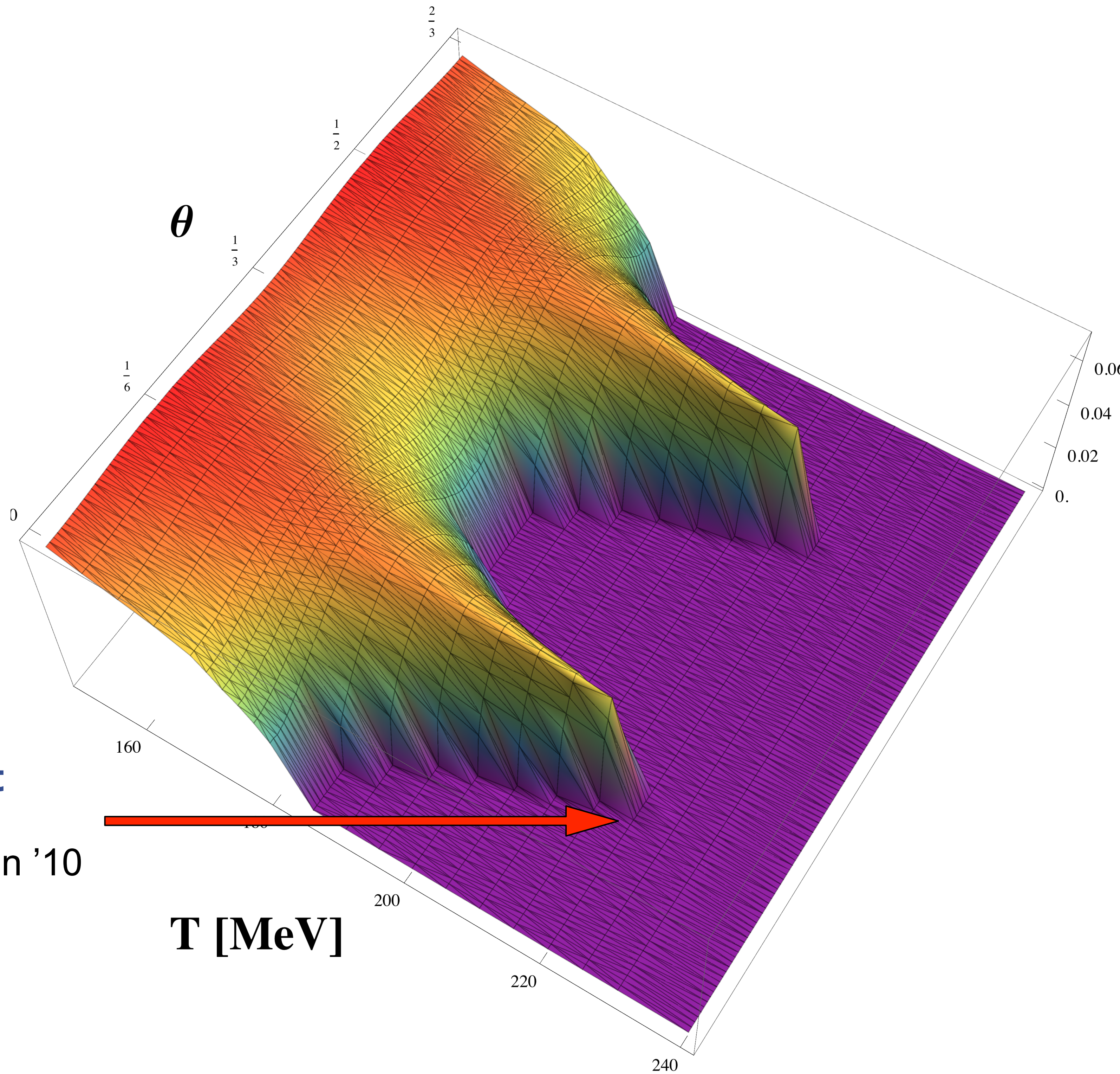
$\mathbf{N_f = 2}$ & chiral limit ⟵ (Functional RG-flows)



Braun, Haas, Marhauser, JMP ‘09

Imaginary chemical potential

$\mathbf{N_f = 2}$ & chiral limit ⟵ (Functional RG-flows)



Nature of RW endpoint

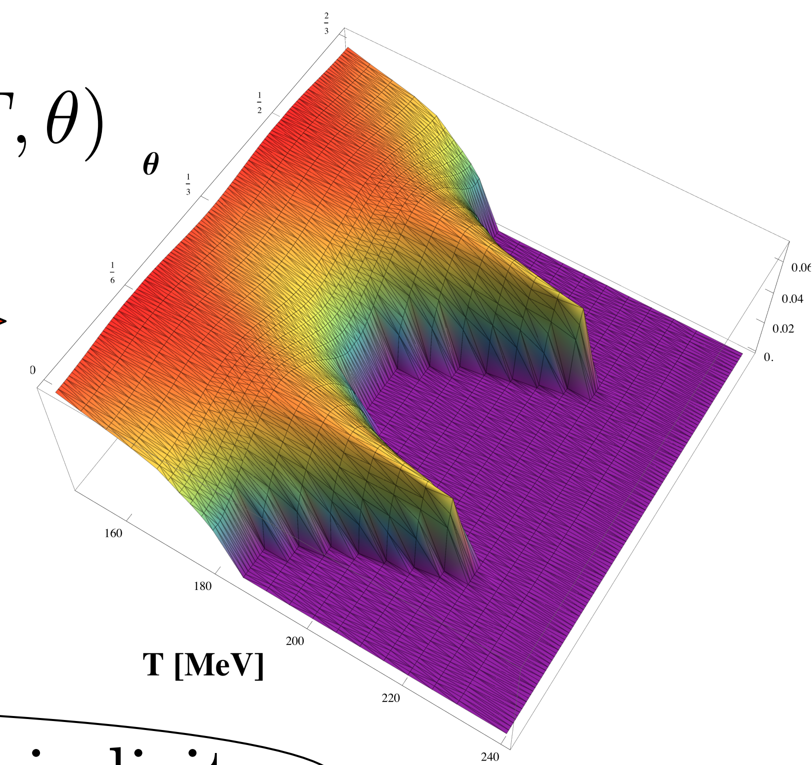
lattice: D'Elia, Sanfilippo '09
de Forcrand, Philipsen '10
PNJL: Sakai et al '10

Braun, Haas, Marhauser, JMP '09

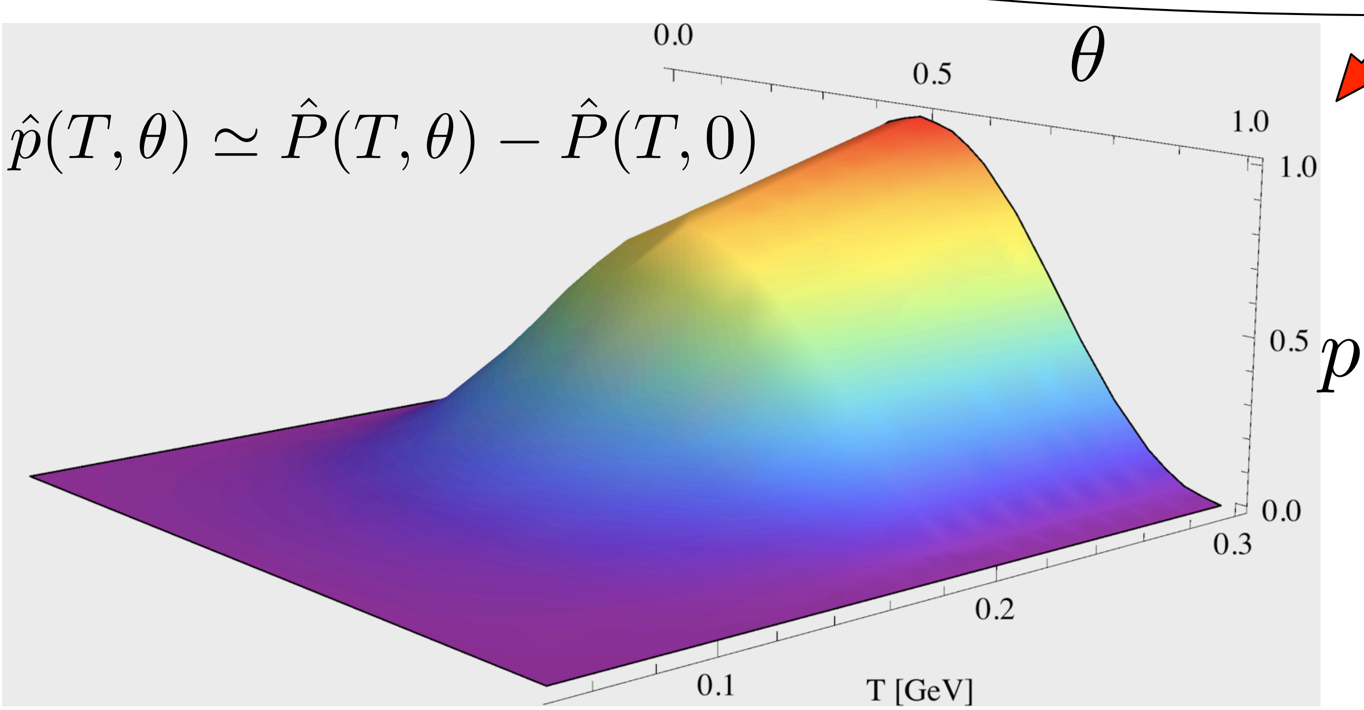
Imaginary chemical potential

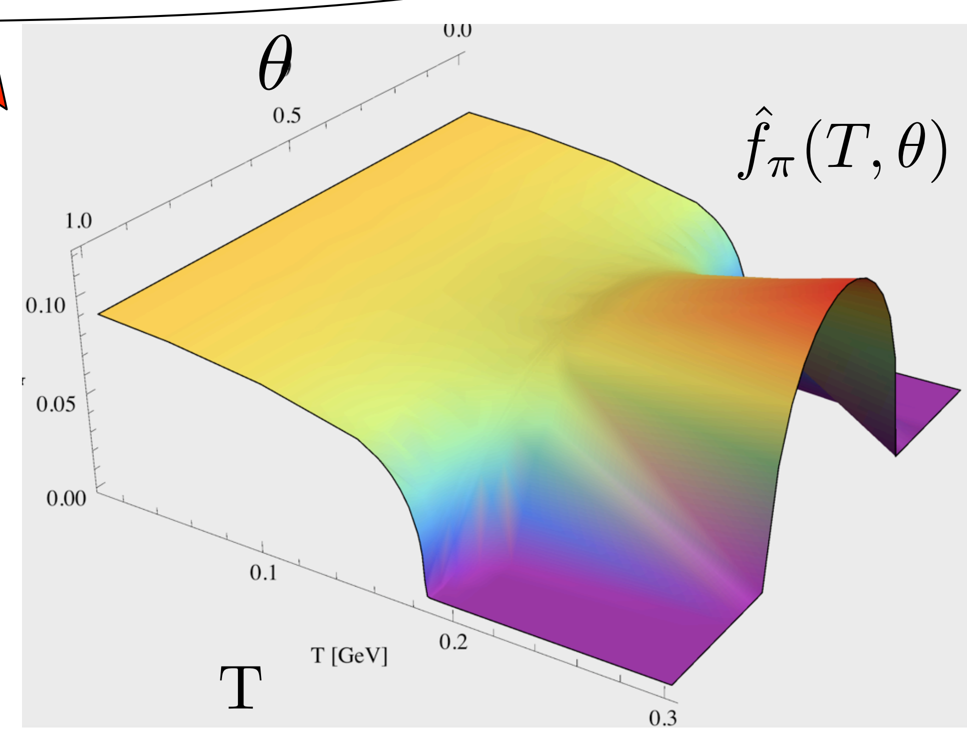
$\mathbf{N_f = 2}$ & chiral limit ⟵ (Functional RG-flows)

A_0 solution of EoM: Roberge-Weiss periodicity →



fixed A_0: no Roberge-Weiss periodicity

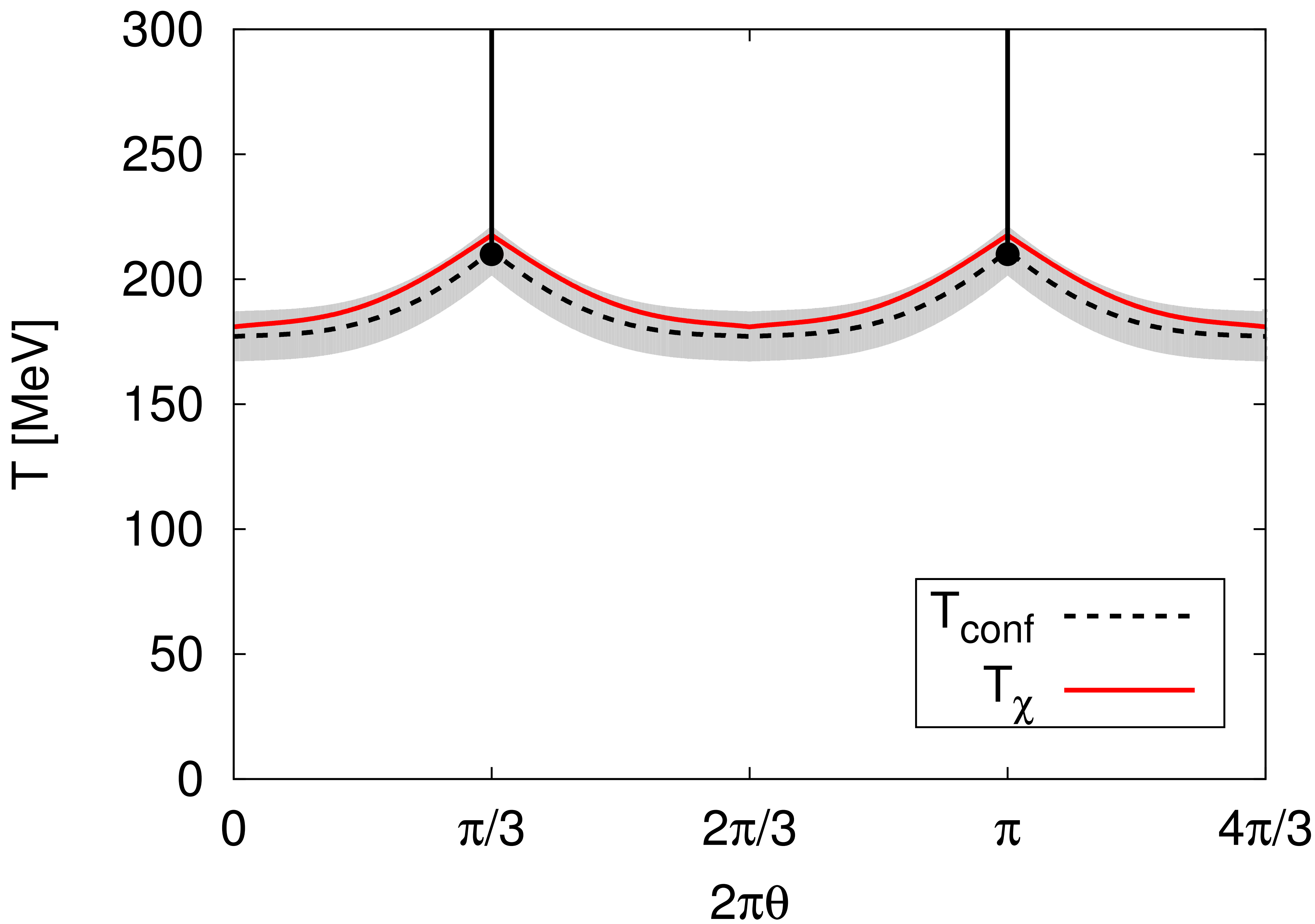




Braun, Haas, Marhauser, JMP '09

Full dynamical QCD: N_f=2

Continuum methods

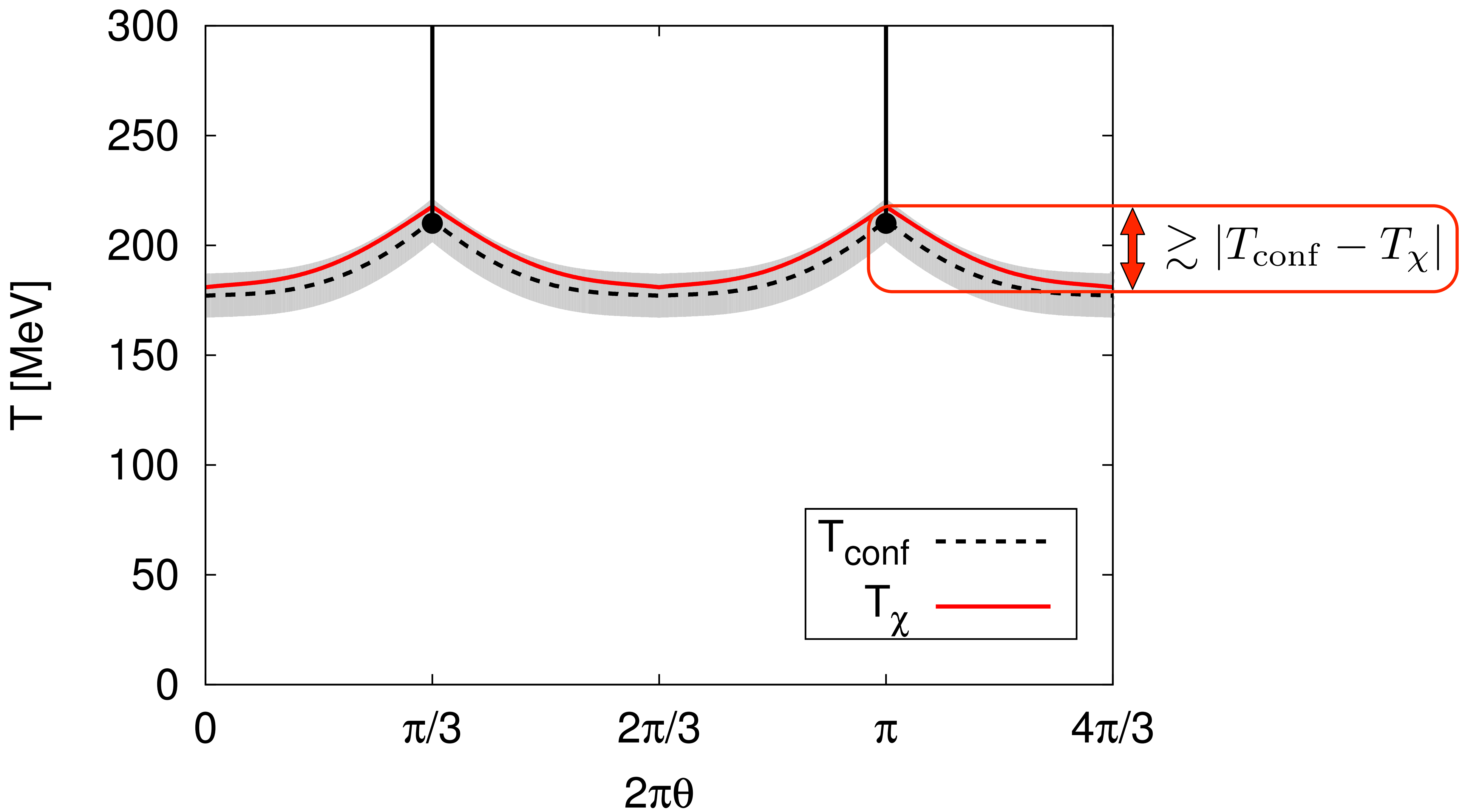


$$\text{chemical potential}: \mu = 2\pi i\, T\, \theta$$

Braun, Haas, Marhauser, JMP '09

Full dynamical QCD: N_f=2

Remark on dual order parameters for confinement

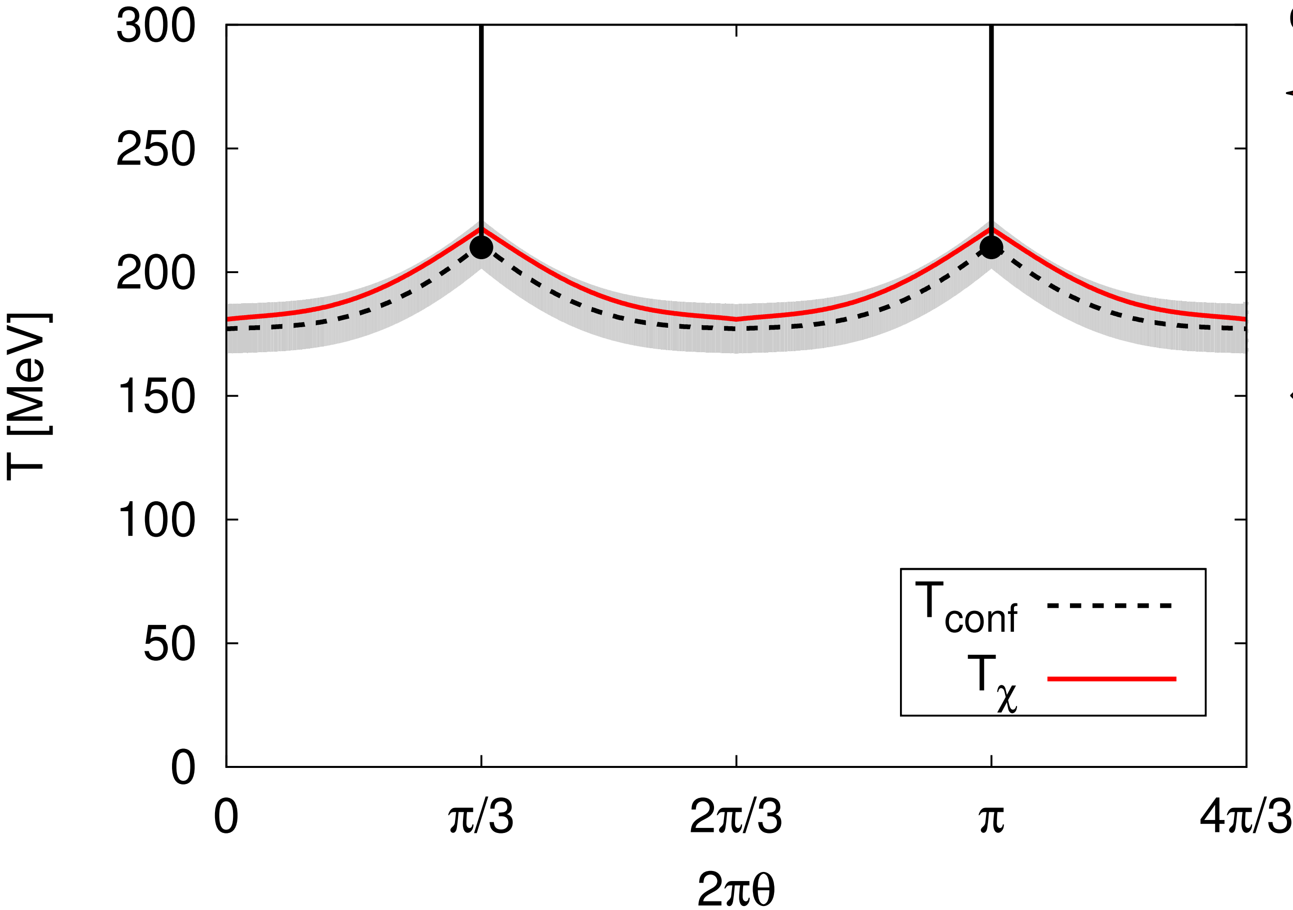


$\text{chemical potential}: \mu = 2\pi i T \theta$

Braun, Haas, Marhauser, JMP '09

Full dynamical QCD: N_f=2

Continuum methods & lattice

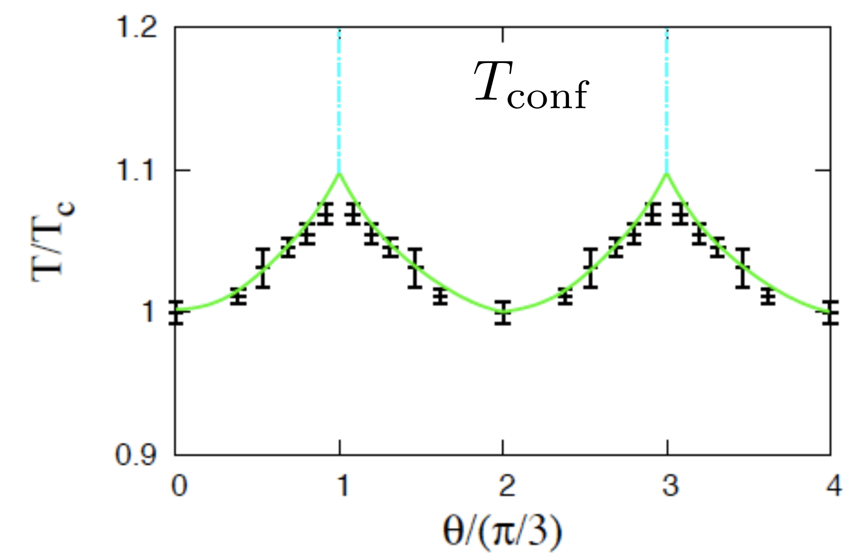


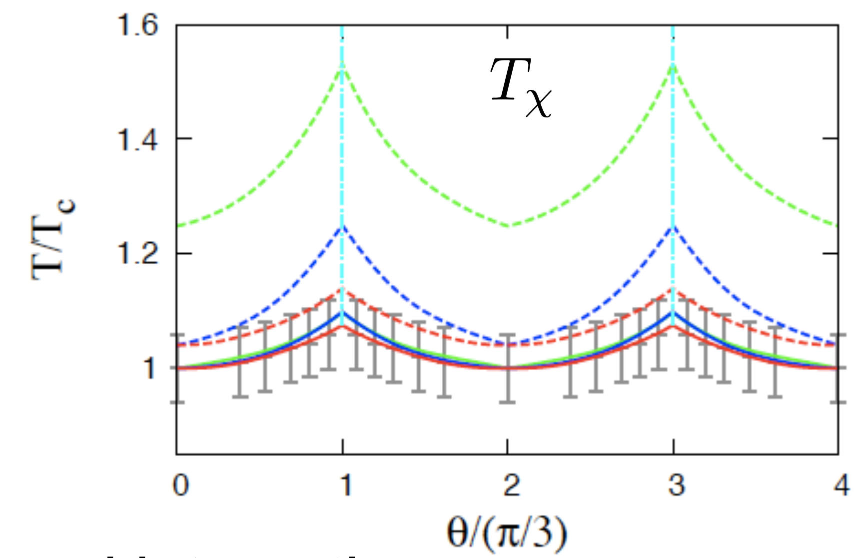
Braun, Haas, Marhauser, JMP ‘09

compatibility

lattice results, e.g.
Kratochvila et al ’06, Wu et al ’06
& D’Elia et al ’07

Polyakov-NJL model
Sakai et al ‘09

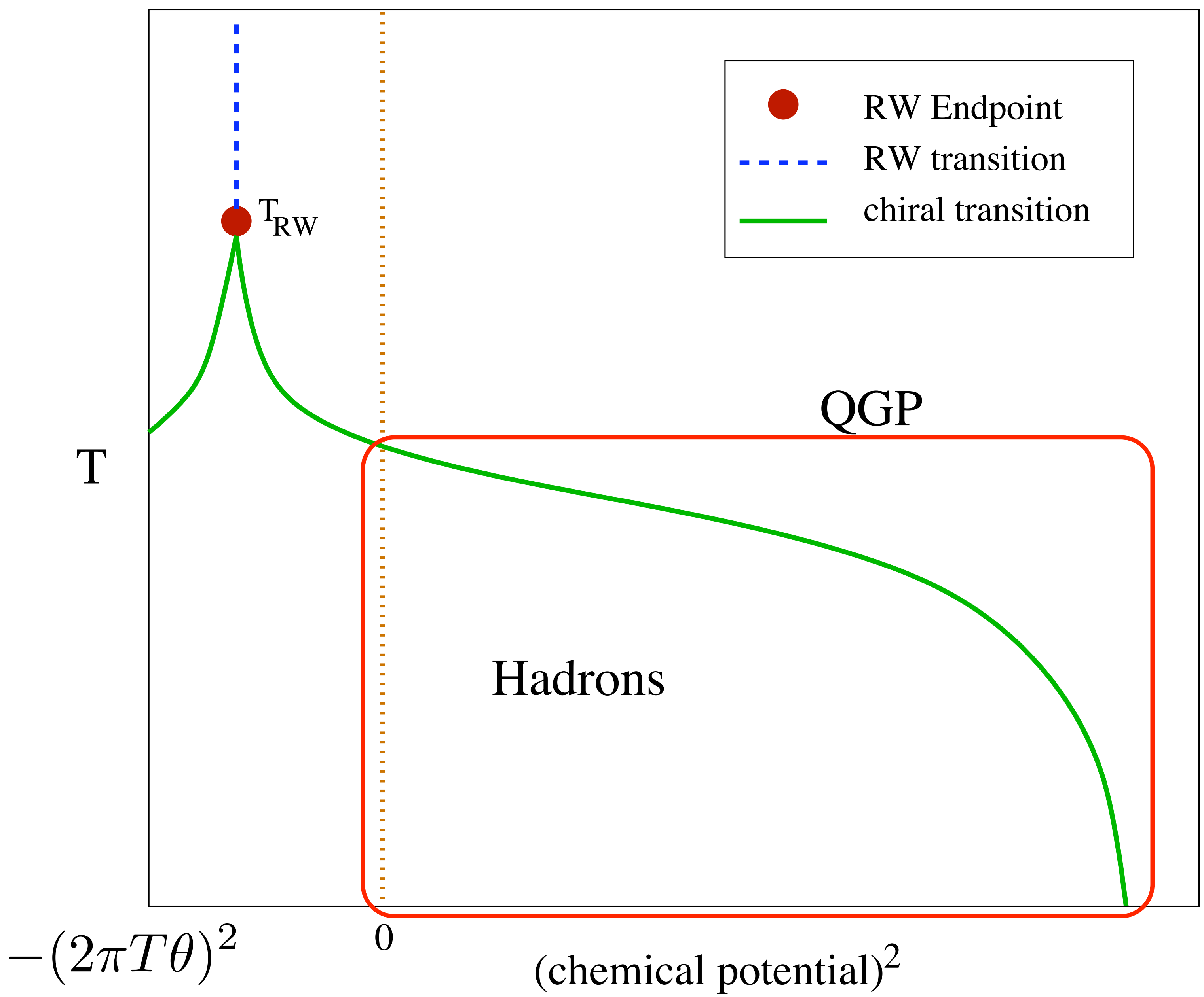




adjust 8-fermi interaction

Real chemical potential

$$\psi_\theta(t+\beta,\vec{x}) = -\psi(t,x)$$



chiral phase structure of one flavour QCD

$$\frac{T_c(\mu_q)}{T_c(0)} = 1 - t_2 \left(\frac{\mu_q}{\pi T_c(0)} \right)^2 + \dots$$

- large N_c expansion: $t_2 \sim \frac{N_f}{N_c}$ (D. Toublan '05, J. Braun '08)
- results from different approaches:

Method	$N_f = 1$	$N_f = 2$	$N_f = 3$	
FRG: QCD flow	0.97 / 0.40*	---	---	(J. Braun '08)
Lattice: imag. μ	0.398(75)	0.50	0.602(9)	(de Forcrand et al. '03, '07)
Lattice: Taylor+Rew.	---	---	1.13(45)	(Karsch et al. '03)

red: obtained from extrapolation

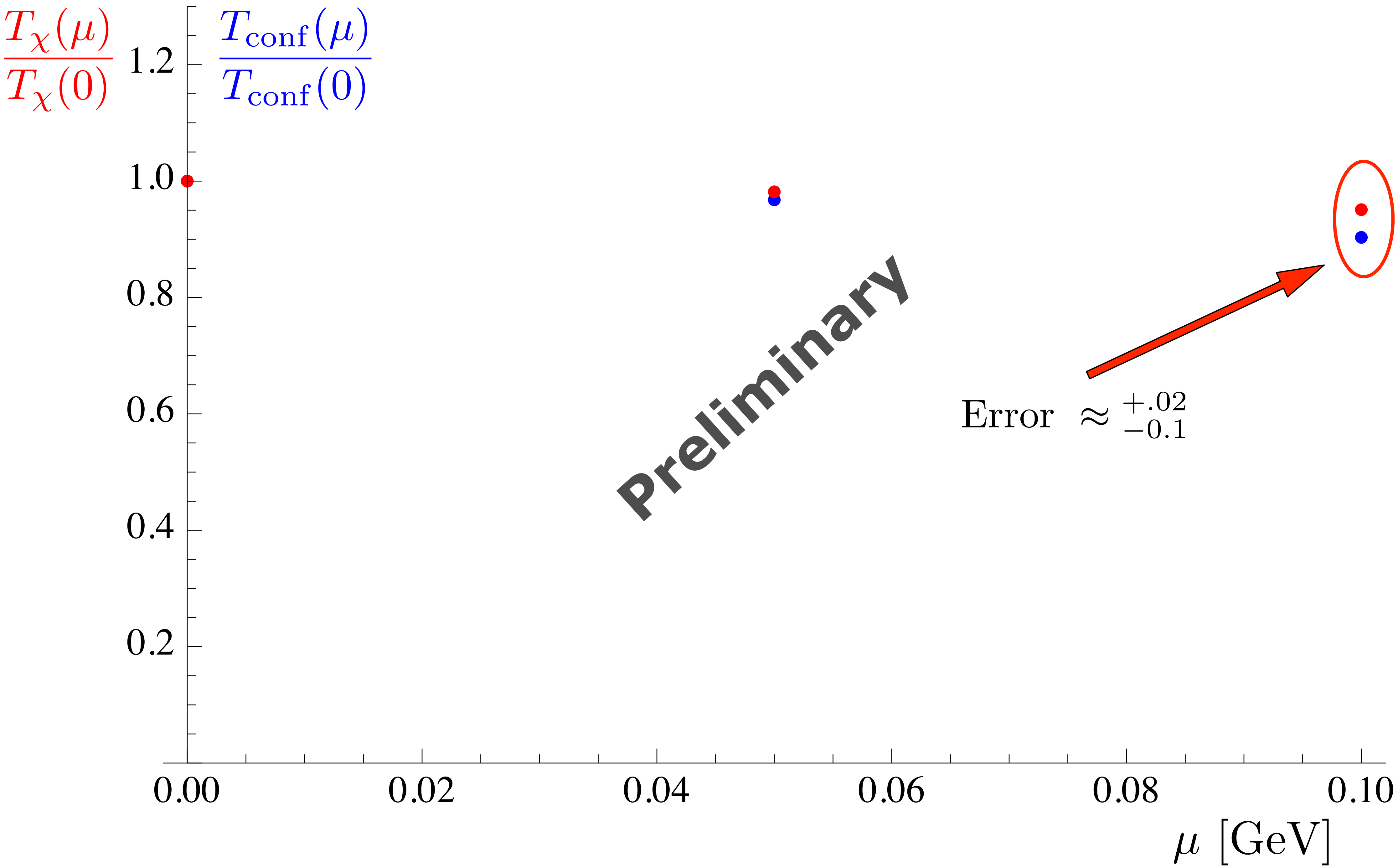
* estimate for lower bound for the curvature with anomaly

- only one single input parameter: $\alpha_s(M_Z)$

Braun '08

Real chemical potential

First results in full dynamical QCD



Braun, Haas, JMP, in prep.

chiral phase structure of two flavour QCD

Braun, Haas, Marhauser, JMP '09

$$\frac{T_c(\mu_q)}{T_c(0)} = 1 - t_2 \left(\frac{\mu_q}{\pi T_c(0)}\right)^2 + \dots$$

- large N_c expansion: $t_2 \sim \frac{N_f}{N_c}$ (D. Toublan '05, J. Braun '08)

chiral limit

Braun, Haas, JMP, in prep.

- results from different approaches:

Method	$N_f = 1$	$N_f = 2$	$N_f = 3$
FRG: QCD flow	0.97 / 0.40*	$\lesssim 1.4$	---
Lattice: imag. μ	0.398(75)	0.50	0.602(9)
Lattice: Taylor+Rew.	---	---	1.13(45)

red: obtained from extrapolation

* estimate for lower bound for the curvature with anomaly

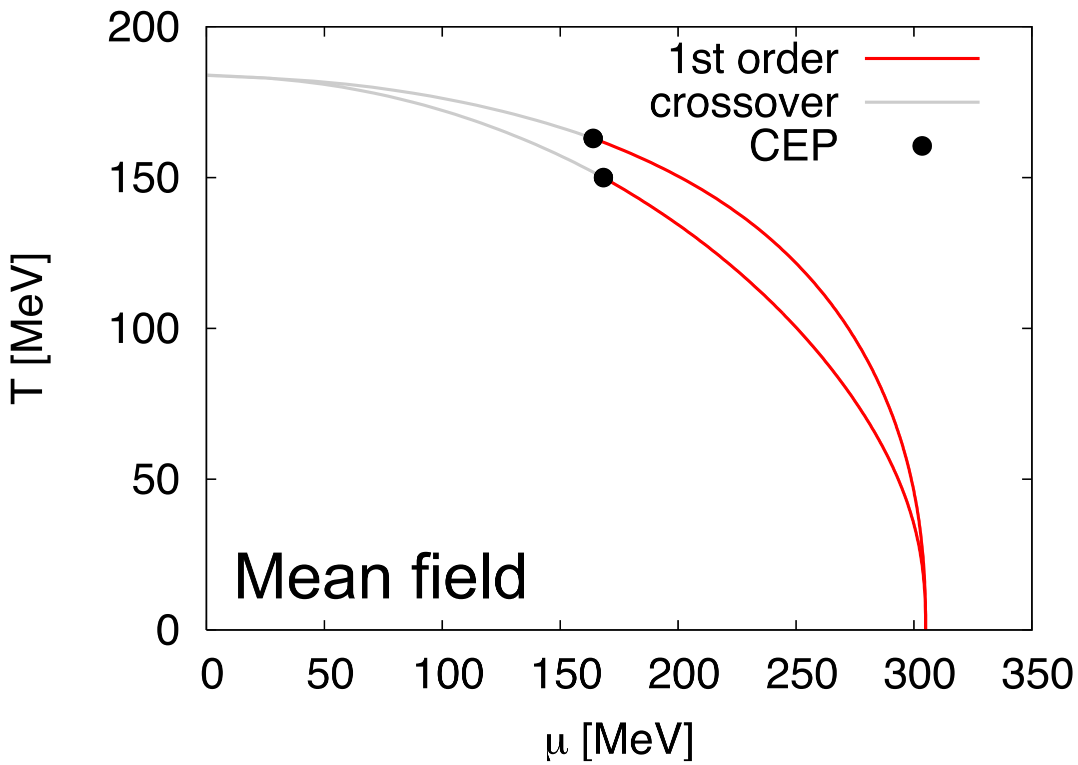
- only one single input parameter: $\alpha_s(M_Z)$

Real chemical potential

Herbst, JMP, Schaefer '10
Skokov, Friman, Redlich '10

→ dynamical Polyakov - Quark-Meson model

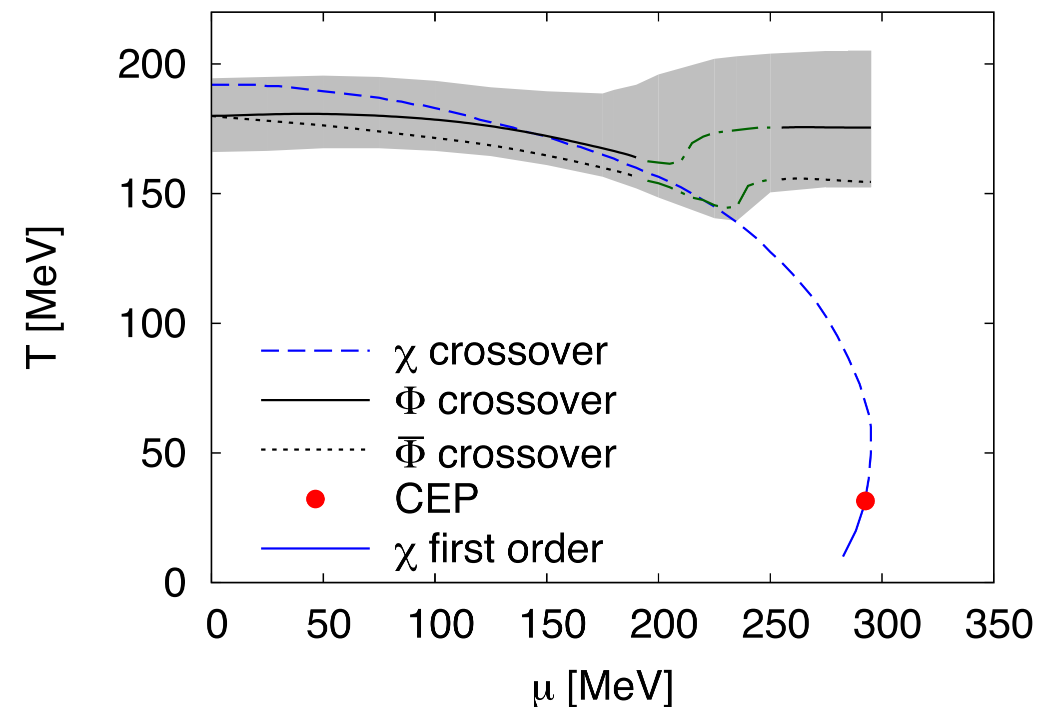
Constrained by QCD dynamics



RG

quark-meson

fluctuations



$N_f = 2$

HTL/HDL

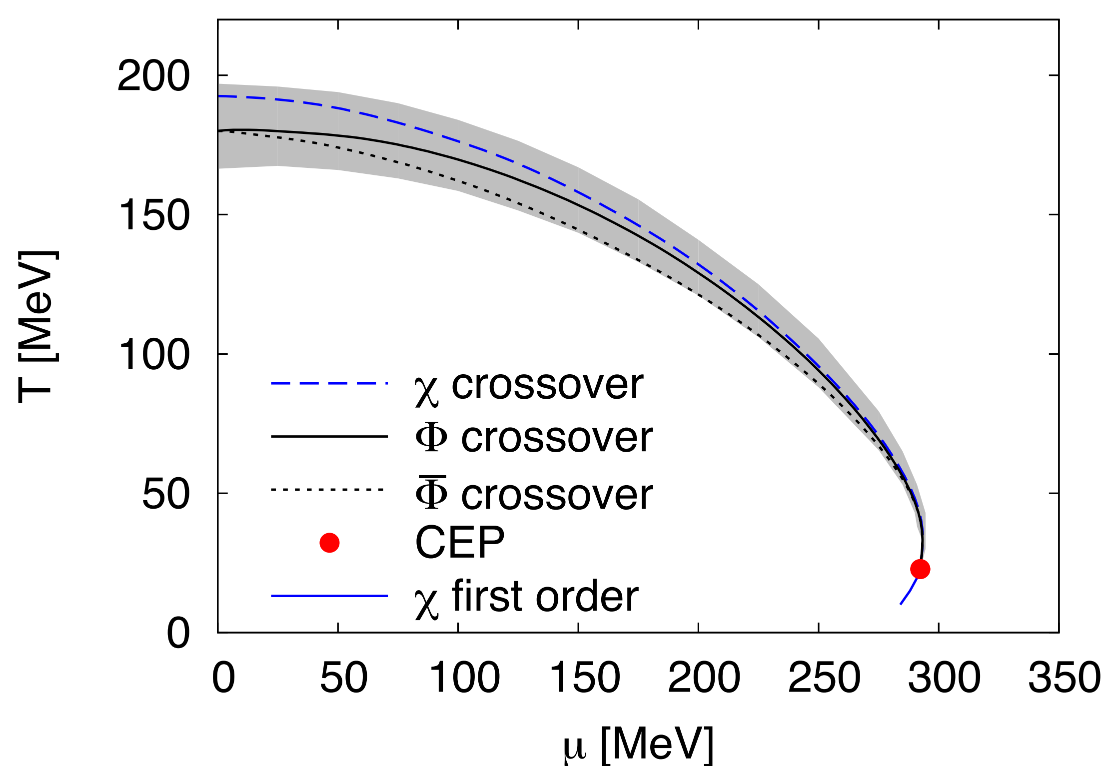
quark fluctuations

in YM sector

Schaefer, JMP, Wambach '07

Critical point unlikely for

$$\frac{\mu_b}{T} < 2$$



Herbst, JMP, Schaefer '10

Summary & Outlook

Summary & outlook

- **Gauge fixed & gauge invariant flows**
- **Yang-Mills flows**
  - **propagators in quantitative agreement with lattice**
  - **Polyakov loop potential & conf-deconf phase transition**
- **QCD**
  - **Many-flavour QCD**
  - **conf-deconf & chiral phase transition at imaginary chemical potential**
  - **first steps at real chemical potential**
- **Outlook**
  - **2+1 flavours, two colour QCD, baryons**