F-theory @ Korfu

Martijn Wijnholt
Max Planck Institute (AEI)

Plan:

- Brief recap of F-theory
- Brief review of Grand Unified Models
- Elliptic fibrations for F-theory SU(5) models
- Matter curves
- Yukawa couplings
- Breaking the GUT group/DT splitting
- Right-handed neutrinos
- Proton decay

Quick review:

F-theory = non-perturbative (in g_s) description of IIb with 7-branes

Axio-dilaton $\tau = a + ie^{-\varphi}$ $\Longrightarrow$ "Elliptic fibration" $y^2 = x^3 + fx + g$

7-branes $\Longrightarrow$ Discriminant locus $\Delta \sim 4f^3 + 27g^2 = 0$

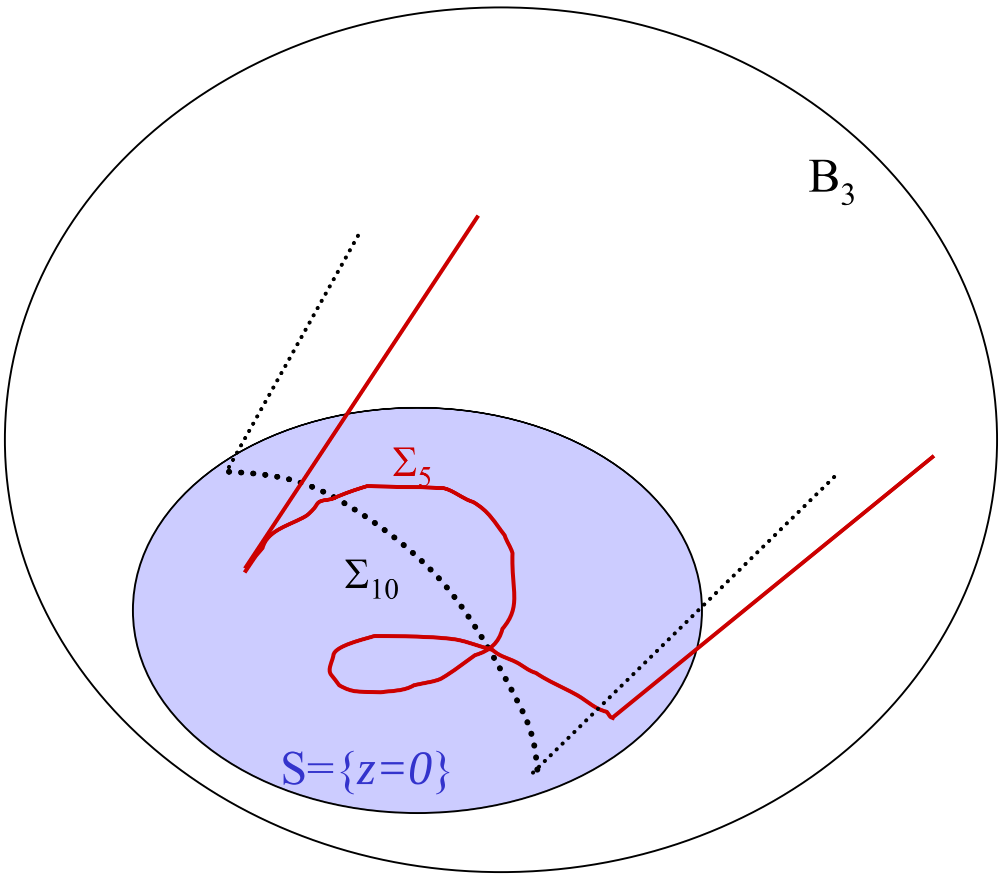
U(1) gauge fields $\Longrightarrow$ Zero modes of $C_3 \sim (B_{RR} - \tau B_{NS}) \wedge dz + c.c$

Non-abelian gauge symmetry $\Longrightarrow$ Singularities of elliptic fibration

$\Longrightarrow$ Kodaira classification

$M_{10} = R^{1,3} \times B_3$

Picture of discriminant locus (i.e. intersecting 7-branes) in B_3, to set the notation.



Kodaira classification: $y^2 = x^3 + fx + g$

ord(f)	*ord (g)*	*ord(Δ)*	*fiber type*	*singularity type*
≥ 0	≥ 0	0	smooth	–
0	0	n	*In*	*An-1*
≥ 1	1	2	*II*	–
1	≥ 2	3	*III*	*A1*
≥ 2	2	4	*IV*	*A2*
2	≥ 3	n + 6	*I*n*	*Dn+4*
≥ 2	3	n + 6	*I*n*	*Dn+4*
≥ 3	4	8	*IV **	*E6*
3	≥ 5	9	*III**	*E7*
4	5	10	*II**	*E8*

Note: exceptional groups!

7-branes intersect over "matter curves" ⟹ *6d* hypermultiplets

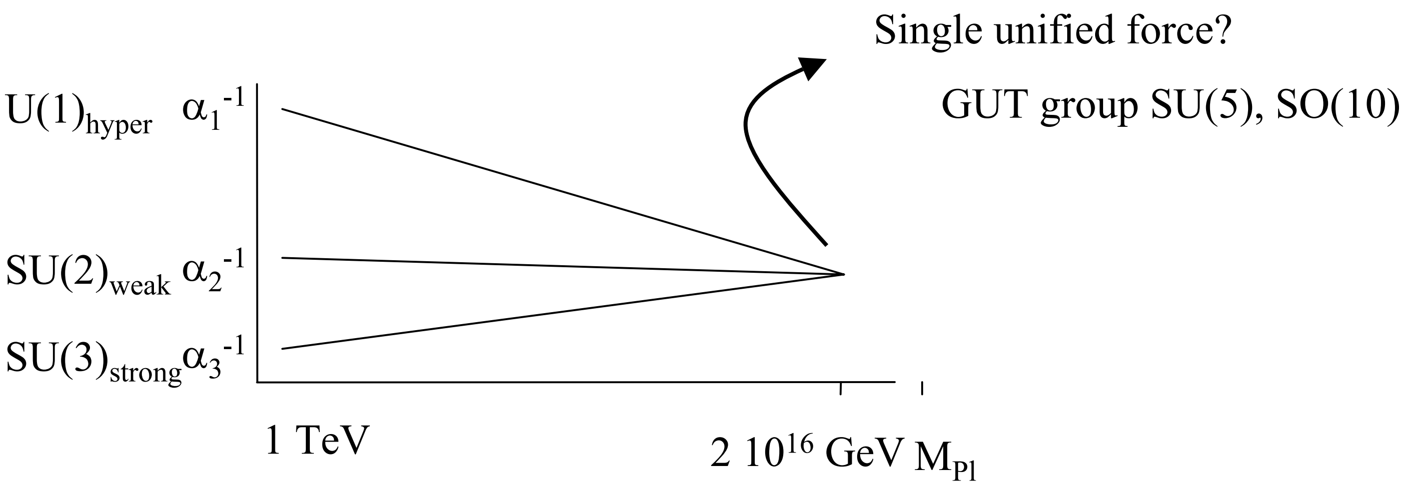
Fluxes on 7-branes ⟹ Hypers yield *4d* chiral fields

Net chiral $= \frac{1}{2\pi}\int_{\Sigma} F_1 - F_2$

Brief Intro to Grand Unified Models.

One of the strongest hints for a new fundamental scale at 10^{16} GeV:

⟹ All three gauge forces seem to unify!



⟹ Points to a unified gauge theory at the "GUT scale"

Many other hints (eg. neutrino masses, charge quantization, 3^{rd} gen Yukawas, …)

Quick review of Grand Unified group theory:

SU(5):

(Q, U, E)	$10 = (2,3)_{1/6} \oplus (1,1)_1 \oplus (1,\bar{3})_{-2/3}$	
(L, D)	$\bar{5}_m = (2,1)_{-1/2} \oplus (1,\bar{3})_{1/3}$	
N	$1 = (1,1)_0$	
(H_d, T_d)	$\bar{5}_d$	$\Rightarrow$ "doublet-triplet splitting problem"
(H_u, T_u)	5_u	

Ugly part in *4d* models: breaking the GUT group. Other issues: unification with gravity, etc.

$\Rightarrow$ This works more naturally if we have extra dimensions at the GUT scale

$\Rightarrow$ "Kaluza-Klein" GUT. Arises in string theory (M/F/Heterotic)

$SU(5) \subset SO(10)$ $\Rightarrow$ $16_m = 10 \oplus \bar{5}_m \oplus 1$ $\quad 10_h = 5_u \oplus \bar{5}_d$

F-theory GUT models

We focus on *SU(5)* GUTs. Pick a suitable B_3, eg. $B_3 = CP^3$

⟹ Want five D7-branes wrapped on divisor $z=0$ in B_3.

Eg. $B_3 = CP^3$, then z = degree 1 ⟹ $\{z=0\} = CP^2$

z = degree 2 ⟹ $\{z=0\} = CP^1 x CP^1$

⟹ Elliptic fibration over B_3 with I_5 singularity along $z=0$.

$$y^2 = x^3 + fx + g$$

⟹ Kodaira classification says:

$\Delta = 4f^3 + 27g^2 \sim z^5$, $\quad f \sim z^0$, $\quad g \sim z^0$, otherwise (almost) generic.

⟹ General expression is given by

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y$$

Here we used the "Tate form" -- can be rewritten in Weierstrass form if desired.

The general SU(5) model:

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y$$

(Note this locally looks like an E_8 ALE space → E_8 Higgs bundle story)

Let's check this yields the expected form of the discriminant.

$$f \sim (b_5^2 + 4 z b_4)^2 - 24(z^2 b_5 b_3 + 2 z^3 b_2)$$

$$g \sim -(b_5^2 + 4 z b_4)^3 + 36(b_5^2 + 4 z b_4)(z^2 b_5 b_3 + 2 z^3 b_2) - 216(z^2 b_3^2 + 4 z^5 b_0)$$

$$\Delta \sim z^5 b_5^4 R + z^6 b_5^2 (\ldots) + z^7 (\ldots)$$

For later use, define $R = b_0 b_5^2 - b_2 b_3 b_5 + b_4 b_3^2$

Precisely as required by Kodaira: $\Delta \sim z^5$, $f \sim z^0$, $g \sim z^0$.

$$\{\Delta = 0\} = \{z^5 = 0\} \cup \{\Delta' = 0\} \qquad \Delta' = \frac{\Delta}{z^5}$$

$\{z^5 = 0\}$ → Five 7-branes wrapped on $z=0$

$\{\Delta' = 0\}$ → additional 7-branes.

The matter curves

Let's call *{z=0}* the "*SU(5)* brane."

The *SU(5)* brane intersects additional "flavour'' 7-branes over the matter curves.

What kind of hypermultiplets do we get on the matter curves?

Matter curves defined by $\{z=0\} \cap \{\Delta' = 0\}$ $\quad \Delta' = \dfrac{\Delta}{z^5}$

$\Longrightarrow$ We find two matter curves: $\{z = R = 0\}$

$\{z = b_5 = 0\}$

First consider $\{z = R = 0\}$

From earlier expressions, $\Delta \sim z^6, \quad f \sim z^0, \quad g \sim z^0.$

⟹ According to the Kodaira, this is type I_6 (i.e. an *SU(6)* ADE singularity).

$$Adj(SU(6)) = 35 = Adj(SU(5)) \oplus 5 \oplus \bar{5} \oplus 1 \qquad SU(6) = \left(\begin{array}{c|c} SU(5) & 5 \\ \hline \bar{5} & \end{array} \right)$$

"off-diagonal" pieces give gauge representation of hypermultiplet.

So we get a *hypermultiplet in the fundamental of SU(5)* over this intersection.

⟹ Call this matter curve $\boxed{\Sigma_5 = \{z = R = 0\}}$

Next consider $\{z = b_5 = 0\}$

From earlier expressions, $\Delta \sim z^7, \quad f \sim z^2, \quad g \sim z^3.$

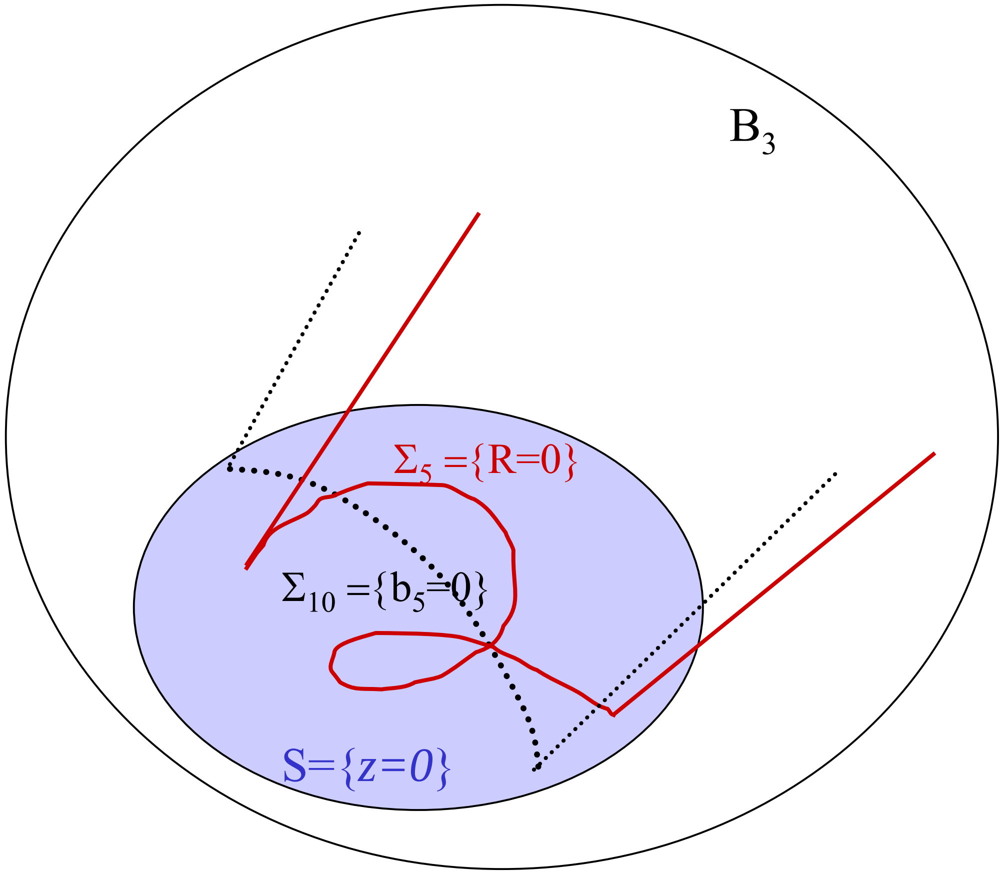
⟹ According to the Kodaira, this is type I_1* (i.e. an *SO(10)* ADE singularity).

$$Adj(SO(10)) = 45 = Adj(SU(5)) \oplus 10 \oplus \overline{10} \oplus 1$$

So we get a *hypermultiplet in the anti-symmetric of SU(5)* over this intersection.

⟹ Call this matter curve $\Sigma_{10} = \{z = b_5 = 0\}$

Summary so far: $M_{10} = R^{1,3} \times B_3$



It follows from the Kodaira classification that all SU(5) models look like this locally.

Charged matter appears `automatically.'

Fluxes: lots

Universal: $$N = \frac{1}{2\pi}\int_{\Sigma_{10}} F = (\tfrac{1}{2} + n)\,(5c_1(S) + [\Sigma_{10}]) \cap_S [\Sigma_{10}]$$

Given this general picture, what can we say about:

- Yukawas (top, down)
- GUT breaking, doublet/triplet splitting
- Neutrinos
- Proton decay?

Yukawas in *SU(5)* models:

$10_m \cdot 10_m \cdot 5_h \Longrightarrow$ up type $\qquad 10_m \cdot \bar{5}_m \cdot \bar{5}_h \Longrightarrow$ down type

Wave function overlap:

$$\lambda_{ijk} = \int_S Tr(\varphi_i \psi_j \psi_k) \quad \Longrightarrow \quad \text{Localized at intersections of } \Sigma_5 \text{ and } \Sigma_{10}.$$

<u>Claim:</u>

$\{b_4 = b_5 = 0\} \Longrightarrow$ up

$\{b_3 = b_5 = 0\} \Longrightarrow$ down

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 xy$$

<u>Case $\{b_4 = b_5 = 0\}$</u> $\Longrightarrow$ E_6 singularity $\Longrightarrow$ local configuration in *8d* E_6 gauge theory

λ_{ijk} inherited from $\int_{7-brane} Tr_{E_6}(\Psi \mathbb{A} \Psi)$ $\qquad E_6$ broken to $SU(5) \times U(1)_a \times U(1)_b$

$$Ad(E_6) = 78 \supset 10_{1,1} \oplus 10_{-1,1} \oplus 5_{0,-2} \quad \Longrightarrow \quad Tr(78^3) \to 10_{1,1} \cdot 10_{-1,1} \cdot 5_{0,2}$$

Similarly $\{b_5 = b_3 = 0\} \Longrightarrow$ SO(12) singularity $\Longrightarrow$ down type

GUT breaking.

Ideas:

4d mechanism 4d adjoints

xd mechanisms
- $U(1)_Y$ fluxes
- Discrete Wilson lines
- Abelian Higgs fields

$U(1)_Y$ fluxes

From mass term $[A, \langle A_Y \rangle]^2$, non-commuting gauge bosons get KK scale masses.

The commutant of $U(1)_Y$ in *SU(5)* is the SM gauge group, *SU(3)*x *SU(2)*x $U(1)_Y$.

⟹ Turn on $F_Y \neq 0$ on 7-brane

This breaks *SU(5)* to the Standard model.

No doublet/triplet splitting problem (no GUT group in *4d*, hence no *4d* colour triplet partner)

Small issue with coupling to RR axions (resolved)

U(1)$_Y$ fluxes (continued)

Easy to use, but some small blemishes:

* universal piece of heavy threshold corrections pushes away from unification
  (non-universal pieces more variable)

* KK scale is lowered compared to GUT scale

$\Longrightarrow$ Dim 5 proton decay must be forbidden

* SO(10) models have exotics

Alternative: internal discrete Wilson lines

(Work in progress, to appear)

* $\langle F_Y \rangle = 0$ but $\langle A_Y \rangle \neq 0$
* requires $\pi_1(S) \neq 1$ (where S is the cycle wrapped by the 7-brane)

* Harder to use, but 3 generation models can be constructed

Right handed (Majorana) neutrinos

Small neutrino masses naturally explained by see-saw mechanism:

$$LH_u N + mN^2$$

Large mass m $\Longrightarrow$ Integrate out N

$\Longrightarrow$ $$\frac{1}{m} LH_u LH_u$$ "Weinberg operator"

$\Longrightarrow$ $$M_{neutrino} \sim M_{weak}^2 / m$$

Note: Dominant contribution from lightest modes

Lightest singlets with such couplings $\Longrightarrow$ complex structure moduli (b_i).

$$y^2 = x^3 + b_0 z^5 + b_2 xz^3 + b_3 yz^2 + b_4 x^2 z + b_5 xy$$

These moduli get masses due to fluxes (OK) or D3-instantons (probably not OK; but may depend on moduli stabilization scenario).

If couplings are absent, use heavier modes (eg. KK modes).

Proton decay

Dim 4: $10_m \cdot \bar{5}_m \cdot \bar{5}_m$ R-parity not guaranteed

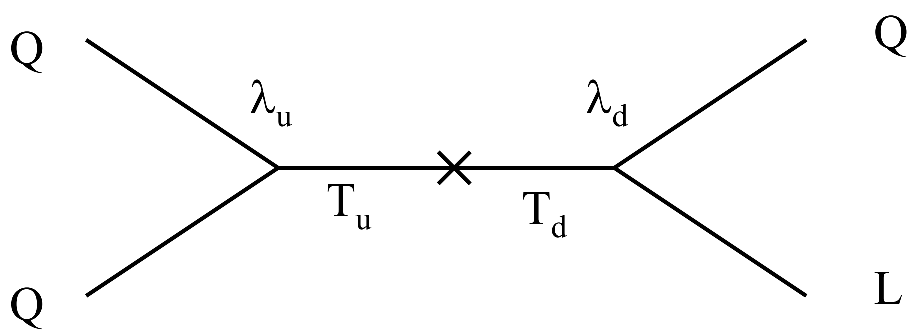
Factorize matter curve: $R \sim (\alpha b_3 + \beta b_5)(\gamma b_3 + \delta b_5) = 0$

$(\alpha b_3 + \beta b_5) \rightarrow \bar{5}_m$ $(\gamma b_3 + \delta b_5) \rightarrow \bar{5}_h$

Higgs/matter propagate on different pieces of Σ_5 $\Longrightarrow$ dangerous couplings absent

Extra U(1) symmetry? Eg. $U(1)_{\text{B-L}}$

Dim 5:



$\Longrightarrow \dfrac{1}{M_{KK}} QQQL$

Proton decay (continued)

To forbid, consider further factorization:

$$(\gamma b_3 + \delta b_5) \sim P_u \; P_d$$

Extra U(1)?

$$P_u \rightarrow 5_{H_u} \qquad P_d \rightarrow \overline{5}_{H_d}$$

Probably need these curves non-intersecting $\rightarrow$ also no classical mu-term

Alternatively, if M_{KK} large enough, consider these interesting possible signatures $p \rightarrow K^+ \overline{\nu}$

Dim 6: Slight enhancement of $p \rightarrow \pi^0 e_L^+$

$$\mathcal{M} \sim -\alpha_{GUT} \, \mathrm{Log}(\alpha_{GUT}) \frac{J_{10}^{\mu} J_{10,\mu}}{M_{GUT}^2}$$

Mu-problem: similar $\quad 5_{H_u} \cdot \overline{5}_{H_d} \cdot 1 \quad \rightarrow \quad H_u H_d S \qquad S$= complex structure modulus

Tune complex structure moduli (fluxes?), put on different pieces of Σ_5, instantons, or use U(1) symmetry.

Outlook on F-theory:

* New arena for model building

* Combines advantages of type IIb (localized branes) and heterotic (GUT structures)

* GUT breaking is nicer than *4d* models

$\Longrightarrow$ Use fluxes or discrete Wilson lines

* Fairly constrained $\Longrightarrow$ underlying E_8 structures

* Variation of F-theory methods can also be applied to M-theory models on G_2

* Still issues with SUSY breaking, strong CP, flavour, …

* As usual, much depends on finding a controlled model of moduli stabilization