LATTICE QCD AND FLAVOUR PHYSICS

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European Twisted Mass Collaboration

FlaviA net

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Prelude

- QCD: a central issue in the Standard Model (SM)
- Weak interaction asymmetries: *CP*-violation under intensive study
- Test subtler properties of SM
- Hope to see signatures of Physics beyond SM
- Experiments (strange sector): CERN, FNAL, ...
- Experiments (bottom sector): CERN, DESY, FNAL, KEK, ...
- Experiments (charm sector): Frascati, FNAL, KEK
- Theory: Dortmund, Dubna, Lund, Montpelier, Munich, Rome, Taipei, Trieste, Valencia, ...
- Main difficulty: control of strong interaction effects at low energies (non-perturbative QCD)

Flavour QCD basics

CP-violation in the Standard Model

- Arises through the interaction of charged matter (current) and gauge bosons:

$$W_\mu^+ J_\mu^+ + W_\mu^- J_\mu^-$$

Current:

$$J_\mu^+ = \bar{U} \quad \gamma_\mu^L \quad V_{CKM} \quad D = (\bar{u} \;\; \bar{c} \;\; \bar{t}) \;\; \gamma_\mu^L \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Interaction:

$$W_\mu^+ \;\; \bar{U} \gamma_L^\mu V_{CKM} D + W_\mu^- \;\; \bar{D} \gamma_L^\mu V_{CKM}^\dagger U$$

Cabibbo-Kobayashi-Maskawa matrix $\quad V_{CKM}$

3×3 , unitary matrix with 4 physical parameters

NB: *CP*-conservation (2 generations) implies $\quad V_{CKM} \in \mathcal{R}$

The CKM matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- 2 X 2 matrix: Cabibbo sector

$$V_{ud} \approx V_{cs} = \cos\theta_C \qquad V_{us} \approx -V_{cd} = \sin\theta_C$$

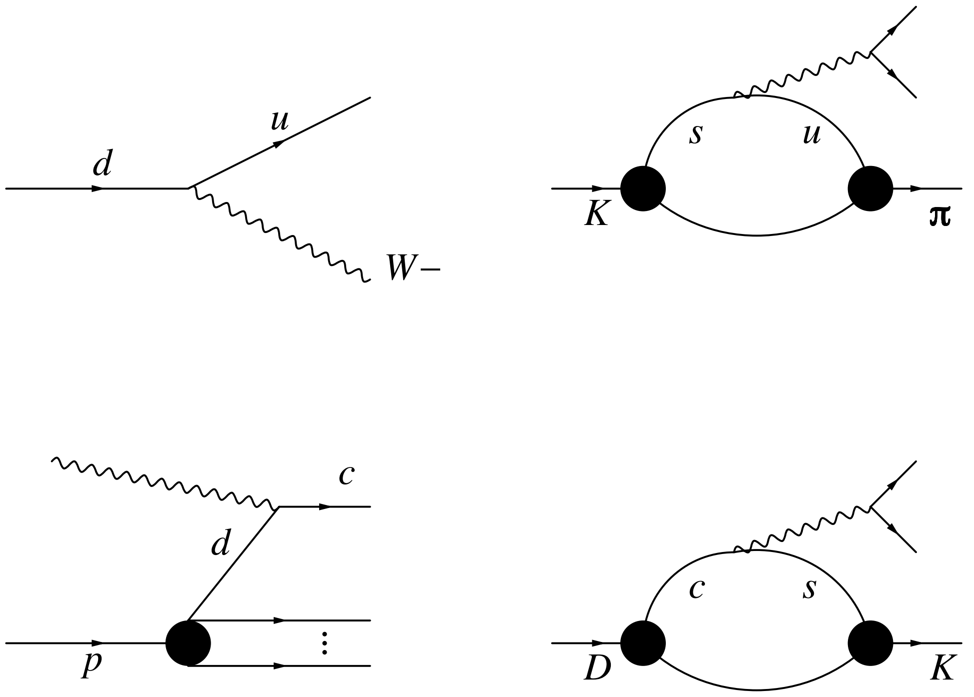
- 3rd column: Bottom sector

$$V_{cb} \approx -V_{ts} \qquad V_{tb} \approx 1$$

- If Cabibbo matrix & Bottom column accurately known, we are done, PROVIDED the SM is the whole story

- Poorest accuracy is due to hadronic NP-effects

The Cabibbo sector



- No *CP* violation in the 2X2 submatrix
- Beta-decays: $V_{ud} = 0.9740 \pm 0.0005$
- Kaon semileptonic decays $V_{us} = \lambda = 0.2256 \pm 0.0022$
- QCD low energy effects: WME $\langle \pi | J_\mu | K \rangle$

The Unitarity Triangle

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Wolfenstein parametrization of V_{CKM} (good to $O(\lambda^4)$)
- $\lambda = \sin(\theta_C) \approx 0.22$
- Unitarity: $[3^{RD}\text{ row}]^\dagger \times [1^{st}\text{ col}]$ implies phaenomenologically useful relation:

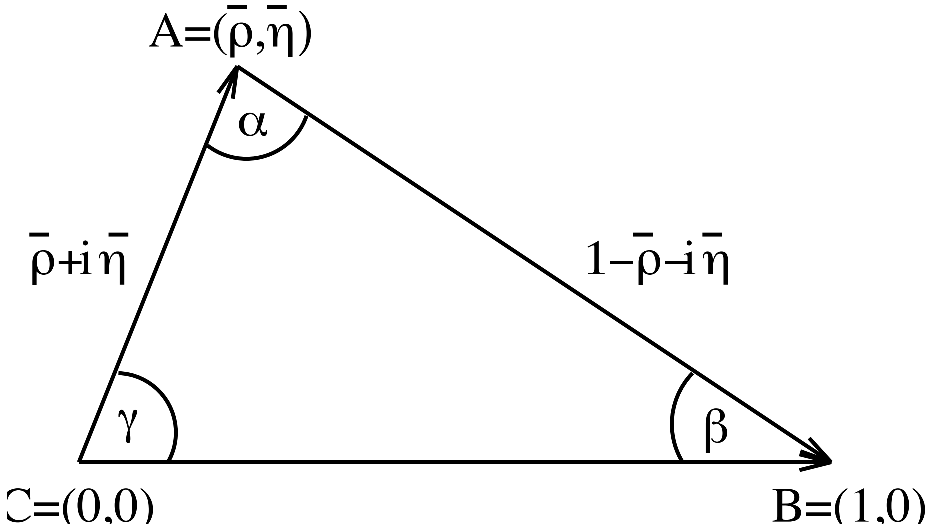
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

In terms of Wolfenstein parameters:

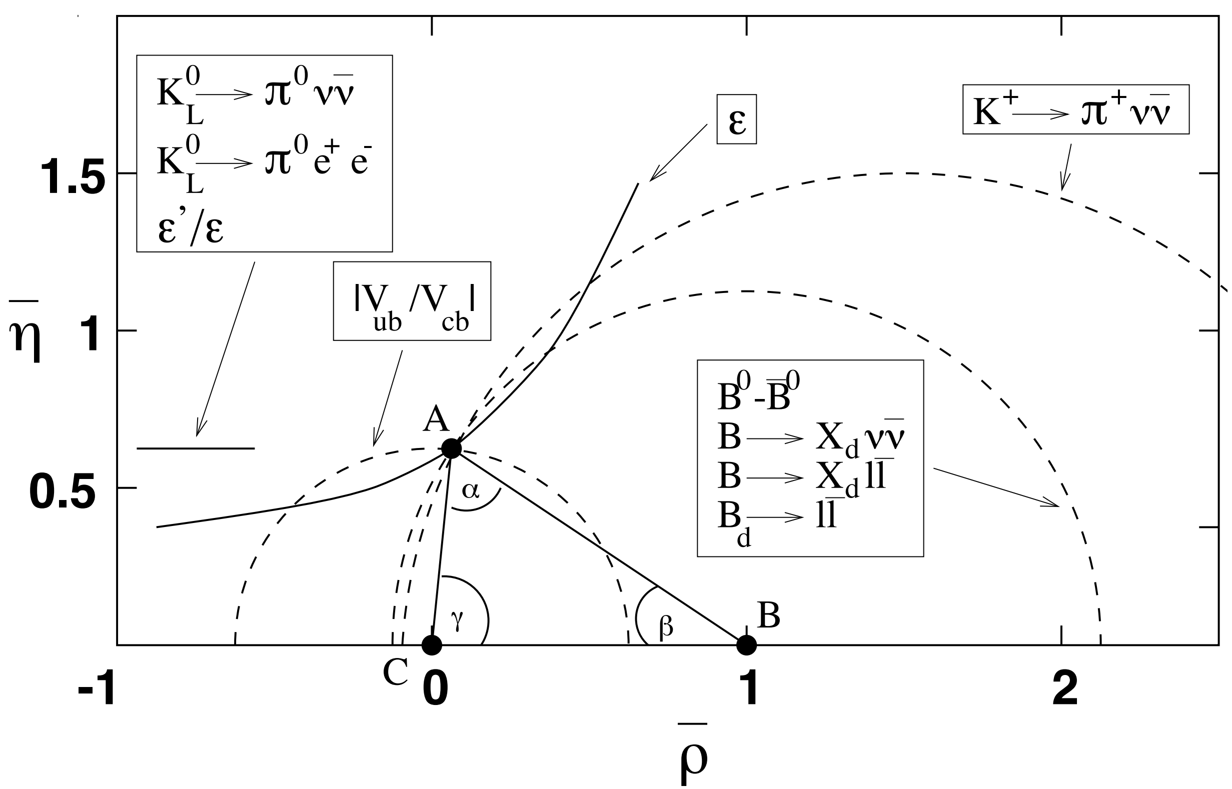
$$V_{ud}V_{ub}^* = A\lambda^3(\overline{\rho} + i\overline{\eta})$$
$$V_{cd}V_{cb}^* = -A\lambda^3$$
$$V_{td}V_{tb}^* = A\lambda^3[1 - (\overline{\rho} + i\overline{\eta})]$$

$$\overline{\rho} = \rho(1 - \lambda^2/2)$$

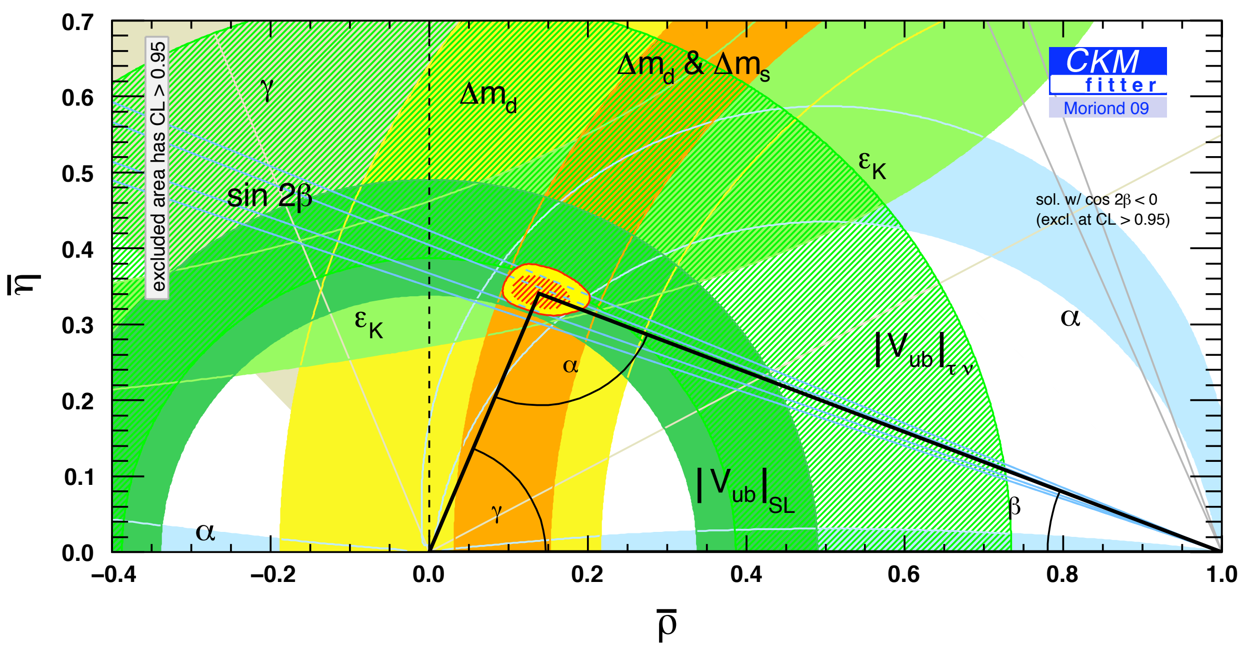
$$\overline{\eta} = \eta(1 - \lambda^2/2)$$

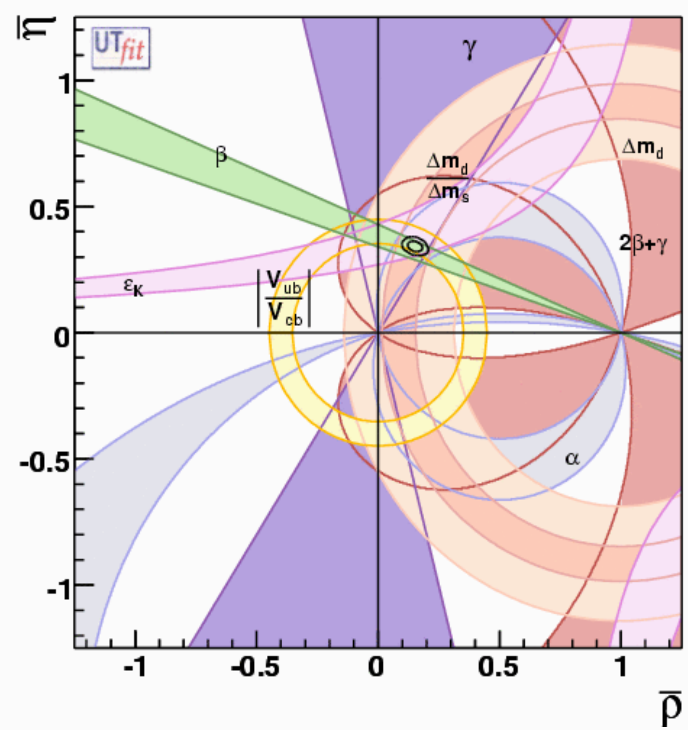


- several processes to check UT
- "Gold plated" decay $B_d \to J/\Psi + K_s$ gives sin(β) [Belle - BaBar] $K^0-\bar{K}^0(\Delta S=2)$
- ε-hyperbola: $B_d^0-\bar{B}_d^0(\Delta B=2)$



ΔS=2 transitions: ε_K





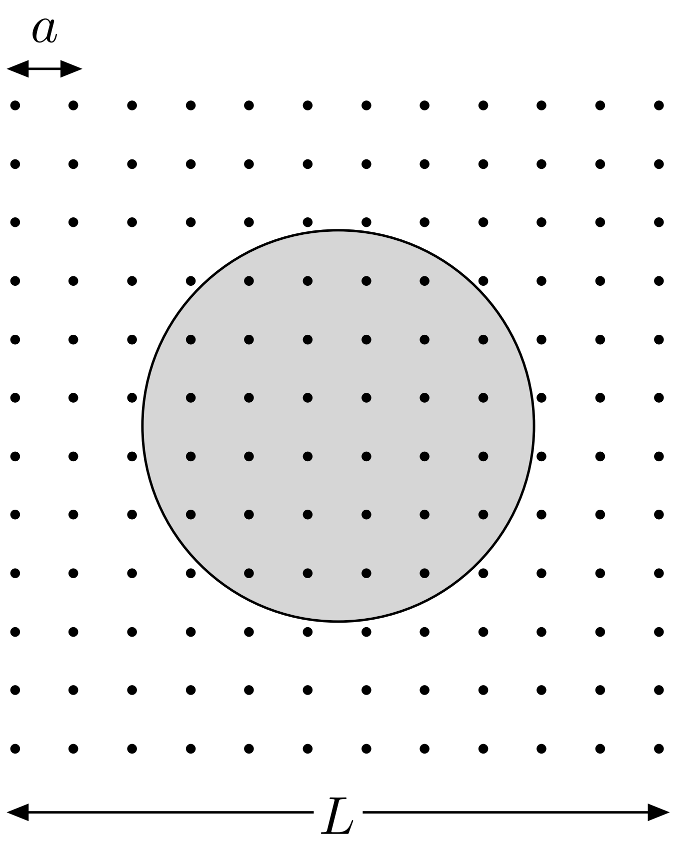
Lattice basics

Lattice themes

- discretization of spacetime and QCD length scales
- hadron masses and WME from the lattice
- lattice actions, fermion doubling
- renormalization & improvement
- heavy flavours on the lattice
    - HQET, NRQCD

Lattice basics

- Regularize QCD by discretizing space-time:
  - hypercube with lattice spacing *a* (UV cutoff) ...
  - ... and linear extension *L* (IR cutoff)
- PI is now well-defined for bare theory and can be computed; we can do experimental QCD at finite UV cutoff



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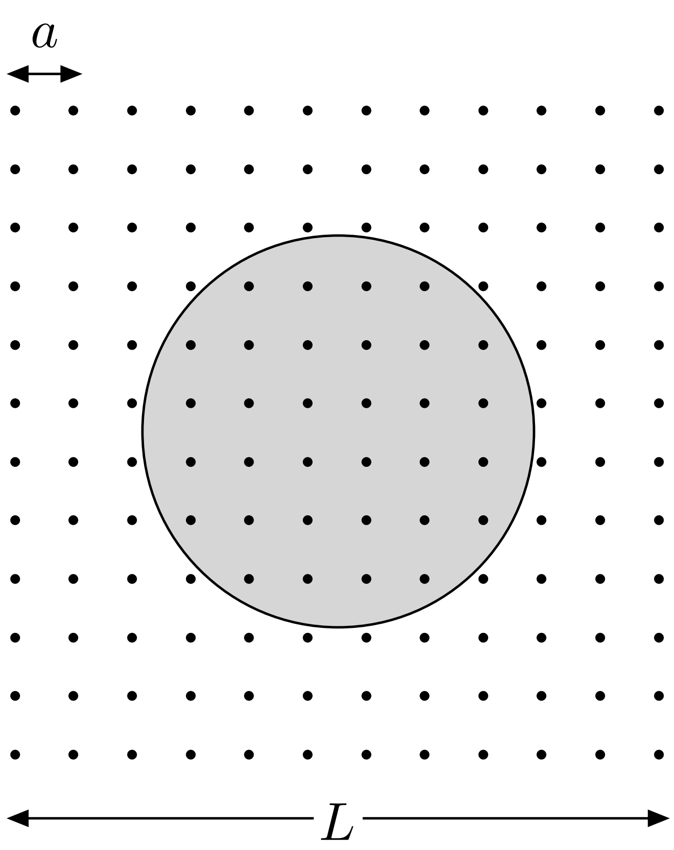


- scales (e.g. hadron masses) must satisfy

$$L^{-1} << m_H << a^{-1}$$

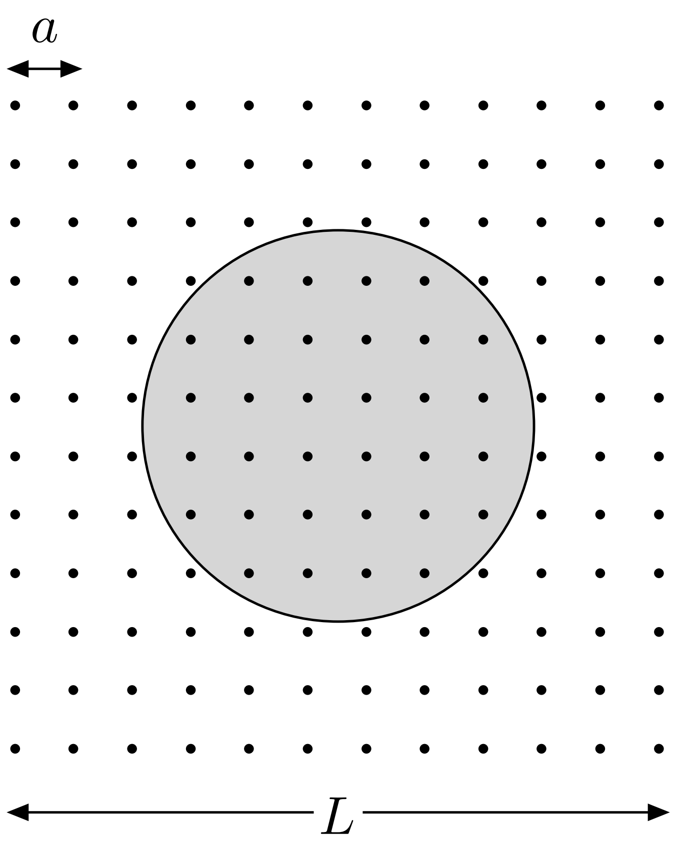
- must also ensure

$$\Lambda_{QCD} << a^{-1}$$

- [N.B. $\Lambda_{QCD} \sim 300$ *MeV*]

Practical difficulties

- suppose computers can tackle $a \sim 0.04$ fm and $L \sim 2$ fm; i.e. $L/a \sim 50$ lattice sites
- we have $O(50^4)$ degrees of freedom
- $a^{-1} \sim 5$ GeV and $L^{-1} \sim 100$ MeV
- OK for strange and charm mesons



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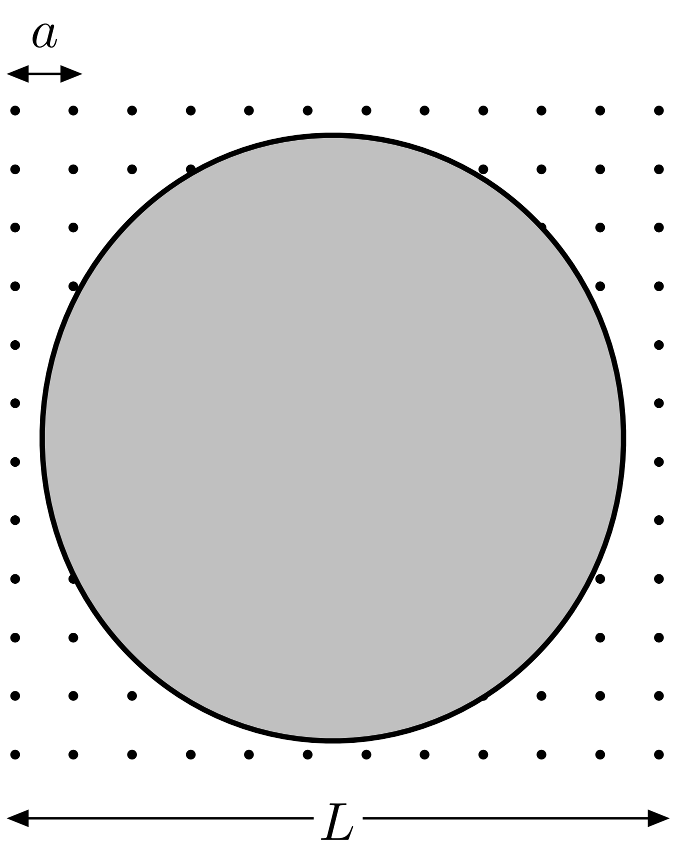
- we have $O(50^4)$ degrees of freedom
- $a^{-1} \sim 5$ *GeV* and $L^{-1} \sim 100$ *MeV*
- "Goldstone" mesons $m_\pi \sim 150$ *MeV* afflicted by finite volume effects



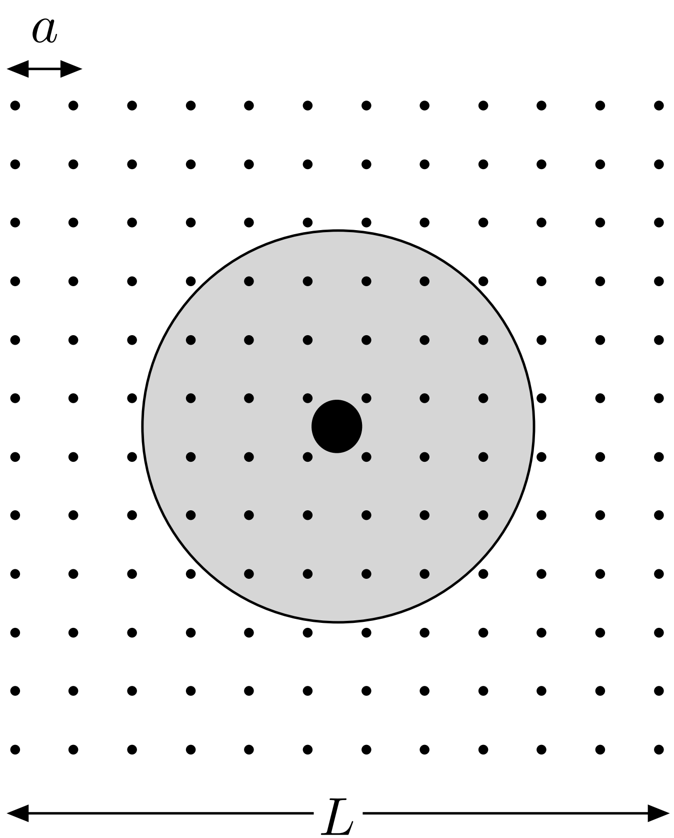
- scales (e.g. hadron masses) must satisfy

$$L^{-1} \ll m_H \ll a^{-1}$$

- compute in range $m_s/8 < m_q < m_s/2$ and extrapolate to light quark values
- use functional form suggested by χPT in the extrapolation
- ensure $m_H L > 4$

Practical difficulties

- present day computers can tackle $a \sim 0.04$ *fm* and $L \sim 2$ *fm*
- we have $O(50^4)$ degrees of freedom
- $a^{-1} \sim 5$ *GeV* and $L^{-1} \sim 100$ *MeV*
- heavy mesons $m_B \sim 5$ *GeV* afflicted by finite size effects



- scales (e.g. hadron masses) must satisfy

$$L^{-1} \ll m_H \ll a^{-1}$$

- compute in range $m_c < m_q < 1.5\, m_c$ and extrapolate to bottom quark values
- using results suggested by HQET or NRQCD interpolate charm up to bottom region

Lattice correlation functions

- in the lattice PI framework, we compute **bare** correlation functions of the form:

$$< 0|\; Q(x_1, \cdots, x_n)\; |0> = \frac{1}{Z} \int \mathcal{D}A_\mu \mathcal{D}\bar{\psi}\mathcal{D}\psi\;\; \exp[-S^{latt}]\; Q(x_1, \cdots, x_n)$$

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- the formalism is set up in Euclidean space-time; i.e. $i\,S \rightarrow -\,S^{latt}$
- this ensures real & bounded exponential factor
- correlation function can be computed numerically (Monte Carlo weighted averages)
- use $\exp[-\,S^{latt}]$ as probability weight to generate a configuration ensemble
- compute observable on this ensemble
- process characterized by **statistical error**; this is the least source of worry
- "easily" controlled by increasing configuration ensemble N_{conf} (NB: $\varepsilon \sim 1 / \sqrt{N_{conf}}$)

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- how does this work with Grassmann (fermionic) variables?

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- lattice (bare QCD) action in general has the form:

$$S^{latt} \quad = \quad a^4 \sum \left\{ [F_{\mu\nu}F_{\mu\nu}]^{latt} + \bar{\psi}[\not{D}^{latt} + m]\psi \right\}$$

- integrate Grassmann degrees of freedom:

$$< 0|\ Q(x_1, \cdots, x_n)\ |0> = \frac{1}{Z}\int \mathcal{D}A_\mu\ \ \exp[-S^{glue}]\ \ \det[\not{D}^{latt}+m]\ \tilde{Q}(x_1, \cdots, x_n)$$

- the non-local determinant is the costly part

Lattice correlation functions

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- the non-local determinant corresponds to internal fermion loops (sea quarks)

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- popular shortcut is to set $\det[D^{latt}+m]=1$; i.e. sea quarks are infinitely heavy.
- This is the QUENCHED APPROXIMATION which has been a principal source of **uncontrolled** errors until recently

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- we are currently at the end of the quenched era, in the middle of *N_f=2 and N_f=2+1*, aiming at *N_f=2+1+1*

- *N_f=2 and N_f=2+1* are the so-called partially quenched lattice theories

From correlation functions to hadron masses and matrix elements

- How do we obtain matrix elements and hadronic masses (i.e. **bare** low energy quantities)?
- Consider the lattice correlation function:

$$
\begin{aligned}
C_Q(t) &= \sum_{\vec{x}} < 0|\ Q(x)\ Q(0)\ |0 > \\
&\sim \sum_{s} < 0|\ Q(0)\ |s > < s|\ Q(0)\ |0 > \exp[-m_s t] \\
&\to \ | < 0|\ Q(0)\ |G > |^2 \exp[-m_G t]\ +\ \cdots
\end{aligned}
$$

- the states $|s>$ are those with the quantum numbers of $Q(x)$
- m_s are the corresponding hadronic masses; m_G the ground state
- $< 0 \,|\, Q \,|\, G >$ is the vacuum-to-G **bare** WME of operator Q
- higher excited states (same quantum numbers) drop out in the large-t limit

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- example: the operator Q is the charged axial current $\quad Q \rightarrow A_0 = \bar{u}\gamma_0\gamma_5 d$
- the state $|G>$ is the charged pion; $m_G \rightarrow m_\pi$
- the matrix element defines the pion decay contant $\quad < 0|\ A_0\ |\pi >= f_\pi\ m_\pi$

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- masses and matrix elements are computed from first principles in a model independent way
- the computation is clean in principle, but systematic errors abound (see later)

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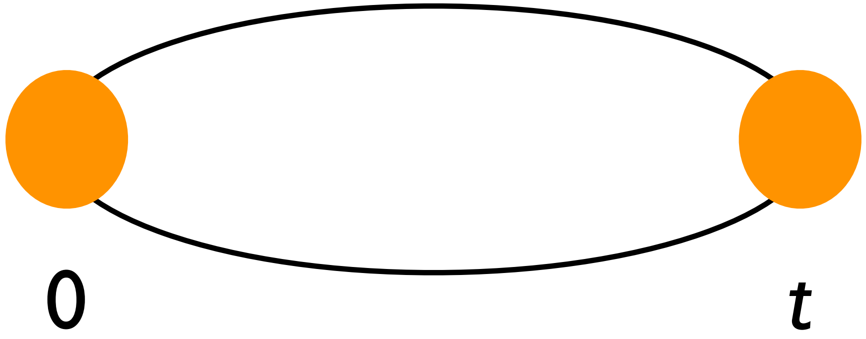
NB: gluon and sea quarks not drawn



Renormalization and improvement

- the lattice formalism is a **bare** QFT
- computation results are **bare** WMEs at fixed UV cutoff *1/a* (i.e. fixed $g_0^2(a)$)
- must renormalize them in order to get continuum physics
- UV cutoff is present at all stages of a computation and it is gradually increased

$$< f|\ Q_R(\mu)\ |i > = \lim_{a\to 0}\Big[\ Z_Q(a\mu, g_0^2)\ \ < f|\ Q(g_0^2)\ |i >\ + \mathcal{O}(a)\ \Big]$$

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bare WME depends on
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renormalized WME depends on dressed coupling, masses and scale

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continuum limit obtained gradually by successive simulations

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bare WME depends on bare coupling and masses

discretization effects due to cutoff finiteness contaminate all computations

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- lattice renormalization can be done either in PT or non-perturbatively (NP)
- lattice PT is tedious and **badly convergent**; at say LO, it introduces large $\mathcal{O}(g_0^4)$ errors in Z_Q
- NP methods introduce $\mathcal{O}(a)$ **discretization errors** is Z_Q ; as also the bare WME has $\mathcal{O}(a)$ effects, this is preferable to PT
- better still: attempt to "help" continuum extrapolation by reducing all discretization errors to $\mathcal{O}(a^2)$ [Symanzik impr.; "automatic" impr. ...]

Lattice actions

Fermion "doubling"

- the problem is general: for any lattice fermion action (free massless case)

$$S^{ferm} = a^4 \sum_{x,y} \bar{\psi}(x)\ D(x-y)\ \psi(y)$$

- the lattice Dirac operator should satisfy:
  - Locality: $D(x\text{-}y) < C \exp[\,-\,\kappa\ |x\text{-}y|\,]$
  - Continuum limit: $D(p) = \gamma_\mu\, p_\mu + \mathcal{O}(a\, p^2)$
  - No doublers: $D(p)$ invertible for $p_\mu \neq 0$
  - chiral symmetry: $D(x)\,\gamma_5 + \gamma_5\, D(x) = 0$
- Nielsen-Ninomyia theorem: all 4 properties cannot be satisfied simultaneously

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- Wilson fermions: introduce irrelevant (D=5) operator in the action, which breaks chiral symmetry, recovered in the **true** continuum limit.

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  - chiral symmetry: $D(x)\, \gamma_5 + \gamma_5\, D(x) = 0$
- Nielsen-Ninomyia theorem: all 4 properties cannot be satisfied simultaneously
- staggered fermions: dilute 16 spinorial degrees of freedom on hypercube points. Retain a reduced U(1) chiral symmetry. Loose "flavour transparency"

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  - Continuum limit: $D(p) = \gamma_\mu\, p_\mu + \mathcal{O}(a\, p^2)$
  - No doublers: $D(p)$ invertible for $p_\mu \neq 0$
  - chiral symmetry: $D(x)\,\gamma_5 + \gamma_5\, D(x) = O(a)$
- Nielsen-Ninomyia theorem: all 4 properties cannot be satisfied simultaneously
- Ginsparg-Wilson fermions: break chirality mildly to $\mathcal{O}(a)$, give up strict locality; costly in practice. Known as overlap fermions

Fermion "doubling"

- the problem is general: for any lattice fermion action (free massless) case)

$$S^{ferm} = a^4 \sum_{x,y} \bar{\psi}(x)\ D(x-y)\ \psi(y)$$

- the lattice Dirac operator should satisfy:
  - Locality: $D(x\text{-}y) < C \exp[- \kappa\ |x\text{-}y|\ ]$
  - Continuum limit: $D(p) = \gamma_\mu\, p_\mu + \mathcal{O}(a\, p^2)$
  - No doublers: $D(p)$ invertible for $p_\mu \neq 0$
  - chiral symmetry: $D(x)\,\gamma_5 + \gamma_5\, D(x) = O(a)$
- Nielsen-Ninomyia theorem: all 4 properties cannot be satisfied simultaneously
- Domain wall fermions: An equivalent formulation to GW fermions: introduce a fifth dimension; the 4-D lattice is a hypersurface (a defect) where both chiralities merge. Fairly costly (computationally); chirality is recovered at infinitely large DW

Renormalization

Hadronic renormalization scheme

- for simplicity consider a lattice theory with isospin symmetry and 3 flavours
- the 3 bare parameters are g_0, $m_q = m_u = m_d < m_s$
- they run with the lattice spacing a in the RG sense
- the hadronic scheme renormalization conditions are simply stated: tune all 3 bare parameters so as to ensure that 3 physical quantities are fixed to their (experimentally) known values

$$\frac{a\,m_P}{m_P^{\exp}} = a(g_0^2; m_q; m_s) \qquad \frac{a\,m_\pi}{am_P} = \frac{m_\pi^{\exp}}{m_P^{\exp}} \qquad \frac{a\,m_K}{am_P} = \frac{m_K^{\exp}}{m_P^{\exp}}$$

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fixed by experiment

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computed non-perturbatively

Hadronic renormalization scheme

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lattice calibration

Hadronic renormalization scheme

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- all other physical quantities (hadronic masses) can now be predicted (i.e. computed) since QCD is a renormalizable theory
- predictions must be repeated at smaller couplings g_0 i.e. smaller lattice spacings (asymptotic freedom)
- NB: this is explicitly non-perturbative and yields QCD mass spectrum

Hadronic renormalization scheme

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- the 3 bare parameters are g_0, $m_q = m_u = m_d > m_s$
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- all other physical quantities (hadronic masses) can now be predicted (i.e. computed) since QCD is a renormalizable theory
- predictions must be repeated at smaller couplings g_0 i.e. smaller lattice spacings (asymptotic freedom)
- several “practical” problems have induced variants of this procedure

Quality Criteria

FLAG: Flavianet Lattice Averaging Group

Quality Criteria

- Lattice simulations performed by different groups involve different choices both at the level of formalism (lattice actions, number of sea flavours etc.) and at the level of resources (lattice volumes, quark masses etc.)

- often this amounts to making different compromises which in turn introduces different systematic effects

- not all lattice results of a given quantity are directly comparable

- FLAG: a group of European lattice practitioners is making an effort to create a compilation of results on a few quantities, which critically summarize the state of the art

- FLAG members: G. Colangelo (Bern), S. Dürr (Jülich), A. Jüttner (Mainz), L. Lellouch (Marseilles), H. Leutwyler (Bern), V. Lubicz (Rome3), S. Necco (CERN), C. Sachrajda (Southampton), S. Simula (Rome3), T. Vladikas (Rome2), U. Wegner (Bern), H. Wittig (Mainz)

Quality Criteria

- a number of criteria have been fixed; these are somewhat subjective and time dependent
- criteria:
  - ★ systematic error estimated in a satisfactory manner and under control
  - ● a reasonable attempt at estimating systematic error; can be improved
  - ■ no attempt or unsatisfactory attempt at controlling a systematic error

Quality Criteria

- chiral extrapolation:

★ $M_{\pi,\mathrm{min}} < 250$ MeV

● 250 MeV $\leq M_{\pi,\mathrm{min}} \leq$ 400 MeV

■ $M_{\pi,\mathrm{min}} \leq$ 400 MeV

NB: at least 3 points requested

- continuum extrapolation:

★ at least 3 lattice spacings, at least two below 0.1 fm

● 2 or more lattice spacings, at least one below 0.1 fm

■ otherwise

NB: action should be O(a)-improved; for non-improved actions an extra point is needed for each criterion

Quality Criteria

- finite volume effects (with $L_{min} > 2$ fm):
  - ★ $[M_\pi\ L\ ]_{min} > 4$ or at least 3 volumes
  - ● $[\ M_\pi\ L\ ]_{min} > 3$ and at least 2 volumes
  - ■ otherwise, and in any case if $L_{min} < 2$ fm

- renormalization (where applicable):
  - ★ non perturbative
  - ● 2-loop perturbation theory
  - ■ otherwise

- renormalization group running (where applicable):
  - ★ non perturbative
  - ● otherwise

Form factor, decay constants and unitarity

Form factor, decay constants and unitarity

- unitarity: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
- experiment: $$|V_{ub}| = 3.93\ (36)\ \cdot 10^{-3}$$
- Kaon decays: $$|V_{us}|\ f_+(0) = 0.21661\ (47)$$

form factor @ zero momentum transfer $K^0 \to \pi^- \nu l^+$

$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599\ (59)$$

- 3 expressions, 4 unknowns; need one more input (e.g. V_{ud} from nuclear β decays)
- lattice provides independent determinations of f_K / f_π and $f_+(0)$

Form factor, decay constants and unitarity

Collaboration	Ref.	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	$f_+(0)$
RBC/UKQCD 07	[37]	2+1	★	●	★	■	0.9644(33)(34)(14)
ETM 09A	[38]	2	●	●	●	●	0.9560(57)(62)
QCDSF 07	[39]	2	■	■	★	■	$0.9647(15)_{stat}$
RBC 06	[40]	2	★	■	★	■	0.968(9)(6)
JLQCD 05	[41]	2	■	■	★	■	0.967(6)

most systematics OK

$$f_+(0) = 0.964\ (3)\ (4) \qquad (N_f = 2+1)$$
$$f_+(0) = 0.956\ (6)\ (6) \qquad (N_f = 2)$$

NB: PRELIMINARY !!!!

Form factor, decay constants and unitarity

Collaboration	Ref.	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	F_K/F_π
MILC 09	[43]	2+1	★	★	★	★	$1.197(3)(^{+6}_{-13})$
ALVdW 08	[44]	2+1	■	★	●	●	1.191(16)(17)
PACS-CS 08, 08B	[3, 45]	2+1	●	★	■	■	1.189(20)
BMW 08	[46]	2+1	■	★	★	★	1.18(1)(1)
HPQCD/UKQCD 08	[47]	2+1	★	★	●	★	1.189(2)(7)
RBC/UKQCD 07	[37]	2+1	★	●	★	■	1.205(18)(62)
NPLQCD 06	[48]	2+1	★	●	■	■	$1.218(2)(^{+11}_{-24})$
ETM 09	[49]	2	★	●	●	★	1.210(6)(15)(9)
QCDSF/UKQCD 07	[50]	2	■	●	★	●	1.21(3)

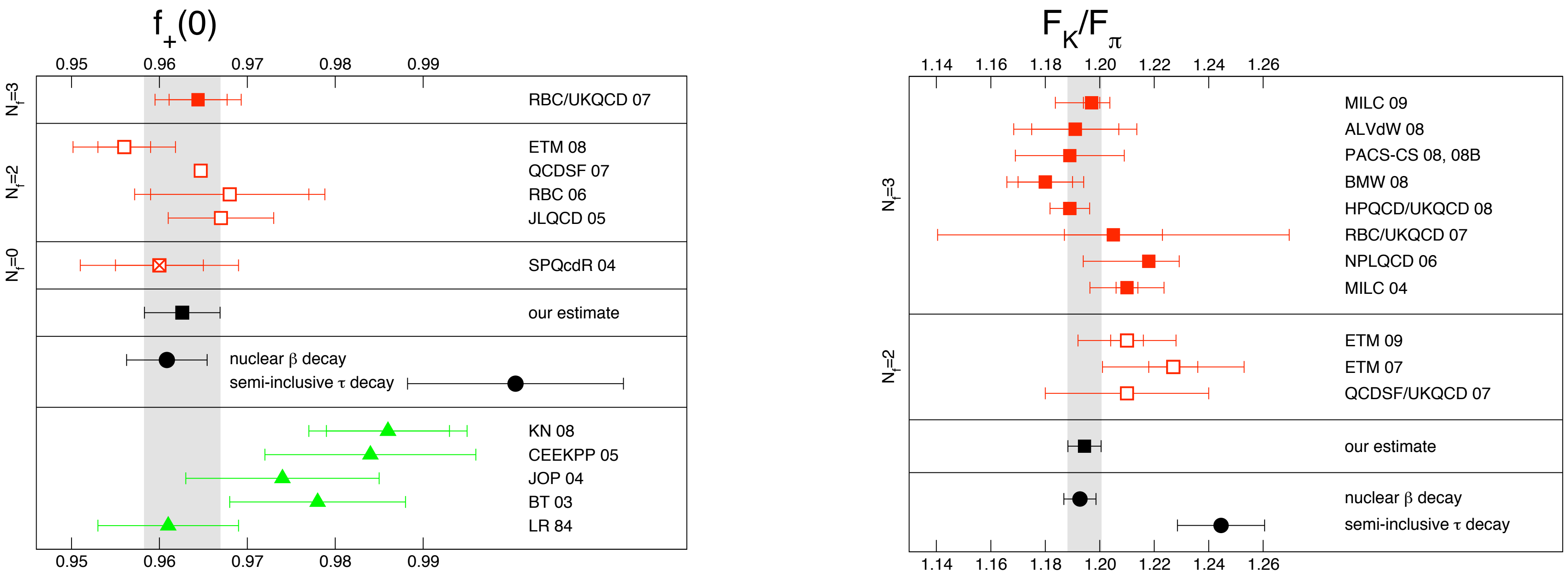
most systematics OK

NB: PRELIMINARY !!!!

$$f_K/f_\pi = 1.190\ (2)\ (10) \qquad (N_f = 2+1)$$

$$f_K/f_\pi = 1.210\ (6)\ (17) \qquad (N_f = 2)$$

Form factor, decay constants and unitarity



use these to extract CKM elements

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.989\ (20) \qquad N_f = 2+1$$

use V_{ud} from β decays and $f_+(0)$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9997\ (7)$$

use V_{ud} from β decays and f_K / f_π

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0002\ (10)$$

Quark Masses

Quark masses

- fundamental parameters of Standard Model
- X-sections & decay rates expressed in formulae with m_{charm} and m_{bottom}
- knowing the quark mass values with good precision for all flavours is an important ingredient of the flavour structure of the Standard Model
- cannot be measured experimentally
- can be calculated theoretically, using some input from hadronic physics
- they are quantities which run with the renormalization scale
- FLAG is centered on $m_{\mathrm{ud}} = 0.5\ (m_{\mathrm{up}} + m_{\mathrm{down}})$ and $\mathrm{m}_{\mathrm{strange}}$
- 3 fundamental QCD quantities (α_S , m_{ud} , $\mathrm{m}_{\mathrm{strange}}$) and lattice spacing a
- fix them through, say, m_π , f_π , m_N , m_K
- FLAG is currently analyzing the lattice data; no averages issued yet
- PRELIMINARY !!!!!!!!!!!

Quark masses

	Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	$m_{ud}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})[\mathrm{MeV}]$	$m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})[\mathrm{MeV}]$
$N_f = 2+1$	PACS-CS 08	[3]	●	★	■	■	■	■	2.527(47)	72.72(78)
	RBC/UKQCD 08	[4]	★	●	■	●	★	●	3.72(16)(33)(18)	107.3(4.4)(9.7)(4.9)
	CP-PACS/ JLQCD 07	[5]	★	■	★	★	■	■	$3.55(19)^{+56}_{-20}$	$90.1(4.3)^{+16.7}_{-4.3}$
	MILC 07	[6]	★	●	★	★	●	●	3.2(0)(1)(2)(0)	88.(0)(3)(4)(0)
	HPQCD/MILC/ UKQCD 05	[7]	★	●	●	●	●	●	3.2(0)(2)(2)(0)	87.(0)(4)(4)(0)
	MILC 04	[8, 9]	★	●	●	●	■	■	2.8(0)(1)(3)(0)	76.(0)(3)(7)(0)
$N_f = 2$	RBC 07	[10]	★	■	■	★	★	●	4.25(23)(26)	119.5(5.6)(7.4)
	ETM 07	[11]	★	●	■	●	★	●	3.85(12)(40)	105(3)(9)
	QCDSF/ UKQCD 04	[12]	★	■	●	★	★	●	4.7(2)(3)	119(5)(8)
	QCDSF/ UKQCD 04	[12]	★	■	●	★	★	●	4.08(23)(19)(23)	111(6)(10)
	SPQcdR 05	[13]	★	■	●	●	★	●	$4.3(4)^{+1.1}_{-0.0}$	$101.(8)^{+25}_{-0}$
	ALPHA 05	[14]	★	■	●	★	★	★		97.(4)(18) §
	JLQCD 02	[15]	★	■	■	●	■	■	$3.223^{+0.046}_{-0.069}$	$84.5^{+12.0}_{-1.7}$
	CP-PACS 01	[16]	★	■	■	★	■	■	$3.45(10)^{+11}_{-18}$	$89.(2)^{+2}_{-6}$ ⋆

Quark masses

$N_f = 2$

Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	$m_{ud}^{\overline{\mathrm{MS}}}$(2 GeV)[MeV]	$m_s^{\overline{\mathrm{MS}}}$(2 GeV)[MeV]
RBC 07	[10]	★	■	■	★	★	●	4.25(23)(26)	119.5(5.6)(7.4)
ETM 07	[11]	★	●	■	●	★	●	3.85(12)(40)	105(3)(9)
QCDSF/ UKQCD 04	[12]	★	■	●	★	★	●	4.7(2)(3)	119(5)(8)
QCDSF/ UKQCD 04	[12]	★	■	●	★	★	●	4.08(23)(19)(23)	111(6)(10)
SPQcdR 05	[13]	★	■	●	●	★	●	$4.3(4)^{+1.1}_{-0.0}$	$101.(8)^{+25}_{-0}$
ALPHA 05	[14]	★	■	●	★	★	★		$97.(4)(18)$ §
JLQCD 02	[15]	★	■	■	●	■	■	$3.223^{+0.046}_{-0.069}$	$84.5^{+12.0}_{-1.7}$
CP-PACS 01	[16]	★	■	■	★	■	■	$3.45(10)^{+11}_{-18}$	$89.(2)^{+2}_{-6}$ ⋆

most systematics OK

ETM: $m_\pi \sim 300$ MeV

ALPHA: mostly OK, but $m_\pi \sim 600$ MeV

Quark masses

$N_f = 2+1$

Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	$m_{ud}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})[\mathrm{MeV}]$	$m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})[\mathrm{MeV}]$
PACS-CS 08	[3]	●	★	■	■	■	■	2.527(47)	72.72(78)
RBC/UKQCD 08	[4]	★	●	■	●	★	●	3.72(16)(33)(18)	107.3(4.4)(9.7)(4.9)
CP-PACS/ JLQCD 07	[5]	★	■	★	★	■	■	$3.55(19)^{+56}_{-20}$	$90.1(4.3)^{+16.7}_{-4.3}$
MILC 07	[6]	★	●	★	★	●	●	3.2(0)(1)(2)(0)	88.(0)(3)(4)(0)
HPQCD/MILC/ UKQCD 05	[7]	★	●	●	●	●	●	3.2(0)(2)(2)(0)	87.(0)(4)(4)(0)
MILC 04	[8, 9]	★	●	●	●	■	■	2.8(0)(1)(3)(0)	76.(0)(3)(7)(0)

most systematics OK

MILC 07: $m_\pi \sim$ 240-300 MeV, mostly OK,
BUT renorm. is 2-loop in 2007; was 1-loop in 2004 and that made a significant difference (cf. HPQCD/MILC/UKQCD)

Quark masses

$N_f = 2+1$

Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	$m_{ud}^{\overline{MS}}(2\,\text{GeV})[\text{MeV}]$	$m_s^{\overline{MS}}(2\,\text{GeV})[\text{MeV}]$
PACS-CS 08	[3]	●	★	■	■	■	■	2.527(47)	72.72(78)
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HPQCD/MILC/ UKQCD 05	[7]	★	●	●	●	●	●	3.2(0)(2)(2)(0)	87.(0)(4)(4)(0)
MILC 04	[8, 9]	★	●	●	●	■	■	2.8(0)(1)(3)(0)	76.(0)(3)(7)(0)

most systematics OK

RBC/UKQCD: $m_\pi \sim$ 240-300 MeV, mostly OK (RI/MOM renormalization), but $a \sim 0.11$ fm, $Lm_\pi \sim 3.3$

Quark masses

$N_f = 2+1$

Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	$m_{ud}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})[\mathrm{MeV}]$	$m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})[\mathrm{MeV}]$
PACS-CS 08	[3]	●	★	■	■	■	■	2.527(47)	72.72(78)
RBC/UKQCD 08	[4]	★	●	■	●	★	●	3.72(16)(33)(18)	107.3(4.4)(9.7)(4.9)
CP-PACS/ JLQCD 07	[5]	★	■	★	★	■	■	$3.55(19)^{+56}_{-20}$	$90.1(4.3)^{+16.7}_{-4.3}$
MILC 07	[6]	★	●	★	★	●	●	3.2(0)(1)(2)(0)	88.(0)(3)(4)(0)
HPQCD/MILC/ UKQCD 05	[7]	★	●	●	●	●	●	3.2(0)(2)(2)(0)	87.(0)(4)(4)(0)
MILC 04	[8, 9]	★	●	●	●	■	■	2.8(0)(1)(3)(0)	76.(0)(3)(7)(0)

most systematics OK

PACS-CS: $m_\pi \sim 160$ MeV !!!

but 1-loop PT renormalization, $a \sim 0.11$ fm, $Lm_\pi \sim 2.3$

systematic errors not estimated (total error underestimated)

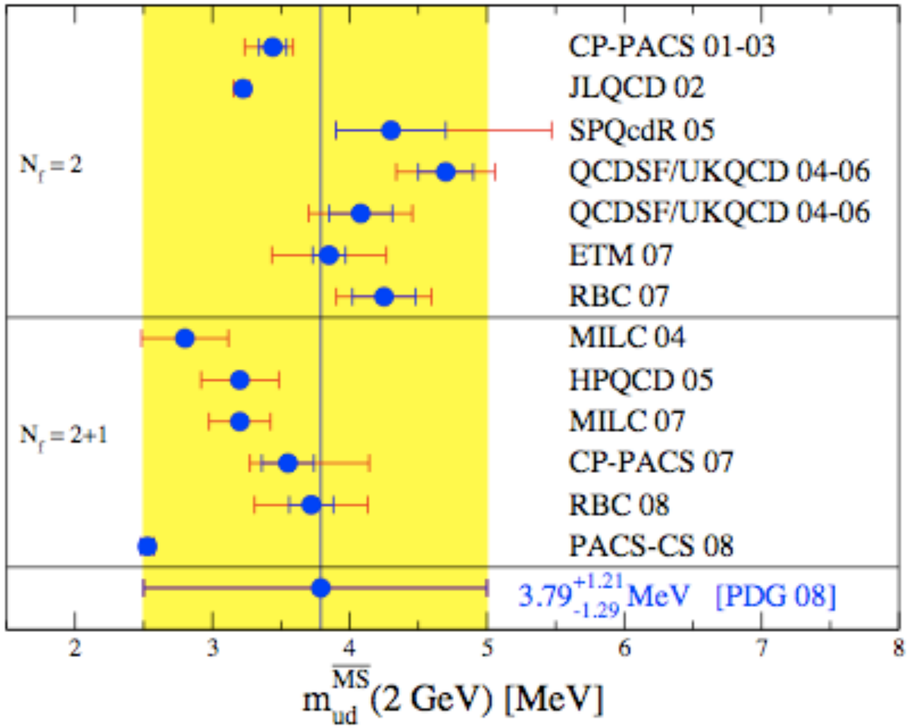
Quark masses

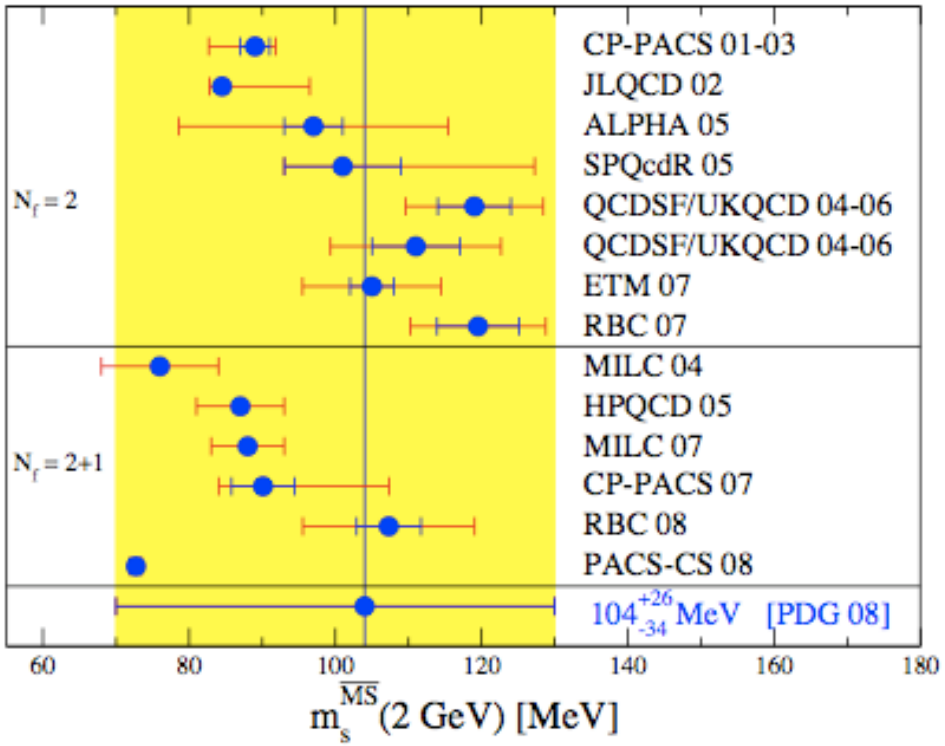
renorm. not really there!

Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	m_s/m_{ud}
PACS-CS 08	[3]	●	★	■	■	■	■	28.8(4)
RBC/UKQCD 08	[4]	★	●	■	●	★	●	$28.8 \pm 0.4 \pm 1.6$
MILC 07	[6]	★	●	★	★	●	●	27.2(1)(3)(0)(0)
MILC 04	[8, 9]	★	●	●	●	■	■	27.4(1)(4)(0)(1)
RBC 07	[10]	★	■	■	★	★	●	28.10(38)
ETM 07	[11]	★	●	■	●	★	●	$27.3 \pm 0.3 \pm 1.2$
QCDSF/ UKQCD 04	[12]	★	■	●	★	★	●	27.2(3.2)

smaller errors because of lack of renormalization

Quark masses





NB: PRELIMINARY !!!!

definitive plots (and averages) by the FLAG group to appear soon...

NB: perturbatively renormalized results are systematically lower than NP ones, irrespective of N_f

ΔS=2 transitions: B_K

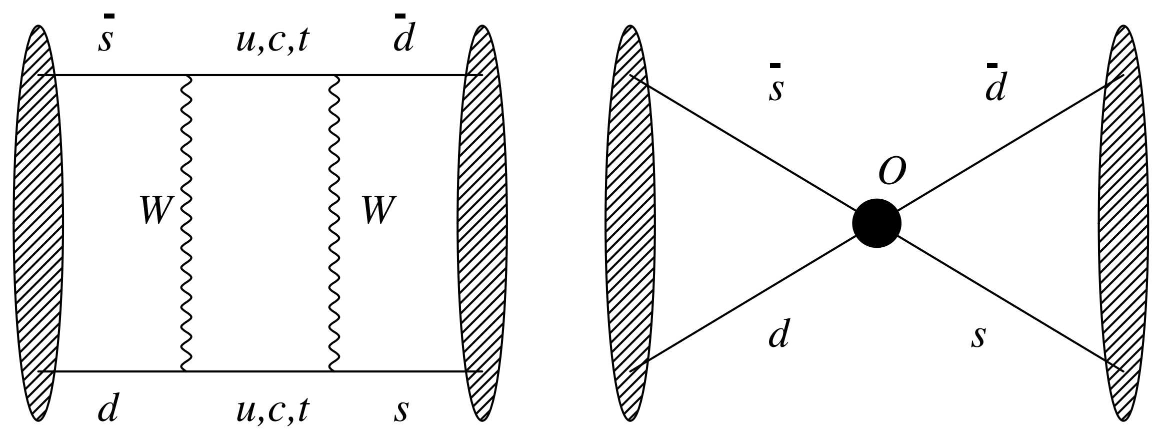
ΔS=2 transitions: B_K

indirect CP-violation

$$\epsilon_K = \frac{\mathcal{A}[K_L \to (\pi\pi)_{I=0}]}{\mathcal{A}[K_S \to (\pi\pi)_{I=0}]} = [2.282(17) \times 10^{-3}]\, \exp(i\pi/4)$$

can also be expressed in terms of K^0 - $\bar{K}^0$ mixing
dominant EW process is FCNC (2 W exchange)

$$|\epsilon_K| \approx C_\epsilon\, \hat{B}_K\, \mathrm{Im}\{V_{td}^* V_{ts}\}\, \{\mathrm{Re}\{V_{cd}^* V_{cs}\}[\eta_1\, S_0(x_c) - \eta_3\, S_0(x_c, x_t)] - \mathrm{Re}\{V_{td}^* V_{ts}\}\eta_2\, S_0(x_t)]\}$$



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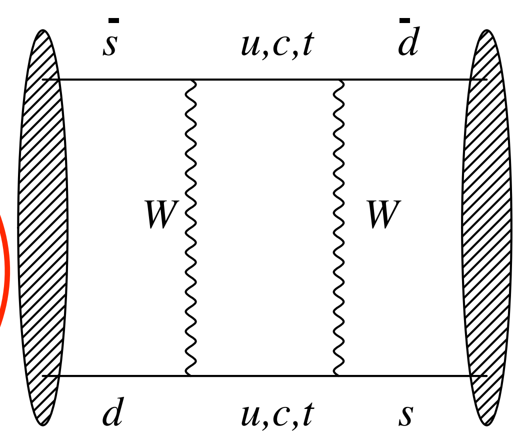
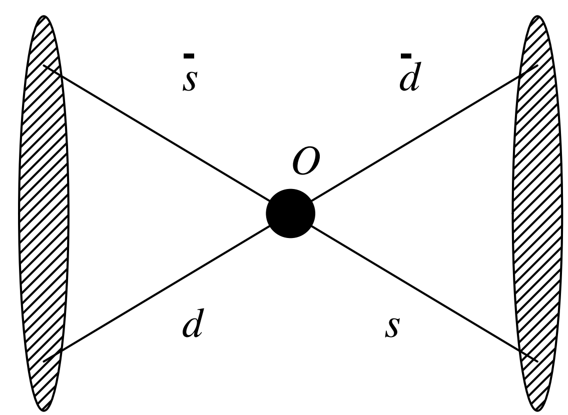
$$C_\epsilon = \frac{G_F^2 f_K^2 m_K M_W^2}{6\sqrt{2}\pi^2(\Delta M_K)} = 3.78 \times 10^4$$

$$\Delta M_K = M_{K_L} - M_{K_S}$$





ΔS=2 transitions: B_K

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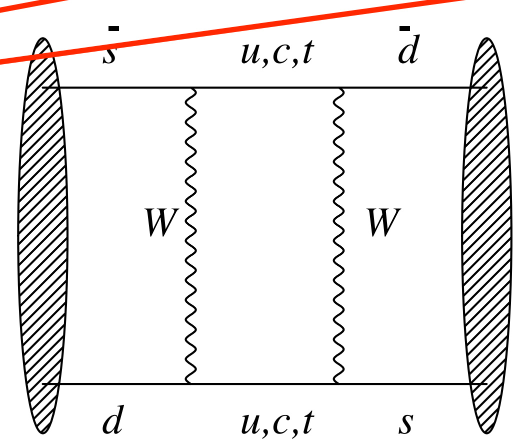
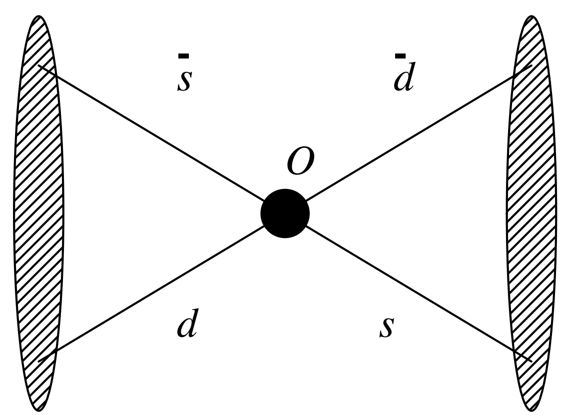
known functions from EW calculation (no QCD)

$$x_k = m_k^2/M_W^2$$





ΔS=2 transitions: B_K

indirect CP-violation

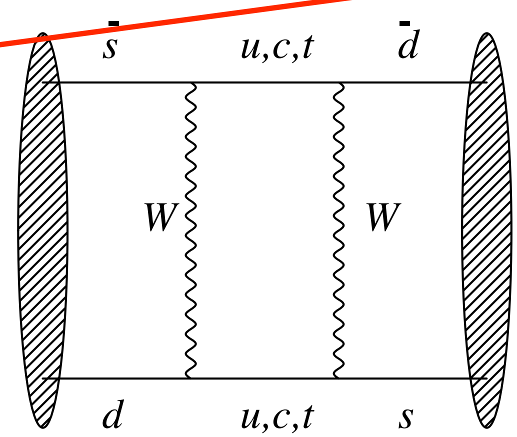
$$\epsilon_K = \frac{\mathcal{A}[K_L \to (\pi\pi)_{I=0}]}{\mathcal{A}[K_S \to (\pi\pi)_{I=0}]} = [2.282(17) \times 10^{-3}]\exp(i\pi/4)$$

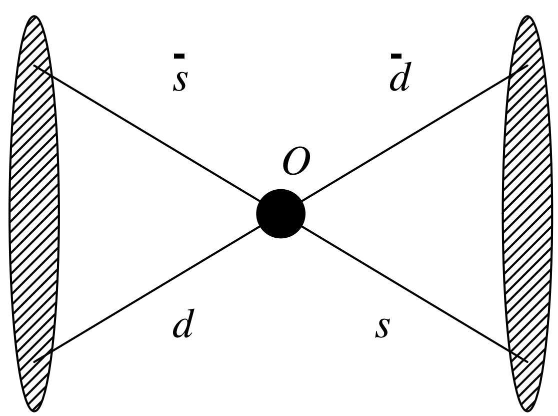
can also be expressed in terms of K^0 - K^0 mixing

dominant EW process is FCNC (2 W exchange)

$$|\epsilon_K| \approx C_\epsilon \hat{B}_K \operatorname{Im}\{V_{td}^* V_{ts}\} \{\operatorname{Re}\{V_{cd}^* V_{cs}\}[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)] - \operatorname{Re}\{V_{td}^* V_{ts}\}\eta_2 S_0(x_t)]\}$$

from NLO PT (with QCD)





ΔS=2 transitions: B_K

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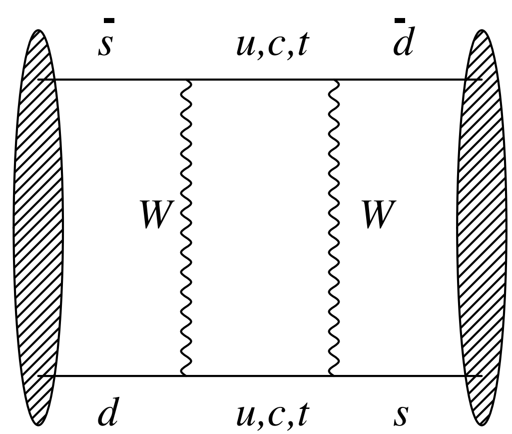
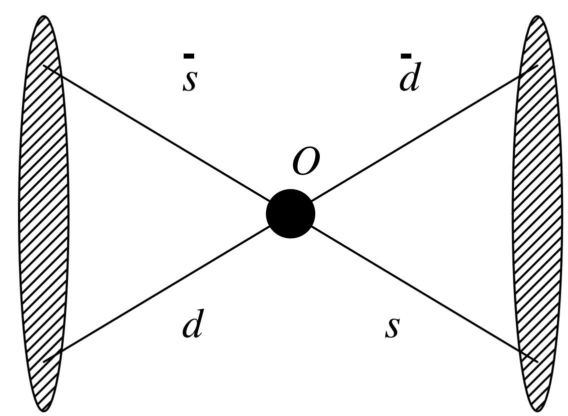
long distance NP

$$\boxed{\hat{B}_K = \frac{\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} F_K^2\, m_K^2}}$$





$$\hat{O}^{\Delta S=2} = [\bar{s}\gamma_\mu^L d]\,[\bar{s}\gamma_\mu^L d]$$

ΔS=2 transitions: ε_K

indirect CP-violation

$$\epsilon_K = \frac{\mathcal{A}[K_L \to (\pi\pi)_{I=0}]}{\mathcal{A}[K_S \to (\pi\pi)_{I=0}]} = [2.282(17) \times 10^{-3}]\exp(i\pi/4)$$

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long distance NP

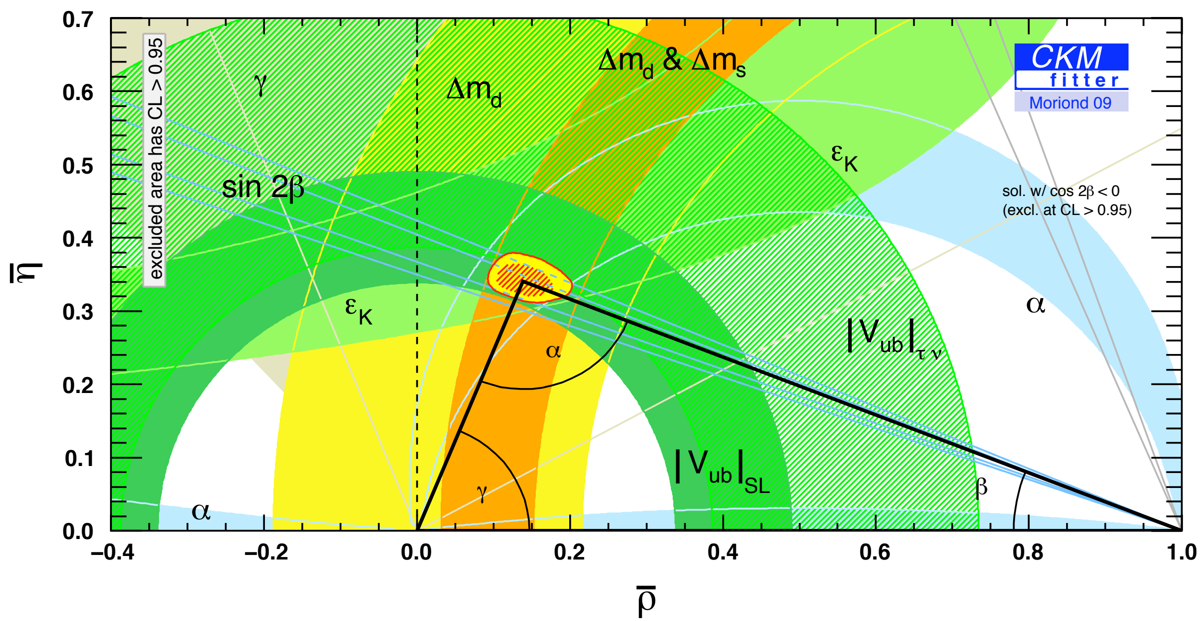
Put in NLO PT + Cabibbo angle + A + $m_{c,t}$:

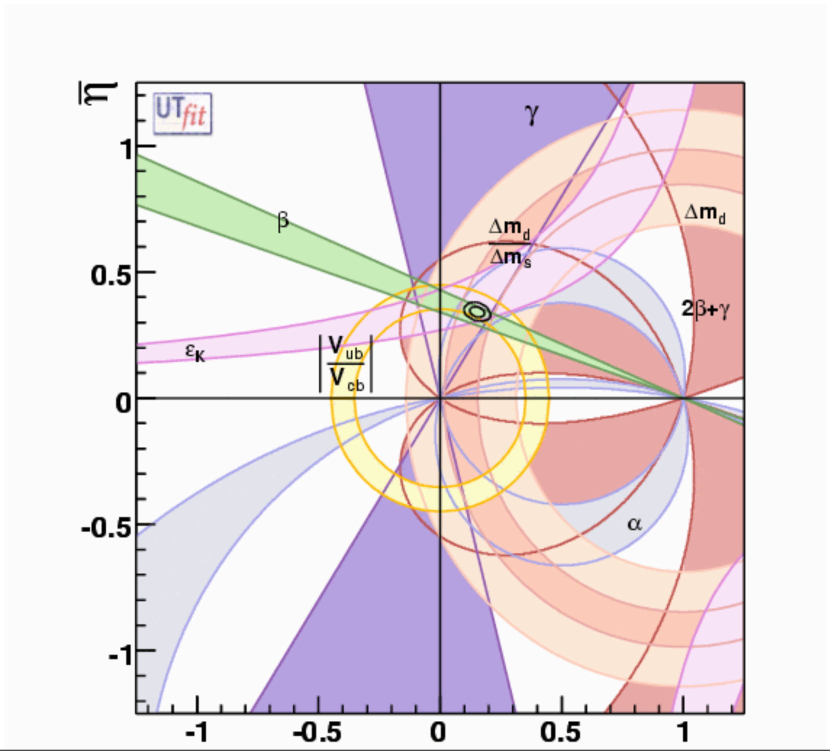
$$\bar{\eta}(1.4 - \bar{\rho})\, \hat{B}_K \approx 0.40$$

$$\boxed{\hat{B}_K = \frac{\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} F_K^2\, m_K^2}}$$

ΔS=2 transitions: B_K

$$|\epsilon_K| = \frac{\mathcal{A}(K_L \to (\pi\pi)_{I=0})}{\mathcal{A}(K_S \to (\pi\pi)_{I=0})} \stackrel{\text{exp}}{=} [2.282(17) \times 10^{-3}]\, e^{i\pi/4}$$





$\Delta S=2$ transitions: B_K

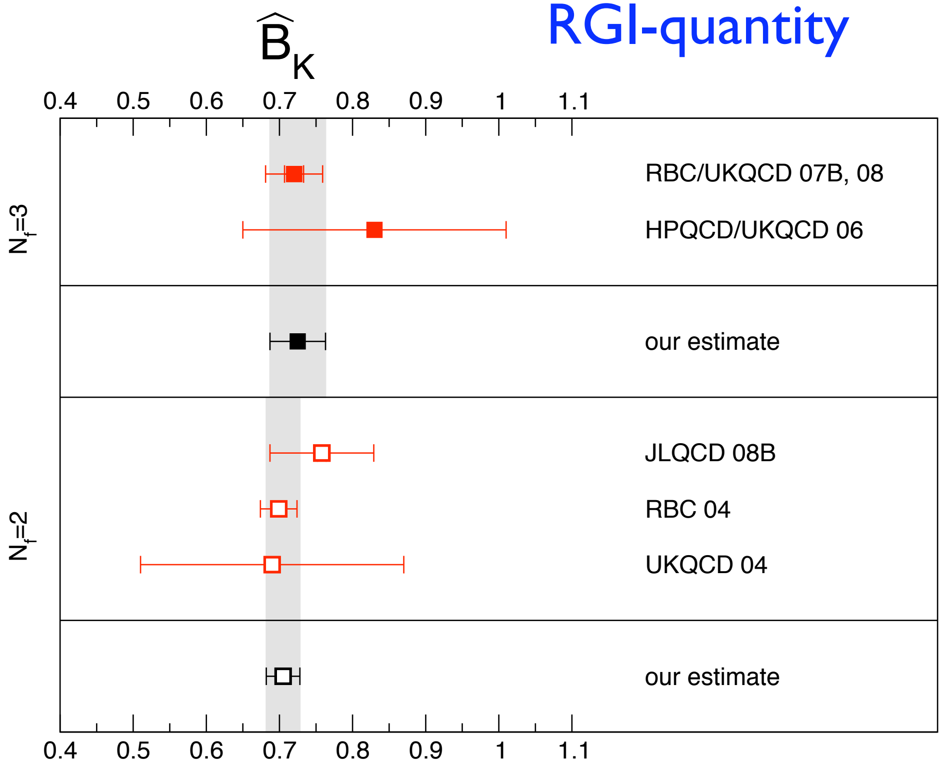
Collaboration	Ref.	N_f	lattice artefacts	smallest pion mass	finite volume errors	renormalization	running	$B_K^{\overline{MS}}(2\,\mathrm{GeV})$	$\hat{B}_K$
RBC/UKQCD 07B, 08	[91, 4]	2+1	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	[92]	2+1	■	●	★	■	●	0.618(18)(135)	0.83(18)
JLQCD 08B	[93]	2	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	[94]	2	■	■	■†	★	●	0.495(18)	0.699(25)
UKQCD 04	[95]	2	■	■	■†	■	●	0.49(13)	0.69(18)

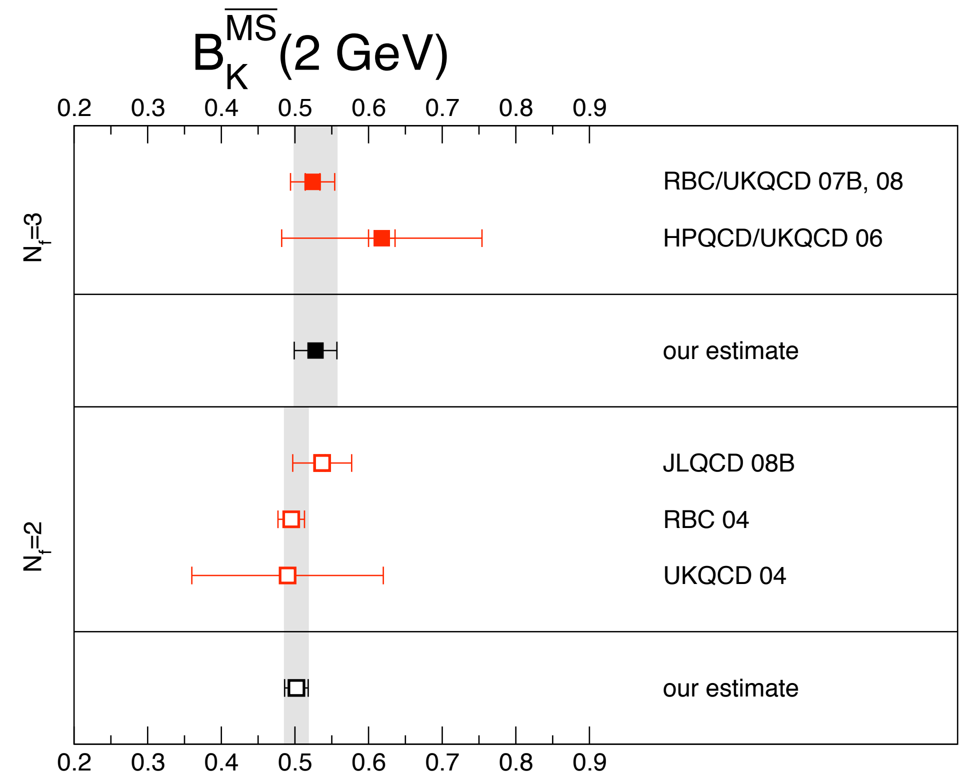
lots of work still to be done

NB: situation much better in quenched approximation (still...)

ΔS=2 transitions: B_K

RGI-quantity



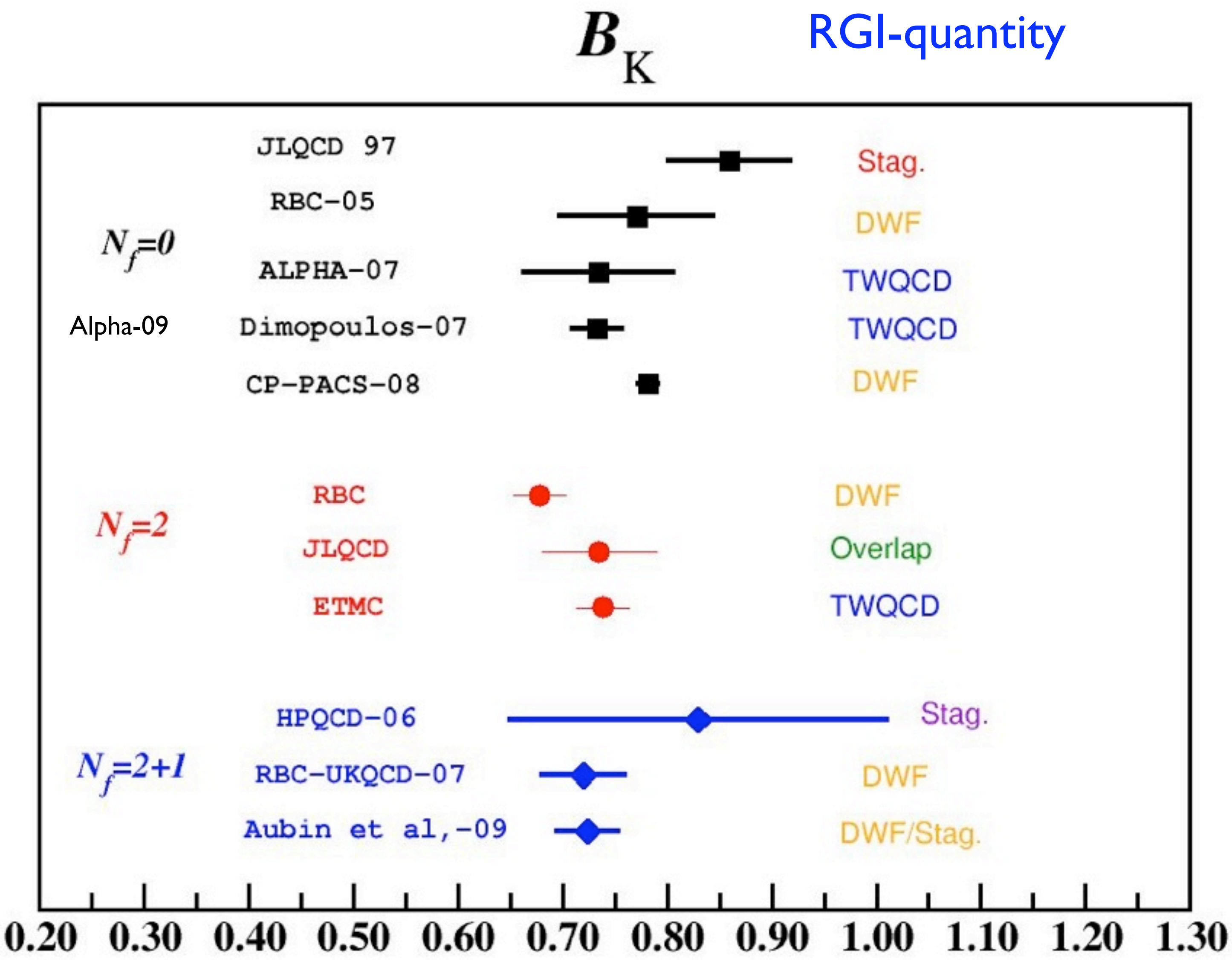


lots of work still to be done

NB: situation much better in quenched approximation (still...)

NB: PRELIMINARY !!!!

ΔS=2 transitions: B_K



Conclusions

- The lattice is a rigorously defined regularization of QCD (the only one?).
- As such, it enables non-pertrubative computations at low energies, from first principles, without any model assumptions.
- The price to pay is the presence of a plethora of systematic effects. They can be kept under control and are being systematically reduced.
- The control of these effects is not just the result of better hardware an software, but principally stems from a better theoretical understanding of non-perturbative QFT at fixed UV cutoff.
- We are currently moving away from uncontrolled approximations (quenching) and approach a realistic situation of $N_f = 2 + 1 + 1$. Moreover, we are approaching the most "critical" areas of the QCD parameter space (chiral limit, heavy flavours).
- The result of this progress is that lattice QCD is a mature field, capable of providing reliably some missing puzzles in Standard Model phenomenology.