STOCHASTIC QUANTIZATION AND HOLOGRAPHY

WORK WITH D.MANSI & A. MAURI: TO APPEAR

TASSOS PETKOU

OUTLINE

OUTLINE

- ☐ CONFORMAL HOLOGRAPHY

OUTLINE

- ☐ CONFORMAL HOLOGRAPHY
- ☐ STOCHASTIC QUANTIZATION

OUTLINE

- ☐ CONFORMAL HOLOGRAPHY
- ☐ STOCHASTIC QUANTIZATION
- ☐ STOCHASTIC QUANTIZATION VS HOLOGRAPHY

OUTLINE

- ☐ CONFORMAL HOLOGRAPHY
- ☐ STOCHASTIC QUANTIZATION
- ☐ STOCHASTIC QUANTIZATION VS HOLOGRAPHY
- ☐ IS HOLOGRAPHY STOCHASTIC?

CONFORMAL HOLOGRAPHY

T. P., S. DE HARO, I. PAPADIMITRIOU (06)

CONFORMAL HOLOGRAPHY

T. P., S. DE HARO, I. PAPADIMITRIOU (06)

THE HOLOGRAPHY OF CONFORMALLY COUPLED SCALAR FIELDS (THE MOST RELEVANT CASE FOR ADS4/CFT3) REDUCES (ESSENTIALLY) TO MASSLESS THEORIES IN FLAT SPACE WITH A BOUNDARY: EXAMPLE

CONFORMAL HOLOGRAPHY

T. P., S. DE HARO, I. PAPADIMITRIOU (06)

THE HOLOGRAPHY OF CONFORMALLY COUPLED SCALAR FIELDS (THE MOST RELEVANT CASE FOR ADS4/CFT3) REDUCES (ESSENTIALLY) TO MASSLESS THEORIES IN FLAT SPACE WITH A BOUNDARY: EXAMPLE

CONFORMAL HOLOGRAPHY

T. P., S. DE HARO, I. PAPADIMITRIOU (06)

THE HOLOGRAPHY OF CONFORMALLY COUPLED SCALAR FIELDS (THE MOST RELEVANT CASE FOR ADS4/CFT3) REDUCES (ESSENTIALLY) TO MASSLESS THEORIES IN FLAT SPACE WITH A BOUNDARY: EXAMPLE

$$I = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4 \right) \qquad ds^2 = \frac{L^2}{r^2} \left(dr^2 + d\vec{x}^2 \right)$$

CONFORMAL HOLOGRAPHY

T. P., S. DE HARO, I. PAPADIMITRIOU (06)

THE HOLOGRAPHY OF CONFORMALLY COUPLED SCALAR FIELDS (THE MOST RELEVANT CASE FOR ADS4/CFT3) REDUCES (ESSENTIALLY) TO MASSLESS THEORIES IN FLAT SPACE WITH A BOUNDARY: EXAMPLE

$$I = \int d^4x\sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4\right) \qquad ds^2 = \frac{L^2}{r^2}\left(dr^2 + d\vec{x}^2\right)$$

$$m^2L^2 = -2 \qquad g_{\mu\nu} = \Omega(x)^{-2}\eta_{\mu\nu}\,, \quad \phi = \Omega(x)f\ , \quad \Omega(x) = \frac{r}{L}$$

CONFORMAL HOLOGRAPHY

T. P., S. DE HARO, I. PAPADIMITRIOU (06)

THE HOLOGRAPHY OF CONFORMALLY COUPLED SCALAR FIELDS (THE MOST RELEVANT CASE FOR ADS4/CFT3) REDUCES (ESSENTIALLY) TO MASSLESS THEORIES IN FLAT SPACE WITH A BOUNDARY: EXAMPLE

$$I = \int d^4x\sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4\right) \qquad ds^2 = \frac{L^2}{r^2}\left(dr^2 + d\vec{x}^2\right)$$

$$m^2L^2 = -2 \qquad g_{\mu\nu} = \Omega(x)^{-2}\eta_{\mu\nu}\,, \quad \phi = \Omega(x)f\ , \quad \Omega(x) = \frac{r}{L}$$

$$I = \cdots = \int_0^\infty dr\int d^3\vec{x}\left(\frac{1}{2}\eta^{\mu\nu}\partial_\mu f\partial_\nu f + \frac{\lambda}{4}f^4\right) + \frac{1}{2}\int d^3\vec{x}\,\frac{1}{r}f^2\Bigg|_0^\infty$$

CONFORMAL HOLOGRAPHY

T. P., S. DE HARO, I. PAPADIMITRIOU (06)

THE HOLOGRAPHY OF CONFORMALLY COUPLED SCALAR FIELDS (THE MOST RELEVANT CASE FOR ADS4/CFT3) REDUCES (ESSENTIALLY) TO MASSLESS THEORIES IN FLAT SPACE WITH A BOUNDARY: EXAMPLE

$$I = \int d^4x\sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4\right) \qquad ds^2 = \frac{L^2}{r^2}\left(dr^2 + d\vec{x}^2\right)$$

$$m^2L^2 = -2 \qquad g_{\mu\nu} = \Omega(x)^{-2}\eta_{\mu\nu}\,, \quad \phi = \Omega(x)f\ , \quad \Omega(x) = \frac{r}{L}$$

$$I = \cdots = \int_0^\infty dr \int d^3\vec{x}\left(\frac{1}{2}\eta^{\mu\nu}\partial_\mu f\partial_\nu f + \frac{\lambda}{4}f^4\right) + \frac{1}{2}\int d^3\vec{x}\,\frac{1}{r}f^2\Big|_0^\infty$$

SUBTRACT-RENORMALIZATION

ASYMPTOTIC BEHAVIOUR $f(r, \vec{x}) \approx \phi_0(\vec{x}) + r\phi_1(\vec{x}) + \cdots$

ASYMPTOTIC BEHAVIOUR $f(r, \vec{x}) \approx \phi_0(\vec{x}) + r\phi_1(\vec{x}) + \cdots$

THE ON-SHELL ACTION YIELDS THE (HOLOGRAPHIC) EFFECTIVE ACTION FOR THE "DUAL" BOUNDARY THEORY I.E. THE THEORY WITH AN OPERATOR OF DIMENSION=1.

ASYMPTOTIC BEHAVIOUR $f(r,\vec{x}) \approx \phi_0(\vec{x}) + r\phi_1(\vec{x}) + \cdots$

THE ON-SHELL ACTION YIELDS THE (HOLOGRAPHIC) EFFECTIVE ACTION FOR THE "DUAL" BOUNDARY THEORY I.E. THE THEORY WITH AN OPERATOR OF DIMENSION=1.

ASYMPTOTIC BEHAVIOUR $f(r,\vec{x}) \approx \phi_0(\vec{x}) + r\phi_1(\vec{x}) + \cdots$

THE ON-SHELL ACTION YIELDS THE (HOLOGRAPHIC) EFFECTIVE ACTION FOR THE "DUAL" BOUNDARY THEORY I.E. THE THEORY WITH AN OPERATOR OF DIMENSION=1.

ASYMPTOTIC BEHAVIOUR $f(r, \vec{x}) \approx \phi_0(\vec{x}) + r\phi_1(\vec{x}) + \cdots$

THE ON-SHELL ACTION YIELDS THE (HOLOGRAPHIC) EFFECTIVE ACTION FOR THE "DUAL" BOUNDARY THEORY I.E. THE THEORY WITH AN OPERATOR OF DIMENSION=1.

$$I \to S[\phi_0] \equiv -\Gamma[\phi_0] \sim \int d^3\vec{x} \left(\frac{1}{\phi_0} \partial_i \phi_0 \partial_i \phi_0 + \kappa \phi_0^3 + \cdots \right)$$

ASYMPTOTIC BEHAVIOUR $f(r,\vec{x}) \approx \phi_0(\vec{x}) + r\phi_1(\vec{x}) + \cdots$

THE ON-SHELL ACTION YIELDS THE (HOLOGRAPHIC) EFFECTIVE ACTION FOR THE "DUAL" BOUNDARY THEORY I.E. THE THEORY WITH AN OPERATOR OF DIMENSION=1.

$$I \to S[\phi_0] \equiv -\Gamma[\phi_0] \sim \int d^3\vec{x} \left(\frac{1}{\phi_0} \partial_i \phi_0 \partial_i \phi_0 + \kappa \phi_0^3 + \cdots \right)$$

$$\phi_0 \mapsto \varphi^2 \ \Rightarrow \ \Gamma[\phi_0] \mapsto \Gamma[\varphi] \sim \int d^3\vec{x} \left(\frac{1}{2} \partial_i \varphi \partial_i \varphi + \frac{\kappa}{8} \varphi^6 + \cdots \right)$$

ASYMPTOTIC BEHAVIOUR $f(r,\vec{x}) \approx \phi_0(\vec{x}) + r\phi_1(\vec{x}) + \cdots$

THE ON-SHELL ACTION YIELDS THE (HOLOGRAPHIC) EFFECTIVE ACTION FOR THE "DUAL" BOUNDARY THEORY I.E. THE THEORY WITH AN OPERATOR OF DIMENSION=1.

$$I \to S[\phi_0] \equiv -\Gamma[\phi_0] \sim \int d^3\vec{x} \left(\frac{1}{\phi_0} \partial_i \phi_0 \partial_i \phi_0 + \kappa \phi_0^3 + \cdots \right)$$

$$\phi_0 \mapsto \varphi^2 \;\Rightarrow\; \Gamma[\phi_0] \mapsto \Gamma[\varphi] \sim \int d^3\vec{x} \left(\frac{1}{2} \partial_i \varphi \partial_i \varphi + \frac{\kappa}{8} \varphi^6 + \cdots \right)$$

CONFORMAL HOLOGRAPHY APPEARS TO RELATE A BULK ACTION IN 4D TO A BOUNDARY ACTION IN 3D - CONTRAST WITH STANDARD HOLOGRAPHY

A SIMPLE WAY TO REPRODUCE THE RESULT: HAMILTONIAN ANALYSIS

A SIMPLE WAY TO REPRODUCE THE RESULT: HAMILTONIAN ANALYSIS

$$I = \int_0^\infty dr \int d^3\vec{x} \left[\dot{f}\pi - \mathcal{H}\right] \; , \;\; \mathcal{H} = \frac{1}{2}\left(\pi^2 - \partial_i f \partial_i f - \frac{\lambda}{2} f^4\right)$$

A SIMPLE WAY TO REPRODUCE THE RESULT: HAMILTONIAN ANALYSIS

$$I = \int_0^\infty dr \int d^3\vec{x} \left[\dot{f}\pi - \mathcal{H}\right] \,, \;\; \mathcal{H} = \frac{1}{2}\left(\pi^2 - \partial_i f \partial_i f - \frac{\lambda}{2} f^4\right)$$

THE 2ND ORDER E.O.M. HAVE A GENERAL INSTANTON SOLUTION $\lambda < 0$

A SIMPLE WAY TO REPRODUCE THE RESULT: HAMILTONIAN ANALYSIS

$$I = \int_0^\infty dr \int d^3\vec{x} \left[\dot{f}\pi - \mathcal{H}\right] \;, \;\; \mathcal{H} = \frac{1}{2}\left(\pi^2 - \partial_i f \partial_i f - \frac{\lambda}{2} f^4\right)$$

THE 2ND ORDER E.O.M. HAVE A GENERAL INSTANTON SOLUTION $\lambda < 0$

$$\ddot{f} + \partial_i \partial_i f = \lambda f^3 \;\Rightarrow\; \hat{f}(r, \vec{x}) = \sqrt{\frac{8}{-\lambda}} \frac{b}{b^2 + r^2 + \vec{x}^2}$$

A SIMPLE WAY TO REPRODUCE THE RESULT: HAMILTONIAN ANALYSIS

$$I = \int_0^\infty dr \int d^3\vec{x} \left[\dot{f}\pi - \mathcal{H}\right] \,,\ \ \mathcal{H} = \frac{1}{2}\left(\pi^2 - \partial_i f \partial_i f - \frac{\lambda}{2} f^4\right)$$

THE 2ND ORDER E.O.M. HAVE A GENERAL INSTANTON SOLUTION $\lambda < 0$

$$\ddot{f} + \partial_i\partial_i f = \lambda f^3 \ \Rightarrow\ \hat{f}(r, \vec{x}) = \sqrt{\frac{8}{-\lambda}} \frac{b}{b^2 + r^2 + \vec{x}^2}$$

GENERICALLY INSTANTONS MAY NOT SIT AT THE ORIGIN

A SIMPLE WAY TO REPRODUCE THE RESULT: HAMILTONIAN ANALYSIS

$$I = \int_0^\infty dr \int d^3\vec{x} \left[\dot{f}\pi - \mathcal{H}\right] \,,\ \mathcal{H} = \frac{1}{2}\left(\pi^2 - \partial_i f \partial_i f - \frac{\lambda}{2} f^4\right)$$

THE 2ND ORDER E.O.M. HAVE A GENERAL INSTANTON SOLUTION $\lambda < 0$

$$\ddot{f} + \partial_i \partial_i f = \lambda f^3 \ \Rightarrow\ \hat{f}(r, \vec{x}) = \sqrt{\frac{8}{-\lambda}} \frac{b}{b^2 + r^2 + \vec{x}^2}$$

GENERICALLY INSTANTONS MAY NOT SIT AT THE ORIGIN

SUCH THAT THE ON-SHELL HAMILTONIAN DENSITY IS

A SIMPLE WAY TO REPRODUCE THE RESULT: HAMILTONIAN ANALYSIS

$$I = \int_0^\infty dr \int d^3\vec{x} \left[\dot{f}\pi - \mathcal{H}\right] \ , \ \ \mathcal{H} = \frac{1}{2}\left(\pi^2 - \partial_i f \partial_i f - \frac{\lambda}{2} f^4\right)$$

THE 2ND ORDER E.O.M. HAVE A GENERAL INSTANTON SOLUTION $\lambda < 0$

$$\ddot{f} + \partial_i \partial_i f = \lambda f^3 \ \Rightarrow \ \hat{f}(r, \vec{x}) = \sqrt{\frac{8}{-\lambda}} \frac{b}{b^2 + r^2 + \vec{x}^2}$$

GENERICALLY INSTANTONS MAY NOT SIT AT THE ORIGIN

SUCH THAT THE ON-SHELL HAMILTONIAN DENSITY IS

$$\hat{\mathcal{H}} = \frac{1}{2}\left(\hat{\pi}^2 - \partial_i \hat{f} \partial_i \hat{f} - \frac{\lambda}{2} \hat{f}^4\right) = -\frac{1}{6}\partial_i \partial_i \hat{f}^2$$

A SIMPLE WAY TO REPRODUCE THE RESULT: HAMILTONIAN ANALYSIS

$$I = \int_0^\infty dr \int d^3\vec{x} \left[\dot{f}\pi - \mathcal{H}\right] , \ \mathcal{H} = \frac{1}{2}\left(\pi^2 - \partial_i f \partial_i f - \frac{\lambda}{2} f^4\right)$$

THE 2ND ORDER E.O.M. HAVE A GENERAL INSTANTON SOLUTION $\lambda < 0$

$$\ddot{f} + \partial_i\partial_i f = \lambda f^3 \ \Rightarrow \ \hat{f}(r,\vec{x}) = \sqrt{\frac{8}{-\lambda}} \frac{b}{b^2 + r^2 + \vec{x}^2}$$

GENERICALLY INSTANTONS MAY NOT SIT AT THE ORIGIN

SUCH THAT THE ON-SHELL HAMILTONIAN DENSITY IS

$$\hat{\mathcal{H}} = \frac{1}{2}\left(\hat{\pi}^2 - \partial_i \hat{f} \partial_i \hat{f} - \frac{\lambda}{2}\hat{f}^4\right) = -\frac{1}{6}\partial_i\partial_i \hat{f}^2$$

$$\Rightarrow \hat{\pi} = \pm\sqrt{\frac{\lambda}{2}}\hat{f}^2 \sqrt{1 + \frac{2}{\lambda \hat{f}^4}\partial_i \hat{f}\partial_i \hat{f} - \frac{2}{3\lambda \hat{f}^4}\partial_i\partial_i \hat{f}^2}$$

CHOOSING THE + SIGN WE FIND THE ON-SHELL ACTION IN A LARGE-LAMBDA EXPANSION, AS:

CHOOSING THE + SIGN WE FIND THE ON-SHELL ACTION IN A LARGE-LAMBDA EXPANSION, AS:

$$I \equiv -\Gamma[\phi_0] = \sqrt{\frac{1}{18\lambda}} \int d^3\vec{x} \left(\lambda \phi_0^3 + \frac{1}{\phi_0} \partial_i \phi_0 \partial_i \phi_0 + \cdots \right)$$

CHOOSING THE + SIGN WE FIND THE ON-SHELL ACTION IN A LARGE-LAMBDA EXPANSION, AS:

$$I \equiv -\Gamma[\phi_0] = \sqrt{\frac{1}{18\lambda}} \int d^3\vec{x} \left(\lambda \phi_0^3 + \frac{1}{\phi_0} \partial_i \phi_0 \partial_i \phi_0 + \cdots \right)$$

SETTING $\phi = \varphi^2$ WE OBTAIN THE STANDARD φ^6 THEORY

CHOOSING THE + SIGN WE FIND THE ON-SHELL ACTION IN A LARGE-LAMBDA EXPANSION, AS:

$$I \equiv -\Gamma[\phi_0] = \sqrt{\frac{1}{18\lambda}} \int d^3\vec{x} \left(\lambda \phi_0^3 + \frac{1}{\phi_0} \partial_i \phi_0 \partial_i \phi_0 + \cdots \right)$$

SETTING $\phi = \varphi^2$ WE OBTAIN THE STANDARD φ^6 THEORY

THIS IS REMARKABLE! ADS/CFT GIVES THE EFFECTIVE ACTION OF COMPOSITE BOUNDARY OPERATORS.

CHOOSING THE + SIGN WE FIND THE ON-SHELL ACTION IN A LARGE-LAMBDA EXPANSION, AS:

$$I \equiv -\Gamma[\phi_0] = \sqrt{\frac{1}{18\lambda}} \int d^3\vec{x} \left(\lambda \phi_0^3 + \frac{1}{\phi_0} \partial_i \phi_0 \partial_i \phi_0 + \cdots \right)$$

SETTING $\phi = \varphi^2$ WE OBTAIN THE STANDARD φ^6 THEORY

THIS IS REMARKABLE! ADS/CFT GIVES THE EFFECTIVE ACTION OF COMPOSITE BOUNDARY OPERATORS.

IT RATHER LOOKS LIKE A TOPOLOGICAL RELATION:

$$\int_{\mathcal{M}} dA \wedge dA = \int_{\partial \mathcal{M}} A \wedge dA$$

STOCHASTIC QUANTIZATION (PARISI-WU (81))

STOCHASTIC QUANTIZATION (PARISI-WU (81))

$$\langle \phi(\vec{x})... \rangle = \frac{1}{Z} \int (\mathcal{D}\phi) e^{-S_{cl}[\phi]} \phi(\vec{x})...$$

STOCHASTIC QUANTIZATION (PARISI-WU (81))

$$\langle \phi(\vec{x})...\rangle = \frac{1}{Z}\int (\mathcal{D}\phi) e^{-S_{cl}[\phi]}\phi(\vec{x})...$$

CONSIDER THE EUCLIDEAN PATH INTEGRAL IN D-DIMENSIONS AS THE STATIONARY DISTRIBUTION OF A STOCHASTIC PROCESS IN D+1-DIMENSIONS; INTRODUCE “STOCHASTIC” TIME.

STOCHASTIC QUANTIZATION (PARISI-WU (81))

$$\langle \phi(\vec{x}) ... \rangle = \frac{1}{Z} \int (\mathcal{D}\phi) e^{-S_{cl}[\phi]} \phi(\vec{x}) ...$$

CONSIDER THE EUCLIDEAN PATH INTEGRAL IN D-DIMENSIONS AS THE STATIONARY DISTRIBUTION OF A STOCHASTIC PROCESS IN D+1-DIMENSIONS; INTRODUCE "STOCHASTIC" TIME.

$$\phi(\vec{x}) \mapsto \phi(t, \vec{x}) \,, \quad \frac{\partial \phi(t, \vec{x})}{\partial t} + k \frac{\delta S_{cl}}{\delta \phi(t, \vec{x})} = \eta(t, \vec{x})$$

STOCHASTIC QUANTIZATION (PARISI-WU (81))

$$\langle \phi(\vec{x}) ... \rangle = \frac{1}{Z} \int (\mathcal{D}\phi) e^{-S_{cl}[\phi]} \phi(\vec{x}) ...$$

CONSIDER THE EUCLIDEAN PATH INTEGRAL IN D-DIMENSIONS AS THE STATIONARY DISTRIBUTION OF A STOCHASTIC PROCESS IN D+1-DIMENSIONS; INTRODUCE "STOCHASTIC" TIME.

$$\phi(\vec{x}) \mapsto \phi(t, \vec{x}) \,, \quad \frac{\partial \phi(t, \vec{x})}{\partial t} + k \frac{\delta S_{cl}}{\delta \phi(t, \vec{x})} = \eta(t, \vec{x}) \text{ “WHITE NOISE”}$$

STOCHASTIC QUANTIZATION (PARISI-WU (81))

$$\langle \phi(\vec{x}) ... \rangle = \frac{1}{Z} \int (\mathcal{D}\phi) e^{-S_{cl}[\phi]} \phi(\vec{x}) ...$$

CONSIDER THE EUCLIDEAN PATH INTEGRAL IN D-DIMENSIONS AS THE STATIONARY DISTRIBUTION OF A STOCHASTIC PROCESS IN D+1-DIMENSIONS; INTRODUCE "STOCHASTIC" TIME.

"LANGEVIN" EQ $\phi(\vec{x}) \mapsto \phi(t, \vec{x})\,, \quad \frac{\partial \phi(t, \vec{x})}{\partial t} + k \frac{\delta S_{cl}}{\delta \phi(t, \vec{x})} = \eta(t, \vec{x})$ "WHITE NOISE"

STOCHASTIC QUANTIZATION (PARISI-WU (81))

$$\langle \phi(\vec{x}) ... \rangle = \frac{1}{Z} \int (\mathcal{D}\phi) e^{-S_{cl}[\phi]} \phi(\vec{x}) ...$$

CONSIDER THE EUCLIDEAN PATH INTEGRAL IN D-DIMENSIONS AS THE STATIONARY DISTRIBUTION OF A STOCHASTIC PROCESS IN D+1-DIMENSIONS; INTRODUCE "STOCHASTIC" TIME.

"LANGEVIN" EQ $\phi(\vec{x}) \mapsto \phi(t, \vec{x})\,,\quad \frac{\partial \phi(t,\vec{x})}{\partial t} + k \frac{\delta S_{cl}}{\delta \phi(t,\vec{x})} = \eta(t,\vec{x})$ "WHITE NOISE"

CONSTANT KERNEL

STOCHASTIC QUANTIZATION (PARISI-WU (81))

$$\langle \phi(\vec{x})...\rangle = \frac{1}{Z}\int (\mathcal{D}\phi) e^{-S_{cl}[\phi]}\phi(\vec{x})...$$

CONSIDER THE EUCLIDEAN PATH INTEGRAL IN D-DIMENSIONS AS THE STATIONARY DISTRIBUTION OF A STOCHASTIC PROCESS IN D+1-DIMENSIONS; INTRODUCE "STOCHASTIC" TIME.

"LANGEVIN" EQ $\phi(\vec{x}) \mapsto \phi(t,\vec{x})\,,\quad \frac{\partial \phi(t,\vec{x})}{\partial t} + k\frac{\delta S_{cl}}{\delta\phi(t,\vec{x})} = \eta(t,\vec{x})$ "WHITE NOISE"

CONSTANT KERNEL

$$S_{cl}[\phi] = \int d^d\vec{x}\left(\frac{1}{2}\partial_i\phi\partial_i\phi + V(\lambda,\phi)\right)$$

STOCHASTIC QUANTIZATION (PARISI-WU (81))

$$\langle \phi(\vec{x})...\rangle = \frac{1}{Z}\int (\mathcal{D}\phi) e^{-S_{cl}[\phi]}\phi(\vec{x})...$$

CONSIDER THE EUCLIDEAN PATH INTEGRAL IN D-DIMENSIONS AS THE STATIONARY DISTRIBUTION OF A STOCHASTIC PROCESS IN D+1-DIMENSIONS; INTRODUCE "STOCHASTIC" TIME.

"LANGEVIN" EQ $\quad \phi(\vec{x}) \mapsto \phi(t,\vec{x})\,, \quad \frac{\partial \phi(t,\vec{x})}{\partial t} + k\frac{\delta S_{cl}}{\delta \phi(t,\vec{x})} = \eta(t,\vec{x})$ "WHITE NOISE"

CONSTANT KERNEL

$$S_{cl}[\phi] = \int d^d\vec{x}\left(\frac{1}{2}\partial_i\phi\partial_i\phi + V(\lambda,\phi)\right)$$

GENERICALLY, FOR SMALL-LAMBDA THE LANGEVIN GIVES A NON-RELATIVISTIC DISPERSION RELATION. HENCE, THE D+1 DIMENSIONAL STOCHASTIC PROCESS IS NON-RELATIVISTIC

UPSHOT: E.G. P. DAMGAARD & H. HUEFFEL PHYS. REP. (87)

UPSHOT: E.G. P. DAMGAARD & H. HUEFFEL PHYS. REP. (87)

$$
\begin{aligned}
Z &= \int (\mathcal{D}\eta) e^{-\frac{1}{4k}\int dt d^d\vec{x}\,\eta^2(t,\vec{x})} \\
&= \int (\mathcal{D}\phi) P[\phi,0] e^{-\int dt \int d^d\vec{x}\left[\frac{1}{4k}\left(\dot{\phi}^2 + k\frac{\delta S_{cl}}{\delta\phi}\right)^2 - \frac{k}{2}\frac{\delta^2 S_{cl}}{\delta\phi^2}\right]} \\
&= \int (\mathcal{D}\phi_0) e^{-\frac{1}{2}\int d^d\vec{x} S_{cl}[\phi_0]} \int (\mathcal{D}\phi) e^{-\int dt d^d\vec{x}\left(\frac{1}{4k}\dot{\phi}^2 + \frac{k}{4}\left(\frac{\delta S}{\delta\phi}\right)^2 - \frac{k}{2}\frac{\delta^2 S}{\delta\phi^2}\right)}
\end{aligned}
$$

UPSHOT: E.G. P. DAMGAARD & H. HUEFFEL PHYS. REP. (87)

$$
\begin{aligned}
Z &= \int (\mathcal{D}\eta) e^{-\frac{1}{4k}\int dt d^d\vec{x}\,\eta^2(t,\vec{x})} \\
&= \int (\mathcal{D}\phi) P[\phi,0] e^{-\int dt \int d^d\vec{x}\left[\frac{1}{4k}\left(\dot{\phi}^2 + k\frac{\delta S_{cl}}{\delta\phi}\right)^2 - \frac{k}{2}\frac{\delta^2 S_{cl}}{\delta\phi^2}\right]} \\
&= \int (\mathcal{D}\phi_0) e^{-\frac{1}{2}\int d^d\vec{x} S_{cl}[\phi_0]} \int (\mathcal{D}\phi) e^{-\int dt d^d\vec{x}\boxed{\left(\frac{1}{4k}\dot{\phi}^2 + \frac{k}{4}\left(\frac{\delta S}{\delta\phi}\right)^2 - \frac{k}{2}\frac{\delta^2 S}{\delta\phi^2}\right)}}
\end{aligned}
$$

FOKKER-PLANCK
LAGRANGIAN

UPSHOT: E.G. P. DAMGAARD & H. HUEFFEL PHYS. REP. (87)

$$
\begin{aligned}
Z &= \int (\mathcal{D}\eta) e^{-\frac{1}{4k}\int dt d^d\vec{x}\, \eta^2(t,\vec{x})} \\
&= \int (\mathcal{D}\phi) P[\phi,0] e^{-\int dt \int d^d\vec{x} \left[\frac{1}{4k}\left(\dot{\phi}^2 + k\frac{\delta S_{cl}}{\delta\phi}\right)^2 - \frac{k}{2}\frac{\delta^2 S_{cl}}{\delta\phi^2}\right]} \\
&= \int (\mathcal{D}\phi_0) e^{-\frac{1}{2}\int d^d\vec{x} S_{cl}[\phi_0]} \int (\mathcal{D}\phi) e^{-\int dt d^d\vec{x} \boxed{\left(\frac{1}{4k}\dot{\phi}^2 + \frac{k}{4}\left(\frac{\delta S}{\delta\phi}\right)^2 - \frac{k}{2}\frac{\delta^2 S}{\delta\phi^2}\right)}}
\end{aligned}
$$

FOKKER-PLANCK
LAGRANGIAN

$$P[\phi,0]: \quad \phi(t \to t_{\partial M}, \vec{x}) \to \phi_0(\vec{x}) \longrightarrow$$ "DIRICHLET" B.C.

UPSHOT: E.G. P. DAMGAARD & H. HUEFFEL PHYS. REP. (87)

$$
\begin{aligned}
Z &= \int (\mathcal{D}\eta) e^{-\frac{1}{4k}\int dt d^d\vec{x}\, \eta^2(t,\vec{x})} \\
&= \int (\mathcal{D}\phi) P[\phi, 0] e^{-\int dt \int d^d\vec{x} \left[\frac{1}{4k}\left(\dot{\phi}^2 + k\frac{\delta S_{cl}}{\delta\phi}\right)^2 - \frac{k}{2}\frac{\delta^2 S_{cl}}{\delta\phi^2}\right]} \\
&= \int (\mathcal{D}\phi_0) e^{-\frac{1}{2}\int d^d\vec{x} S_{cl}[\phi_0]} \int (\mathcal{D}\phi) e^{-\int dt d^d\vec{x} \left(\frac{1}{4k}\dot{\phi}^2 + \frac{k}{4}\left(\frac{\delta S}{\delta\phi}\right)^2 - \frac{k}{2}\frac{\delta^2 S}{\delta\phi^2}\right)}
\end{aligned}
$$

FOKKER-PLANCK LAGRANGIAN

$$P[\phi, 0] : \quad \phi(t \to t_{\partial M}, \vec{x}) \to \phi_0(\vec{x}) \longrightarrow \text{“DIRICHLET” B.C.}$$

$$det \frac{\delta\eta}{\delta\phi} = e^{\frac{k}{2}\int dt \int d^d\vec{x} \frac{\delta^2 S_{cl}}{\delta\phi^2}}$$

UPSHOT: E.G. P. DAMGAARD & H. HUEFFEL PHYS. REP. (87)

$$
\begin{aligned}
Z &= \int (\mathcal{D}\eta) e^{-\frac{1}{4k}\int dt d^d\vec{x}\,\eta^2(t,\vec{x})} \\
&= \int (\mathcal{D}\phi) P[\phi,0] e^{-\int dt \int d^d\vec{x}\left[\frac{1}{4k}\left(\dot{\phi}^2 + k\frac{\delta S_{cl}}{\delta\phi}\right)^2 - \frac{k}{2}\frac{\delta^2 S_{cl}}{\delta\phi^2}\right]} \\
&= \int (\mathcal{D}\phi_0) e^{-\frac{1}{2}\int d^d\vec{x} S_{cl}[\phi_0]} \int (\mathcal{D}\phi) e^{-\int dt d^d\vec{x}\left(\frac{1}{4k}\dot{\phi}^2 + \frac{k}{4}\left(\frac{\delta S}{\delta\phi}\right)^2 - \frac{k}{2}\frac{\delta^2 S}{\delta\phi^2}\right)}
\end{aligned}
$$

FOKKER-PLANCK LAGRANGIAN

$$P[\phi,0]: \quad \phi(t \to t_{\partial M}, \vec{x}) \to \phi_0(\vec{x}) \longrightarrow \text{“DIRICHLET” B.C.}$$

$$det\frac{\delta\eta}{\delta\phi} = e^{\frac{k}{2}\int dt \int d^d\vec{x}\,\frac{\delta^2 S_{cl}}{\delta\phi^2}}$$

I CAN USE **Z** TO CALCULATE CORRELATION FUNCTIONS OF $\phi(t,\vec{x})$

WHICH FOR LARGE “TIMES” RELAX TO THE EUCLIDEAN CORRELATION

FUNCTIONS OF THE D-DIMENSIONAL THEORY.

HOLOGRAPHY VS STOCHASTIC QUANTIZATION

HOLOGRAPHY VS STOCHASTIC QUANTIZATION

THE S.Q. PARTITION FUNCTION IS FORMALLY EQUIVALENT TO THE HOLOGRAPHIC FORMULA UPON INTEGRATION OVER THE BOUNDARY DATA (MODULO "RENORMALIZATIONS")

HOLOGRAPHY VS STOCHASTIC QUANTIZATION

THE S.Q. PARTITION FUNCTION IS FORMALLY EQUIVALENT TO THE HOLOGRAPHIC FORMULA UPON INTEGRATION OVER THE BOUNDARY DATA (MODULO "RENORMALIZATIONS")

$$\begin{aligned} Z_{hol}[\phi_0] &= \int (\mathcal{D}\phi)_{\phi\to\phi_0} e^{-S_{d+1}[\phi]} \equiv e^{\Gamma_d[\phi_0]} \\ Z &= \int (\mathcal{D}\phi_0) e^{-S_d[\phi_0]+\Gamma_d[\phi_0]} \end{aligned}$$

G. COMPERE & D. MAROLF (08)

HOLOGRAPHY VS STOCHASTIC QUANTIZATION

THE S.Q. PARTITION FUNCTION IS FORMALLY EQUIVALENT TO THE HOLOGRAPHIC FORMULA UPON INTEGRATION OVER THE BOUNDARY DATA (MODULO "RENORMALIZATIONS")

"FOKKER-PLANCK"

$$
\begin{aligned}
Z_{hol}[\phi_0] &= \int (\mathcal{D}\phi)_{\phi\to\phi_0} e^{-S_{d+1}[\phi]} \equiv e^{\Gamma_d[\phi_0]} \\
Z &= \int (\mathcal{D}\phi_0) e^{-S_d[\phi_0]+\Gamma_d[\phi_0]}
\end{aligned}
$$

G. COMPERE & D. MAROLF (08)

HOLOGRAPHY VS STOCHASTIC QUANTIZATION

THE S.Q. PARTITION FUNCTION IS FORMALLY EQUIVALENT TO THE HOLOGRAPHIC FORMULA UPON INTEGRATION OVER THE BOUNDARY DATA (MODULO "RENORMALIZATIONS")

"FOKKER-PLANCK"

$$Z_{hol}[\phi_0] = \int (\mathcal{D}\phi)_{\phi\to\phi_0} e^{-S_{d+1}[\phi]} \equiv e^{\Gamma_d[\phi_0]}$$

$$\longleftarrow \quad Z = \int (\mathcal{D}\phi_0) e^{-S_d[\phi_0]+\Gamma_d[\phi_0]}$$

G. COMPERE & D. MAROLF (08)

HOLOGRAPHY VS STOCHASTIC QUANTIZATION

THE S.Q. PARTITION FUNCTION IS FORMALLY EQUIVALENT TO THE HOLOGRAPHIC FORMULA UPON INTEGRATION OVER THE BOUNDARY DATA (MODULO "RENORMALIZATIONS")

"FOKKER-PLANCK"

$$Z_{hol}[\phi_0] = \int (\mathcal{D}\phi)_{\phi\to\phi_0} \boxed{e^{-S_{d+1}[\phi]}} \equiv e^{\Gamma_d[\phi_0]}$$

1, UP TO BULK LOOPS $\longleftarrow$

$$Z = \int (\mathcal{D}\phi_0) e^{-S_d[\phi_0]+\Gamma_d[\phi_0]}$$

G. COMPERE & D. MAROLF (08)

HOLOGRAPHY VS STOCHASTIC QUANTIZATION

THE S.Q. PARTITION FUNCTION IS FORMALLY EQUIVALENT TO THE HOLOGRAPHIC FORMULA UPON INTEGRATION OVER THE BOUNDARY DATA (MODULO "RENORMALIZATIONS")

"FOKKER-PLANCK"

$$Z_{hol}[\phi_0] \quad = \quad \int (\mathcal{D}\phi)_{\phi\to\phi_0} \boxed{e^{-S_{d+1}[\phi]}} \equiv e^{\Gamma_d[\phi_0]}$$

1, UP TO BULK LOOPS $\longleftarrow$

$$Z \quad = \quad \int (\mathcal{D}\phi_0) e^{-S_d[\phi_0]+\Gamma_d[\phi_0]}$$

G. COMPERE & D. MAROLF (08)

STOCHASTIC QUANTIZATION

HOLOGRAPHY VS STOCHASTIC QUANTIZATION

THE S.Q. PARTITION FUNCTION IS FORMALLY EQUIVALENT TO THE HOLOGRAPHIC FORMULA UPON INTEGRATION OVER THE BOUNDARY DATA (MODULO "RENORMALIZATIONS")

"FOKKER-PLANCK"

$$Z_{hol}[\phi_0] = \int (\mathcal{D}\phi)_{\phi\to\phi_0} e^{-S_{d+1}[\phi]} \equiv e^{\Gamma_d[\phi_0]}$$

1, UP TO BULK LOOPS $\longleftarrow$

$$Z = \int (\mathcal{D}\phi_0) e^{-S_d[\phi_0]+\Gamma_d[\phi_0]}$$

G. COMPERE & D. MAROLF (08)

STOCHASTIC QUANTIZATION

$$S_{cl} = 2\sqrt{\frac{1}{18\lambda}} \int d^3\vec{x} \left(\lambda\phi^3 + \frac{1}{\phi}\partial_i\phi\partial_i\phi \right)$$

HOLOGRAPHY VS STOCHASTIC QUANTIZATION

THE S.Q. PARTITION FUNCTION IS FORMALLY EQUIVALENT TO THE

HOLOGRAPHIC FORMULA UPON INTEGRATION OVER THE BOUNDARY DATA

(MODULO "RENORMALIZATIONS")

"FOKKER-PLANCK"

$$Z_{hol}[\phi_0] = \int (\mathcal{D}\phi)_{\phi\to\phi_0} \boxed{e^{-S_{d+1}[\phi]}} \equiv e^{\Gamma_d[\phi_0]}$$

1, UP TO BULK LOOPS $\longleftarrow$

$$Z = \int (\mathcal{D}\phi_0) e^{-S_d[\phi_0]+\Gamma_d[\phi_0]}$$

G. COMPERE & D. MAROLF (08)

STOCHASTIC QUANTIZATION

$$S_{cl} = 2\sqrt{\frac{1}{18\lambda}} \int d^3\vec{x} \left(\lambda\phi^3 + \frac{1}{\phi}\partial_i\phi\partial_i\phi\right)$$

CALCULATIONS..

HOLOGRAPHY VS STOCHASTIC QUANTIZATION

THE S.Q. PARTITION FUNCTION IS FORMALLY EQUIVALENT TO THE HOLOGRAPHIC FORMULA UPON INTEGRATION OVER THE BOUNDARY DATA (MODULO "RENORMALIZATIONS")

"FOKKER-PLANCK"

$$Z_{hol}[\phi_0] = \int (\mathcal{D}\phi)_{\phi\to\phi_0} e^{-S_{d+1}[\phi]} \equiv e^{\Gamma_d[\phi_0]}$$

1, UP TO BULK LOOPS $\longleftarrow$

$$Z = \int (\mathcal{D}\phi_0) e^{-S_d[\phi_0]+\Gamma_d[\phi_0]}$$

G. COMPERE & D. MAROLF (08)

STOCHASTIC QUANTIZATION

$$S_{cl} = 2\sqrt{\frac{1}{18\lambda}} \int d^3\vec{x} \left(\lambda\phi^3 + \frac{1}{\phi}\partial_i\phi\partial_i\phi \right)$$

CALCULATIONS..

$$S_{3+1}^{F.P.} = \int dt \int d^3\vec{x} \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\partial_i\phi\partial_i\phi + \frac{\lambda}{4}\phi^4 + \cdots \right)$$

HOLOGRAPHY VS STOCHASTIC QUANTIZATION

THE S.Q. PARTITION FUNCTION IS FORMALLY EQUIVALENT TO THE HOLOGRAPHIC FORMULA UPON INTEGRATION OVER THE BOUNDARY DATA (MODULO "RENORMALIZATIONS")

"FOKKER-PLANCK"

$$Z_{hol}[\phi_0] = \int (\mathcal{D}\phi)_{\phi\to\phi_0} e^{-S_{d+1}[\phi]} \equiv e^{\Gamma_d[\phi_0]}$$

1, UP TO BULK LOOPS ⟵

$$Z = \int (\mathcal{D}\phi_0) e^{-S_d[\phi_0]+\Gamma_d[\phi_0]}$$

G. COMPERE & D. MAROLF (08)

STOCHASTIC QUANTIZATION

$$S_{cl} = 2\sqrt{\frac{1}{18\lambda}} \int d^3\vec{x} \left(\lambda\phi^3 + \frac{1}{\phi}\partial_i\phi\partial_i\phi\right)$$

CALCULATIONS..

$$S^{F.P.}_{3+1} = \int dt \int d^3\vec{x} \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\partial_i\phi\partial_i\phi + \frac{\lambda}{4}\phi^4 + \cdots\right)$$

FOR LARGE-LAMBDA!

RECAPITULATE: I HAVE SHOWN THAT

RECAPITULATE: I HAVE SHOWN THAT

$$S_{cl} \sim \int d^3\vec{x} \left(\lambda\phi^3 + \frac{1}{\phi}\partial_i\phi\partial_i\phi \right)$$

RECAPITULATE: I HAVE SHOWN THAT

$$S_{cl} \sim \int d^3\vec{x} \left(\lambda\phi^3 + \frac{1}{\phi}\partial_i\phi\partial_i\phi \right)$$

STOCHASTIC QUANTIZATION

RECAPITULATE: I HAVE SHOWN THAT

$$S_{cl} \sim \int d^3\vec{x} \left(\lambda\phi^3 + \frac{1}{\phi}\partial_i\phi\partial_i\phi \right)$$

STOCHASTIC QUANTIZATION

$$\Big\downarrow$$

$$S_{3+1} \sim \int dt \int d^3\vec{x} \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\partial_i\phi\partial_i\phi + \frac{\lambda}{4}\phi^4 \right)$$

RECAPITULATE: I HAVE SHOWN THAT

$$S_{cl} \sim \int d^3\vec{x} \left(\lambda\phi^3 + \frac{1}{\phi}\partial_i\phi\partial_i\phi \right)$$

STOCHASTIC QUANTIZATION

HOLOGRAPHY

$$S_{3+1} \sim \int dt \int d^3\vec{x} \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\partial_i\phi\partial_i\phi + \frac{\lambda}{4}\phi^4 \right)$$

RECAPITULATE: I HAVE SHOWN THAT

$$S_{cl} \sim \int d^3\vec{x} \left(\lambda\phi^3 + \frac{1}{\phi}\partial_i\phi\partial_i\phi \right)$$

STOCHASTIC QUANTIZATION

LARGE-LAMBDA INSTANTONS

HOLOGRAPHY

$$S_{3+1} \sim \int dt \int d^3\vec{x} \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\partial_i\phi\partial_i\phi + \frac{\lambda}{4}\phi^4 \right)$$

RECAPITULATE: I HAVE SHOWN THAT

$$S_{cl} \sim \int d^3\vec{x} \left(\lambda\phi^3 + \frac{1}{\phi}\partial_i\phi\partial_i\phi \right)$$

STOCHASTIC QUANTIZATION

LARGE-LAMBDA INSTANTONS

HOLOGRAPHY

$$S_{3+1} \sim \int dt \int d^3\vec{x} \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\partial_i\phi\partial_i\phi + \frac{\lambda}{4}\phi^4 \right)$$

CAN WE UNDERSTAND WHAT IS GOING ON?

IS HOLOGRAPHY STOCHASTIC?

IS HOLOGRAPHY STOCHASTIC?

A GEOMETRIC PICTURE OF STOCHASTIC HOLOGRAPHY AND CONFORMAL HOLOGRAPHY:

IS HOLOGRAPHY STOCHASTIC?

A GEOMETRIC PICTURE OF STOCHASTIC HOLOGRAPHY AND CONFORMAL HOLOGRAPHY:

$$I_{EH} = \frac{-1}{16\pi G}\int d^4x\sqrt{g}\,(R-2\Lambda)$$

IS HOLOGRAPHY STOCHASTIC?

A GEOMETRIC PICTURE OF STOCHASTIC HOLOGRAPHY AND CONFORMAL HOLOGRAPHY:

$$I_{EH} = \frac{-1}{16\pi G}\int d^4x\sqrt{g}\,(R-2\Lambda)$$

CONSIDER CONFORMALLY FLAT METRICS

IS HOLOGRAPHY STOCHASTIC?

A GEOMETRIC PICTURE OF STOCHASTIC HOLOGRAPHY AND CONFORMAL HOLOGRAPHY:

$$I_{EH} = \frac{-1}{16\pi G} \int d^4x \sqrt{g}\,(R - 2\Lambda)$$

CONSIDER CONFORMALLY FLAT METRICS

$$g_{\mu\nu}(x) = \varphi^2(x)\eta_{\mu\nu} \Rightarrow I_{EH} = -\frac{3}{4\pi} \int d^4x \left(\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{6}\phi^4 \right) \;, \;\; \phi = \frac{\varphi}{\sqrt{G}} \,, \;\; \lambda = \frac{\Lambda}{G}$$

IS HOLOGRAPHY STOCHASTIC?

A GEOMETRIC PICTURE OF STOCHASTIC HOLOGRAPHY AND CONFORMAL HOLOGRAPHY:

$$I_{EH} = \frac{-1}{16\pi G}\int d^4x\sqrt{g}\,(R-2\Lambda)$$

CONSIDER CONFORMALLY FLAT METRICS

$$g_{\mu\nu}(x) = \varphi^2(x)\eta_{\mu\nu} \Rightarrow I_{EH} = -\frac{3}{4\pi}\int d^4x\left(\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{6}\phi^4\right)\,,\;\; \phi = \frac{\varphi}{\sqrt{G}}\,,\;\; \lambda = \frac{\Lambda}{G}$$

FOR POSITIVE C.C. THE INSTANTON SOLUTION IS JUST THE 4-SPHERE

IS HOLOGRAPHY STOCHASTIC?

A GEOMETRIC PICTURE OF STOCHASTIC HOLOGRAPHY AND CONFORMAL HOLOGRAPHY:

$$I_{EH} = \frac{-1}{16\pi G}\int d^4x\sqrt{g}\,(R-2\Lambda)$$

CONSIDER CONFORMALLY FLAT METRICS

$$g_{\mu\nu}(x) = \varphi^2(x)\eta_{\mu\nu} \Rightarrow I_{EH} = -\frac{3}{4\pi}\int d^4x\left(\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{6}\phi^4\right)\,,\;\; \phi = \frac{\varphi}{\sqrt{G}}\,,\;\; \lambda = \frac{\Lambda}{G}$$

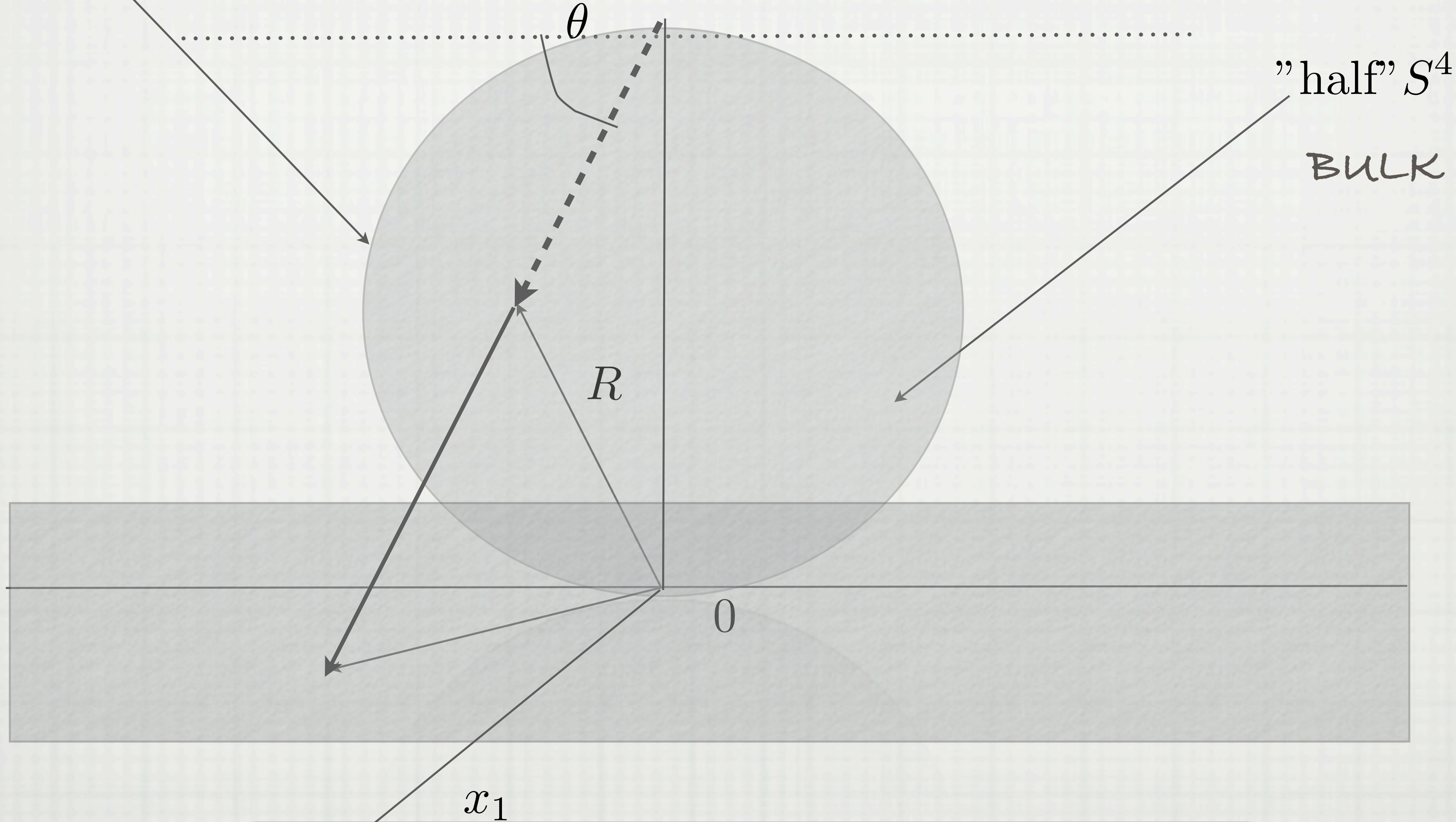
FOR POSITIVE C.C. THE INSTANTON SOLUTION IS JUST THE 4-SPHERE

$$\phi(x) = \sqrt{\frac{12}{\lambda}}\frac{R}{R^2+x^2} \to ds^2 = \frac{4R^4}{(R^2+x^2)^2}dx^2\,,\;\; \Lambda = \frac{3}{R^2}$$

BOUNDARY

S^3

$$R\phi(x_1,\vec{x}) \sim \cos^2\theta\Big|_{S^4} \quad \begin{array}{l} \xrightarrow{x_1 \to 0} \cos\theta\Big|_{S^3} \\ \xrightarrow[x_1 \to \infty]{} 0 \end{array}$$



INSTANTONS SATISFY

$$\frac{d}{dx_1}\partial_i \log\phi = 2\partial_i \log\phi \;:\; \partial_i \log\phi \mapsto \vec{v} \;:\; x_1 \mapsto t$$

LANGEVIN (NO WHITE-NOISE)

GEOMETRIC PICTURE:

THE 4D EFFECTIVE E-H ACTION FOR CONFORMALLY FLAT METRICS, AS A PATH INTEGRAL MEASURE, RELAXES FOR LARGE "TIMES" TO A THREE-DIMENSIONAL E-H ACTION.

GEOMETRIC PICTURE:

THE 4D EFFECTIVE E-H ACTION FOR CONFORMALLY FLAT METRICS, AS A PATH INTEGRAL MEASURE, RELAXES FOR LARGE "TIMES" TO A THREE-DIMENSIONAL E-H ACTION.

$$
\begin{aligned}
I_{EH} &= \frac{-1}{16\pi G}\int d^3\vec{x}\sqrt{g}\,(R-2\Lambda) \\
g_{\mu\nu} &= \varphi^2(\vec{x})\eta_{\mu\nu} \Rightarrow I_{EH} = \frac{-1}{4\pi}\int d^3\vec{x}\left(\frac{1}{\phi}\partial_i\phi\partial_i\phi - \lambda\phi^3\right)\,, \quad \phi = \frac{\varphi}{G}\,,\ \lambda = \frac{\Lambda}{G^2}
\end{aligned}
$$

GEOMETRIC PICTURE:

THE 4D EFFECTIVE E-H ACTION FOR CONFORMALLY FLAT METRICS, AS A PATH INTEGRAL MEASURE, RELAXES FOR LARGE "TIMES" TO A THREE-DIMENSIONAL E-H ACTION.

$$
\begin{aligned}
I_{EH} &= \frac{-1}{16\pi G}\int d^3\vec{x}\sqrt{g}\,(R-2\Lambda) \\
g_{\mu\nu} &= \varphi^2(\vec{x})\eta_{\mu\nu} \Rightarrow I_{EH} = \frac{-1}{4\pi}\int d^3\vec{x}\left(\frac{1}{\phi}\partial_i\phi\partial_i\phi - \lambda\phi^3\right)\,,\quad \phi = \frac{\varphi}{G}\,,\ \ \lambda = \frac{\Lambda}{G^2}
\end{aligned}
$$

$$
\phi = f^2 \Rightarrow I_{EH} = \frac{-2}{\pi}\int d^3\vec{x}\left(\frac{1}{2}\partial_i f\partial_i f - \frac{\lambda}{8}f^6\right)
$$

GEOMETRIC PICTURE:

THE 4D EFFECTIVE E-H ACTION FOR CONFORMALLY FLAT METRICS, AS A PATH INTEGRAL MEASURE, RELAXES FOR LARGE "TIMES" TO A THREE-DIMENSIONAL E-H ACTION.

$$\begin{aligned} I_{EH} &= \frac{-1}{16\pi G}\int d^3\vec{x}\sqrt{g}\,(R-2\Lambda) \\ g_{\mu\nu} &= \varphi^2(\vec{x})\eta_{\mu\nu} \Rightarrow I_{EH} = \frac{-1}{4\pi}\int d^3\vec{x}\left(\frac{1}{\phi}\partial_i\phi\partial_i\phi - \lambda\phi^3\right)\,,\quad \phi = \frac{\varphi}{G}\,,\ \lambda = \frac{\Lambda}{G^2} \end{aligned}$$

$$\phi = f^2 \Rightarrow I_{EH} = \frac{-2}{\pi}\int d^3\vec{x}\left(\frac{1}{2}\partial_i f\partial_i f - \frac{\lambda}{8}f^6\right)$$

FOR $\lambda > 0$ (I.E. POSITIVE C.C.) THIS HAS INSTANTON SOLUTIONS (3-SPHERE)

GEOMETRIC PICTURE:

THE 4D EFFECTIVE E-H ACTION FOR CONFORMALLY FLAT METRICS, AS A PATH INTEGRAL MEASURE, RELAXES FOR LARGE "TIMES" TO A THREE-DIMENSIONAL E-H ACTION.

$$\begin{aligned} I_{EH} &= \frac{-1}{16\pi G}\int d^3\vec{x}\sqrt{g}\,(R-2\Lambda) \\ g_{\mu\nu} &= \varphi^2(\vec{x})\eta_{\mu\nu} \Rightarrow I_{EH} = \frac{-1}{4\pi}\int d^3\vec{x}\left(\frac{1}{\phi}\partial_i\phi\partial_i\phi - \lambda\phi^3\right)\,,\quad \phi = \frac{\varphi}{G}\,,\ \lambda = \frac{\Lambda}{G^2} \end{aligned}$$

$$\phi = f^2 \Rightarrow I_{EH} = \frac{-2}{\pi}\int d^3\vec{x}\left(\frac{1}{2}\partial_i f\partial_i f - \frac{\lambda}{8}f^6\right)$$

FOR $\lambda > 0$ (I.E. POSITIVE C.C.) THIS HAS INSTANTON SOLUTIONS (3-SPHERE)

$$f(\vec{x}) = \left(\frac{4}{\lambda}\right)^{1/4}\frac{R^{1/2}}{(R^2+\vec{x}^2)^{1/2}}$$

GEOMETRIC PICTURE:

THE 4D EFFECTIVE E-H ACTION FOR CONFORMALLY FLAT METRICS, AS A PATH INTEGRAL MEASURE, RELAXES FOR LARGE "TIMES" TO A THREE-DIMENSIONAL E-H ACTION.

$$
\begin{aligned}
I_{EH} &= \frac{-1}{16\pi G}\int d^3\vec{x}\sqrt{g}\,(R-2\Lambda) \\
g_{\mu\nu} &= \varphi^2(\vec{x})\eta_{\mu\nu} \Rightarrow I_{EH} = \frac{-1}{4\pi}\int d^3\vec{x}\left(\frac{1}{\phi}\partial_i\phi\partial_i\phi - \lambda\phi^3\right)\,, \quad \phi = \frac{\varphi}{G}\,,\ \ \lambda = \frac{\Lambda}{G^2}
\end{aligned}
$$

$$
\phi = f^2 \Rightarrow I_{EH} = \frac{-2}{\pi}\int d^3\vec{x}\left(\frac{1}{2}\partial_i f\partial_i f - \frac{\lambda}{8}f^6\right)
$$

FOR $\lambda > 0$ (I.E. POSITIVE C.C.) THIS HAS INSTANTON SOLUTIONS (3-SPHERE)

$$
f(\vec{x}) = \left(\frac{4}{\lambda}\right)^{1/4}\frac{R^{1/2}}{(R^2+\vec{x}^2)^{1/2}}
$$

THESE ARE THE BOUNDARY VALUES
OF THE BULK INSTANTONS IF

GEOMETRIC PICTURE:

THE 4D EFFECTIVE E-H ACTION FOR CONFORMALLY FLAT METRICS, AS A PATH INTEGRAL MEASURE, RELAXES FOR LARGE "TIMES" TO A THREE-DIMENSIONAL E-H ACTION.

$$\begin{aligned} I_{EH} &= \frac{-1}{16\pi G}\int d^3\vec{x}\sqrt{g}\,(R-2\Lambda) \\ g_{\mu\nu} &= \varphi^2(\vec{x})\eta_{\mu\nu} \Rightarrow I_{EH} = \frac{-1}{4\pi}\int d^3\vec{x}\left(\frac{1}{\phi}\partial_i\phi\partial_i\phi - \lambda\phi^3\right)\ ,\quad \phi = \frac{\varphi}{G}\,,\ \ \lambda = \frac{\Lambda}{G^2} \end{aligned}$$

$$\phi = f^2 \Rightarrow I_{EH} = \frac{-2}{\pi}\int d^3\vec{x}\left(\frac{1}{2}\partial_i f\partial_i f - \frac{\lambda}{8}f^6\right)$$

FOR $\lambda > 0$ (I.E. POSITIVE C.C.) THIS HAS INSTANTON SOLUTIONS (3-SPHERE)

$$f(\vec{x}) = \left(\frac{4}{\lambda}\right)^{1/4}\frac{R^{1/2}}{(R^2+\vec{x}^2)^{1/2}}$$

THESE ARE THE BOUNDARY VALUES OF THE BULK INSTANTONS IF

$$\lambda_4 = 3\lambda_3 \Rightarrow \frac{\Lambda_3}{\Lambda_4} = \frac{G_3}{2G_4}$$

TOWARDS A STOCHASTIC HOLOGRAPHY?

TOWARDS A STOCHASTIC HOLOGRAPHY?

- ✓ I HAVE SHOWN AN EXPLICIT EXAMPLE OF 4D AND 3D THEORIES THAT ARE RELATED: 1) HOLOGRAPHICALLY, 2) VIA STOCHASTIC QUANTIZATION AND 3) GEOMETRICALLY.

TOWARDS A STOCHASTIC HOLOGRAPHY?

- ✓ I HAVE SHOWN AN EXPLICIT EXAMPLE OF 4D AND 3D THEORIES THAT ARE RELATED: 1) HOLOGRAPHICALLY, 2) VIA STOCHASTIC QUANTIZATION AND 3) GEOMETRICALLY.

- ✓ THE CRUCIAL POINT WAS THE "RELATIVISTIC" LANGEVIN EQ. SATISFIED BY THE INSTANTON CONFIGURATIONS.

TOWARDS A STOCHASTIC HOLOGRAPHY?

- ✓ I HAVE SHOWN AN EXPLICIT EXAMPLE OF 4D AND 3D THEORIES THAT ARE RELATED: 1) HOLOGRAPHICALLY, 2) VIA STOCHASTIC QUANTIZATION AND 3) GEOMETRICALLY.

- ✓ THE CRUCIAL POINT WAS THE “RELATIVISTIC” LANGEVIN EQ. SATISFIED BY THE INSTANTON CONFIGURATIONS.

- ✓ THIS IN NOT A STRONG/WEAK COUPLING CORRESPONDENCE. THE BULK AND BOUNDARY THEORIES ARE IN THE SAME REGIME (I.E. D-INSTANTONS/YM INSTANTONS IN ADS5/CFT4).

- ✓ RELATED TO RICCI-FLOW ? (1ST-ORDER EQUATIONS)

- ✓ RELATED TO RICCI-FLOW ? (1ST-ORDER EQUATIONS)

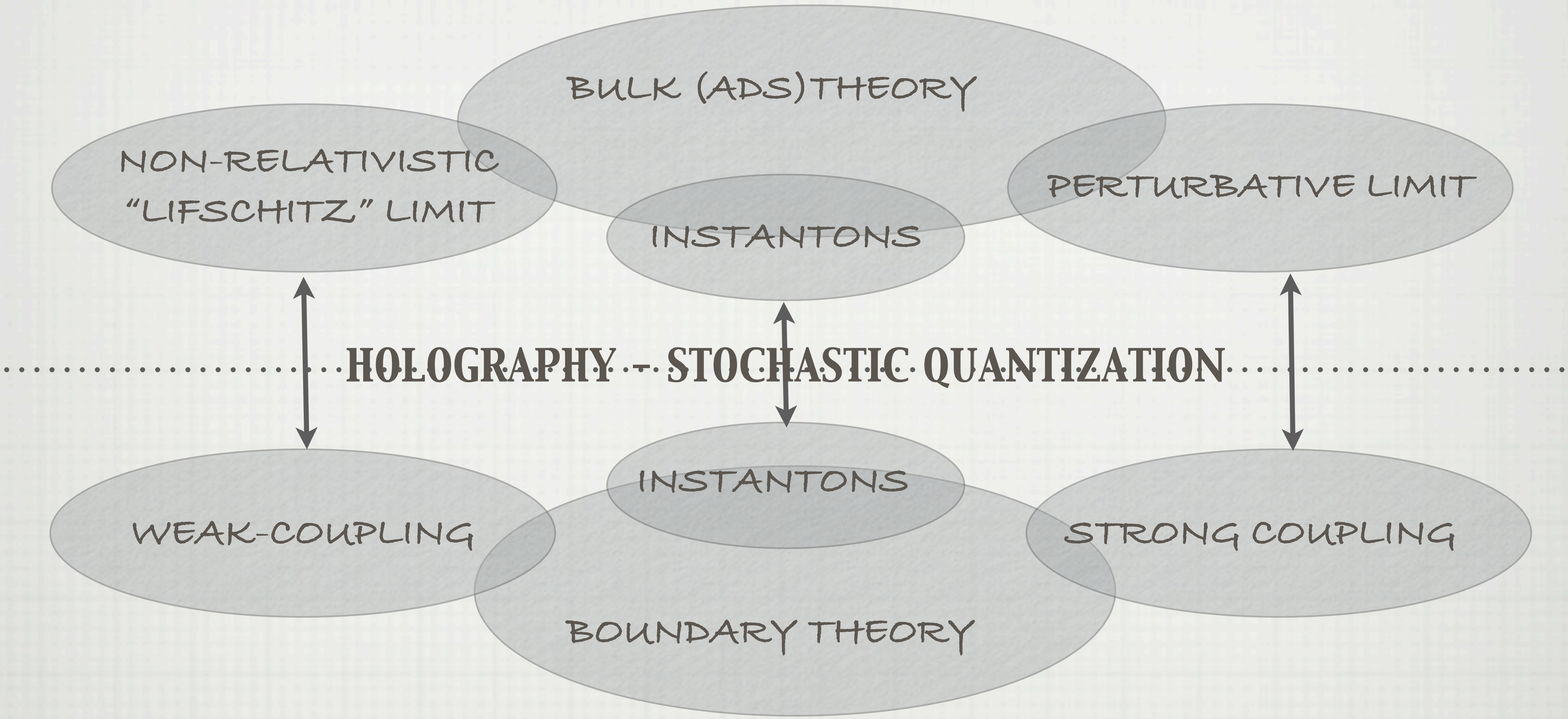
- ✓ INTIMATELY RELATED TO THE WELL-KNOWN STOCHASTIC QUANTIZATION RESULT: TOPOLOGIGAL YANG-MILLS/ CHERN-SIMONS. THIS ARISES FROM THE BRST GAUGE FIXING OF THE TOPOLOGICAL "GAUGE" INVARIANCE. IT IS A GENERALIZATION OF THE STOKES FORMULA.

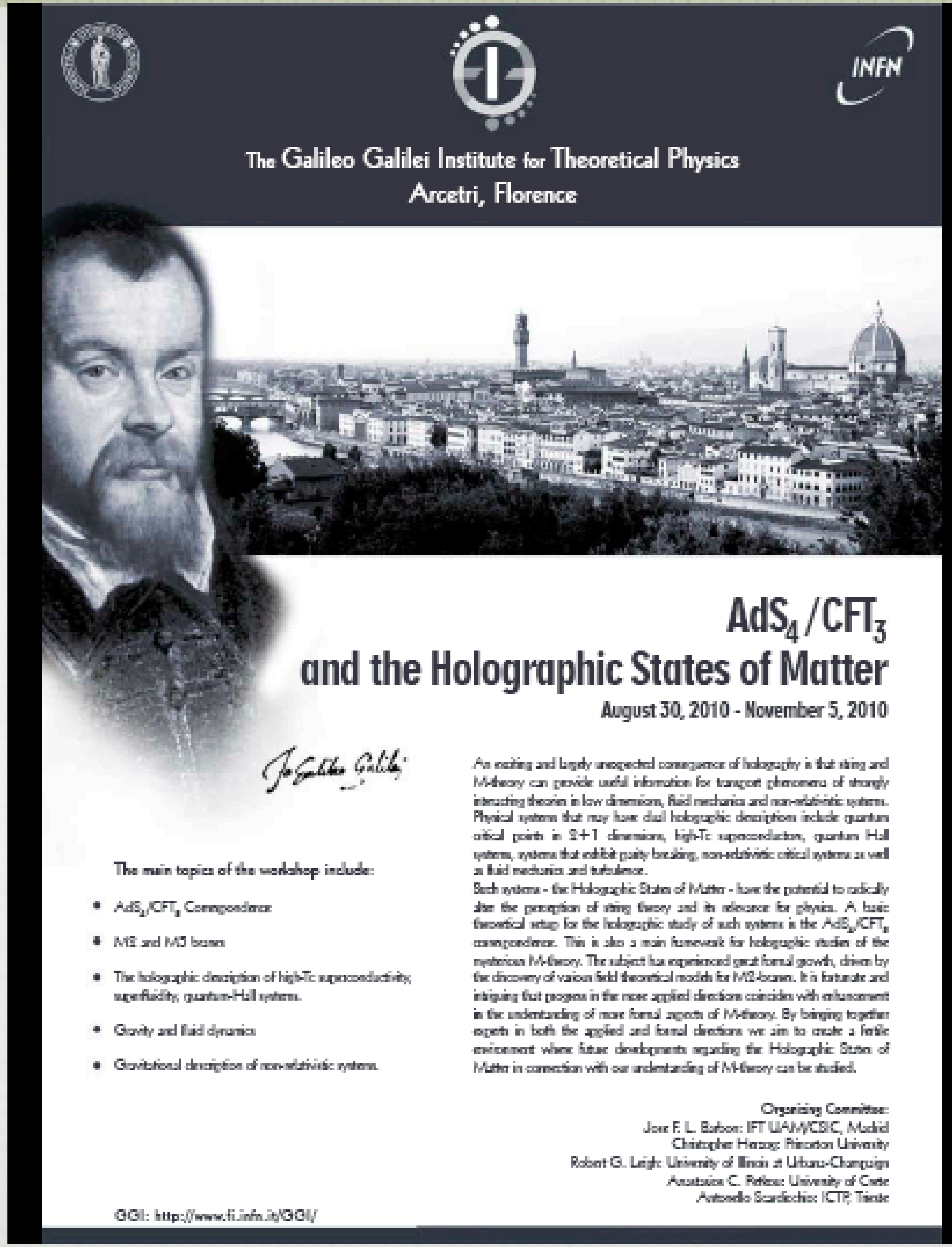
- ✓ RELATED TO RICCI-FLOW ? (1ST-ORDER EQUATIONS)

- ✓ INTIMATELY RELATED TO THE WELL-KNOWN STOCHASTIC QUANTIZATION RESULT: TOPOLOGIGAL YANG-MILLS/ CHERN-SIMONS. THIS ARISES FROM THE BRST GAUGE FIXING OF THE TOPOLOGICAL "GAUGE" INVARIANCE. IT IS A GENERALIZATION OF THE STOKES FORMULA.

D. MANSI & A. MAURI: TO APPEAR

THE HOLOGRAPHIC WEB




The Galileo Galilei Institute for Theoretical Physics
Arcetri, Florence
INFN
AdS4 /CFT3
and the Holographic States of Matter
August 30, 2010 - November 5, 2010
An exciting and largely unexpected consequence of holography is that string and M-theory can provide useful information for transport phenomena of strongly interacting theories in low dimensions, fluid mechanics and non-relativistic systems. Physical systems that may have dual holographic descriptions include quantum critical points in 2+1 dimensions, high-Tc superconductors, quantum Hall systems, systems that exhibit parity breaking, non-relativistic critical systems as well as fluid mechanics and turbulence.
Such systems - the Holographic States of Matter - have the potential to radically alter the perception of string theory and its relevance for physics. A basic theoretical setup for the holographic study of such systems is the AdS4/CFT3 correspondence. This is also a main framework for holographic studies of the mysterious M-theory. The subject has experienced great formal growth, driven by the discovery of various field theoretical models for M2-branes. It is fortunate and intriguing that progress in the more applied directions coincides with enhancement in the understanding of more formal aspects of M-theory. By bringing together experts in both the applied and formal directions we aim to create a fertile environment where future developments regarding the Holographic States of Matter in connection with our understanding of M-theory can be studied.
The main topics of the workshop include:
AdS4/CFT3 Correspondence
M2 and M5 branes
The holographic description of high-Tc superconductivity, superfluidity, quantum-Hall systems.
Gravity and fluid dynamics
Gravitational description of non-relativistic systems.
Organizing Committee:
Jose F. L. Barbon: IFT UAM/CSIC, Madrid
Christopher Herzog: Princeton University
Robert G. Leigh: University of Illinois at Urbana-Champaign
Anastasios C. Petkou: University of Crete
Antonello Scardicchio: ICTP, Trieste
GGI: http://www.fi.infn.it/GGI/