Open & Closed vs. Pure Open String Disk Amplitudes

Stephan Stieberger, MPP München

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Corfu Summer Institute
9th Hellenic School and Workshops
on Elementary Particle Physics and Gravity

Outline

Open & closed vs. pure open string disk amplitudes

St. St., `arXiv:0907.2211`

- Sort of generalized KLT on the disk
- Open string subamplitude relations
- Relation between brane and bulk couplings

Recap: Disk scattering of open strings

N–point open string tree–level disk amplitudes in backgrounds with CFT description

Motivation:

- Recent results in YM in spinor basis:

  Compact expressions, Recursion relations, . . .

  St.St., Taylor, 2006–2008

- Impact on the perturbative structure of string theory:

  Recursion relations, . . . , Impact on open/closed sector, . . .

- YM amplitudes give rise to gravity amplitudes (KLT)

  Transcendentality properties, possible constraints on counter terms, . . .

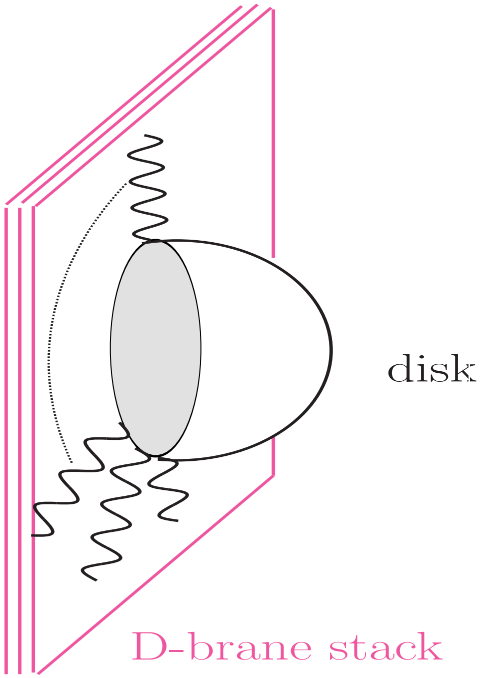
  St.St. arXiv: 0910.0180

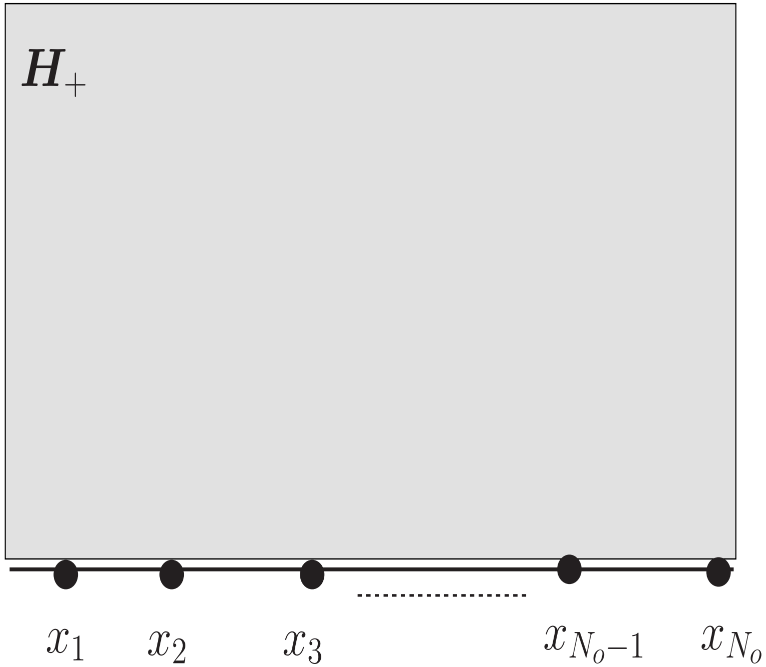
- Parton amplitudes are important for (collider) phenomenology:

  LHC: Multijet production is dominated by tree–level QCD–scattering

  Härtl, Lüst, Schlotterer, St.St., Taylor

Recap: Disk scattering of open strings





$$\mathcal{A}(1,2,\ldots,N) = g_{YM}^{N-2} \sum_{\sigma\in S_N/\mathbf{Z}_N} \mathrm{Tr}(T^{a_1}T^{a_{\sigma(2)}}\ldots T^{a_{\sigma(N)}})\ A(1,\sigma(2),\ldots,\sigma(N))$$

$A(1,2,\ldots,N)$ tree–level color–ordered N–leg partial amplitude

Recap: Disk scattering of open strings

E.g.: $N = 3$: $\text{Tr}(T^1T^2T^3) = \text{Tr}(T^2T^3T^1) = \text{Tr}(T^3T^1T^2)$

$$\text{Tr}(T^1T^3T^2) = \text{Tr}(T^3T^2T^1) = \text{Tr}(T^2T^1T^3)$$

The $(N-1)!$ subamplitudes are not all independent:

In addition to **cyclic symmetries** by applying

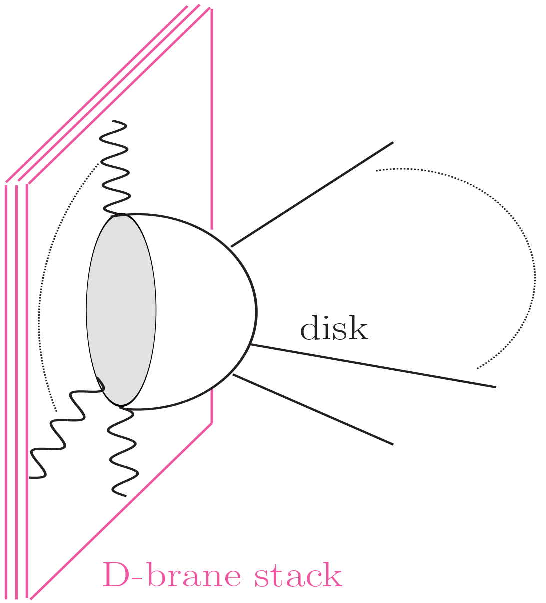
reflection and **parity symmetries**

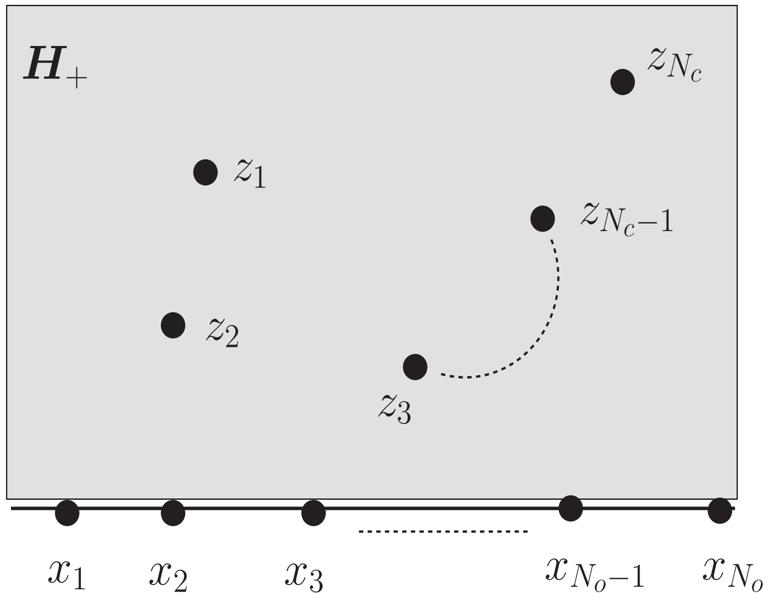
$$\boxed{A(1,2,\ldots,N) = (-1)^N \; A(N,N-1,\ldots,2,1)}$$

reduce the number of independent partial amplitudes
from $(N-1)!$ to $\frac{1}{2}(N-1)!$

(properties of the string world–sheet)

Disk scattering of open and closed strings





$$\mathcal{A} = \sum_{\sigma \in S_{N_o}/\mathbf{Z}_{N_o}} V_{\mathsf{CKG}}^{-1} \left(\int_{\mathcal{I}_\sigma} \prod_{j=1}^{N_o} dx_j \prod_{i=1}^{N_c} \int_{H_+} d^2 z_i \right) \langle \prod_{j=1}^{N_o} : V_o(x_j) : \prod_{i=1}^{N_c} : V_c(\overline{z}_i, z_i) : \rangle$$

$V_o(x_i) =$ open string vertex operators inserted at x_i on the boundary of the disk

$V_c(\overline{z}_i, z_i) =$ closed string vertex operators inserted at z_i inside the disk

Disk scattering of open and closed strings

$$\mathcal{A}(N_o,N_c)=\sum_{\sigma\in S_{N_o}/\mathbf{Z}_{N_o}}\sum_I \mathcal{K}_I\ \mathcal{I}_I^\sigma$$

$\mathcal{I}_I^\sigma =$ complex disk integral involving open and closed positions

<u>*Result:*</u> Integrand looks like a pure open string amplitude

involving N_o+2N_c open strings,

with integrations over N_o+2N_c-3 real positions x_l,ξ_i,η_j

$$\mathcal{A}(N_o,N_c)=\left(\frac{i}{2}\right)^{N_c}\sum_{\Sigma\in\mathcal{P}}\Pi(\Sigma)\ \ A(1,\Sigma(2),\ldots,\Sigma(N_0+2N_c))\ ,$$

Disk scattering of open and closed strings

$$
\begin{aligned}
\mathcal{I}_I^\sigma \;=\; & V_{\mathrm{CKG}}^{-1} \left(\int_{\mathcal{I}_\sigma} \prod_{j=1}^{N_o} dx_j \;\; \prod_{i=1}^{N_c} \int_{\mathbf{C}} d^2 z_i \right) \prod_{j_1<j_2}^{N_o} |x_{j_1}-x_{j_2}|^{4p_{j_1}p_{j_2}} \; (x_{j_1}-x_{j_2})^{n^I_{j_1j_2}} \\
\times \; & \prod_{i=1}^{N_c} |z_i-\overline{z}_i|^{2q^2_{i\|}} \; (z_i-\overline{z}_i)^{r^I_{ii}} \;\; \prod_{j=1}^{N_o}\prod_{i=1}^{N_c} |x_j-z_i|^{4p_jq_i} \; (x_j-z_i)^{m^I_{ij}} \; (x_j-\overline{z}_i)^{\overline{m}^I_{ij}} \\
\times \; & \prod_{i_1<i_2}^{N_c} |z_{i_1}-z_{i_2}|^{2q_{i_1}q_{i_2}} \; |z_{i_1}-\overline{z}_{i_2}|^{2q_{i_1}Dq_{i_2}} \\
\times \; & (z_{i_1}-z_{i_2})^{r^I_{i_1i_2}} \;\; (\overline{z}_{i_1}-\overline{z}_{i_2})^{\overline{r}^I_{i_1i_2}} \; (z_{i_1}-\overline{z}_{i_2})^{\tilde{r}^I_{i_1i_2}} \; (\overline{z}_{i_1}-z_{i_2})^{\overline{\tilde{r}}^I_{i_1i_2}}
\end{aligned}
$$

with some integers $n^I_{ij}, m^I_{ij}, \overline{m}^I_{ij}, r^I_{ij}, \tilde{r}^I_{ij}, \overline{r}^I_{ij}, \overline{\tilde{r}}^I_{ij} \in \mathbf{Z}$ referring to the kinematical factor $\mathcal{K}_I$.
Integers and the kinematical factor $\mathcal{K}_I$ do not depend on the ordering σ.

Disk scattering of open and closed strings

Split complex integral into holomorphic and anti–holomorphic pieces:

$$\xi_i = z_{1i} + i\ z_{2i} \equiv z_i$$
$$\eta_j = z_{1j} - i\ z_{2j} \equiv \overline{z}_j\ ,\ \ i,j = 1,\ldots,N_c$$

integrand becomes an analytic function in ξ_i, η_j

$$\Pi(x_l,\xi_i,\eta_j) \ = \ e^{2\pi i p_l q_i \theta[-(x_l-\xi_i)(x_l-\eta_i)]}\ e^{i\pi q_i q_j \theta[-(\xi_i-\xi_j)(\eta_i-\eta_j)]}$$
$$\times \ \ e^{i\pi q_i D q_j \theta[-(\xi_i-\eta_j)(\eta_i-\xi_j)]}\ e^{2\pi i q_{i\|}^2 \theta(\eta_i-\xi_i)}$$

Phase factor $\Pi(x_l,\xi_i,\eta_j)$ accounts for correct branch of the integrand

The factor $\Pi(\Sigma)$ is the phase factor from for the insertions x_l, ξ_i, η_j with the ordering Σ of the $N_o + 2N_c$ open string positions.

disentangle

$$\Pi(\xi,\eta,\rho) = \begin{cases} e^{i\pi\gamma_2} & , \quad (\xi-\rho)\ (\eta+\rho) < 0\ , \\ e^{i\pi\beta_2} & , \quad (\xi+\rho)\ (\eta-\rho) < 0\ , \\ e^{i\pi\lambda_2} & , \quad (1-\xi)\ (1-\eta) < 0\ , \\ e^{i\pi\alpha_2} & , \quad |x| > 1 \end{cases}$$

$$\begin{aligned}
\mathcal{A}(1,2;3,4) \;=\;& [\ A(1,6,5,3,4,2) + A(1,5,6,3,4,2)\ ] + e^{i\pi\gamma_2}\ A(1,5,3,6,4,2) \\
+\;& e^{i\pi\gamma_2}\ e^{i\pi\beta_2}\ A(1,5,3,4,6,2) + e^{i\pi\lambda_2}\ e^{i\pi\gamma_2} e^{i\pi\beta_2}\ A(1,5,3,4,2,6) + e^{i\pi\beta_2}\ A(1,6,3,5,4,2) \\
+\;& e^{i\pi\gamma_2} e^{i\pi\beta_2}\ [\ A(1,3,6,5,4,2) + A(1,3,5,6,4,2)\ ] + e^{i\pi\gamma_2}\ A(1,3,5,4,6,2) \\
+\;& e^{i\pi\lambda_2} e^{i\pi\gamma_2}\ A(1,3,5,4,2,6) + e^{i\pi\gamma_2} e^{i\pi\beta_2}\ A(1,6,3,4,5,2) + e^{i\pi\beta_2}\ A(1,3,6,4,5,2) \\
+\;& [\ A(1,3,4,6,5,2) + A(1,3,4,5,6,2)\ ] + e^{i\pi\lambda_2}\ A(1,3,4,5,2,6) \\
+\;& e^{i\pi\lambda_2} e^{i\pi\beta_2} e^{i\pi\gamma_2}\ A(1,6,3,4,2,5) + e^{i\pi\lambda_2} e^{i\pi\beta_2}\ A(1,3,6,4,2,5) + e^{i\pi\lambda_2}\ A(1,3,4,6,2,5) \\
+\;& [\ A(1,3,4,2,6,5) + A(1,3,4,2,5,6)\ ] + [\ A(1,6,5,4,3,2) + A(1,5,6,4,3,2)\ ] \\
+\;& e^{i\pi\beta_2}\ A(1,5,4,6,3,2) + e^{i\pi\gamma_2} e^{i\pi\beta_2}\ A(1,5,4,3,6,2) + e^{i\pi\lambda_2} e^{i\pi\gamma_2} e^{i\pi\beta_2}\ A(1,5,4,3,2,6) \\
+\;& e^{i\pi\gamma_2}\ A(1,6,4,5,3,2) + e^{i\pi\gamma_2} e^{i\pi\beta_2}\ [\ A(1,4,6,5,3,2) + A(1,4,5,6,3,2)\ ] \\
+\;& e^{i\pi\beta_2}\ A(1,4,5,3,6,2) + e^{i\pi\lambda_2} e^{i\pi\beta_2}\ A(1,4,5,3,2,6) + e^{i\pi\gamma_2} e^{i\pi\beta_2}\ A(1,6,4,3,5,2) \\
+\;& e^{i\pi\gamma_2}\ A(1,4,6,3,5,2) + [A(1,4,3,6,5,2) + A(1,4,3,5,6,2)] \\
+\;& e^{i\pi\lambda_2}\ A(1,4,3,5,2,6) + e^{i\pi\lambda_2} e^{i\pi\beta_2} e^{i\pi\gamma_2}\ A(1,6,4,3,2,5) + e^{i\pi\lambda_2} e^{i\pi\gamma_2}\ A(1,4,6,3,2,5) \\
+\;& e^{i\pi\lambda_2}\ A(1,4,3,6,2,5) + [\ A(1,4,3,2,6,5) + A(1,4,3,2,5,6)\ ] \\
+\;& e^{i\pi\alpha_2}\ \{\ [\ A(1,6,5,3,2,4) + A(1,5,6,3,2,4)\ ] + e^{i\pi\gamma_2}\ A(1,5,3,6,2,4) \\
+\;& e^{i\pi\lambda_2} e^{i\pi\gamma_2}\ A(1,5,3,2,6,4) + e^{i\pi\lambda_2} e^{i\pi\gamma_2} e^{i\pi\beta_2}\ A(1,5,3,2,4,6) + e^{i\pi\beta_2}\ A(1,6,3,5,2,4) \\
+\;& e^{i\pi\beta_2} e^{i\pi\gamma_2}\ [\ A(1,3,6,5,2,4) + A(1,3,5,6,2,4)\ ] + e^{i\pi\lambda_2} e^{i\pi\gamma_2} e^{i\pi\beta_2}\ A(1,3,5,2,6,4) \\
+\;& e^{i\pi\lambda_2} e^{i\pi\gamma_2}\ A(1,3,5,2,4,6) + e^{i\pi\lambda_2} e^{i\pi\beta_2}\ A(1,6,3,2,5,4) + e^{i\pi\lambda_2} e^{i\pi\gamma_2} e^{i\pi\beta_2}\ A(1,3,6,2,5,4) \\
+\;& e^{i\pi\beta_2} e^{i\pi\gamma_2}\ [A(1,3,2,6,5,4) + A(1,3,2,5,6,4)] + e^{i\pi\gamma_2}\ A(1,3,2,5,4,6) \\
+\;& e^{i\pi\lambda_2} e^{i\pi\gamma_2} e^{i\pi\beta_2}\ A(1,6,3,2,4,5) + e^{i\pi\lambda_2} e^{i\pi\beta_2}\ A(1,3,6,2,4,5) + e^{i\pi\beta_2}\ A(1,3,2,6,4,5) \\
+\;& [\ A(1,3,2,4,6,5) + A(1,3,2,4,5,6)\ ]\ \}
\end{aligned}$$

Disk scattering of open and closed strings

$\Longrightarrow$ Basic ingredients of open & closed disk amplitude:

(color) ordered open string amplitudes $A(1,\ldots,N)$

<u>After inspecting phase $\Pi(x_l,\xi_i,\eta_j)$:</u>

- many different contributions (open string orderings) $A(a,b,c,d,e,f)$
- many striking relations

Open string disk amplitudes

Recall: *The full open string tree–level N–gluon amplitude $\mathcal{A}$:*

$$\mathcal{A}(1,2,\ldots,N) = g_{YM}^{N-2} \sum_{\sigma \in S_{N-1}} \mathrm{Tr}(T^{a_1} T^{a_{\sigma(2)}} \ldots T^{a_{\sigma(N)}})\ A(1,\sigma(2),\ldots,\sigma(N))$$

runs over $\frac{1}{2}(N-1)!$ *subamplitudes.*

Field–theory $D = 4$

Moreover in $D = 4$ FT further relations found by:

- Kleiss, Kuijf, 1989 $(N-2)!$
  Del Duca, Dixon, Maltoni, 2000
- Bern, Carrasco, Johannson, 2008 $(N-3)!$

E.g.: Subcyclic property (photon-decoupling identity: $T^{a_1} \to 1$)

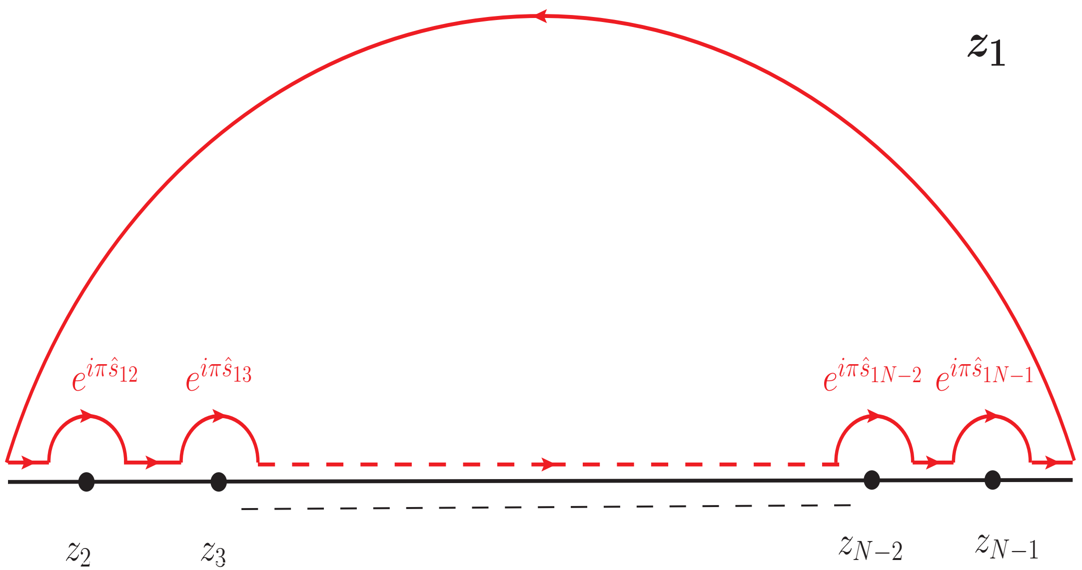
$$\sum_{\sigma \in S_{N-1}} A_{FT}(1, \sigma(2), \sigma(3), \ldots, \sigma(N)) = 0$$

In STTH these relations **do not** hold beyond FT order !

World–sheet derivation of amplitude relations

By applying world–sheet string techniques $\Longrightarrow$ new algebraic identities

$$A(1,\ldots,N) = V_{\rm CKG}^{-1} \int\limits_{z_1<\ldots<z_N} \left(\prod_{j=1}^N \; dz_j \right) \sum_{\mathcal{K}_I} \mathcal{K}_I \prod_{i<j}^N |z_i - z_j|^{s_{ij}} \; (z_i - z_j)^{n_{ij}^I}$$



by analytically continuing
the z_1–integration to
the whole complex plane
and integrating z_1
along the contour integral

$$A(1,2,\ldots,N) \;\; + e^{i\pi s_{12}} \; A(2,1,3,\ldots,N-1,N) + e^{i\pi(s_{12}+s_{13})} \; A(2,3,1,\ldots,N-1,N)$$

$$+ \ldots + e^{i\pi(s_{12}+s_{13}+\ldots+s_{1N-1})} \; A(2,3,\ldots,N-1,1,N) = 0$$

World–sheet derivation of amplitude relations

- proof does not rely on any kinematic properties of the subamplitudes
- for any open string state: boson or fermion
- these relations hold in any space–time dimensions D
- for any amount of supersymmetry

World–sheet derivation of amplitude relations

E.g. $N=4$:

$$\frac{A(1,2,4,3)}{A(1,2,3,4)}=\frac{\sin(\pi u)}{\sin(\pi t)}\quad,\quad\frac{A(1,3,2,4)}{A(1,2,3,4)}=\frac{\sin(\pi s)}{\sin(\pi t)}$$

As a result these relations allow to express all six partial amplitudes in terms of **one**, say $A(1,2,3,4)$:

$$\begin{aligned}A(1,4,3,2)&=A(1,2,3,4)\ ,\\ A(1,2,4,3)&=A(1,3,4,2)=\frac{\sin(\pi u)}{\sin(\pi t)}\,A(1,2,3,4)\ ,\\ A(1,3,2,4)&=A(1,4,2,3)=\frac{\sin(\pi s)}{\sin(\pi t)}\,A(1,2,3,4)\ .\end{aligned}$$

Clearly, in the field–theory limit the relations simply reduce to the well–known identities:

$$\frac{A_{FT}(1,2,4,3)}{A_{FT}(1,2,3,4)}=\frac{u}{t}\quad,\quad\frac{A_{FT}(1,3,2,4)}{A_{FT}(1,2,3,4)}=\frac{s}{t}$$

Subcyclic property $A_{FT}(1,2,3,4)+A_{FT}(1,3,4,2)+A_{FT}(1,4,2,3)=0$

E.g. $N = 5$: Relations:

$$\sin[\pi(s_2 - s_4)]\ A(1,2,3,4,5) + \{\sin[\pi(s_1 + s_2 - s_4)] - \sin(\pi s_1)\}\ A(1,3,4,5,2)$$
$$+\ \sin[\pi(s_2 - s_4)]\ A(1,4,5,2,3) + \{\sin(\pi s_5) + \sin[\pi(s_2 - s_4 - s_5)]\}\ A(1,5,2,3,4) = 0$$

$$[\sin(\pi s_1) + \sin(\pi s_5)]\ A(1,2,3,4,5) + \sin[\pi(s_1 + s_5)]\ A(1,3,4,5,2)$$
$$+\ \{\sin[\pi(s_1 + s_2 - s_4)] - \sin[\pi(s_2 - s_4 - s_5)]\}\ A(1,4,5,2,3) + \sin[\pi(s_1 + s_5)]\ A(1,5,2,3,4) = 0$$

As a result these relations allow to express all six partial amplitudes in terms of **two**, say $A(1,2,3,4,5)$ and $A(1,3,2,4,5)$, e.g.:

$$\begin{aligned} A(1,2,5,4,3) &= -A(1,3,4,5,2) = \sin[\pi(s_3 - s_1 - s_5)]^{-1} \\ &\times \{\ \sin[\pi(s_3 - s_5)]\ A(1,2,3,4,5) + \sin[\pi(s_2 + s_3 - s_5)]\ A(1,3,2,4,5)\ \}\ , \\ A(1,3,4,2,5) &= -A(1,5,2,4,3) = \sin[\pi(s_3 - s_1 - s_5)]^{-1} \\ &\times \{\ \sin(\pi s_1)\ A(1,2,3,4,5) - \sin[\pi(s_1 + s_2)]\ A(1,3,2,4,5)\ \}\ ,\ \ldots \end{aligned}$$

Clearly, in the field theory limit, these two relations boil down to the subcyclic identity:

$$A_{FT}(1,2,3,4,5) + A_{FT}(1,3,4,5,2) + A_{FT}(1,4,5,2,3) + A_{FT}(1,5,2,3,4) = 0.$$

World–sheet derivation of amplitude relations

- These relations allow for a **complete reduction** of the full string subamplitudes to a **minimal basis of** $(N-3)!$ **subamplitudes** just like in field–theory
- **Reproduce** Kleiss–Kuijf and Bern–Carrasco–Johanson identitities in field–theory limit

$\Longrightarrow$ Basic ingredients of **open & closed** disk amplitude are $(N-3)!$ (color) ordered **open** string amplitudes $A(1,\ldots,N)$

Two open & two closed strings versus six open strings on the disk

After inspecting phase $\Pi(\rho,\xi,\eta)$:

- many different contributions (open string orderings) $A(a,b,c,d,e,f)$
- many striking relations:

$$\begin{aligned}
A(1,5,3,6,4,2) &= A(1,2,3,5,4,6)\ ,\\
A(1,5,4,6,3,2) &= A(1,2,4,5,3,6)\ ,\\
A(1,2,3,6,5,4) &= \frac{\cos\left[\frac{\pi}{2}(s+t)\right]}{\sin\left(\frac{\pi\ t}{2}\right)}\ A(1,2,3,4,6,5) + \frac{\cos\left[\frac{\pi}{2}(s+t)\right]}{\sin\left(\frac{\pi\ t}{2}\right)}\ A(1,2,4,3,6,5)\\
&+ \frac{\cos\left[\frac{\pi}{4}(s+2t)\right]}{\sin\left(\frac{\pi t}{2}\right)}\ A(1,2,4,5,3,6)
\end{aligned}$$

$$\boxed{\begin{aligned}
\mathcal{A}(1,2;3,4) &= \sin\left(\frac{\pi s}{2}\right)\ \sin(\pi s)\ A(1,6,3,5,4,2)\\
&- \sin\left(\frac{\pi s}{2}\right)\ \sin(\pi t)\ A(1,3,5,4,2,6)\ ,
\end{aligned}}$$

Open & closed vs. pure open string disk amplitude

General:

Disk amplitude involving N_o open and N_c closed strings is mapped to disk amplitudes of $N_o + 2N_c$ open strings

E.g.:

$N_o = 2,\ N_c = 1 \implies$ four open strings

$N_o = 3,\ N_c = 1 \implies$ five open strings

$N_0 = 4,\ N_c = 1\ ,\ N_o = 2,\ N_c = 2 \implies$ six open strings

$\vdots \qquad \vdots$

Open string subamplitude relations $\Longleftrightarrow$ Analysis of contour integrals in the complex plane yield $(N_0 + 2N_c - 3)!$ – dimensional basis of functions

Open & closed vs. pure open string disk amplitude

Examples:

$$\begin{aligned}
\mathcal{A}(1,2) &= A(1,2,3,4)\ , \\
\mathcal{A}(1,2;3) &= \sin(\pi t)\ A(1,2,3,4)\ , \\
\mathcal{A}(1,2,3;4) &= \sin(\pi t)\ A(1,5,2,4,3)\ , \\
\mathcal{A}(1,2;3,4) &= \sin\left(\frac{\pi s}{2}\right)\ \sin(\pi s)\ A(1,6,3,5,4,2) \\
&- \sin\left(\frac{\pi s}{2}\right)\ \sin(\pi t)\ A(1,3,5,4,2,6)\ , \\
\mathcal{A}(1,2,3,4;5) &= \sin(\pi s_4)\ A(1,6,4,5,3,2) \\
&- \sin\pi\left(\frac{s_1}{2}-\frac{s_3}{2}+s_5\right)\ A(1,4,3,5,2,6)\ , \\
&\ldots\ .
\end{aligned}$$

Open & closed vs. pure open string disk amplitudes

This map reveals

important relations between

open & closed string disk amplitudes

and pure open string disk amplitudes !

Open & closed vs. pure open string disk amplitudes

Two vectors and two massless Neveu–Schwarz closed string states:

$$\langle A_{\mu_1}(x_1)\ A_{\mu_2}(x_2)\ G_{\mu_3\mu_4}(\overline{z}_1, z_1)\ G_{\mu_5\mu_6}(\overline{z}_2, z_2)\rangle\ \zeta^{\mu_1}\ \zeta^{\mu_2}\ \epsilon_1^{\mu_3\mu_4}\ \epsilon_2^{\mu_5\mu_6}$$

$$\simeq \langle A_{\mu_1}(x_1)\ A_{\mu_2}(x_2)\ A_{\mu_3}(x_3)\ A_{\mu_4}(x_4)\ A_{\mu_5}(x_5)\ A_{\mu_6}(x_6)\rangle\ \xi^{\mu_1}\xi^{\mu_2}\xi^{\mu_3}\xi^{\mu_4}\xi^{\mu_5}\xi^{\mu_6}$$

with the identifications and assignment of momenta

$$\xi_1 = \zeta_1 \quad , \quad \xi_2 = \zeta_2 \quad , \quad \xi_3 \otimes \xi_4 = \epsilon_1 \quad , \quad \xi_5 \otimes \xi_6 = \epsilon_2$$

Two vectors and two massless Ramond $p-$ and $q-$forms

$$\langle A_{\mu_1}(x_1)\ A_{\mu_2}(x_2)\ F_{\alpha\beta}(\overline{z}_1, z_1)\ F_{\gamma\delta}(\overline{z}_2, z_2)\rangle$$

$$\times \zeta^{\mu_1}\ \zeta^{\mu_2}\ f^1_{\mu_0\ldots\mu_p}\ \left(P_-\Gamma^{[\mu_0}\ldots\Gamma^{\mu_p]}\right)^{\alpha\beta}\ f^2_{\nu_0\ldots\nu_q}\ \left(P_-\Gamma^{[\nu_0}\ldots\Gamma^{\nu_q]}\right)^{\gamma\delta}$$

$$\simeq \langle A_{\mu_1}(x_1)\ A_{\mu_2}(x_2)\ \chi_\alpha(x_3)\ \chi_\beta(x_4)\ \chi_\gamma(x_5)\ \chi_\delta(x_6)\rangle\ \xi^{\mu_1}\xi^{\mu_2}\ u^\alpha\ v^\beta\ u^\gamma\ v^\delta$$

Open & closed vs. pure open string disk amplitudes

Sort of generalized KLT on the disk

$$V_{\text{closed}}(\overline{z}_i, z_i) \;\simeq V_{\text{open}}(\overline{z}_i)\; V_{\text{open}}(z_i) \simeq \; V_{\text{open}}(\eta_i)\; V_{\text{open}}(\xi_i)$$

$$z_i \in \mathbf{C} \qquad\qquad \eta_i,\ \xi_i \in \mathbf{R}$$

Recall KLT:

$$\begin{aligned} M_4(1,2,3,4)_{S^2} &= -i\ \sin(\alpha' s_{12})\ A_4(1,2,3,4)_{D_2}\ A_4(1,2,4,3)_{D_2} \\ M_5(1,2,3,4,5)_{S^2} &= i\ \sin(\alpha' s_{12})\ \sin(\alpha' s_{34})\ A_5(1,2,3,4,5)_{D_2}\ A_5(2,1,4,3,5)_{D_2} \\ &+ \sin(\alpha' s_{13})\ \sin(\alpha' s_{24})\ A_5(1,3,2,4,5)_{D_2}\ A_5(3,1,4,2,5)_{D_2} \\ &\ \vdots \end{aligned}$$