

Aspects of Higgs Boson Physics beyond the SM in the Linear Colliders

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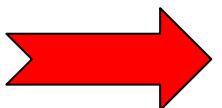
HEP Group
Departament d'Estructura i Constituents de la Matèria
Institut de Ciències del Cosmos, Univ. Barcelona

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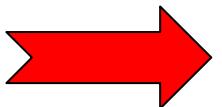
Guidelines of the Talk



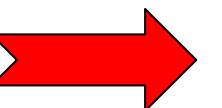
SM, Supersymmetry and the MSSM



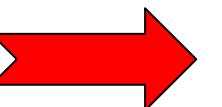
Higgs bosons in the MSSM and the 2HDM



Higgs boson self interactions



**Quantum effects in $e^+e^- \rightarrow$ Higgs bosons
a window to 2HDM physics?**



Conclusions

Works and collaborators:

Recent works:

- D.López-Val, JS, Neutral Higgs-pair production at Linear Colliders within the general 2HDM: quantum effects and triple Higgs boson self-interactions, (subm. to Phys. Rev. D) arXiv:0908.2898 [hep-ph]
- N. Bernal, D.López-Val, JS Single Higgs-boson production through $\gamma\gamma$ -scattering within the general 2HDM, Phys. Lett. B677 (2009) 39, arXiv:0901.2257 [hep-ph]
- R. N. Hodgkinson, D.López-Val, JS Higgs boson pair production through gauge boson fusion at linear colliders within the general 2HDM., Phys. Lett. B673 (2009) 47, arXiv:0903.4978 [hep-ph]
- G. Ferrera, J. Guasch, D.López-Val, JS Triple Higgs boson production in the Linear Collider., Phys.Lett. B659 (2008) 297, arXiv:0801.3907 [hep-ph].

Standard Model

principle of local gauge invariance



symmetry group $SU(2) \times U(1) \times SU(3)_C$



Higgs mechanism and Yukawa interactions
→ masses $M_W, M_Z, m_{\text{fermion}}$

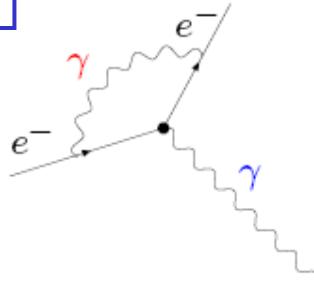
SM { renormalizable quantum field theory
 ↓
 accurate theoretical predictions

detect deviations → “new physics” ?

Precision Physics in the SM

QED:

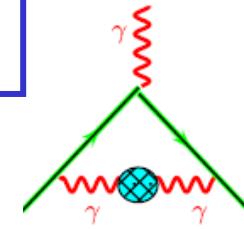
g - 2:



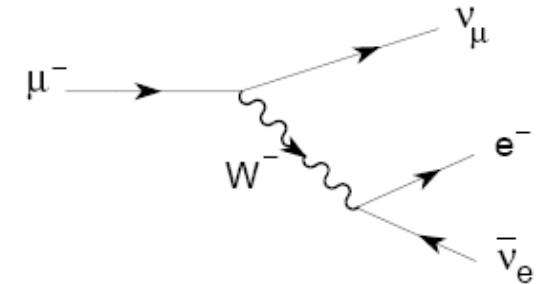
$$a = \frac{1}{2}(g - 2)$$

$$a_{\text{exp}} = 1159\,652\,188(\pm 4) \times 10^{-12}$$

$$a_{\text{theo}} = 1159\,652\,157(\pm 28) \times 10^{-12}$$

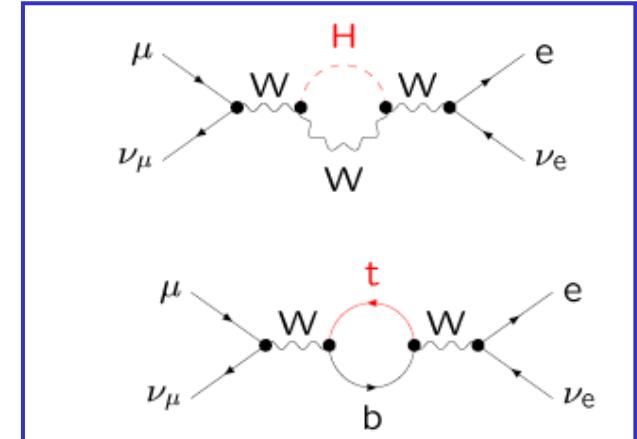


EW: $\left\{ \begin{array}{l} G_F \\ M_Z, \Gamma_Z, g_V, g_A, \sin^2 \theta_{\text{eff}}, \dots \\ M_W, m_t \end{array} \right.$



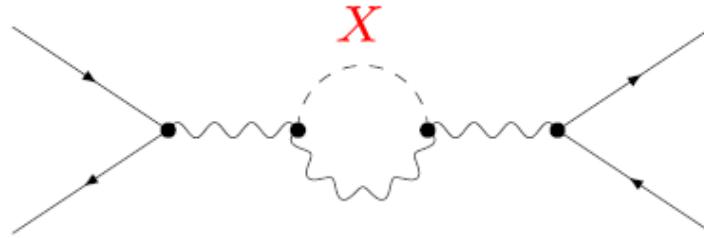
$$\frac{G_F}{\sqrt{2}} = \frac{\pi \alpha}{M_W^2 (1 - M_W^2/M_Z^2)} (1 + \Delta r)$$

$$\Delta r = \Delta r(m_t, M_H) \quad \rightarrow$$



Precision Physics Beyond the SM

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2 (1 - M_W^2/M_Z^2)} (1 + \Delta r)$$



X = Higgs bosons, SUSY particles

$$\Delta r = \Delta r (M_W, m_t, M_X)$$

↑
new physics!

First calculations in SUSY:

- J. Grifols, J. Solà, Phys.Lett.B137:257,1984.
J. Grifols, J. Solà, Nucl.Phys.B253:47,1985.
D. Garcia, J. Solà, Mod.Phys.Lett.A9:211-224,1994.
P.Chankowski, A. Dabelstein, W. Hollik, W. Mosle, S. Pokorski, J. Rosiek, Nucl.Phys.B417:101-129,1994.

State of the art in SUSY:

- S. Heinemeyer, W. Hollik, D. Stockinger, A.M. Weber, G. Weiglein, JHEP 0608:052,2006.

Theory versus Data

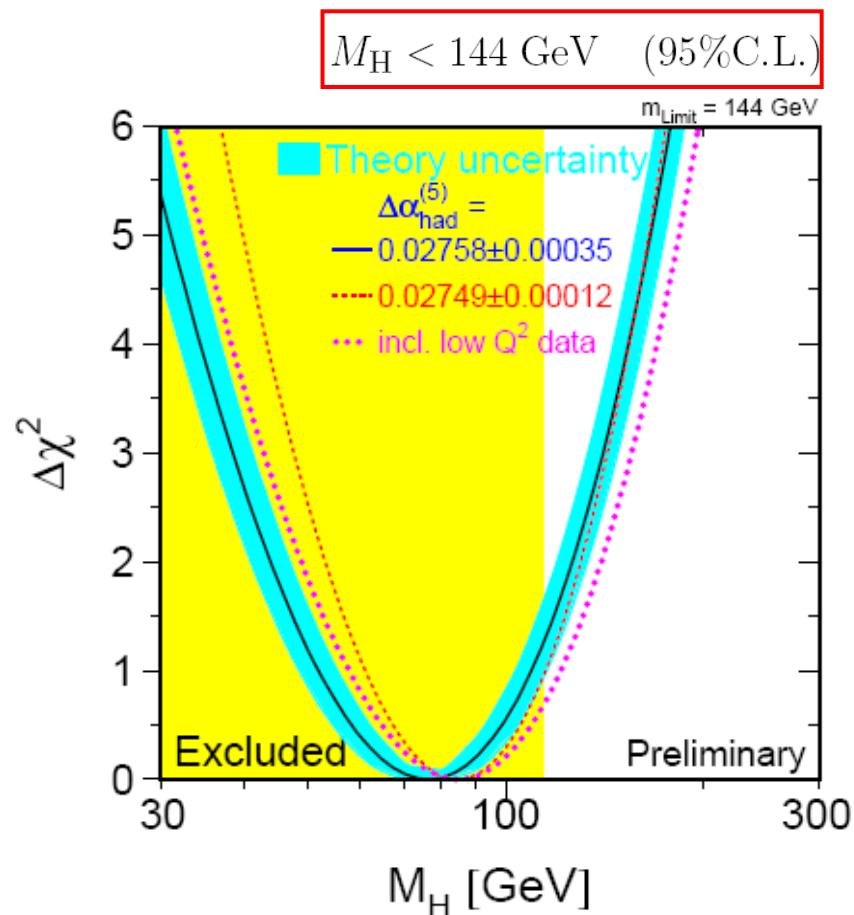
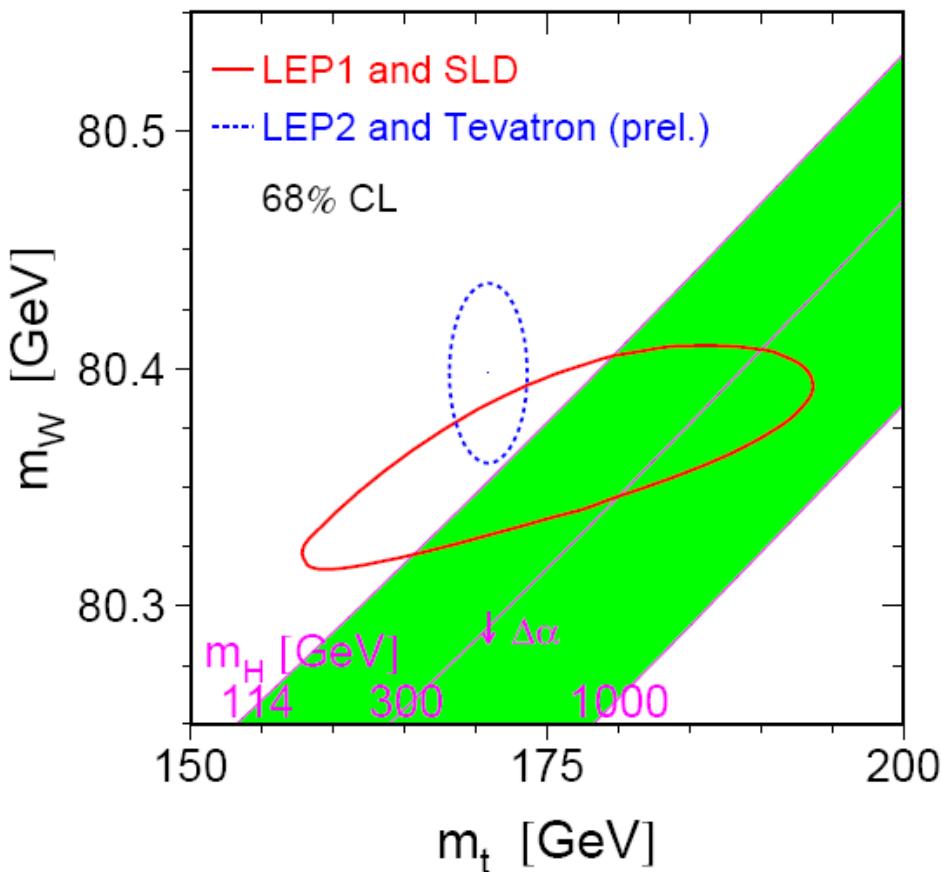
experimental results

New physics?...

M_Z [GeV]	$= 91.1875 \pm 0.0021$	0.002%
Γ_Z [GeV]	$= 2.4952 \pm 0.0023$	0.09%
$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	$= 0.23148 \pm 0.00017$	0.07%
M_W [GeV]	$= 80.392 \pm 0.029$	0.04%
m_t [GeV]	$= 170.9 \pm 1.8$	1.05%
G_F [GeV^{-2}]	$= 1.16637(1)10^{-5}$	0.001%

quantum effects at least one order of magnitude larger than experimental uncertainties

LEP Electroweak Working Group



Direct exp. Search: $M_H > 114$ GeV



$M_H < 182$ GeV (95% C.L.)

SUSY: Supersymmetry

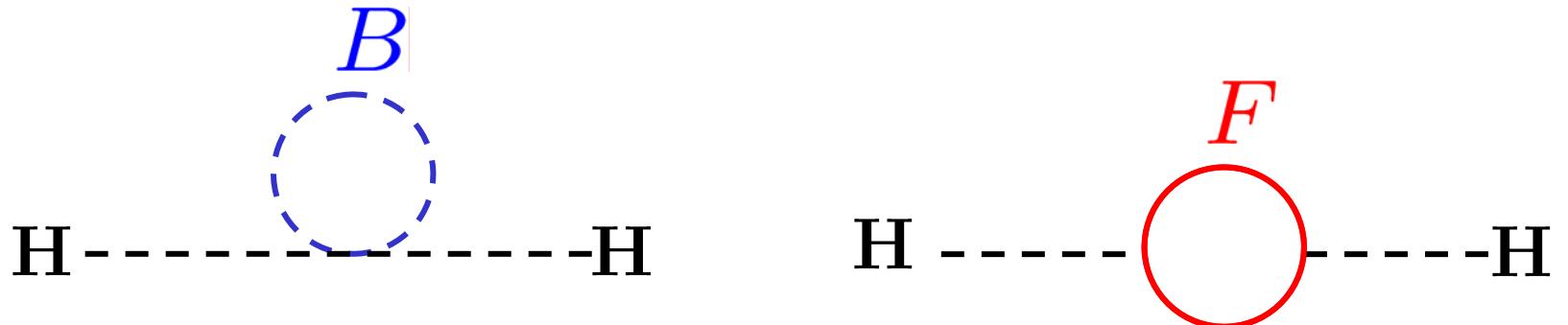
$$Q^{(S=\frac{1}{2})} |Fermion\rangle = |Boson\rangle \quad Q^{(S=\frac{1}{2})} |Boson\rangle = |Fermion\rangle$$

$$\{Q_\alpha^A, \bar{Q}_{\dot{\beta}B}\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu \delta_B^A \quad (\textbf{GUT+gravity!})$$

$\psi_i (S = \frac{1}{2})$	\longleftrightarrow	$\phi_i (S = 0)$
fermion		sfermion
quark (u, c, t)	\longleftrightarrow	squark ($\tilde{u}, \tilde{c}, \tilde{t}$)

$v_i (S = 1)$	\longleftrightarrow	$\lambda_i (S = \frac{1}{2})$
gauge boson		gaugino
g, γ	\longleftrightarrow	$\tilde{g}, \tilde{\gamma}$

SUSY and the Hierarchy Problem



$$\delta M_H^2 \sim \frac{g^2}{16\pi^2} \left[\int_{m_B^2}^{\Lambda^2} dk^2 - \int_{m_F^2}^{\Lambda^2} dk^2 \right]$$

$$\sim \frac{g^2}{16\pi^2} \int_{m_B^2}^{m_F^2} dk^2 \sim \frac{g^2}{16\pi^2} |m_F^2 - m_B^2|$$

$$\delta M_H^2 \lesssim M_H^2 \lesssim 1 \text{ TeV}^2$$

$M_{\text{SUSY}} \lesssim (1 - 10) \text{ TeV}$

LHC ??

MSSM:

Minimal Supersymmetric SM

	fields			gauge group		
	superfield	fermion	boson	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Matter sector	<i>sfermions</i>			$(Q = T_3 + \frac{Y}{2})$		
Quarks	\tilde{Q}_i	$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L$	$\begin{pmatrix} \tilde{u}_i \\ \tilde{d}_i \end{pmatrix}_L$	3	2	$\frac{1}{3}$
Squarks	\hat{U}_i \hat{D}_i	$u_{i\,R}^c$ $d_{i\,R}^c$	$\tilde{u}_{i\,R}^*$ $\tilde{d}_{i\,R}^*$	$\bar{3}$	1	$-\frac{4}{3}$
Leptons	\hat{L}_i	$\begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L$	$\begin{pmatrix} \tilde{\nu}_i \\ \tilde{e}_i \end{pmatrix}_L$	1	2	-1
Sleptons	\hat{E}_i	$e_{i\,R}^c$	$\tilde{e}_{i\,R}^*$	1	1	2
Gauge sector	<i>gauginos</i>			<i>gauge</i>		
$SU(3)_C$	\hat{G}^a	$\tilde{\lambda}_g^a$	g_μ^a	8	1	0
$SU(2)_L$	\hat{W}^i	$\tilde{\lambda}_W^i$	W_μ^i	1	3	0
$U(1)_Y$	\hat{B}	$\tilde{\lambda}_B$	B_μ	1	1	0

Higgs bosons and Higgsinos

	fields		gauge group		
superfield	fermion	scalar	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Higgs Sector	Higgsino	Higgs doublets			
\hat{H}_1	$\begin{pmatrix} \tilde{H}_1^1 \\ \tilde{H}_1^2 \end{pmatrix}$	$\begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix}$	1	2	-1
\hat{H}_2	$\begin{pmatrix} \tilde{H}_2^1 \\ \tilde{H}_2^2 \end{pmatrix}$	$\begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix}$	1	2	1

As in the SM 

$$H_2 = \begin{pmatrix} H_2^+ \\ v_2 + H_2^0 \end{pmatrix}, \quad H_1 = \begin{pmatrix} v_1 + H_1^0 \\ H_1^- \end{pmatrix} \quad (Q = T_3 + \frac{Y}{2})$$

couples to u

couples to d

$$\tan \beta = \frac{v_2}{v_1}$$

➤ Higgs bosons in the general 2HDM

- It is the minimal extension of the SM which adds new phenomena – e.g. H^\pm
- It can easily embody new sources of CP violation
- It provides a useful setup for building models of Baryogenesis
- Extended Higgs sectors based on $SU_L(2)$ -Doublet structure easily circumvent the major phenomenological constraints.
-
-
-
- A Two-Higgs Doublet structure arises naturally as a low-energy realization of some more fundamental theories (e.g. in SUSY).

Higgs bosons in the **MSSM** and the **2HDM**

2HDM \mathcal{CP} -conserving, gauge invariant, renormalizable potential

$$V(\Phi_1, \Phi_2) = \lambda_1(\Phi_1^\dagger \Phi_1 - v_1^2)^2 + \lambda_2(\Phi_2^\dagger \Phi_2 - v_2^2)^2 + \lambda_3 [(\Phi_1^\dagger \Phi_1 - v_1^2) + (\Phi_2^\dagger \Phi_2 - v_2^2)]^2 \\ + \lambda_4 [(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) - (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)] + \lambda_5 [Re(\Phi_1^\dagger \Phi_2) - v_1 v_2]^2 + \lambda_6 [Im(\Phi_1^\dagger \Phi_2)]^2$$

$$\Phi_1 = \begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix} \quad (Y = +1), \quad \Phi_2 = \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix} \quad (Y = +1)$$

MSSM

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \equiv \epsilon \Phi_1^* = \begin{pmatrix} \Phi_1^{0*} \\ -\Phi_1^- \end{pmatrix} \quad (Y = -1) \quad \epsilon = i \sigma_2$$

MSSM with soft-breaking terms. **SUSY** highly restricts the potential:

$$V_H = (|\mu|^2 + m_{H_1}^2)|H_1|^2 + (|\mu|^2 + m_{H_2}^2)|H_2|^2 - \mu B \epsilon_{ij} (H_1^i H_2^j + \text{h.c.}) \\ + \frac{g_2^2 + g_1^2}{8} (|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g_2^2 |H_1^\dagger H_2|^2$$

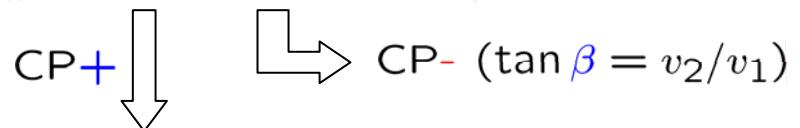
only gauge self-couplings !!

Physical parameters and fields in the 2HDM

λ_i , ($i = 1 \dots 6$) dimensionless real parameters and $v_{1,2} = \langle \Phi_{1,2}^0 \rangle$

$$\left(v_1^2 + v_2^2 = \frac{G_F^{-1}}{2\sqrt{2}} = 2M_W^2/g^2 \right)$$

$$(M_{h^0}, M_{H^0}, M_{A^0}, M_{H^\pm}, \alpha, \tan \beta, \lambda_5)$$



$$\mathcal{M}^{CP+} = \begin{pmatrix} 4v_1^2(\lambda_1 + \lambda_3) + v_2^2\lambda_5 & (4\lambda_3 + \lambda_5)v_1v_2 \\ (4\lambda_3 + \lambda_5)v_1v_2 & 4v_1^2(\lambda_2 + \lambda_3) + v_1^2\lambda_5 \end{pmatrix}$$

CP-even sector $\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \text{Re}(\Phi_1^0) - v_1 \\ \text{Re}(\Phi_2^0) - v_2 \end{pmatrix}$

CP-odd $\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \text{Im}(\Phi_1^0) \\ \text{Im}(\Phi_2^0) \end{pmatrix}$

charged $\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \Phi_1^+ \\ \Phi_2^+ \end{pmatrix}$ G^0, G^\pm
Goldstone bosons

- In the MSSM the CP-even and CP-odd angles are related:

$$\alpha = \frac{1}{2} \arctan \left(\tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} \right), \quad -\frac{\pi}{2} \leq \alpha \leq 0$$

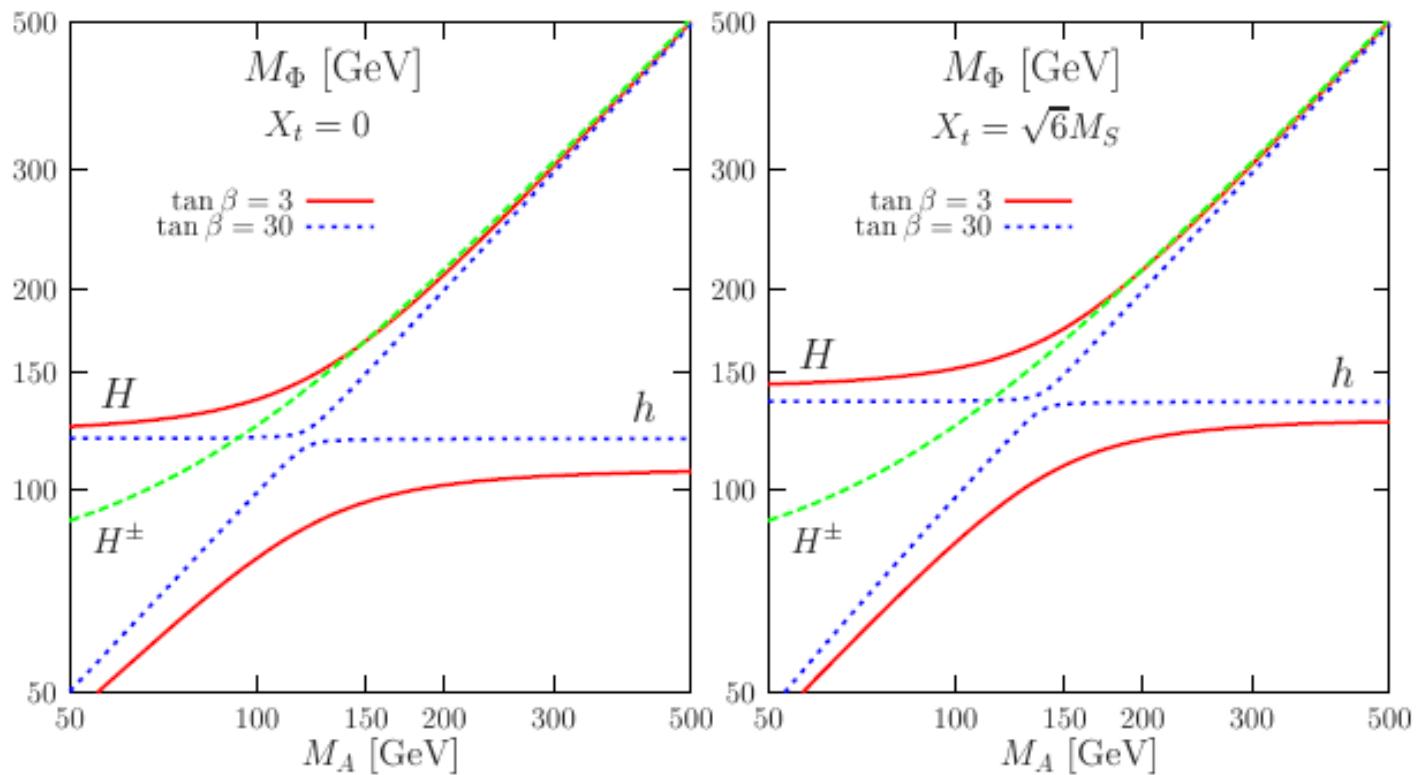
- In the MSSM the only free parameters (at the tree level) are

$$(M_A, \tan \beta)$$

- The **decoupling limit** occurs when $M_A \rightarrow \infty$

In practice, when $M_A \gtrsim 300 \text{ GeV}$ for $\tan \beta < 10$
 and when $M_A \gtrsim M_h^{\max}$ for high $\tan \beta$

M_h^{\max} is the limiting mass of the lightest CP-even state in the MSSM



$$X_t = A_t - \mu / \tan \beta, \quad M_S = (m_{\tilde{t}_1} + m_{\tilde{t}_2})/2$$

$$M_h^{\max} \simeq \begin{cases} 135 \text{ GeV for high } \tan \beta \text{ and maximal mixing} \\ 110 \text{ GeV for low } \tan \beta \text{ and no mixing} \end{cases}$$

➤ Higgs couplings

- Extended Higgs sector \Rightarrow large source of new quantum effects and also of larger Higgs boson production cross-sections
- However, one has to be careful with tree-level FCNC \Rightarrow two models or types of 2HDM: Glashow-Weinberg-Paschos Theorem (1977)
 - In type I 2HDM, Φ_2 couples to all the SM fermions; Φ_1 does not couple to them at all
 - In type II 2HDM, Φ_2 couples only to up-like quarks; Φ_1 couples to down-like only.



Type 2HDM	H^0_{uu}	H^0_{dd}	h^0_{uu}	h^0_{dd}	A^0_{uu}	A^0_{dd}
Model I	$\frac{\sin \alpha}{\sin \beta}$		$\frac{\cos \alpha}{\sin \beta}$		$\pm \frac{1}{\tan \beta}$	
Model II	$\frac{\sin \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\cos \beta}$	$\frac{\cos \alpha}{\sin \beta}$	$-\frac{\sin \alpha}{\cos \beta}$	$\frac{1}{\tan \beta}$	$\tan \beta$

Trilinear Higgs couplings ($h = h^0, H^0, A^0$)

- In the case of the MSSM, and due to the SUSY invariance, the Higgs self-couplings are of pure gauge nature. This is the reason for the tiny triple-Higgs boson production rates obtained within the framework of the MSSM

$$\lambda_1 = \lambda_2 ,$$

$$\lambda_3 = \frac{\pi \alpha_{em}}{2 s_W^2 c_W^2} - \lambda_1 ,$$

$$\lambda_4 = 2\lambda_1 - \frac{2\pi \alpha_{em}}{c_W^2} ,$$

$$\lambda_5 = \lambda_6 = 2\lambda_1 - \frac{2\pi \alpha_{em}}{s_W^2 c_W^2} .$$

- In contrast, the general 2HDM accommodates trilinear Higgs couplings with great potential enhancement.

After **SSB**:

$$(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, v)$$



$$(M_{h^0}, M_{H^0}, M_{A^0}, M_{H^\pm}, \alpha, \tan \beta, \lambda_5)$$

λ_5 is a free parameter
that remains unrelated
to any physical mass or
mixing angle!

$$\begin{aligned} \lambda_1 &= \frac{\lambda_5 (1 - \tan^2 \beta)}{4} + \frac{\alpha_{em} \pi}{2 M_W^2 s_W^2 \cos^2 \beta} \\ &\quad \times [M_{H^0}^2 \cos^2 \alpha + M_{h^0}^2 \sin^2 \alpha \\ &\quad - \frac{1}{2} (M_{H^0}^2 - M_{h^0}^2) \sin 2\alpha \cot \beta], \end{aligned}$$

$$\begin{aligned} \lambda_2 &= \frac{\lambda_5 (1 - 1/\tan^2 \beta)}{4} + \frac{\alpha_{em} \pi}{2 M_W^2 s_W^2 \sin^2 \beta} \\ &\quad \times [M_{h^0}^2 \cos^2 \alpha + M_{H^0}^2 \sin^2 \alpha \\ &\quad - \frac{1}{2} (M_{H^0}^2 - M_{h^0}^2) \sin 2\alpha \tan \beta], \end{aligned}$$

$$\lambda_3 = -\frac{\lambda_5}{4} + \frac{\alpha_{em} \pi \sin 2\alpha}{2 M_W^2 s_W^2 \sin 2\beta} (M_{H^0}^2 - M_{h^0}^2)$$

$$\lambda_4 = \frac{2 \alpha_{em} \pi}{M_W^2 s_W^2} M_{H^\pm}^2,$$

$$\lambda_6 = \frac{2 \alpha_{em} \pi}{M_W^2 s_W^2} M_{A^0}^2,$$

➤ Triple Higgs self-couplings

$h^0 h^0 h^0$	$-\frac{3ie}{2M_W \sin 2\beta s_W} \left[M_{h^0}^2 (2 \cos(\alpha + \beta) + \sin 2\alpha \sin(\beta - \alpha)) - \cos(\alpha + \beta) \cos^2(\beta - \alpha) \frac{4\lambda_5 M_W^2 s_W^2}{e^2} \right]$
$h^0 h^0 H^0$	$-\frac{ie \cos(\beta - \alpha)}{2M_W \sin 2\beta s_W} \left[(2M_{h^0}^2 + M_{H^0}^2) \sin 2\alpha - (3 \sin 2\alpha - \sin 2\beta) \frac{2\lambda_5 M_W^2 s_W^2}{e^2} \right]$
$h^0 H^0 H^0$	$\frac{ie \sin(\beta - \alpha)}{2M_W \sin 2\beta s_W} \left[(M_{h^0}^2 + 2M_{H^0}^2) \sin 2\alpha - (3 \sin 2\alpha + \sin 2\beta) s_W^2 \frac{2\lambda_5 M_W^2}{e^2} \right]$
$H^0 H^0 H^0$	$-\frac{3ie}{2M_W \sin 2\beta s_W} \left[M_{H^0}^2 (2 \sin(\alpha + \beta) - \cos(\beta - \alpha) \sin 2\alpha) - \sin(\alpha + \beta) \sin^2(\beta - \alpha) s_W^2 \frac{4\lambda_5 M_W^2}{e^2} \right]$
$h^0 A^0 A^0$	$-\frac{ie}{2M_W s_W} \left[\frac{\cos(\alpha + \beta)}{\sin 2\beta} \left(2M_{h^0}^2 - \frac{4\lambda_5 M_W^2 s_W^2}{e^2} \right) - \sin(\beta - \alpha) (M_{h^0}^2 - 2M_{A^0}^2) \right]$

➤ ... more trilinear couplings

$h^0 A^0 G^0$	$\frac{ie}{2M_W s_W} (M_{A^0}^2 - M_{h^0}^2) \cos(\beta - \alpha)$
$h^0 G^0 G^0$	$-\frac{ie}{2M_W s_W} M_{h^0}^2 \sin(\beta - \alpha)$
$H^0 A^0 A^0$	$-\frac{ie}{2M_W s_W} \left[\frac{\sin(\alpha+\beta)}{\sin 2\beta} \left(2M_{H^0}^2 - \frac{4\lambda_5 M_W^2 s_W^2}{e^2} \right) - \cos(\beta - \alpha) (M_{H^0}^2 - 2M_{A^0}^2) \right]$
$H^0 A^0 G^0$	$-\frac{ie}{2M_W s_W} (M_{A^0}^2 - M_{H^0}^2) \sin(\beta - \alpha)$
$H^0 G^0 G^0$	$-\frac{ie}{2M_W s_W} M_{H^0}^2 \cos(\beta - \alpha)$
$h^0 H^+ H^-$	$-\frac{ie}{2M_W s_W} \left[\frac{\cos(\alpha+\beta)}{\sin 2\beta} \left(2M_{h^0}^2 - \frac{4\lambda_5 M_W^2 s_W^2}{e^2} \right) - (M_{h^0}^2 - 2M_{H^-}^2) \sin(\beta - \alpha) \right]$
$H^0 H^+ H^-$	$-\frac{ie}{2M_W s_W} \left[\frac{\sin(\alpha+\beta)}{\sin 2\beta} \left(2M_{H^0}^2 - \frac{4\lambda_5 M_W^2 s_W^2}{e^2} \right) - \cos(\beta - \alpha) (M_{H^0}^2 - 2M_{H^-}^2) \right]$

➤ Decoupling limit

It corresponds to $\alpha \rightarrow \beta - \frac{\pi}{2}$ and with all masses much larger than M_{h^0}

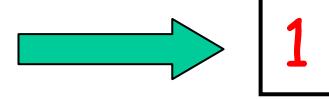
(In the particular case of the MSSM, this limit is correlated with $M_{A^0} \rightarrow \infty$)

In this limit, it is easy to see that

$$\lambda_{h^0 h^0 h^0} \rightarrow \lambda_{HHH}^{\text{SM}} = -\frac{3 e M_H^2}{2 M_W s_W} = \boxed{-\frac{3 g M_H^2}{2 M_W}}$$

and similarly with the Yukawa couplings of the h^0 :

	type I	type II
$h^0 t \bar{t}$	$\cos \alpha / \sin \beta$	$\cos \alpha / \sin \beta$
$h^0 b \bar{b}$	$\cos \alpha / \sin \beta$	$-\sin \alpha / \cos \beta$



➤ Constraints on 2HDM models

- Maintain the (approximate) $SU(2)$ custodial symmetry: $\rho = \rho_0 + \delta\rho$ where $\rho_0 = M_W^2/M_Z^2 \cos^2 \theta_W = 1$. $|\delta\rho_{2HDM}| \leq 10^{-3}$ from experimental constrain.

$$\delta\rho = \left. \frac{\Sigma_Z(k^2) - \Sigma_W(k^2)}{M_Z^2 - M_W^2} \right|_{k^2=0}.$$

$$|\delta\rho_{bSM}| \lesssim 10^{-3} \text{ (exp.)}$$

$$\begin{aligned} \delta\rho_{2HDM} &= \frac{G_F}{8\sqrt{2}\pi^2} \left\{ M_{H^\pm}^2 \left[1 - \frac{M_{A^0}^2}{M_{H^\pm}^2 - M_{A^0}^2} \ln \frac{M_{H^\pm}^2}{M_{A^0}^2} \right] \right. \\ &\quad + \cos^2(\beta - \alpha) M_{h^0}^2 \left[\frac{M_{A^0}^2}{M_{A^0}^2 - M_{h^0}^2} \ln \frac{M_{A^0}^2}{M_{h^0}^2} - \frac{M_{H^\pm}^2}{M_{H^\pm}^2 - M_{h^0}^2} \ln \frac{M_{H^\pm}^2}{M_{h^0}^2} \right] \\ &\quad \left. + \sin^2(\beta - \alpha) M_{H^0}^2 \left[\frac{M_{A^0}^2}{M_{A^0}^2 - M_{H^0}^2} \ln \frac{M_{A^0}^2}{M_{H^0}^2} - \frac{M_{H^\pm}^2}{M_{H^\pm}^2 - M_{H^0}^2} \ln \frac{M_{H^\pm}^2}{M_{H^0}^2} \right] \right\} \end{aligned}$$

This can be achieved e.g. when $M_{A^0} \rightarrow M_{H^\pm}$

Restrictions: $\mathcal{B}(b \rightarrow s\gamma)$

- We have strong constraints coming from flavor physics
 - $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) \sim (3.55 \pm 0.25) \cdot 10^{-4}$ from BaBar and Belle
 - $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) \sim (3.15 \pm 0.23) \cdot 10^{-4}$ SM NNLO prediction
- The good agreement between the SM prediction and the experimental result puts severe constraints on the flavor structure of NP models.

New charged-particles contribute to this rare decay.

The charged Higgs bosons contribution:

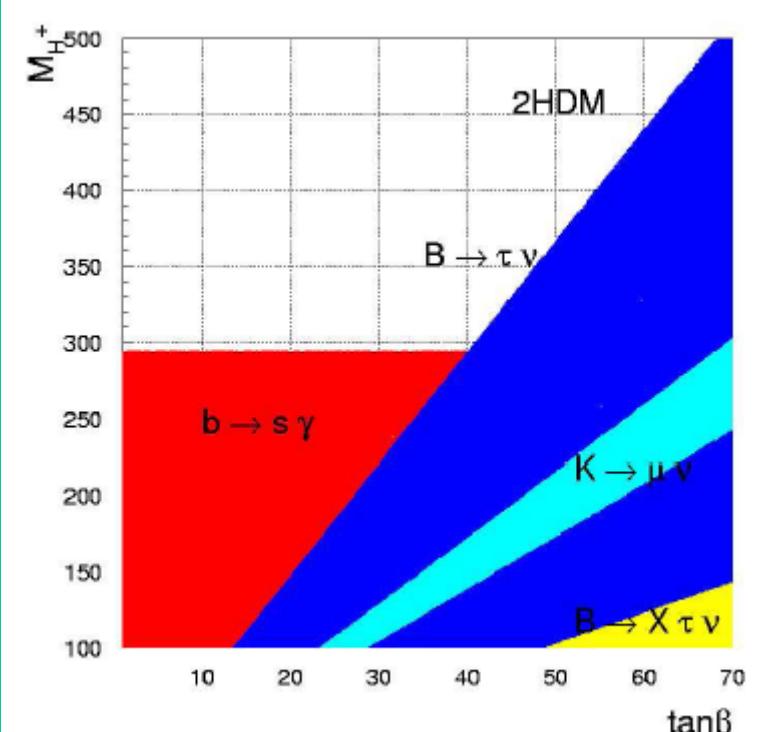
- positive
- increases when M_{H^\pm} decreases

Type-I 2HDM: Couplings $H^\pm q q' \propto 1/\tan\beta$
 Couplings highly suppressed for $\tan\beta > 1$

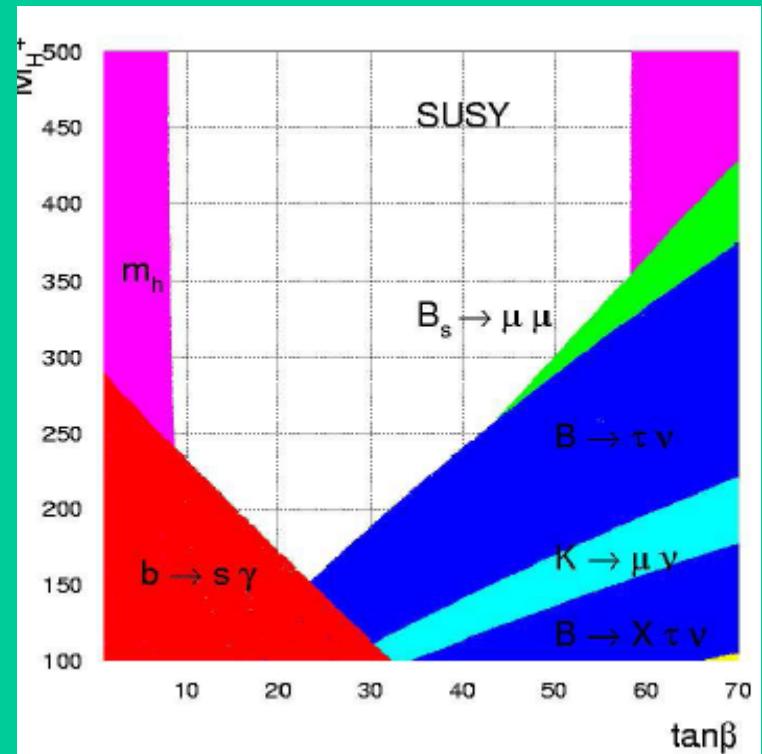
Type-II 2HDM: Couplings $H^\pm q q' \propto \tan\beta$
 Couplings enhanced for $\tan\beta > 1$
 Restriction $M_{H^\pm} > 295$ GeV

Misiak et al., 2006

Present Constraints from Flavour Physics!



$$m_{H^+} > 295 \text{ GeV}$$



$$m_{H^+} \gtrsim 130 \text{ GeV}$$

F. Mescia (UB), private communication

$$m_{SUSY} = -2A_{\tilde{t}} = 1 \text{ TeV}$$

Joan Solà (UB) Corfu 09

➤ Perturbativity and unitarity

- Keep the theory within a perturbative regime: $0.1 \lesssim \tan \beta \lesssim 60$

$$h_t = \frac{gm_t}{\sqrt{2}M_W \sin \beta}, \quad h_b = \frac{gm_b}{\sqrt{2}M_W \cos \beta} \rightarrow \frac{gm_b \tan \beta}{\sqrt{2}M_W}$$

- Unitarity bounds: we bound the size of the trilinear Higgs boson couplings by the value of their single SM counterpart at the scale of 1 TeV.

$$|C(HHH)| \leq \left| \lambda_{HHH}^{(SM)}(M_H = 1 \text{ TeV}) \right| = \left. \frac{3 e M_H^2}{2 \sin \theta_W M_W} \right|_{M_H=1 \text{ TeV}}.$$

(This would be at least the simplest unitarity requirement to start with)

➤ More strict unitarity bounds

- ♣ Asymptotic “flatness” of the scattering amplitudes \leftrightarrow Unitarity of the S-matrix
- ♣ **Standard tree-level unitarity condition:** $|a_0| < 1/2$
- ♣ Lee-Quigg-Thacker bound in the SM:
$$M_H^2 < 8\pi v^2 = \frac{8\pi}{\sqrt{2} G_F} \simeq (1.2 \text{ TeV})^2$$

Similarly with the **2HDM** :

Higgs and Goldstone boson $2 \rightarrow 2$ S-matrix elements, S_{ij} \Rightarrow Unitarity condition over its eigenvalues $|\alpha_i| < 1/2 \forall i$.

e.g.

$$a_{\pm} = \frac{1}{16\pi} \left\{ 3(\lambda_1 + \lambda_2 + 2\lambda_3) \pm \left(\sqrt{9(\lambda_1 - \lambda_2)^2 + (4\lambda_3 + \lambda_4 + \frac{\lambda_5}{2} + \frac{\lambda_6}{2})^2} \right) \right\}$$

Kanemura, Kubota & Takasugi, 1993; Akeroyd, Arhrib & Naimi, 2000;
Horejsi & Kladiva, 2006

$$a_{\pm} = \frac{1}{16\pi} \left\{ 3(\lambda_1 + \lambda_2 + 2\lambda_3) \right. \\ \left. \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + (4\lambda_3 + \lambda_4 + \frac{\lambda_5}{2} + \frac{\lambda_6}{2})^2} \right\},$$

$$b_{\pm} = \frac{1}{16\pi} \left\{ \lambda_1 + \lambda_2 + 2\lambda_3 \right. \\ \left. \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{(-2\lambda_4 + \lambda_5 + \lambda_6)^2}{4}} \right\},$$

$$c_{\pm} = d_{\pm} = \frac{1}{16\pi} \left\{ \lambda_1 + \lambda_2 + 2\lambda_3 \right. \\ \left. \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{(\lambda_5 - \lambda_6)^2}{4}} \right\},$$

$$e_1 = \frac{1}{16\pi} \left\{ 2\lambda_3 - \lambda_4 - \frac{\lambda_5}{2} + 5\frac{\lambda_6}{2} \right\},$$

$$e_2 = \frac{1}{16\pi} \left\{ 2\lambda_3 + \lambda_4 - \frac{\lambda_5}{2} + \frac{\lambda_6}{2} \right\},$$

$$f_+ = \frac{1}{16\pi} \left\{ 2\lambda_3 - \lambda_4 + 5\frac{\lambda_5}{2} - \frac{\lambda_6}{2} \right\},$$

$$f_- = \frac{1}{16\pi} \left\{ 2\lambda_3 + \lambda_4 + \frac{\lambda_5}{2} - \frac{\lambda_6}{2} \right\},$$

$$f_1 = f_2 = \frac{1}{16\pi} \left\{ 2\lambda_3 + \frac{\lambda_5}{2} + \frac{\lambda_6}{2} \right\},$$

$$p_1 = \frac{1}{16\pi} \left\{ 2(\lambda_3 + \lambda_4) - \frac{\lambda_5 + \lambda_6}{2} \right\}.$$

➤ Vacuum stability bounds

We assume that the quartic interaction terms in the potential do not give negative contribution for all directions of scalar fields at each energy scale up to Λ

Require a Higgs potential bounded from below

$$\lambda_1 + \lambda_3 > 0$$

$$\lambda_2 + \lambda_3 > 0$$

$$2\sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)} + 2\lambda_3 + \lambda_4 + \frac{1}{2} \text{Min}\left(0, \lambda_5 + \lambda_6 - 2\lambda_4 - |\lambda_5 - \lambda_6|\right) > 0$$

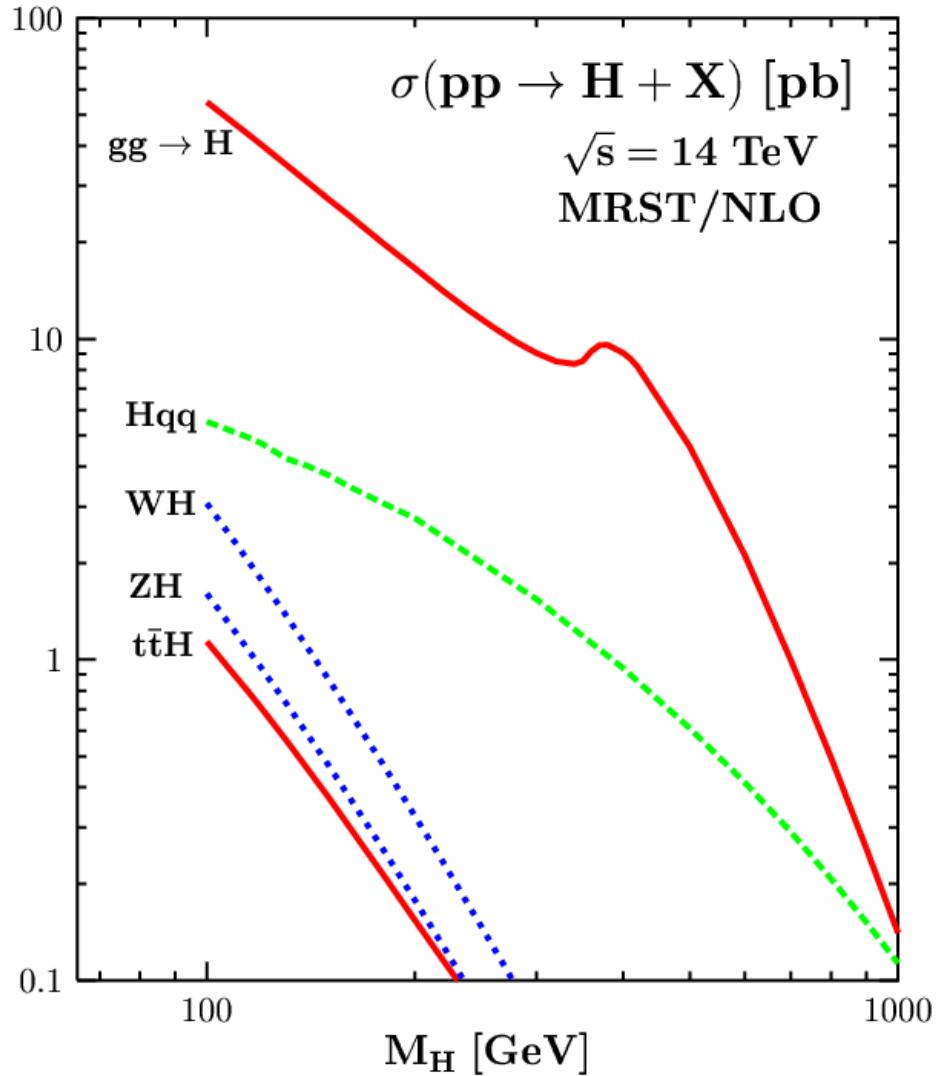
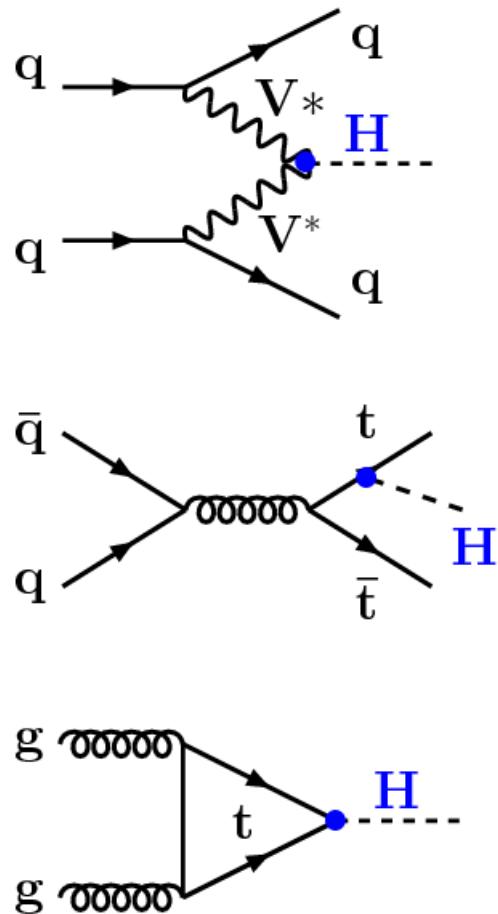
Kanemura, Kasai & Okada, 1999

With all these restrictions in mind
we are **ready** for particular calculations
of 2HDM production at e.g.

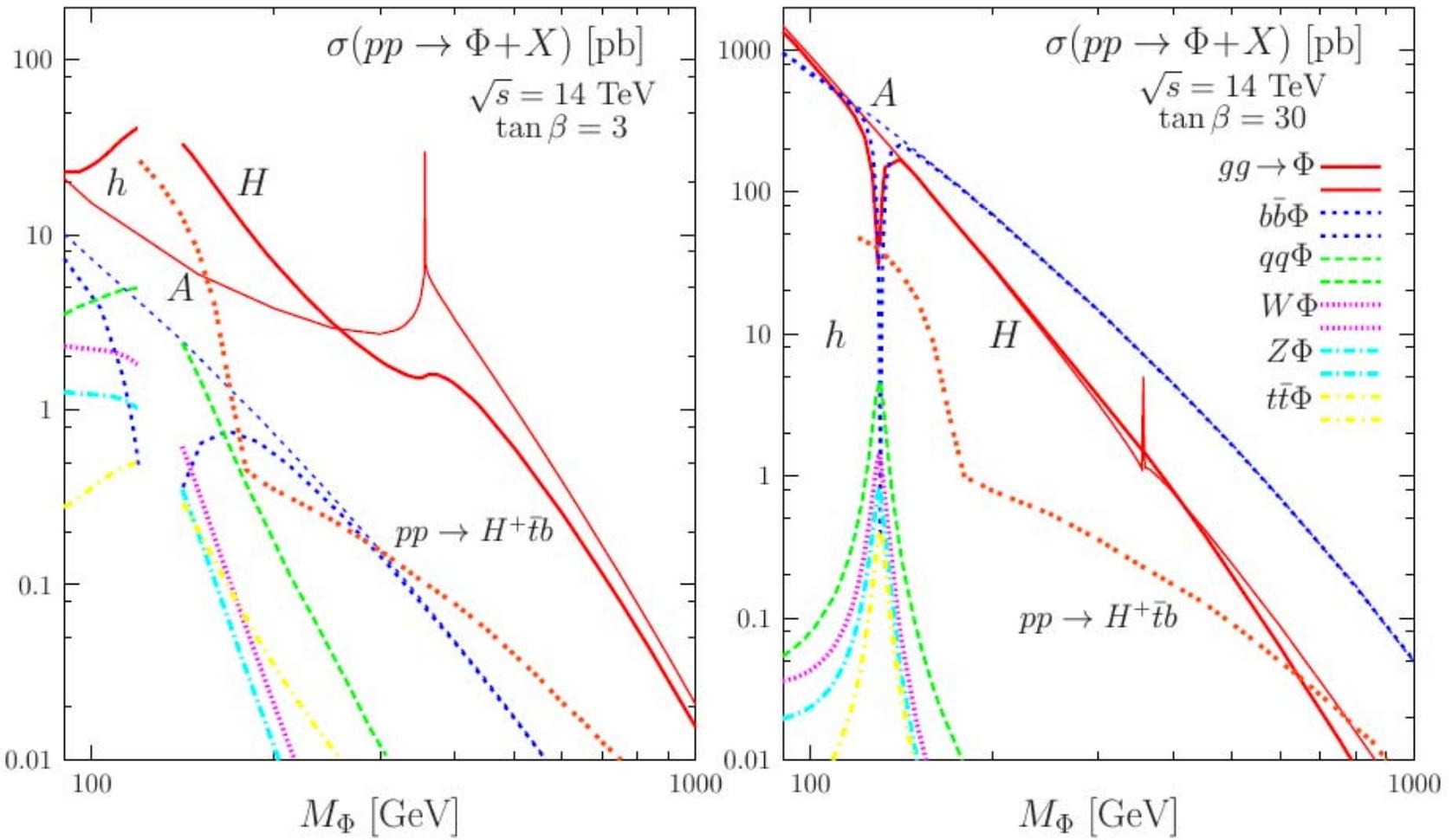
➤ LHC

➤ Linear Colliders

➤ Higgs boson production at the LHC in the SM



➤ Higgs boson production at the LHC in the MSSM



➤ Higgs boson production
at the Linear Colliders
ILC/CLIC
within the general 2HDM



- $e^+e^- \rightarrow 2H$ (at the quantum level)
D. López-Val, JS, arXiv:0908.2898 [hep-ph]
- $e^+e^- \rightarrow 3H$
- $e^+e^- \rightarrow 2H + X$

}

(very promising too!)

N. Hodgkinson, D, López-Val, JS,
Phys. Lett. B677 (2009) 39

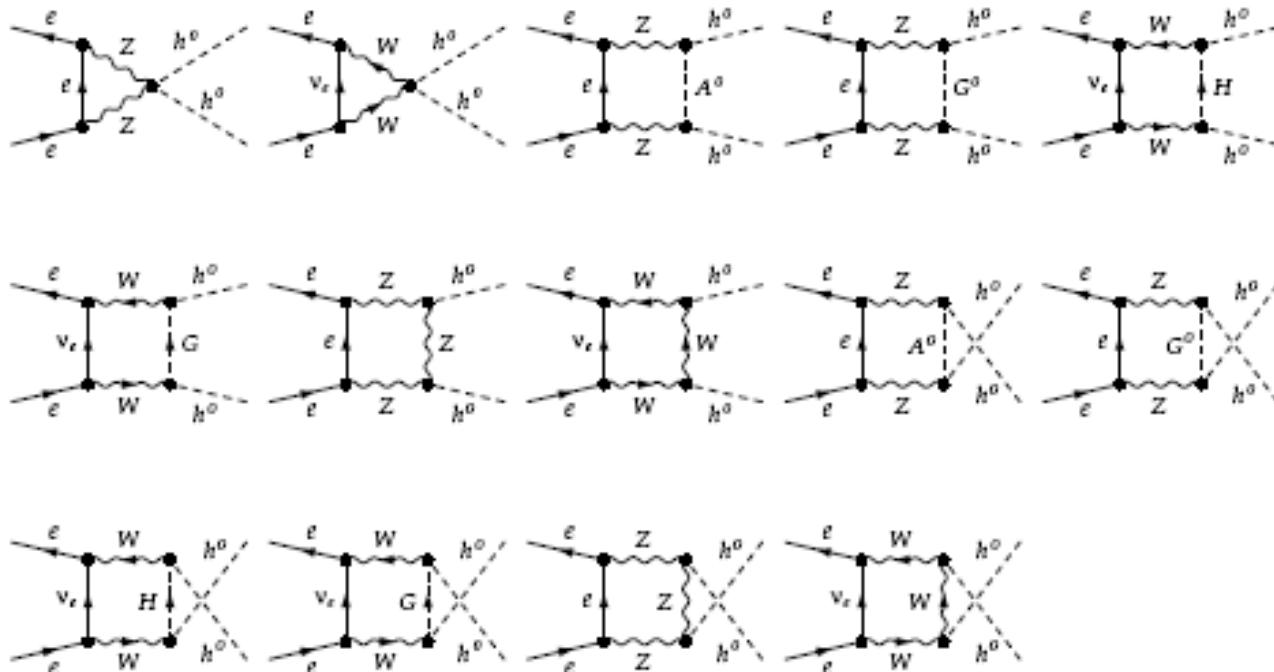
G. Ferrera, J. Guasch, D, López-Val, JS,
Phys.Lett.B659: (2008) 297

CP forbidden channels

G. Ferrera, J. Guasch, D. López-Val, J. Solà,
Phys.Lett.B659: (2008) 297

- CP conservation will be assumed

$$\begin{array}{lcl} e^+e^- & \rightarrow & h^0h^0 \\ e^+e^- & \rightarrow & H^0H^0 \\ e^+e^- & \rightarrow & A^0A^0 \end{array} \quad \left. \right\} \text{LO: 1-loop } CP\text{-conserving boxes } (\mathcal{O}(\alpha_{ew}^4))$$



♠ $\sigma \sim 10^{-5}$ pb – at the same level than $\sigma(2H_{SM})$

CP allowed channels

$$\left. \begin{array}{l} e^+e^- \rightarrow H^\pm H^\mp \\ e^+e^- \rightarrow h^0 A^0 \\ e^+e^- \rightarrow H^0 A^0 \end{array} \right\} \text{LO: tree-level diagrams } (\mathcal{O}(\alpha_{ew}^2))$$

- $C(HHV)$ couplings are of purely gauge nature

$$C_{\text{MSSM}}(h^0 A^0 Z) = \frac{e \cos(\beta - \alpha)}{2 \sin \theta_W \cos \theta_W}$$

- ♠ There is no dynamical distinction between the general 2HDM and the MSSM
- ♠ Dedicated studies on radiative corrections in 2H processes are hence mandatory: Chankowski, Pokorski, Rosiek (94, 95); Driesen, Hollik, Rosiek (95, 96); Arhrib, Moltaka (98); Kraft (PhD Thesis, 99), Guasch, Hollik, Kraft (01); Heinemeyer et al (01)

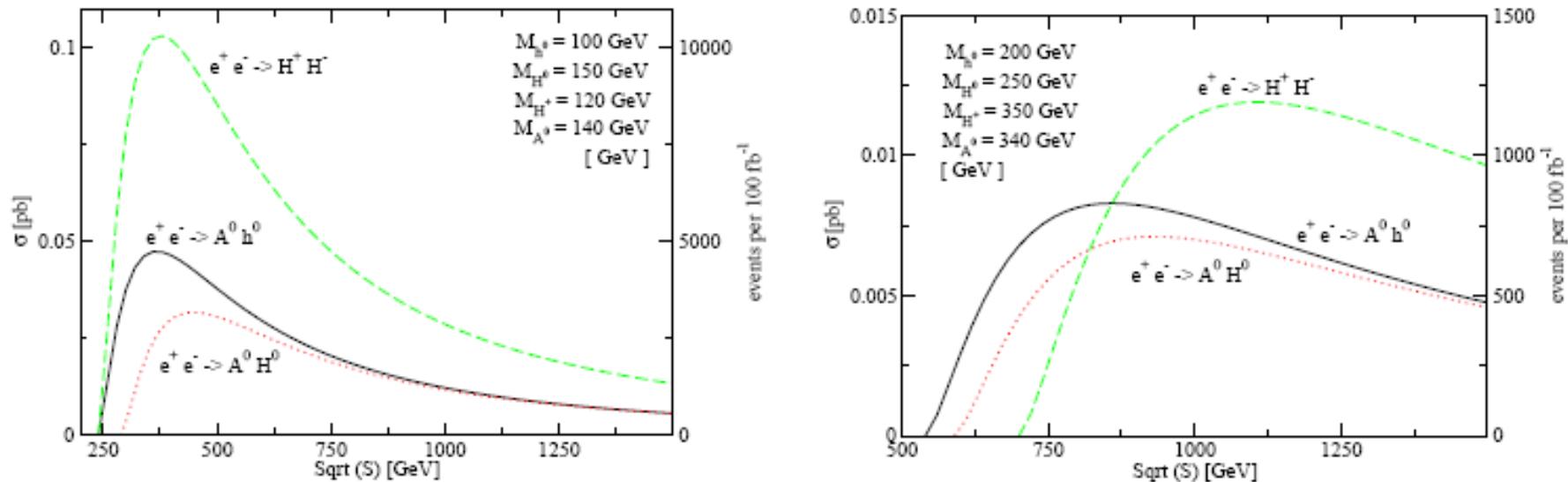


Figure 1: Total cross section σ (in pb) and number of events per 100 fb^{-1} as a function of $E_{\text{cm}} = \sqrt{s}$ for the tree-level Higgs boson pair production channels in the general 2HDM within two representative regimes of Higgs boson masses

♠ Up to $10^3 - 10^4$ events per 100 fb^{-1}

➤ Quantum effects on $e^+e^- \rightarrow 2H$ in the 2HDM

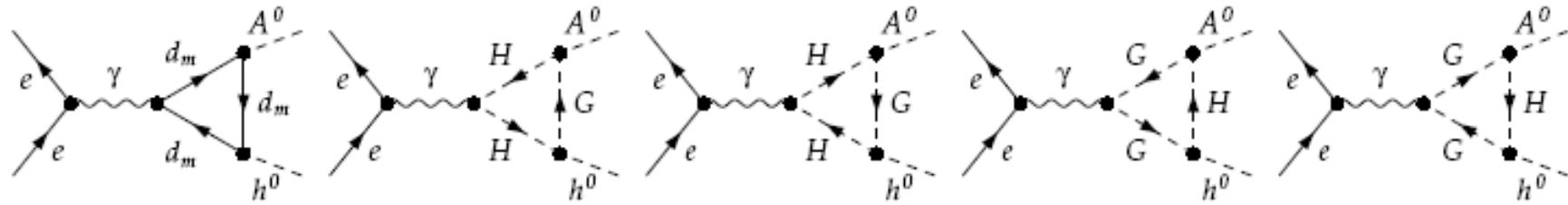
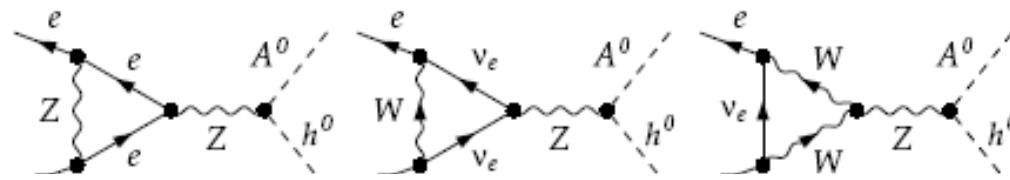
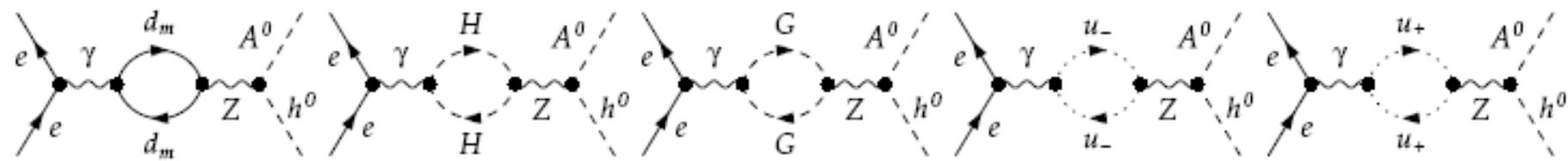
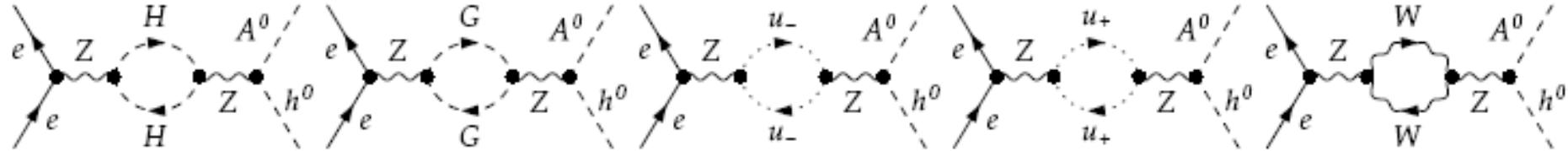
$$e^+ e^- \rightarrow A^0 h^0 \quad (\text{similarly with } e^+ e^- \rightarrow A^0 H^0)$$

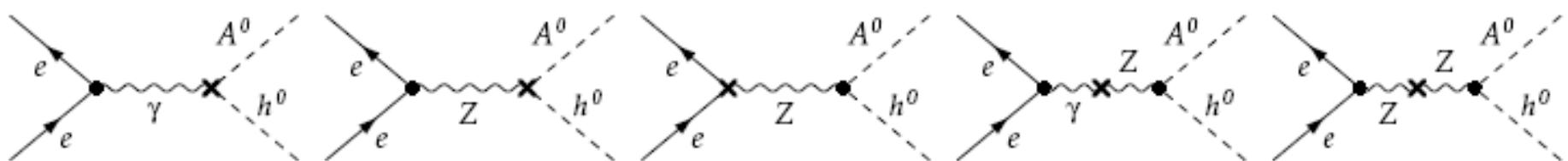
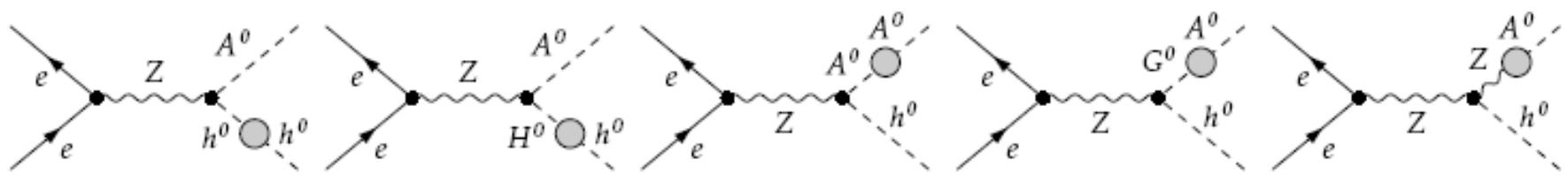
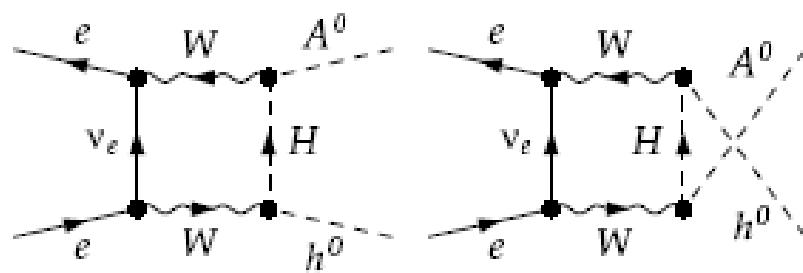
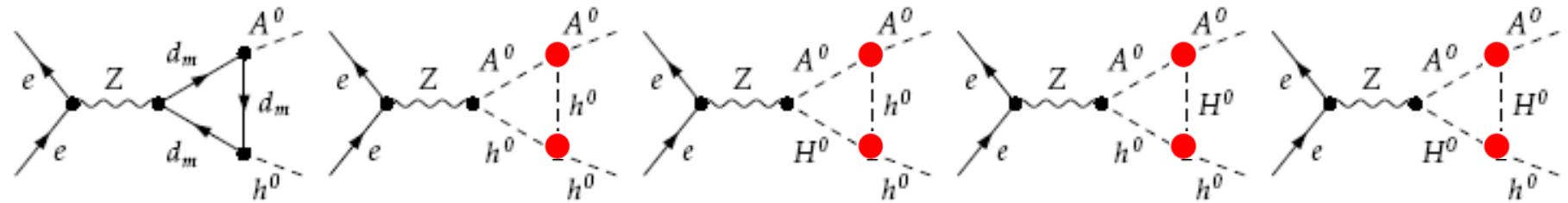
Basic one-loop amplitude:

$$\begin{aligned} M_{e^+ e^- \rightarrow A^0 h^0}^1 &= (M^{1,Z-Z} + M^{1,\gamma-Z} \\ &\quad + M^{1,e^+e^-Z} + M^{1,e^+e^-\gamma} \\ &\quad + M^{1,Z^0 A^0 h^0} + M^{1,\text{box}} + M^{1,\text{WF}} + \delta M^1) \end{aligned}$$

The loop correction appears from the interference with the tree-level amplitude: $2\Re e \mathcal{M}^{(0)} \mathcal{M}^{(1)}$

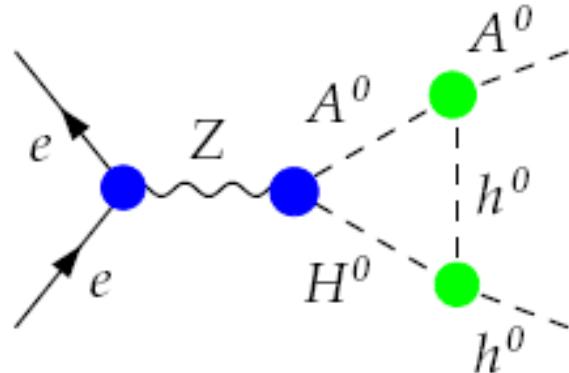
Some of the diagrams involved...





For the complete list see: D, López-Val, JS,
arXiv:0908.2898 [hep-ph]

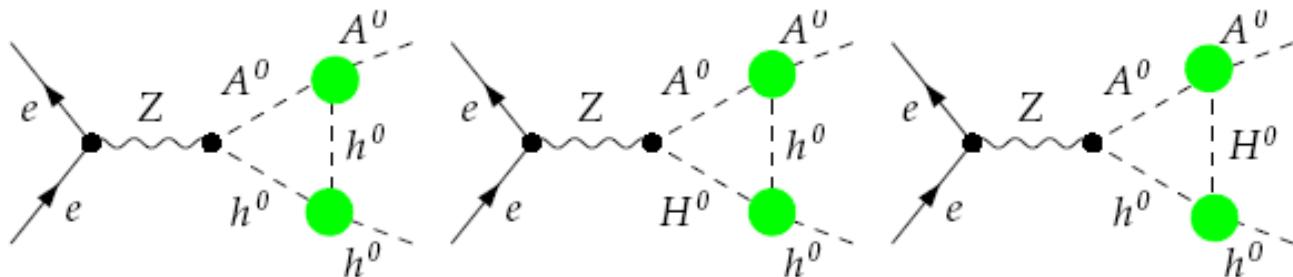
Why we expect important effects in the 2HDM?



whereas in the MSSM,

$$C_{\text{MSSM}}[h^0 h^0 H^0] = \frac{ie M_W}{2 \sin \theta_W \cos \theta_W} (\cos 2\alpha \cos(\alpha + \beta) - 2 \sin 2\alpha \sin(\alpha + \beta))$$

The $Z^0 A^0 h^0$ interaction at 1-loop



- ♠ Due to its sensitivity to 3H self-couplings, the strength of the $Z^0 A^0 h^0$ interaction (which is purely gauge-like at the leading-order) may be largely enhanced at the quantum level:

$$\begin{aligned}\Gamma_{A^0 h^0 Z^0}^0 &\rightarrow Z^{1/2} \left(\Gamma_{A^0 h^0 Z^0}^0 + \Gamma_{A^0 h^0 Z^0}^1 \right) \\ &\simeq Z^{1/2} \left(\Gamma_{A^0 h^0 Z^0}^0 + \Gamma_{A^0 h^0 Z^0}^{1,3H} \right) \\ \Gamma_{A^0 h^0 Z^0}^{eff} &\sim \Gamma_{A^0 h^0 Z^0}^0 \frac{\lambda_{3H}^2}{16\pi^2 s} f(M_{h^0}/s, M_{A^0}/s)\end{aligned}$$

➤ Renormalization conditions and counterterms

Renormalization of the Higgs sector OS (on-shell) scheme

♠ Higgs fields: 1 WF constant per $SU_L(2)$ doublet

$$\begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix} \rightarrow Z_{\Phi_1}^{1/2} \begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix}, \quad \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix} \rightarrow Z_{\Phi_2}^{1/2} \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix},$$

$$(Z_{\Phi_i} = 1 + \delta Z_{\Phi_i})$$

$$\begin{aligned} \spadesuit \quad \tan \beta: \quad & \left. \begin{aligned} \frac{\delta v_1}{v_1} &= \frac{\delta v_2}{v_2} \\ t_{h^0 H^0} + \delta t_{h^0 H^0} &= 0 \end{aligned} \right\} \quad \frac{\delta \tan \beta}{\tan \beta} = \frac{\delta v_2}{v_2} - \frac{\delta v_1}{v_1} + \frac{1}{2} (\delta Z_{\Phi_2} - \delta Z_{\Phi_1}) \\ &= \frac{1}{2} (\delta Z_{\Phi_2} - \delta Z_{\Phi_1}) . \end{aligned}$$

Higgs masses (OS scheme)

$$\text{Re } \hat{\Sigma}(M_{h^0}^2) = 0; \text{ Re } \hat{\Sigma}(M_{H^0}^2) = 0; \text{ Re } \hat{\Sigma}(M_{A^0}^2) = 0; \text{ Re } \hat{\Sigma}(M_{H^\pm}^2) = 0$$

$$\text{Re } \hat{\Sigma}'_{A^0 A^0}(k^2) \Big|_{k^2=M_{A^0}^2} = 0 \quad , \quad \text{Re } \hat{\Sigma}_{A^0 Z^0}(k^2) \Big|_{k^2=M_{A^0}^2} = 0$$

$$\delta Z_{\Phi_1} = -\text{Re } \Sigma'_{A^0 A^0}(M_{A^0}^2) - \frac{1}{M_Z \tan \beta} \text{Re } \Sigma_{A^0 Z^0}(M_{A^0}^2)$$

$$\delta Z_{\Phi_2} = -\text{Re } \Sigma'_{A^0 A^0}(M_{A^0}^2) + \frac{\tan \beta}{M_Z} \text{Re } \Sigma_{A^0 Z^0}(M_{A^0}^2)$$



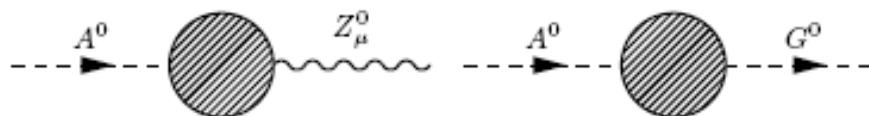
$$\frac{\delta \tan \beta}{\tan \beta} = \frac{1}{2} (\delta Z_{\Phi_2} - \delta Z_{\Phi_1})$$

$$\boxed{\frac{\delta \tan \beta}{\tan \beta} = \frac{1}{M_Z \sin 2\beta} \text{Re } \Sigma_{A^0 Z^0}(M_{A^0}^2)}$$

Physical content of the OS conditions:

- On-shell A^0 renormalized propagators have unit residue,
 $1/[1 + \text{Re } \Sigma'_{A^0}(M_{A^0}^2)] = 1$
- No $A^0 - Z^0$ mixing occurs for on-shell A^0 bosons at any order in perturbation theory.
- Likewise, the absence of $A^0 - G^0$ mixing is guaranteed by the Slavnov-Taylor identity: (Feynman gauge)

$$q^2 \hat{\Sigma}_{A^0 Z^0}(q^2) + M_Z \hat{\Sigma}_{A^0 G^0}(q^2) \Big|_{q^2 = M_{A^0}^2} = 0$$



Renormalization of the mixing angle α

- ♠ As usual, we introduce a 1-loop counterterm $\alpha^{(0)} = \alpha + \delta \alpha$
- ♠ Renormalization condition: $\text{Re } \hat{\Sigma}_{h^0 H^0}(q^2 = M_{h^0}^2) = 0 \Rightarrow$

$$\delta m_{h^0 H^0}^2 = \Sigma_{h^0 H^0}(M_{h^0}^2) + \frac{1}{2} \delta Z_{h^0 H^0} (M_{h^0}^2 - M_{H^0}^2)$$

$$\delta Z_{h^0 H^0} = \sin 2\alpha (\delta \tan \beta / \tan \beta)$$

$$\delta \alpha = \frac{\delta m_{h^0 H^0}^2}{M_{h^0}^2 - M_{H^0}^2}$$

- ♠ The same condition ensures that the CP -even Higgs final state is on-shell, as the *physical* masses must fulfill

$$(p^2 - M_{H^0}^2 + \hat{\Sigma}_{H^0 H^0}) (p^2 - M_{h^0}^2 + \hat{\Sigma}_{h^0 h^0}) - \hat{\Sigma}_{h^0 H^0}^2 = 0.$$

$e^+ e^- \rightarrow A^0 h^0$ phenomenology in a nutshell

- Two main scenarios of phenomenological interest are identified:
 - ♣ Regions with small $|\lambda_5| \sim \mathcal{O}(0.1)$ and moderate $\tan \beta$ ($\tan \beta \sim \mathcal{O}(10)$)

Quantum effects turn out to be ...

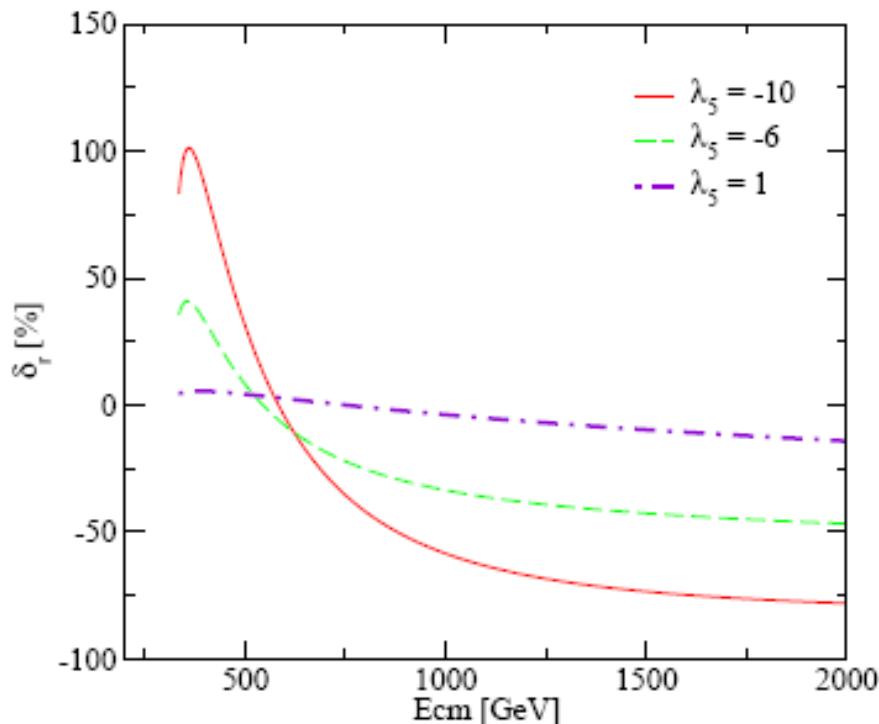
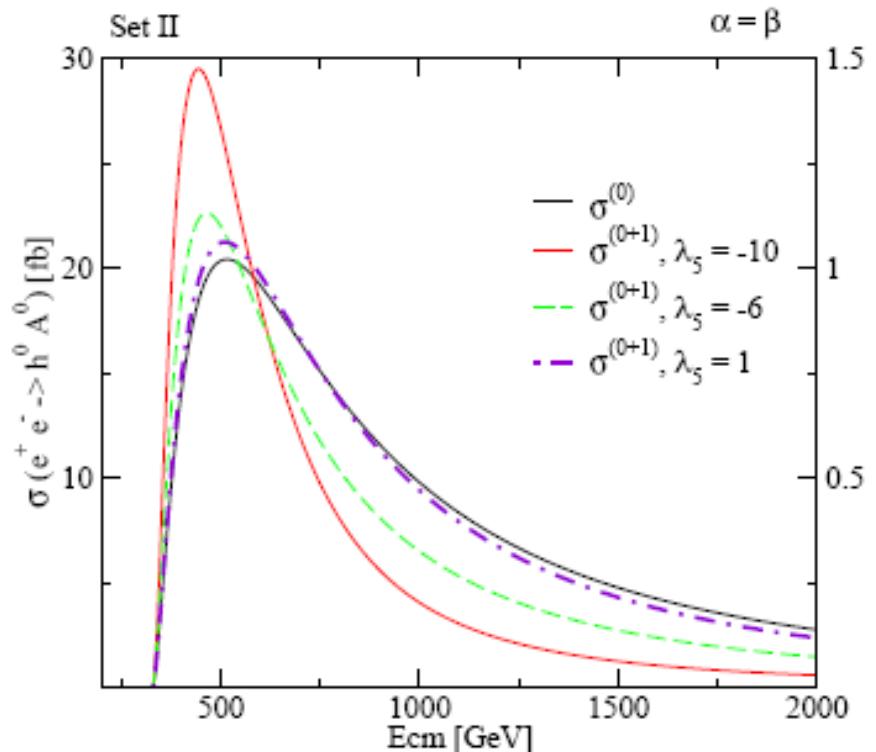
- Subleading, with respect to the former scenario.
- Typically not larger than $\sim 5\% - 10\%$ at the topmost.
- Driven by the interplay of quark, Higgs-boson and gauge-boson mediated 1-loop contributions

$$\delta_r = \frac{\sigma^{(0+1)-\sigma^{(0)}}}{\sigma^{(0)}} \left\{ \begin{array}{l} \sqrt{s} = 0.5 \text{ TeV} : 100\% \\ \sqrt{s} = 1.0 \text{ TeV} : -80\% \end{array} \right.$$

- Typical maximum cross sections:

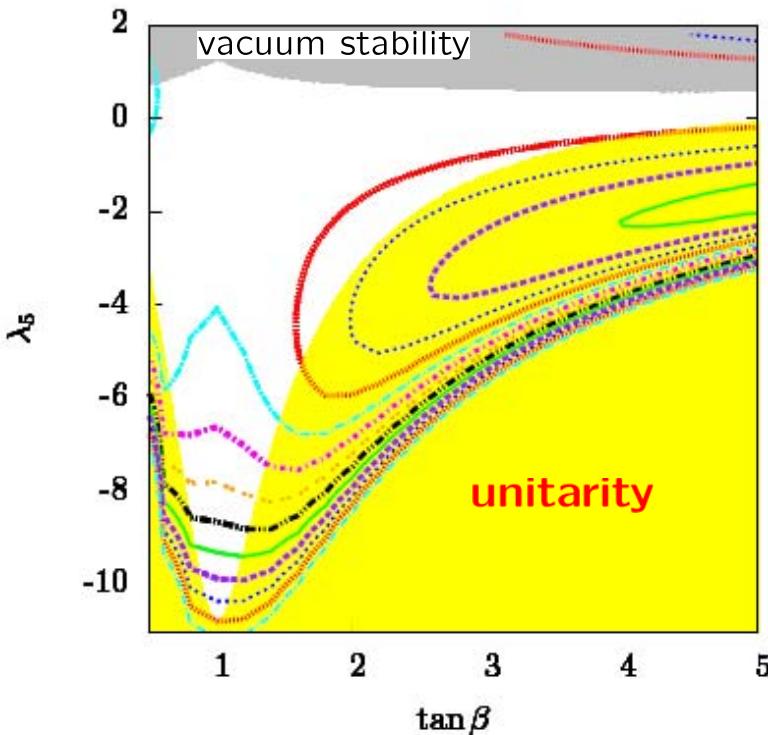
$$\sigma(e^+e^- \rightarrow A^0 h^0) (\sqrt{s} = 0.5 \text{ TeV}) \sim \mathcal{O}(10 \text{ fb}) \quad \sim 10^3 \text{ events per } 100 \text{ fb}^{-1}$$

$\tan \beta$	α	$M_{h^0} [\text{ GeV}]$	$M_{H^0} [\text{ GeV}]$	$M_{A^0} [\text{ GeV}]$	$M_{H^\pm} [\text{ GeV}]$
1	β	130	150	200	160

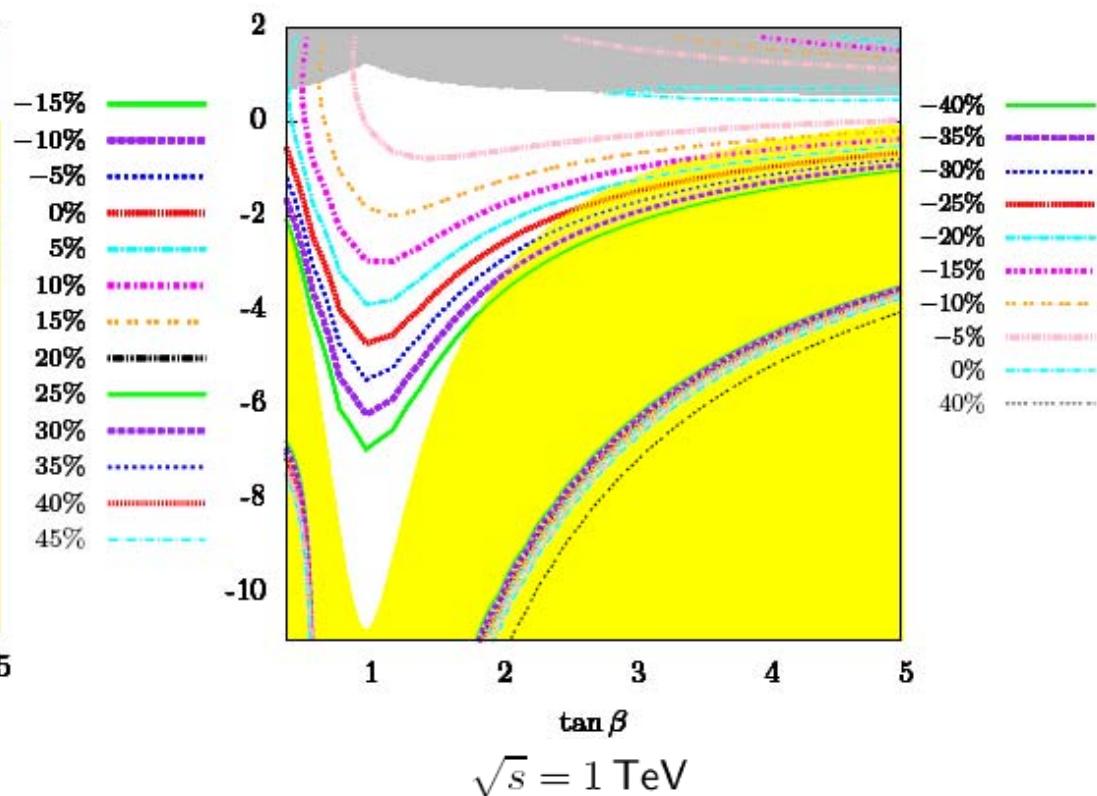


♣ Radiative corrections over the $\tan \beta - \lambda_5$ plane and its interplay with the unitarity and vacuum stability constraints.

Set II



$\sqrt{s} = 500$ GeV



$\sqrt{s} = 1$ TeV

Dependence of σ with masses, angles etc is studied in detail in our work: D. López-Val, JS, arXiv:0908.2898 [hep-ph]

➤ Mass sets for the numerical analysis

MSSM-like
masses at
one-loop
FeynHiggs
(Heinemeyer et al)

	M_{h^0} [GeV]	M_{H^0} [GeV]	M_{A^0} [GeV]	M_{H^\pm} [GeV]	
Set I	100	150	140	120	type-I
	130	150	200	160	
	150	200	260	300	
	95	205	200	215	
	115	220	220	235	
	130	285	285	300	

♠ $\sqrt{s} = 500$ GeV

		$\alpha = \beta$	$\alpha = \beta - \pi/3$	$\alpha = \beta - \pi/6$	$\alpha = \pi/2$
Set II	σ_{max} [fb]	26.71	7.34	20.05	13.10
	δ_r [%]	31.32	44.43	31.42	28.81
Set III	σ_{max} [fb]	11.63	3.60	9.08	6.36
	δ_r [%]	35.17	67.59	40.68	47.86
Set IV	σ_{max} [fb]	27.44	12.12	18.37	15.41
	δ_r [%]	12.86	99.42	0.76	26.81

MSSM-like
masses

Compared predictions for 2H and 3H events

Combined **strategy** for Higgs
boson search at **Linear Colliders**

$$\left\{ \begin{array}{lll} \bullet & e^+e^- \rightarrow \textcolor{red}{2H} & 0.5 \text{ TeV} \\ \bullet & e^+e^- \rightarrow \textcolor{red}{3H} & \sim 1 \text{ TeV} \\ \bullet & e^+e^- \rightarrow \textcolor{red}{2H} + X & \gtrsim 1 \text{ TeV} \end{array} \right.$$

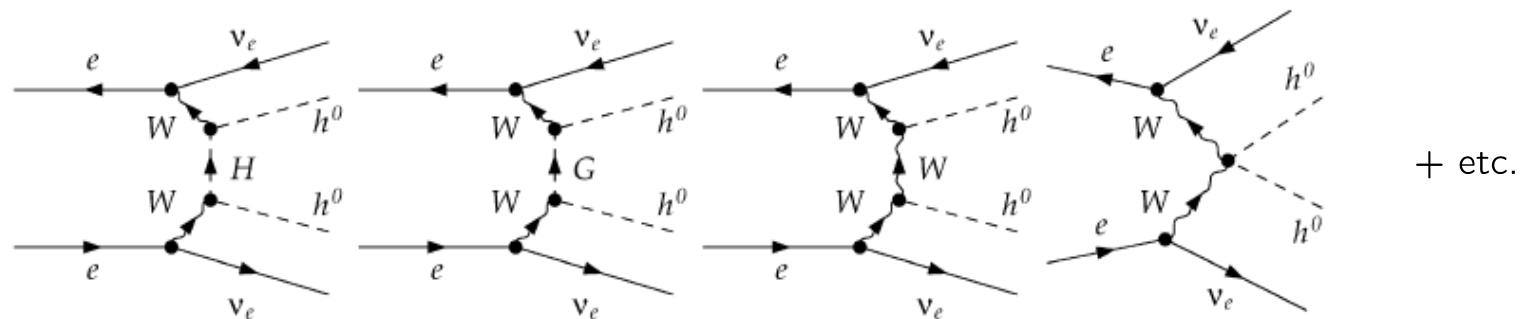
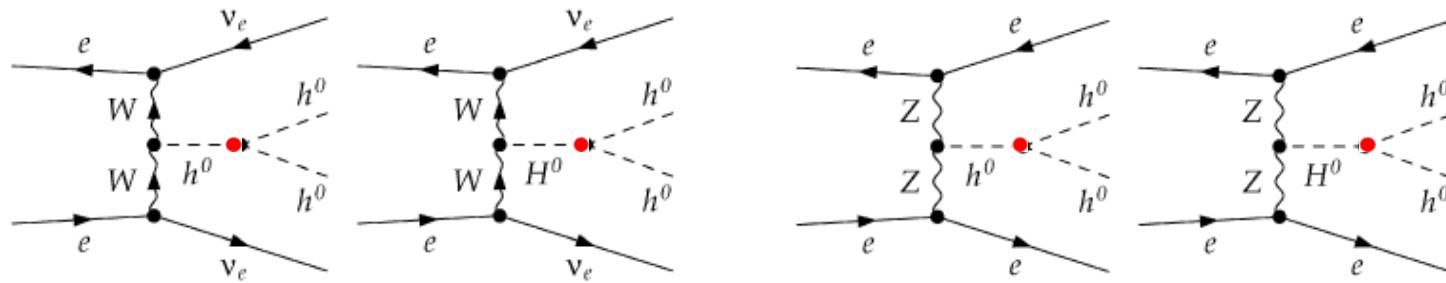
- ♣ For the $\textcolor{red}{2H}$ channel, analysis of **quantum effects** is essential!!
- ♣ For the $\textcolor{red}{3H}$ and $\textcolor{red}{2H} + X$ channels, analysis at leading order may be sufficient

e.g. $e^+e^- \rightarrow 2H + X$

➤ Double-Higgs production through gauge boson fusion

$$e^+ e^- \rightarrow V^* V^* \rightarrow h h + X$$

N. Hodgkinson, D. López-Val, J. Solà,
 Phys. Lett. B677 (2009) 39,
 arXiv:0901.2257 [hep-ph]



Numerical example:

$\tan \beta$	α	λ_5	M_{h^0} [GeV]	M_{H^0} [GeV]	M_{A^0} [GeV]	M_{H^\pm} [GeV]
1	β	-10	130	150	200	160

σ (in fb)	\sqrt{s} [TeV]	0.5	1.0	1.5
	$h^0 A^0$	26.71	4.07	1.27
	$h^0 H^0 A^0$	0.02	5.03	3.55
	$H^0 H^+ H^-$	0.17	11.93	8.39
	$h^0 h^0 + X$	1.47	17.36	38.01

Great complementarity is observed between the different channels at different energy ranges.

➤ Conclusions

- We have presented a complete $\mathcal{O}(\alpha_{ew}^3)$ calculation of the pairwise production of neutral Higgs bosons ($A^0 h^0 \{H^0\}$) at Linear Colliders within the general 2HDM.
- Our analysis identifies sizable quantum effects:
 - up to 100%
 - in regions with $\tan \beta \sim 1$ and $|\lambda_5| \sim 5 - 10$ ($\lambda_5 < 0$)
 - either positive ($\sqrt{s} = 0.5$ TeV) and negative ($\sqrt{s} = 1.0$ TeV)
 - sourced by the Higgs-mediated vertex corrections at one-loop, which are sensitive to the potentially enhanced 3H self-couplings
- The production rates at $\sqrt{s} = 0.5$ TeV may amount up to **a few thousands** of 2H events per 100 fb^{-1}

- The combined study of 2H and 3H processes at different energy ranges may provide a manifold of complementary signatures, which might reveal strong hints of (non-SUSY) Higgs physics BSM.

- **Build the Linear Collider !!**

both **ILC** and **CLIC!!**

Thank you!!