Deformations of branes and exact CFTs

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Based on:

- JHEP **02** (2008) 087, arXiv:0712.1912 [hep-th],
  with: A. Fotopoulos, M. Petropoulos and N. Prezas
- JHEP **06** (2008) 080, arXiv:0804.3062 [hep-th],
  with: N. Prezas (Mostly on this).
- JHEP, to appear, arXiv:0905.1623 [hep-th],
  with: M. Petropoulos and N. Prezas

General remarks-Motivation

- In string theory we want to go beyond (super)gravity.
- Obtain exact CFT description of solutions.
- Branes are important in many developments: From the formulation of string theory, to black hole physics, to ads/cft.

Brane solutions admitting exact string description-CFTs

Only a tiny subset:

- NS5-branes at a point: $\mathbb{R}^{5,1} \times \mathbb{R}_\phi \times SU(2)$ [Callan-Harvey-Strominger, 91].
- NS5-branes at a circle: $\mathbb{R}^{5,1} \times \frac{SU(2,\mathbb{R})}{U(1)} \times \frac{SU(2)}{U(1)}$ [K.S., 98].
- Intersecting F1- and NS5-branes: $\mathbb{R}^4 \times AdS_3 \times SU(2)$.

Main motivation

What about backgrounds that do not admit an exact CFT interpretation?

- Understand deformations with exact CFT operators:
  - On transverse space, world-volume. Could preserve SUSY.
  - Adding angular momentum etc. Do not preserve SUSY.
- Interplay between exact string theory and σ-model (geometrical) approaches.
  Exact operators and their semiclassical realizations.
- When does a perturbation have a clear σ-model interpretation?
- Aspects of holography when the exact string theory is known.

Outline

- *NS5-branes: Supergravity and basic CFT*
  - General considerations on supergravity backgrounds.
  - NS5-branes at a point and $\mathbb{R}_\phi \times SU(2)_k$.

- *Dictionary for CFT deformations*
  - General idea on how the dictionary is built up.
  - Holographic and non-holographic deformations.
  - Relocating the brane's, symmetry considerations, classification.

- *Examples*
  - Branes on circles and on a line segment.
  - Conformal to Egutchi–Hanson.
  - From near horizon to asymptotic flatness.

- *Summary, concluding remarks*

NS5-branes: Supergravity and basic CFT

Supergravity

Consider k parallel NS5-branes, with:

- Worldvolume, parameterized by $x^\mu, \mu = 0, 1, \ldots, 5$,
- transverse space $\mathbb{R}^4$ parametrized by $\mathbf{x} = \{x^i, i = 6, 7, 8, 9\}$.
  They can be spread out in $\mathbb{R}^4$ with

$$\text{density} \quad \rho(\mathbf{x}) : \qquad \int_{\mathbb{R}^4} d^4x \, \rho(\mathbf{x}) = 1 \ .$$

which defines

$$H(\mathbf{x}) = 1 + \alpha' k \int_{\mathbb{R}^4} d^4x' \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^2} \ ,$$

a harmonic function in $\mathbb{R}^4$.

The 1/2-susy preserving solitonic solution [Duff-Lu, 91], has:

- A metric

$$ds^2 = \underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{6-\text{dim flat}} + \underbrace{H(\mathbf{x})\delta_{ij} dx^i dx^j}_{4-\text{dim non-trivial part}} \quad .$$

- A three-form NS–NS field

$$H_{ijk} = \epsilon_{ijk}{}^{l} \partial_l H \ ,$$

  with indices raised and lowered with the flat metric of $\mathbb{R}^4$ and

- A dilaton field

$$e^{2(\Phi - \Phi_0)} = H \ ,$$

  where Φ_0 is related to the asymptotic string coupling $g_s = e^{\Phi_0}$ far from the NS5-branes.

Basic exact CFT

Branes at a point in the near horizon limit (1 dropped in H) [Callan-Harvey-Strominger, 91].

- $\rho(\mathbf{x}) = \delta^{(4)}(\mathbf{x})$ (every localized distribution far from sources).
- Letting $r = \sqrt{2k}e^{\Phi_0 + \phi/\sqrt{2k}}$ $(a' = 2)$

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + d\phi^2 + 2k \underbrace{d\Omega_3^2}_{\theta,\tau,\psi} , \quad H = 2\mathrm{Vol}_{S^3} \ .$$

- Linear [Myers, 87; Antoniadis-Bachas-Ellis-Nanopoulos, 89] dilaton

$$\Phi = -\frac{q}{2}\phi \ , \qquad q = \sqrt{2/k} \ .$$

This is nothing but the exact supersymmetric theory ($\mathcal{N} = 4$) for

$$\mathbb{R}^{5,1} \times \mathbb{R}_\phi \times SU(2)_k \ .$$

Operators for deformations of the NS5-branes at a point

In the world-volume of NS5's Little string theory (LST) [Seiberg, 97].
Asymptotically linear dilaton backgrounds provide holographic duals [Aharony-Berkooz-Kutasov-Seiberg, 98].
We restrict to Lorentz-invariant operators in the NS5 world-volume

$$\text{symmetric, traceless of} \quad \widetilde{\mathrm{tr}}(X^{i_1} X^{i_2} \cdots X^{i_{2j+2}}) \ , \quad i = 6, 7, 8, 9 \ ,$$

where the $SO(4)$ scalars X^i are $k \times k$ traceless matrices.

- Their eigenvalues parametrize the transverse positions NS5's.

The correspondence in the CFT side should involve:

- The $\Phi^{\mathrm{su}}_{j;m,\bar{m}}$ primaries of the bosonic $SU(2)_{k-2}$.
  Semiclassically realized in terms of the Euler angles θ, τ, ψ.
- The fermions $\psi_\pm$ and ψ_3 in the adjoint of $SU(2)$.
- The boson ϕ and the corresponding fermion ψ_ϕ.
- The vertex operator $e^{-qa_j\phi}$ of $\mathbb{R}_\phi$.

Holographic deformations

$$\overbrace{\widetilde{\mathrm{tr}}(X^{i_1}X^{i_2}\cdots X^{i_{2j+2}})}^{\text{LST}} \quad \Longleftrightarrow \quad \overbrace{(\psi\bar{\psi}\Phi_j^{\mathrm{su}})_{j+1;m,\bar{m}}e^{-qa_j\phi}}^{\text{CFT}} \; .$$

Scalars in $(\frac{1}{2},\frac{1}{2})$ rep. of $SU(2)_L \times SU(2)_R$, l.h.s. has spin $j+1$.

- Quite generally

$$(\psi\bar{\psi}\Phi_j^{\mathrm{su}})_{j+1;m,\bar{m}} = \left(\mu_3\psi^3\Phi^{\mathrm{su}}_{j,m_3} + \mu_+\psi^+\Phi^{\mathrm{su}}_{j,m_+} + \mu_-\psi^3\Phi^{\mathrm{su}}_{j,m_-}\right) e^{-a_jq\phi} \; .$$

- Spin $j+1$: $\mu_{3,\pm}$ Clebsch-Gordan and $m_3 = m$, $m_\pm = m \mp 1$.
- m and $\bar{m}$ will be associated with $U(1)$'s symmetries in space.
- CFT for $SU(2)_{k-2} \rightarrow j = 0, \frac{1}{2}, \ldots, (k-2)/2$. The number of matrices on the l.h.s. is k, same as the dim of the matrix X^i.

Non-Holographic deformations

- Relax condition on coefficients (no spin is specified).
- Use also the fermion ψ_ϕ.

$\mathcal{N} = 1$ preserving perturbation: We perturb the Lagrangian with

$$\sum_{j=0}^{(k-2)/2} \sum_{m,\bar{m}=-(j+1)}^{j+1} \left(\lambda_{j;m,\bar{m}} G_{-\frac{1}{2}} \bar{G}_{-\frac{1}{2}} (\psi \bar{\psi} \Phi_j^{\mathrm{su}})_{j+1;m,\bar{m}} e^{-qa_j\phi} + \mathrm{c.c.} \right) .$$

- G is the $\mathcal{N} = 1$ supercurrent, realized in terms of $SU(2)_{k-2}$ currents, ϕ and the fermions.
- Giving Vev's to the X^i's (dictates the density $\rho(\mathbf{x})$) determines

$$\lambda_{j;m,\bar{m}} \sim \widetilde{\mathrm{tr}}(X^{i_1} X^{i_2} \cdots X^{i_{2j+2}}) .$$

- Perturbation produces a purely bosonic term, and two fermionic terms, one quadratic and one quartic.
- Should be cast in the form of an supersymmetric σ-model [Zumino, 79; Alvarez-Gaume-Freedman, 81; Howe-Sierra, 84]

$$\begin{aligned} \mathcal{L} &= e^{\imath}_{-} e^{\imath}_{+} + i \Psi^{\imath}_{+} \left(\partial_{-} \Psi^{\imath}_{+} + \omega^{\imath}{}_{-k} \Psi^{k}_{+} \right) \\ &\quad + i \Psi^{\imath}_{-} \left(\partial_{+} \Psi^{\imath}_{-} + \omega^{\imath}{}_{+k} \Psi^{k}_{-} \right) + \frac{1}{2} R_{\imath\jmath kl} \Psi^{\imath}_{+} \Psi^{\jmath}_{+} \Psi^{k}_{-} \Psi^{l}_{-} , \end{aligned}$$

- For a marginal perturbation $h = 1$, we should have, either
  - $a_j = j + 1$: Normalizable operators (vanish as $\phi \to \infty$).
  - $a_j = -j$: Non-normalizable operators (diverge as $\phi \to \infty$).

Summary, classification

Relating m to j and paying attention exceptional cases we arrive at:

operator	chiral	primary	$\mathcal{N}=2$	$\mathcal{N}=4$	spacetin
$\psi^{+}\Phi_{j;j}e^{-q(j+1)\phi}$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
$\psi^{+}\Phi_{j;j}e^{qj\phi}$		$\surd$			
$(\psi\Phi_j)_{j+1;m}e^{-q(j+1)\phi}$			$\surd$	$\surd$	$\surd$
$(\psi\Phi_j)_{j+1;m}e^{qj\phi}$					
$\psi_3 e^{-q\phi}$		$\surd$	$\surd$	$\surd$	$\surd$
ψ_3			$\surd$		
$(\psi\Phi_j)_m e^{-q(j+1)\phi}$					
$(\psi\Phi_j)_m e^{qj\phi}$			$\surd$	$\surd$	$\surd$
$\psi\Phi_{j;-j}e^{-q(j+1)\phi}$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
$\psi\Phi_{j;-j}e^{qj\phi}$	$\surd$		$\surd$		
$\psi_\phi e^{-q\phi}$		$\surd$	$\surd$	$\surd$	$\surd$
ψ_ϕ			$\surd$	$\surd$	$\surd$

Deforming the brane's position

To describe deformation of the brane's position and not a flow to a different theory:

- Should preserve $\mathcal{N}=4$, but only in the normalizable branch.

Using the operators of $\mathcal{N}=4$ realized in terms of the $SU(2)_{k-2}$ currents, ϕ and the fermions [Kounnas-Porrati-Rostand, 91], we find:

- $a_j = j+1$: Only the normalizable branch preserves $\mathcal{N}=4$.
- Not all values of m, for fixed j, are allowed, but

$$m = j+1: \qquad \mathcal{V}_{j;j+1} = \psi^{+}\Phi^{\rm su}_{j,j} e^{-q(j+1)\phi} \text{ chiral primary },$$

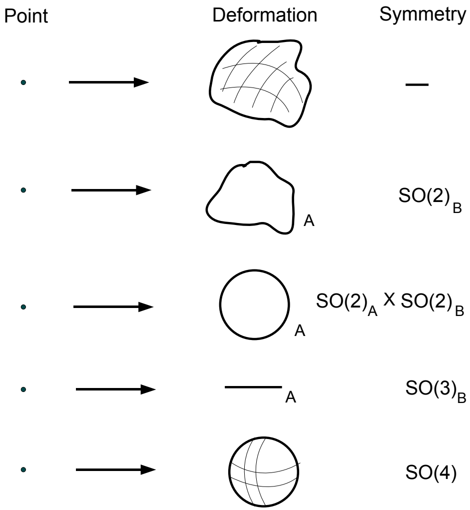
$$m = -j-1: \qquad \mathcal{V}_{j;-j-1} = \psi^{-}\Phi^{\rm su}_{j,-j} e^{-q(j+1)\phi} \text{ anti-chiral primary }.$$

  Chiral (anti-) primaries [Lerche-Vafa-Warner, 89] have $h = \pm q/2$.

- An exception

$$\psi_3 e^{-q\phi} \text{ primary, but not chiral }.$$

NS5-branes configurations and their symmetries in the continuum



Holographic examples

NS5-branes on a circle

The background for NS5-branes on a circle is [K.S., 98]

$$\begin{aligned} ds^2 &= k\left[d\rho^2 + d\theta^2 + \frac{1}{\Sigma}(\tanh^2\rho\ d\tau^2 + \tan^2\theta\ d\psi^2)\right] \ , \\ B_{\tau\psi} &= \frac{k}{\Sigma} \ , \qquad \Sigma = \tanh^2\rho\tan^2\theta + 1 \ , \end{aligned}$$

plus a dilaton.

- Expanding for large ρ ($\phi = \sqrt{2k}\ \rho$)

$$\begin{aligned} ds^2 &= ds^2_{(0)} + 4e^{-2\rho}(\sin^4\theta\ d\psi^2 - \cos^4\theta\ d\tau^2) + \mathcal{O}\left(e^{-4\rho}\right) \ , \\ B_{\tau\psi} &= B^{(0)}_{\tau\psi} + 4e^{-2\rho}\sin^2\theta\cos^2\theta + \mathcal{O}\left(e^{-4\rho}\right) \ , \end{aligned}$$

- The perturbation is just

$$J_3\bar{J}_3 e^{-q\phi} \qquad \Longleftrightarrow \qquad \psi^3\bar{\psi}^3 e^{-q\phi} \ ,$$

NS5-branes on a finite line segment

- Background has an $SO(3) \subset SO(4)$ symmetry.
- The bosonic part of the perturbation is

$$(J^+ \bar{J}^+ + J^- \bar{J}^- + 2J^3 \bar{J}^3) e^{-q\phi} ,$$

- This commutes with the $SO(3)$ generated by

$$J^3 - \bar{J}^3 , \qquad J^+ - \bar{J}^- , \qquad J^- - \bar{J}^+ ,$$

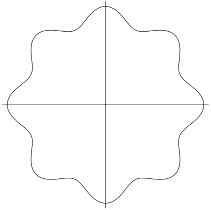
in accordance with the symmetry of the deformation.

Deformation of a finite size circle

The appropriate CFT with $\mathcal{N} = 4$ SUSY is [K.S., 98]

$$\mathbb{R}^{5,1} \times \frac{SU(2)_k}{U(1)} \times \frac{SL(2,\mathbb{R})_k}{U(1)} \ ,$$

orbifolded under a $\mathbb{Z}_k$ discrete symmetry.



- Perturbation in terms of compact parafermions dressed with primaries of the non-compact theory.
- Fermions enter through their bosonized versions.
- New feature: For finite k, no purely bosonic or fermionic terms.
- A clear semiclassical σ-model picture in the limit $k \gg 1$.

Non-holographic examples

Conformal to Egutchi–Hanson metric

Consider the 4-dim background conformal to Hyperkähler metric.

- If the conformal factor is harmonic, then it has generically $\mathcal{N} = (4,1)$ supersymmetry.
- For the Egutchi–Hanson Hyperkähler metric

$$\nabla^2_{\mathrm{EH}} H = 0 \implies H = \frac{1}{r^2}\left[1 + \frac{a^4}{3r^4} + \mathcal{O}\left(\frac{a^8}{r^8}\right)\right] .$$

- To leading order background is $SU(2) \times \mathbb{R}_\phi$.
- The bosonic part of the perturbation is $(r = e^{q\phi/2})$

$$e^{-2q\phi}\left[\left(\Phi_{1;1,-1} J^- - \Phi_{1;-1,-1} J^+ + 4\Phi_{1;0,-1} J^3\right) \bar{J}^- + \cdots\right] .$$

- The holomorphic part of the deformation preserves $\mathcal{N} = 4$, but the non-holomorphic none.

Beyond the near-horizon

Want to "restore" the "1" in the harmonic function H.
Perturbation should:

- Be non-normalizable, since the asymptotic behavior changes.
- Preserve $(4,4)$ susy, since there is no-susy enhancement.

Perturbation is written is two equivalent ways

- Manifestly $\mathcal{N}=4$ for the holomorphic part

$$e^{q\phi}\Big[\left(\Phi_{1;1,-1}J^- - \Phi_{1;-1,-1}J^+ - 2\Phi_{1;0,-1}J^3\right)\bar{J}^- + \cdots\Big]\ .$$

- Manifestly $\mathcal{N}=4$ for the anti-holomorphic part

$$e^{q\phi}\Big[\left(\Phi_{1;-1,1}\bar{J}^- - \Phi_{1;-1,-1}\bar{J}^+ - 2\Phi_{1;-1;0}\bar{J}^3\right)J^- + \cdots\Big]\ .$$

Therefore it preserves $(4,4)$ susy.

Summary, concluding remarks

We studied NS5-background deformations in two different ways:

- By using the proposed holography, relating NS5-brane world-volume to CFT marginal operators preserving $\mathcal{N} = 4$.
- by computing the deformation of the supergravity solution and of the corresponding σ-model.
- Comparison and complete agreement at the semiclassical level. Non-trivial confirmation of the holographic correspondence.
- Susy breaking deformations (adding angular momentum) [K.S., unpublished].

Some related future work:

- Check the correspondence for operators on the NS5-brane world-volume, in relation also to thermodynamics of LST.
- CFT deformations on the wold-volume. Appropriate supergravity solution seems to exist [G. Papadopoulos].