# Asymptotic Safety in Quantum Einstein Gravity

Martin Reuter

(I) The Effective Average Action approach to Quantum Einstein Gravity and Asymptotic Safety

(II) The "Reconstruction Problem": From the effective RG flow to a regularized functional integral

Standard quantization of gravity $\widehat{=}$

| | |
|---|---|
| degrees of freedom carried by: | $g_{\mu\nu}(x)$ |
| bare action: | $\int d^4x \sqrt{-g}\, R$ |
| calculational method: | $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{8\pi G}\, h_{\mu\nu}$, perturbative quantization, renormalization |

What should be given up in order to arrive at a "fundamental" or "microscopic" quantum theory of gravity?

String Theory: d.o.f., action, calc. meth.

Loop Quantum Gravity: d.o.f., calc. meth.

Asymptotic Safety: calc. meth., action

# Asymptotic Safety Approach:

- degrees of freedom carried by $g_{\mu\nu}$
- quantization/renormalization is non-perturbative in an essential way
- bare action $\Gamma_*$ is not an ad hoc assumption, but a <u>prediction</u>:

  $\Gamma_* \sim \int d^4x \sqrt{-g}\, R$ + "more" is a non-Gaussian fixed point of the ($\infty$-dimensional, non-pert.) Wilsonian renormalization group flow

- fixed point "controls" UV divergences

# The Effective Average Action $\Gamma_k[g_{\mu\nu}, \dots]$

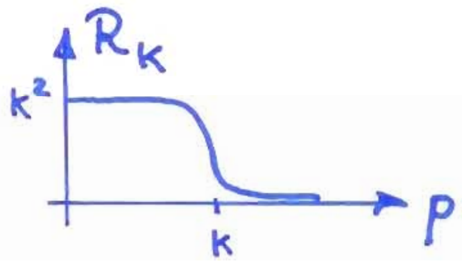
- Scale-dependent (coarse grained) effective action functional for the metric

- Defines family of effective field theories:
  $\{\Gamma_k \mid 0 \leq k < \infty\}$

- Built-in IR cutoff: Only metric fluctuations with cov. momentum $p > k$ are integrated out fully.

  Modes with $p < k$ are suppressed by "mass" term added to the bare action:

  $$(\text{mass})^2 = R_k(p^2)$$

- $$\Gamma_{k\to\infty} = S = \text{bare action}$$
  $$\Gamma_{k\to 0} = \Gamma = \text{standard eff. action}$$

- $\Gamma_k$ satisfies a FRGE; symbolically:

  $$k\partial_k \Gamma_k = \frac{1}{2}\, \mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\partial_k R_k\right]$$

- Natural (nonperturbative) approximation scheme: project RG flow onto truncated theory space

# Construction of $\Gamma_k$ for Gravity

M.R. 1996

- starting point : $\int \mathcal{D}\gamma_{\mu\nu}\, e^{-S[\gamma_{\mu\nu}]}$

- decompose $\gamma_{\mu\nu} = \underbrace{\bar{g}_{\mu\nu}}_{\substack{\text{arbitrary} \\ \text{backgrd. metric}}} + h_{\mu\nu}$

- add background gauge fixing $S_{gf}[h;\bar{g}]$ + ghost terms

- expand $h_{\mu\nu}$ in $\bar{D}^2$-eigenmodes, and introduce IR cutoff $k^2$ : only modes with generalized momenta ($\bar{D}^2$-eigenvalues) $> k$ are integrated out.

- add sources : generating fctl. $W_k[\text{sources};\bar{g}]$

$$\Big\downarrow \text{Legendre transf.}$$

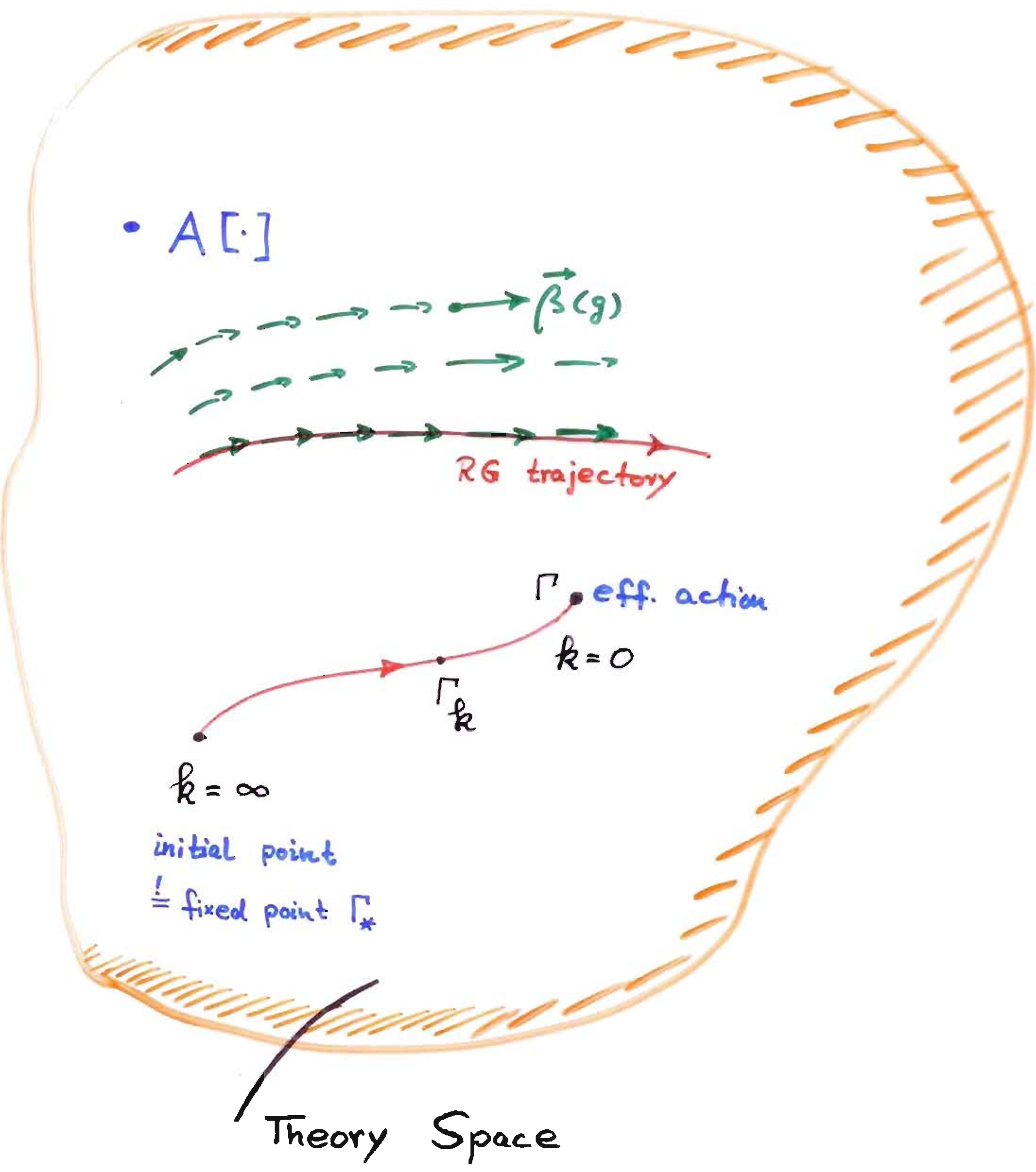
$g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle$ $\qquad \Gamma_k[g_{\mu\nu}, \bar{g}_{\mu\nu}, \text{ghosts}]$

- derive exact RG equation from path integral:

$$k \frac{\partial}{\partial k} \Gamma_k[g, \bar{g}, \cdots] = \mathrm{Tr}(\cdots)$$

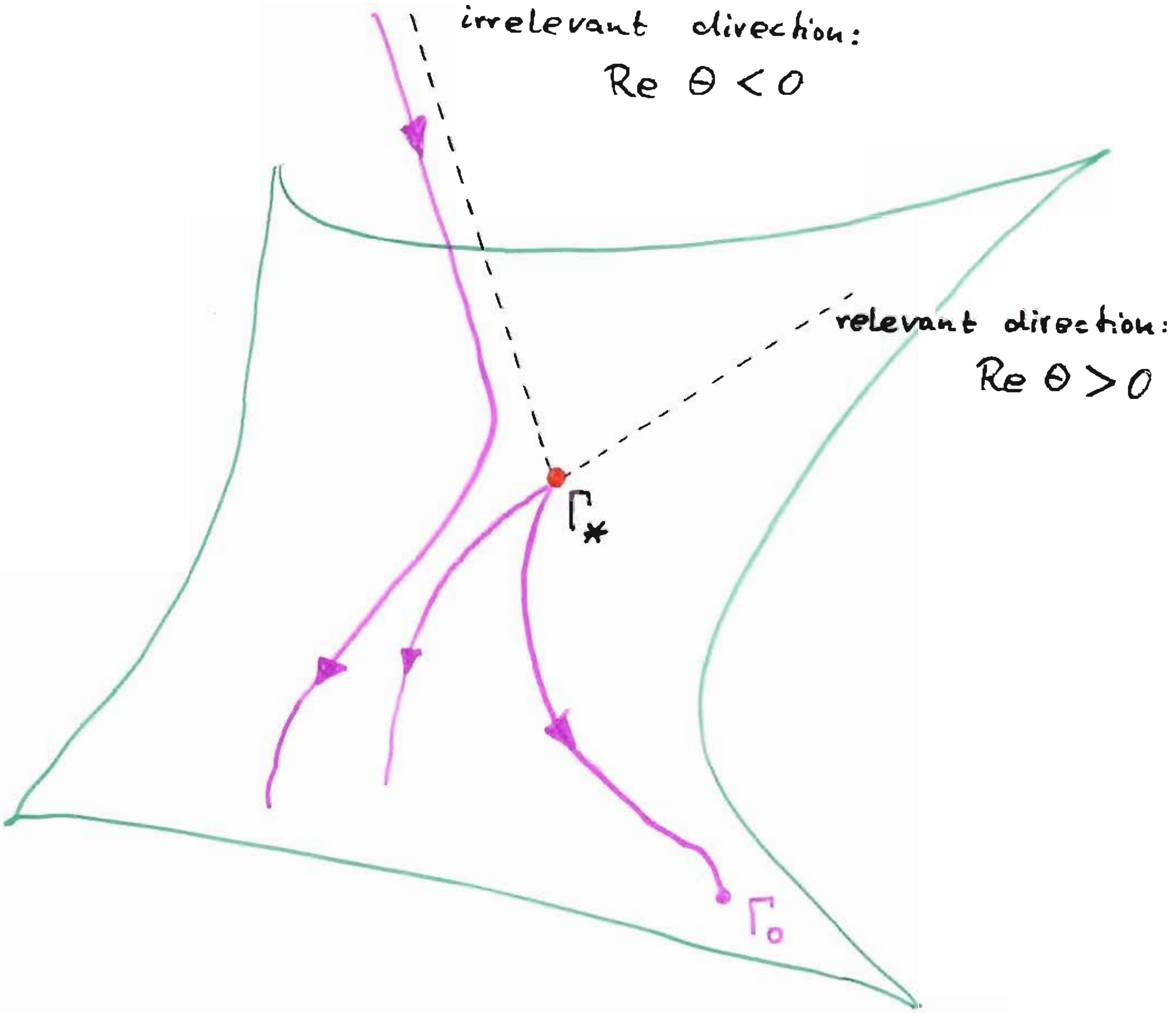
- "Ordinary" diffeomorphism invariant action:

$$\Gamma_k[g] = \Gamma_k[g, \bar{g} = g, \text{ghosts} = 0]$$



• A[·]
β(g)
RG trajectory
Γ eff. action
k = 0
Γ_k
k = ∞
initial point
≟ fixed point Γ_*
Theory Space

## The UV-critical hypersurface $\mathcal{S}_{UV}$ :




$\Delta_{UV} \equiv \dim \mathcal{S}_{UV}$ = # relevant directions

= # free parameters in the a.s. quantum field theory

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UV $\longrightarrow$ IR

$\Theta$ : critical exponent (neg. eigenvalue of lin. flow)

# Taking the UV-limit in QEG

If there exists a non-Gaussian Fixed Point $\Gamma_*$, $\beta_i(\Gamma_*) = 0$, Quantum Einstein Gravity is nonperturbatively renormalizable ("asymptotically safe").

Weinberg 1979

Quantum theory is defined by a RG trajectory running inside the UV-critical hypersurface of the FP, with

initial point = $\Gamma_{k\to\infty} \stackrel{!}{=}$ action infinitesimally close to $\Gamma_*$

end point = $\Gamma_0 \equiv \Gamma$

# The Einstein-Hilbert Truncation

(M.R., 1996)

ansatz:

$$\Gamma_k = -\frac{1}{16\pi G_k} \int d^dx \sqrt{g} \, \{ R - 2\Lambda_k \}$$

+ classical gauge fixing and ghost terms

two running parameters:

Newton constant $G_k$ , dimensionless: $g(k) = k^{d-2} G_k$

cosmological constant $\Lambda_k$ , dimensionless: $\lambda(k) = \Lambda_k / k^2$

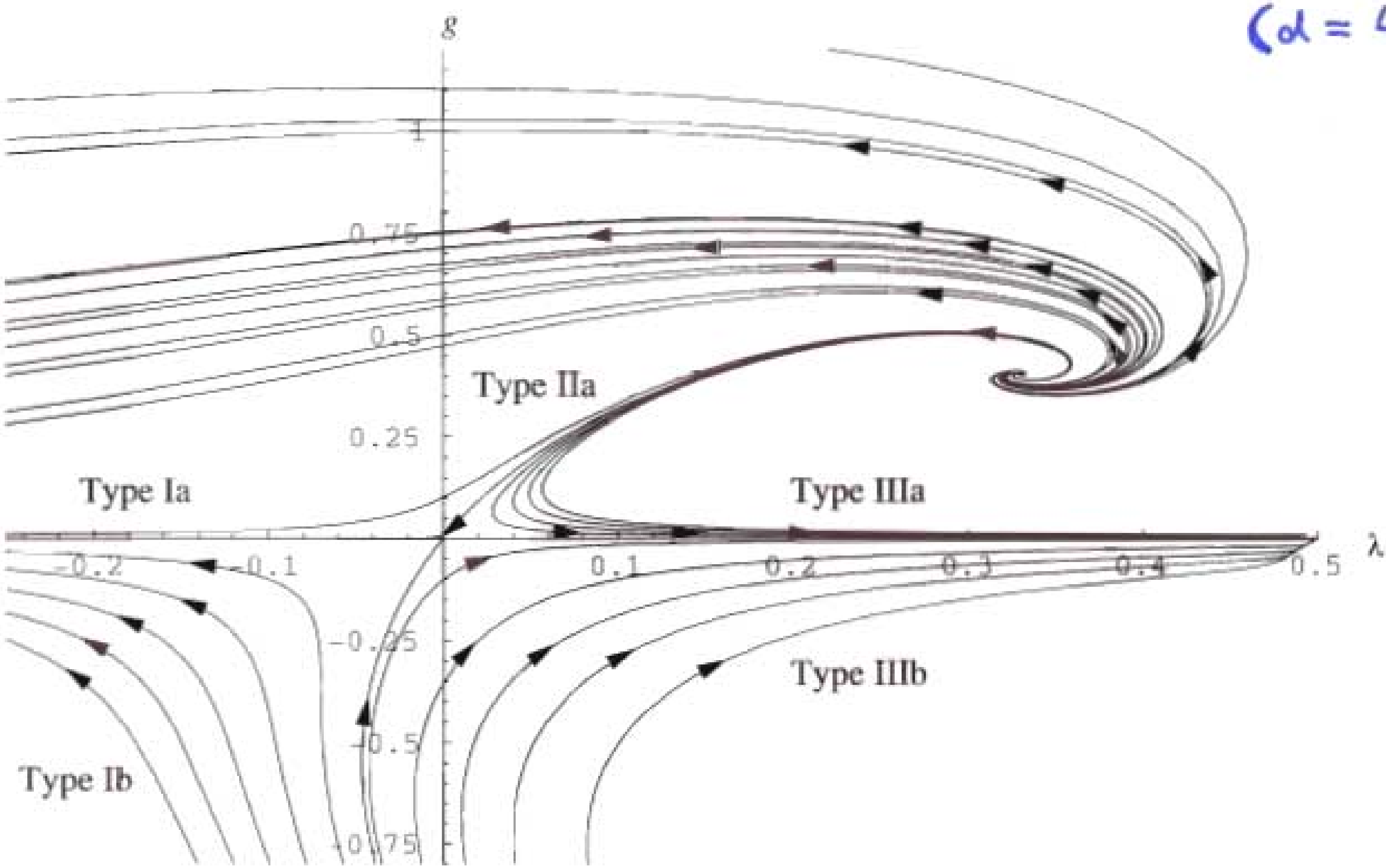
insert ansatz into flow equation, expand

$$\mathrm{Tr}[\cdots] = (\cdots) \int \sqrt{g} \; + (\cdots) \int \sqrt{g} R \; + \cdots$$

$$\leadsto \quad \boxed{\begin{aligned} k \partial_k g(k) &= \beta_g(g, \lambda) \\ k \partial_k \lambda(k) &= \beta_\lambda(g, \lambda) \end{aligned}}$$

# RG - Flow in the Einstein - Hilbert Truncation

(d = 4)




M.R., F. Saueressig, hep-th/0110054

# The Reconstruction Problem

E. Manrique, M.R.

$$\int \mathcal{D}\gamma_{\mu\nu}\, e^{-S[\gamma_{\mu\nu}]}$$

only formal: not well defined in the UV

$\Big\downarrow$ gauge fix, add sources, add IR cutoff, $\partial_k$

FRGE

well defined in the UV

$$\int \mathcal{D}\gamma_{\mu\nu}\, e^{-S[\gamma_{\mu\nu}]}$$

$\Big\downarrow$ add UV regulator, UV cutoff at $\Lambda$

$$\int \mathcal{D}_{\Lambda}\gamma_{\mu\nu}\, e^{-S_{\Lambda}[\gamma_{\mu\nu}]}$$

$\Big\downarrow$ ...
...
...

FRGE with UV regulator

$\Big\downarrow$ $\Lambda \to \infty$

FRGE without UV regulator

- Flow defined by FRGE without UV-regulator investigated so far:
  Likely to have NGFP,
  used for primary definition of the quantum field theory

- This flow, by itself, does not imply a regularized path integral.

- Extra ingredient to be specified:

  UV regularization scheme (only for path int.)

  (e.g. "finite mode" regularization in the measure, discretization, ... ; $\mathcal{R}_k \to \mathcal{R}_{k,\Lambda}$ not sufficient!)

- The "reconstruction problem":

  Which regularized integral $\int \mathcal{D}_\Lambda \gamma \, e^{-S_\Lambda[\gamma]}$ reproduces in the limit $\Lambda \to \infty$ a given trajectory $k \mapsto \Gamma_k$ ?

  When $\mathcal{D}_\Lambda \gamma$ fixed:

  Which "bare trajectory" $\Lambda \mapsto S_\Lambda$ reproduces a given "effective traj." $k \mapsto \Gamma_k$ ?

Well defined path integral representation is probably not indispensable for a consistent theory, but useful, if it exists:

- Yields UV completion of path integral-based calculations, as dictated by a given $\Gamma$-trajectory.
- Comparison with other approaches (Regge, CDT, canonical quantization, LQG, models from symmetry reduction, ...)
- Reconstruction of the hamiltonian description:

$$\Gamma_* \longrightarrow S_* \longrightarrow H_*$$

(à la Ostrogradski)

$\rightsquigarrow$ Identification of the dof.s which actually were quantized.

⋮

Functional differential eq. for $\tilde{S}_\Lambda[h, \bar{g}, C, \bar{C}]$

( "finite mode" regularization in the UV ) :

$$\boxed{\tilde{S}_\Lambda + \frac{1}{2}\, \mathrm{STr}_\Lambda \ln\left[\left(\tilde{S}^{(2)}_\Lambda + \mathcal{R}_\Lambda\right) M^{-2}\right] = \Gamma_\Lambda}$$

Given solution $\Gamma_k \big|_{k=\Lambda}$ of the FRGE without UV cutoff

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Result for QEG in Einstein-Hilbert truncation for both $S$ and $\Gamma_k$ :

- $S$-flow has NGFP with $\left(g_*^{\text{bare}}, \lambda_*^{\text{bare}}\right) \neq (g_*, \lambda_*)$
- $S$- and $\Gamma$-flows diffeomorphic near NGFP
- identical critical exponents
- $g_*^{\text{bare}} \cdot \lambda_*^{\text{bare}}$ is not universal (Running bare action has no effective field theory properties.)
- Near the GFP: $S$-flow is not diffeomorphic to $\Gamma$-flow, has log-corrections to power law scaling

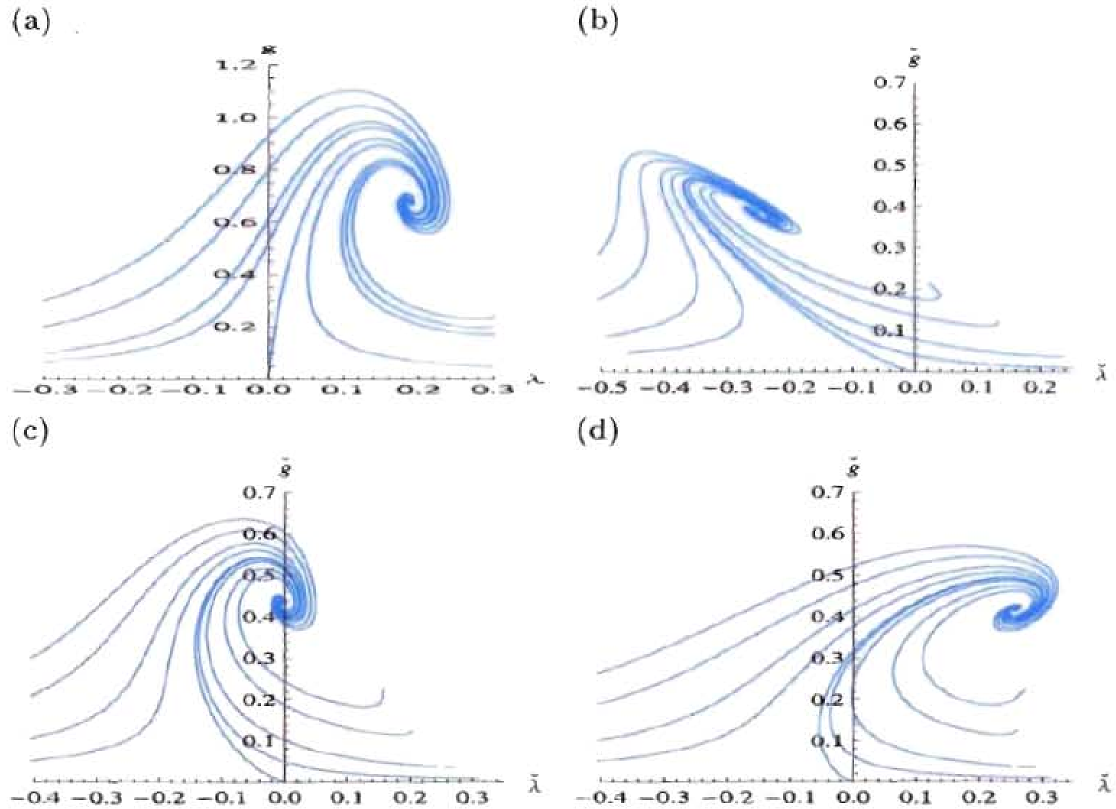


Figure 4: The diagram (a) shows the phase portrait of the effective RG flow on the $(g, \lambda)$-plane. The other diagrams are its image on the $(\check{g}, \check{\lambda})$-plane of bare parameters for three different values of $Q$, namely (b) $Q = +1$ , (c) $Q = -0.1167$ where $\check{\lambda}_* = 0$, and (d) $Q = -1$, respectively.

E. Manrique, M.R., 0811.3888

# Conclusion

Worthwhile to shift attention from the FRGE to the underlying path integral:

- "reconstruction problem"
- Which dof.s did we actually quantize?
- Phase space description
- UV-completion of "old" computations
- Contact with other approaches:

$$S_\Lambda + \left[\text{UV regulator dependent term}\right] = \Gamma_{k=\Lambda}$$