Title :

Yukawa Couplings and Right-Handed Neutrinos in F -Theory Compactifications

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Outline

- 1. Yukawa Couplings and Bundles in Heterotic String
- 2. Yukawa Couplings and Singularities in F-theory
- 3. Extending Heterotic - F-theory Duality
- 3. Right Handed Neutrinos as Complex Structure Moduli
- 4. Yukawa Couplings for Neutral Fields

- SU(5): up-type quarks Yukawa couplings $10^{ij}\ 10^{kl}\ H(5)^m \epsilon_{ijklm}$
- ϵ^{ijklm} cannot be realised in brane configurations
- Can get this term from a coset $E_6/SU(5)$ and then embed E_6 into E_8.
- Need the E_8 group obtained in Heterotic, M theory and F-theory
- We need to describe the breaking of E_8 to $SU(5)_{GUT}$. This can be done in both Heterotic and F-theory
- In heterotic $E_8 \times E_8$ turn on an $SU(5)$ vector bundle within one E_8
- The correspondence between the representations $\rho(V)$ and and those of the unbroken $SU(5)_{GUT}$ is:
- $V \leftrightarrow 10$ (V is the 5 of vector bundle SU(5)), $\wedge^2 V \leftrightarrow \bar{5}$
- How to explicitly build the vector bundle?

- Consider an elliptic fibered Calabi-Yau 3-fold Z, $\pi_Z : Z \longrightarrow B_2$ and a rank-N vector bundle V on Z.

  Need to build a spectral surface of degree N over B_2 and a line bundle $\mathcal{N}_{\mathcal{V}}$ over C_V

- The chiral multiplets in the low energy theory are identified with $H^1(Z, \rho(V))$.

  This is non-zero only along the matter curves

  $\bar{c}_{\rho(V)} = C_{\rho(V)}\ \sigma$

  - Specific example: Rank-5 Bundles

  Spectral surface C_V of rank-5 bundle V is given by

  $s = a_0(u,v) + a_2(u,v)x + a_3(u,v)y + a_4(u,v)x^2 + a_5(u,v)xy = 0;$

  The matter curve of the fundamental representation $\bar{c}_V = C_V \cdot \sigma$ is given by the zero locus of a_5.

The matter curve of $\wedge^2 V$: defining equation of the spectral surface factorises locally as

$$s = (Ax + B)(Py + Qx + R)$$

The factorisation condition is equivalent to

$$P^{(5)} := a_0 a_5^2 - a_2 a_3 a_5 + a_4 a_3^2 = 0$$

The two matter curves $\bar{c}_V$ and $\bar{c}_{\wedge^2 V}$ intersect in B_2 with two different types of intersections:

(a)Multiplicity 1: $a_5 = 0$ and $a_4 = 0$, and hence $P^{(5)} = 0$

(d)Multiplicity 2: $a_5 = 0$ and $a_3 = 0$, and hence $P^{(5)} = 0$.

The form of $P^{(5)}$ reveals that $\bar{c}_{\wedge^2 V}$ forms a double point at each type (d) intersection point.

The covering matter curve $\tilde{\bar{c}}_{\wedge^2 V}$ is obtained by blowing up the double points of the matter curve $\bar{c}_{\wedge^2 V}$, and the map $\tilde{\pi}_D : D \longrightarrow \tilde{\bar{c}}_{\wedge^2 V}$ becomes a degree-2 cover.

Lesson: use the covering matter curves to describe Yukawa couplings

Yukawa couplings in F-theory

- Hints from Heterotic String but valid for models with no Heterotic dual
- $\mathcal{N} = 1$ supersymmetry: F-theory is compactified on an elliptic fibered Calabi–Yau 4-fold $\pi : X \to B_3$
- The discriminant Δ of this elliptic fibration may have several irreducible components

  $\Delta = \sum_i n_i S_i$, S_i are divisors of B_3, n_i their multiplicities

  For $\Delta = nS + D'$, matter multiplets charged under the gauge group on S at $S \cdot D'$
- F-theory phenomenology: Beasley-Heckmann-Vafa and Donagi-Weijnholt
- Many aspects of gauge theory associated with the discriminant locus S, only on the geometry of X around S.

- The study of F-theory on X reduces to the study of an 8-dimensional field theory on S times Minkowski.

  The matter multiplets "see" only the geometry along the $S \cdot D'$ codimension-2 loci of B_3

- The Yukawa couplings –from codimension-3 loci of B_3. Thus, one can go a long way in phenomenology by studying only the local geometry of F-theory compactification.

- Suppose a zero mode exists for $\phi_{mn}(u_1, u_2) du_m \wedge du_n$ on S - transverse fluctuation of D7-branes in Type IIB orientifold compactification on a Calabi–Yau 3-fold X ((u_1, u_2) coordinates of S).

- This corresponds to deforming geometry of X, $\Delta = n'' S'' + S' + D'$

- Singularity along the irreducible discriminant locus S is reduced from g to the commutant g'' of ϕ in g.

Example: Generic Rank-2 Deformation of A_{N+1} Singularity

The most generic form of deformation to A_{N-1} is given by two parameters, s_1 and s_2:

$$Y^2 = X^2 + Z^N(Z^2 + s_1 Z + s_2). \tag{1}$$

An alternative parametrization of deformation

$$2\alpha\phi_{12}(u_1, u_2) = (0, \cdots, 0^N, \tau_{N+1}, \tau_{N+2}), \tag{2}$$

Easy case: $s_1(u_1, u_2) = F_1 u_1 + F_2 u_2, \qquad s_2(u_1, u_2) = F_1 F_2 u_1 u_2$ so $2\alpha\phi_{12} = (0, \cdots, 0^N, F_1 u_1, F_2 u_2)$,

The irreducible decomposition of $su(N+2)$ is
$su(N+2)\text{-}\mathbf{adj}. \to su(N)\text{-}\mathbf{adj} + \left[N^{(-,0)} + N^{(0,-)} + \mathbf{1}^{(+,-)}\right] + \mathrm{h.c.}$

Zero-mode equations give solutions:

- For the $N^{(-,0)}$ and $\bar{N}^{(+,0)}$ components,

$$\tilde{\chi}_{\mp} = c_{\mp} \exp\left[-F_1|u_1|^2\right], \qquad \tilde{\psi}_{\bar{1}\mp} = \pm c_{\mp} \exp\left[-F_1|u_1|^2\right], \qquad \tilde{\psi}_{\bar{2}} = 0. \tag{3}$$

- For the $N^{(0,-)}$ component,

$$\tilde{\chi} = c(u_1) \exp\left[-F_2|u_2|^2\right], \qquad \tilde{\psi}_{\bar{2}} = c(u_1) \exp\left[-F_2|u_2|^2\right], \qquad \tilde{\psi}_{\bar{1}} = 0. \tag{4}$$

- This simple case has a IIB interpretation in terms of open strings between

$N - - - N+1$ D7-branes

$N - - - N+2$ D7 branes respectively

Complicated Case:

$s_1 = 2u_1, \qquad s_2 = u_2,$

This second case of the deformation of A_{N+1} singularity is described by the field theory on a local patch of S with $2\alpha\phi_{12}$ given by

$$\tau_+ \equiv \tau_{N+1} = -u_1 + \sqrt{u_1^2 - u_2}, \qquad \tau_- \equiv \tau_{N+2} = -u_1 - \sqrt{u_1^2 - u_2}. \tag{5}$$

The decomposition is

$$su(N+2)\text{-}\mathbf{adj}. \rightarrow su(N)\text{-}\mathbf{adj}. + su(2)\text{-}\mathbf{adj}. + (\mathbf{2}, N) + (\mathbf{2}, \bar{N}) \tag{6}$$

Resolve A_{N+1} with $N+1$ cycles, 2 of them being

$$C_\pm : \ (x,y,z) = (r(z) i \cos\theta, r(z)\sin\theta, z) \ \ z \in [0, z_\pm] \quad \theta \in [0, 2\pi], \tag{7}$$
$$r(z) \equiv \sqrt{z^N (z - z_-)(z - z_+)} \tag{8}$$

The vev of $2\alpha\phi_{12}$,

$$\left(+\sqrt{u_1^2 - u_2}, -\sqrt{u_1^2 - u_2} \right) \tag{9}$$

becomes $\times(-1)$ of its own around the branch locus $u_1^2 - u_2 = 0$.

Overall, we need to introduce a branch cut extending out from the branch locus $u_1^2 - u_2 = 0$

$su(N+2)$-**adj.** fields are glued to themselves after twisting by Weyl group of $su(2) \subset su(N+2)$ algebra—across the branch cut (cf. Katz-Morrison identification).

This is not a simple theory of fields in the $su(N+2)$-**adj.** representation.

Introduce a new surface: $C(u,x)$, $(u,x) = (u_1, \sqrt{u_1^2 - u_2})$ as the space of all possible values for ϕ. This is a covering space similar to the Heterotic covering matter curve.

Apply this observation to $E_6, D_6 \longrightarrow A_4$:

$A_4 : y^2 = x^3 + a_5xy + a_4zx^2 + a_3z^2y + ...$, and consider case when

$a_4 \longrightarrow 0, \quad a_5 \longrightarrow 0$ i.e. $E_6 \longrightarrow A_4$: up-type Yukawa

$a_3 \longrightarrow 0, \quad a_5 \longrightarrow 0$ i.e. $D_6 \longrightarrow A_4$: down-type Yukawa

The zero-mode wavefunctions of SU(5) - 10 representation are determined as diag$(-a_4/2 \pm \sqrt{a_4/2)^2 - a_5})$ and at small a_4 is goes like $e^{-|a_5|^{3/2}}$

The zero-mode wavefunctions of SU(5) - 5 representation are determined as $-a_4$ and has a Gaussian normal to the matter curve $e^{-|a_4|^2}$

For the $D_6 \longrightarrow A_4$ there is no branch-cut and it can be represented by flat D7-branes intersecting at angles.

These are local models with wavefunctions defined on the surface S. The 0-modes should also be defined as line bundles of globally defined curves.

Main observation: the charged matter multiplets in F-theory are sheaves on spectral covers and not on matter curves.

The spectral surfaces are key notion to generalize objects like D-branes and gauge bundles on them.

Supersymmetric compactification of F-theory is described by 8-dim. field theory with a Higgs bundle (F, ϕ) as background.

$$\bar{\partial}_{\bar{m}}\phi = 0, \partial_m\overline{\phi} = 0, F - i[\phi, \overline{\phi}] = 0$$

are Hitchin equations for Higgs bundles.

For Higgs bundles, the techniques are very similar as the ones for spectral covers and one needs to build a pair $(C_V, \mathcal{N}_V)$ denoted $(C_V, \mathcal{N}_V)^{\mathrm{F}}$

Heterotic - F Theory duality. The duality map is simply stated:

$$(C_V, \mathcal{N}_V)^{\text{Het}} = (C_V, \mathcal{N}_V)^{\text{F}}. \tag{10}$$

The spectral data is used in both sides of the duality and is mapped under duality.

Avoids some subtleties related to del Pezzo fibrations usually used in discussing Heterotic-F theory duality.

Important to map the heterotic $\mathcal{N}_V = \mathcal{O}\left(\frac{1}{2}r + \gamma\right)$ into F -theory

γ corresponds to four-form fluxes in F-theory.

Four Form Fluxes and Neutrino Masses

Right-handed neutrinos $\bar{N}$ are not charged under $SU(5)_{GUT}$

$$\Delta\mathcal{L} = \lambda^{(\nu)}_{ij}\bar{N}_i l_j h_u + \text{h.c.}$$

l_j are lepton doublets and h_u the Higgs doublet.

Any moduli chiral multiplet in supersymmetric string compactification can be identified with chiral multiplets of right-handed neutrinos, as long as they have the trilinear interactions.

For measured value of atmospheric neutrino oscillation

$$\Delta m^2 \sim 2\text{–}3 \times 10^{-3}$$

the lightest right-handed neutrino is not heavier than about

$$\frac{(v\lambda_\nu)^2}{\sqrt{\Delta m^2}} = \lambda_\nu^2 \times (5.5\text{–}6.7) \times 10^{14}$$

Here, λ_ν is the neutrino Yukawa couplings.

The complex structure moduli have interactions in the superpotential

$$\Delta W = W_{\text{GVW}} = \int_X \Omega \wedge G.$$

A generic flux G determine a mass for all the complex structure moduli from the Gukov–Vafa–Witten superpotential.

In Type IIB string compactification on Calabi–Yau orientifolds, complex structure moduli acquire masses

$$m_{cs}^2 \sim m_{\text{KK}}^6 l_s^4 = \left[m_{\text{KK}} \times \left(\frac{l_s}{R_6} \right)^2 \right]^2 .$$

The complex structure moduli of F-theory compactifications contain both complex structure moduli and D7-brane moduli of Type IIB orientifolds.

The 4-form fluxes of F-theory correspond both to the 3-form fluxes and to gauge bundles on D7-branes in Type IIB orientifolds

By using $i\frac{1}{g_{s,\mathrm{IIB}}} = i\frac{\rho_\beta}{\rho_\alpha} \qquad M_* = \frac{1}{g_s l_s^4} = \frac{\rho^2}{l_{11}^6}$

we find that $m_{cs} \sim \frac{1}{R_6^3 M_*^2}$ is valid as an estimate of all the complex structure moduli masses in F-theory compactification.

Set

$$\epsilon \equiv \left(\frac{R_{\mathrm{GUT}}}{R_6}\right)^3 = \frac{\sqrt{4\pi} M_{\mathrm{GUT}}}{\alpha_{\mathrm{GUT}} c M_{\mathrm{Pl}}} \sim 0.35 \times (\frac{M_{\mathrm{GUT}}}{c\; 10^{16} GeV})$$

with R_6 the size of B_3 and R_{GUT}^4 the volume of the locus of A_4 singularity, the masses of complex structure moduli become

$$m_{cs} \sim M_{\mathrm{GUT}} \times \frac{\sqrt{\alpha_{\mathrm{GUT}}}}{c} \times \left(\epsilon^{\gamma=1}\right)^{0-3}$$

with $\frac{1}{\alpha_{GUT}} = 24, M_{GUT} = 10^{16}GeV$,
$M_{Pl}^2 = 4\pi R_6^6 M_*^8 = (2.4 \times 10^{18}GeV)^2$.

The $SU(5)_{GUT}$ singlet field in the Yukawa couplings - fluctuations from the vacuum in $H^{1,2}(X,C)$ and $H^{3,1}(X,C)$

The $H^{1,2}(X,C)$ fluctuations are calculated by the overlap integral $\int_S tr(\chi_U \wedge \psi_{adj(U)} \wedge \psi_{\bar{U}})$ with χ_U, ψ_U coming from the fluctuations of the chiral matter multiplet min the $(U, \bar{5})$ of $G \times SU(5)_{GUT}$

The $H^{3,1}(X,C)$ fluctuations are calculated by the overlap integral $\int_S tr(\psi_U \wedge \chi_{adj(U)} \wedge \psi_{\bar{U}})$.

Conclusions

1. Heterotic Strings: It is important to extend the spectral sequence constructions to include other representations of V besides the fundamental.

Singularities need to resolved by considering the covering matter curves

2. F-theory picture: based on the 8-dimensional field theory plus the uplift of S to covering spaces.

Extend the Heterotic - F theory duality beyond orientifold limit. Use Higgs Bundles.

4. Right-hand handed neutrinos - complex moduli space