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A Bayesian approach to supersymmetry phenomenology

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There are three kinds of lies: lies, damned lies, and statistics.
(Mark Twain, reportedly quoting Benjamin Disraeli)

The problem

Introduction

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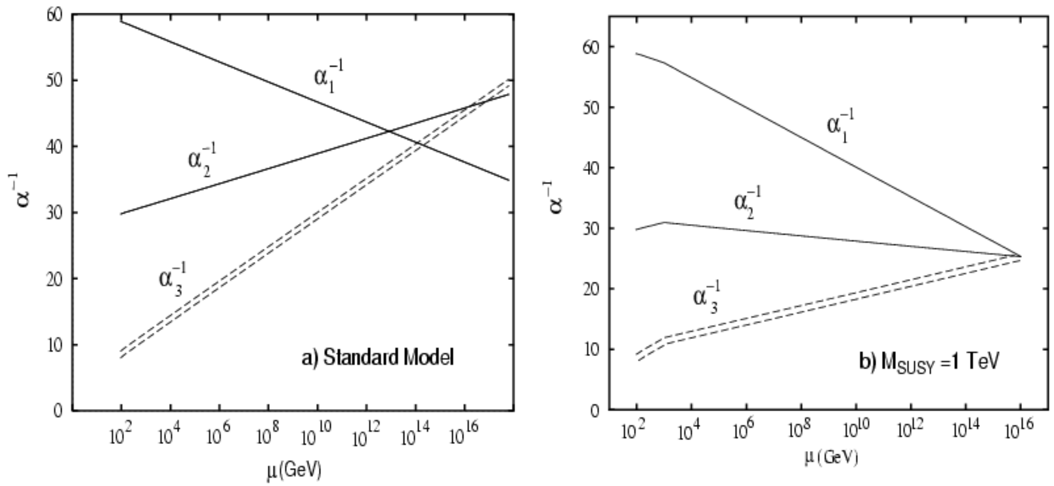
- Why supersymmetry? (SUSY)



Amaldi et al (1991)

The particle content of SUSY

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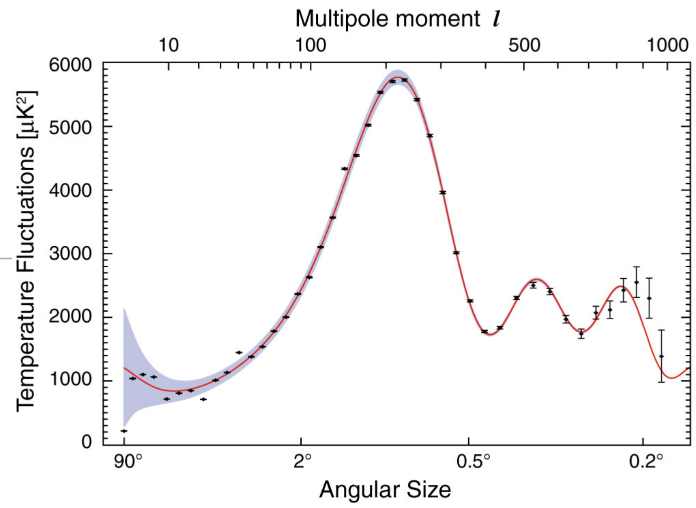
Standard Model particles and fields		Supersymmetric partners			
		Interaction eigenstates		Mass eigenstates	
Symbol	Name	Symbol	Name	Symbol	Name
$q = d, c, b, u, s, t$	quark	$\tilde{q}_L, \tilde{q}_R$	squark	$\tilde{q}_1, \tilde{q}_2$	squark
$l = e, \mu, \tau$	lepton	$\tilde{l}_L, \tilde{l}_R$	slepton	$\tilde{l}_1, \tilde{l}_2$	slepton
$\nu = \nu_e, \nu_\mu, \nu_\tau$	neutrino	$\tilde{\nu}$	sneutrino	$\tilde{\nu}$	sneutrino
g	gluon	$\tilde{g}$	gluino	$\tilde{g}$	gluino
$W^\pm$	W-boson	$\tilde{W}^\pm$	wino	$\tilde{\chi}^\pm_{1,2}$	chargino
H^-	Higgs boson	$\tilde{H}_1^-$	higgsino		
H^+	Higgs boson	$\tilde{H}_2^+$	higgsino		
B	B-field	$\tilde{B}$	bino	$\tilde{\chi}^0_{1,2,3,4}$	neutralino
W^3	W^3-field	$\tilde{W}^3$	wino		
H_1^0	Higgs boson	$\tilde{H}_1^0$	higgsino		
H_2^0	Higgs boson	$\tilde{H}_2^0$	higgsino		
H_3^0	Higgs boson				

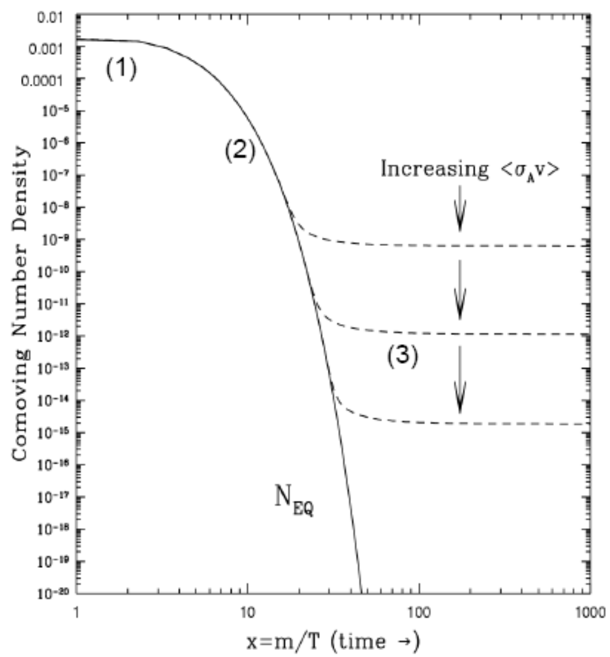
From Bertone, Hooper & Silk (2005)

Links with cosmology

- Strong cosmological and astrophysical evidence for the existence of Cold Dark Matter (CDM)
- The lightest SUSY particle (the neutralino) is a natural candidate for the CDM
- Cosmological DM abundance constrained (with a few assumptions) within ~ 10%:
  WMAP 5 yrs data:
  $\Omega_{CDM} h^2 = 0.1099 \pm 0.0062$



WMAP team



Courtesy Sabine Kraml

Links with astrophysics

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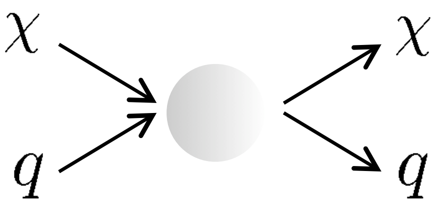
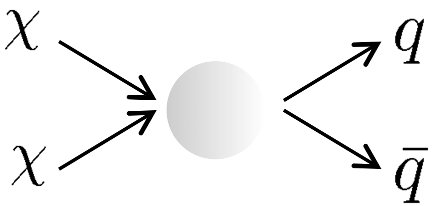


Direct detection

Underground detectors looking for neutralino from local halo
scattering off nuclei
(complicated by local WIMP distribution)

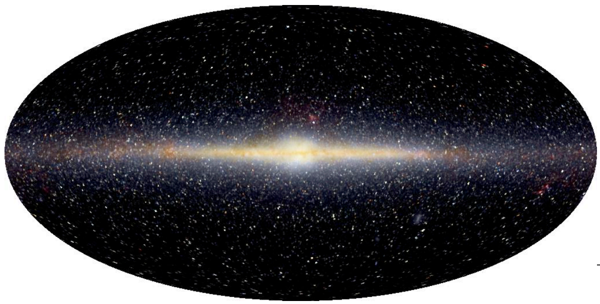
CRESST



Indirect detection

Look for neutralino-neutralino annihilation products:
gamma ray (continuum and & lines)
antimatter (e.g., positrons)
neutrinos (from Sun & Earth)
(complicated by gastrophysical nuisance parameters)

Fermi



The model & data

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- The general Minimal Supersymmetric Standard Model (MSSM): **105 free parameters!**
- Need some (pretty strong) simplifying assumption: **the Constrained MSSM** (CMSSM) reduces the free parameters to just **4 continous variables plus a discrete one** (sign(μ)).
- Clearly a highly constrained model (probably not the end of the story!)
- **Present-day data:** collider measurements of rare processes, CDM abundance (WMAP), sparticle masses lower limits, EW precision measurements. Soon, LHC sparticle spectrum measurements.
- **Astrophysical direct and indirect detection techniques might also be competitive:** neutrino (IceCUBE), gamma-rays (Fermi), antimatter (PAMELA), direct detection (XENON, CDMS, Eureca, Zeplin)

Analysis pipeline

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SCANNING ALGORITHM

4 CMSSM parameters
$\theta = \{m_0, m_{1/2}, A_0, \tan\beta\}$
(fixing sign(μ) > 0)

4 SM "nuisance parameters"
$\Psi=\{m_t, m_b, \alpha_S, \alpha_{EM}\}$

Data:
Gaussian likelihoods for each of the Ψ_j (j=1...4)

RGE

Non-linear numerical function
via SoftSusy 2.0.18
DarkSusy 4.1
MICROMEGAS 2.2
FeynHiggs 2.5.1
Hdecay 3.102

Observable quantities
$f_i(\theta, \Psi)$

CDM relic abundance
BR's
EW observables
g-2
Higgs mass
sparticle spectrum
(gamma-ray, neutrino, antimatter flux, direct detection x-section)

Physically acceptable?
EWSB, no tachyons, neutralino CDM

NO → Likelihood = 0

YES → Joint likelihood function

Data:
Gaussian likelihood
(CDM, EWO, g-2, $b \to s\gamma$, ΔM_{Bs})
other observables have only lower/upper limits

The accessible "surface"

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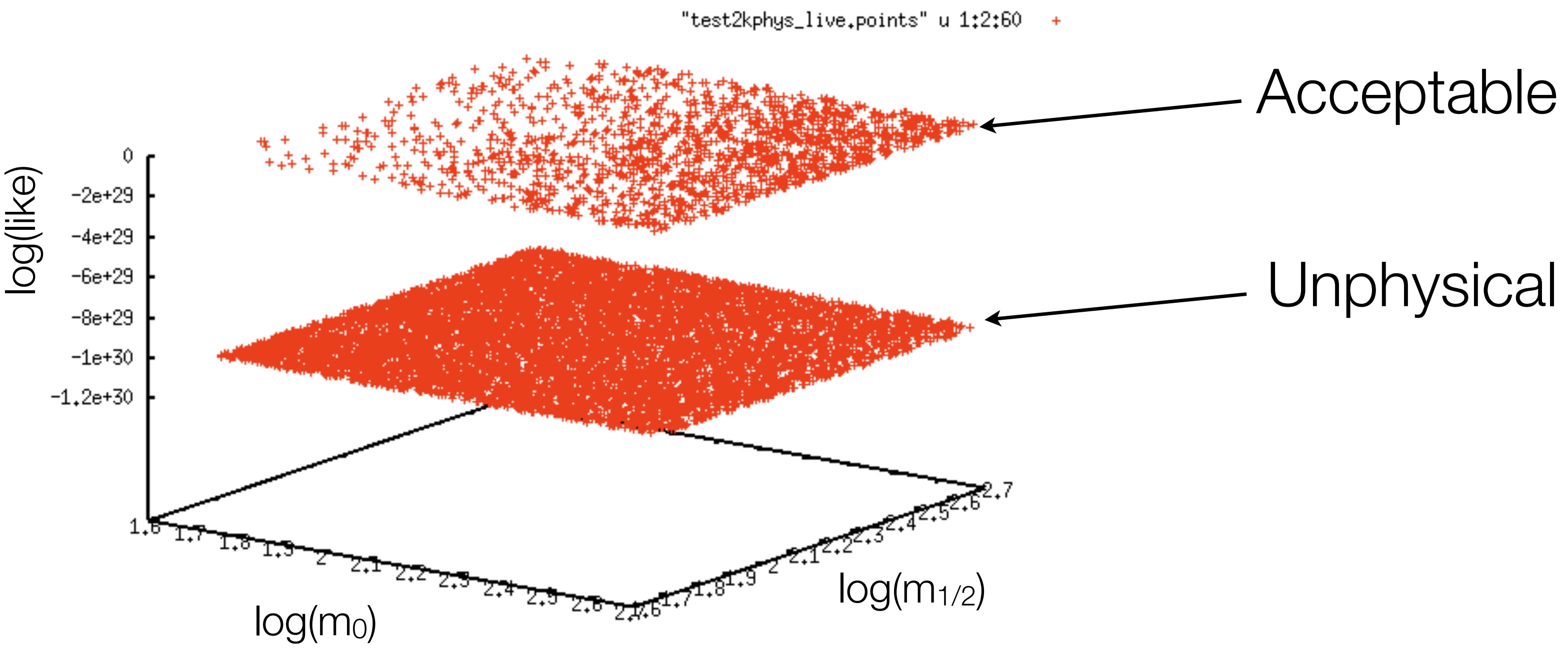
Scan from the prior with no likelihood except physicality constraints



Courtesy F. Feroz & M. Bridges

Data included

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Indirect observables

Observable	Mean value μ	Uncertainties σ (exper.)	τ (theor.)	ref.
M_W	80.398 GeV	25 MeV	15 MeV	[30]
$\sin^2\theta_{\rm eff}$	0.23153	16×10^{-5}	15×10^{-5}	[30]
$\delta a_\mu^{\rm SUSY} \times 10^{10}$	29.5	8.8	1.0	[31]
$BR(\overline{B} \to X_s\gamma) \times 10^4$	3.55	0.26	0.21	[32]
ΔM_{B_s}	17.77 ps^{-1}	0.12 ps^{-1}	2.4 ps^{-1}	[33]
$BR(\overline{B}_u \to \tau\nu) \times 10^4$	1.32	0.49	0.38	[32]
$\Omega_\chi h^2$	0.1099	0.0062	$0.1\,\Omega_\chi h^2$	[34]
	Limit (95% CL)		τ (theor.)	ref.
$BR(\overline{B}_s \to \mu^+\mu^-)$	$< 5.8 \times 10^{-8}$		14%	[35]
m_h	$>$ 114.4 GeV (SM-like Higgs)		3 GeV	[36]
ζ_h^2	$f(m_h)$ (see text)		negligible	[36]
$m_{\tilde{q}}$	$>$ 375 GeV		5%	[25]
$m_{\tilde{g}}$	$>$ 289 GeV		5%	[25]
other sparticle masses	As in table 4 of ref. [6].			

SM parameters

SM (nuisance) parameter	Mean value μ	Uncertainty σ (exper.)	Ref.
M_t	172.6 GeV	1.4 GeV	[24]
$m_b(m_b)^{\overline{MS}}$	4.20 GeV	0.07 GeV	[25]
$\alpha_s(M_Z)^{\overline{MS}}$	0.1176	0.002	[25]
$1/\alpha_{\rm em}(M_Z)^{\overline{MS}}$	127.955	0.03	[26]

Why is this a difficult problem?

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- **Inherently 8-dimensional:** reducing the dimensionality over-simplifies the problem. Nuisance parameters (in particular m_t) cannot be fixed!
- **Likelihood discontinuous** and multi-modal due to physicality conditions
- RGE connect input parameters to observables in highly non-linear fashion: **only indirect (sometimes weak) constraints on the quantities of interest (-> prior volume effects are difficult to keep under control)**
- **Mild discrepancies between observables** (in particular, g-2 and $b \rightarrow s\gamma$) tend to pull constraints in different directions

2 dimensional slices

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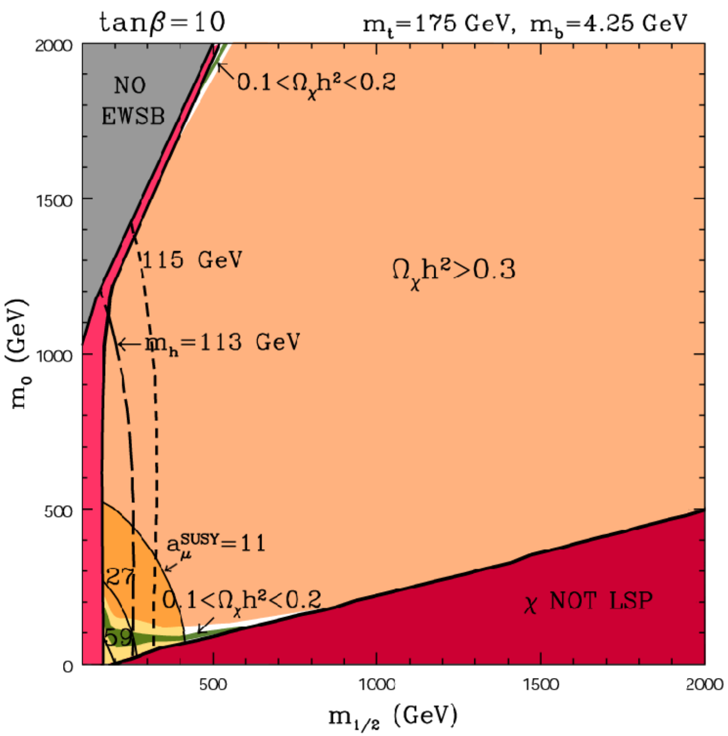

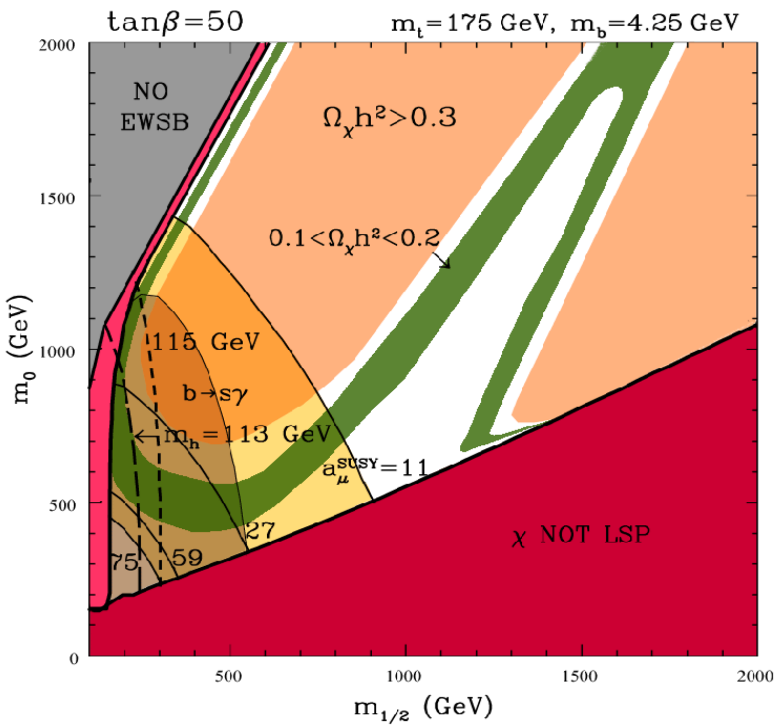

Roszkowski et al (2001)

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The “WMAP strips”

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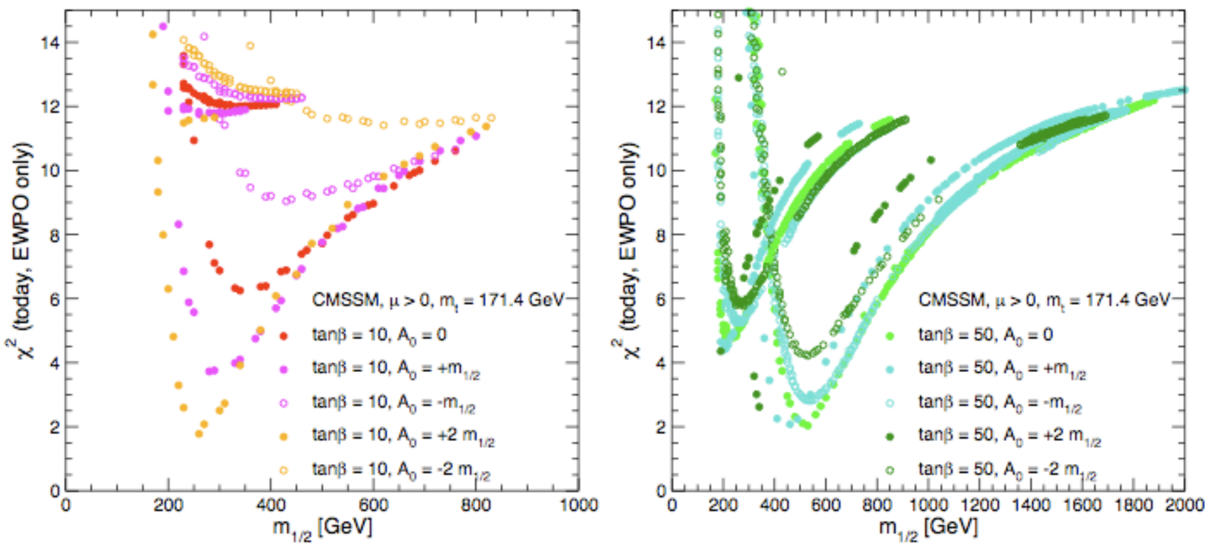
m_0 fixed by requiring that the CDM abundance matches the WMAP value.
All nuisance parameters fixed.



Ellis et al (2007)

Impact of nuisance parameters

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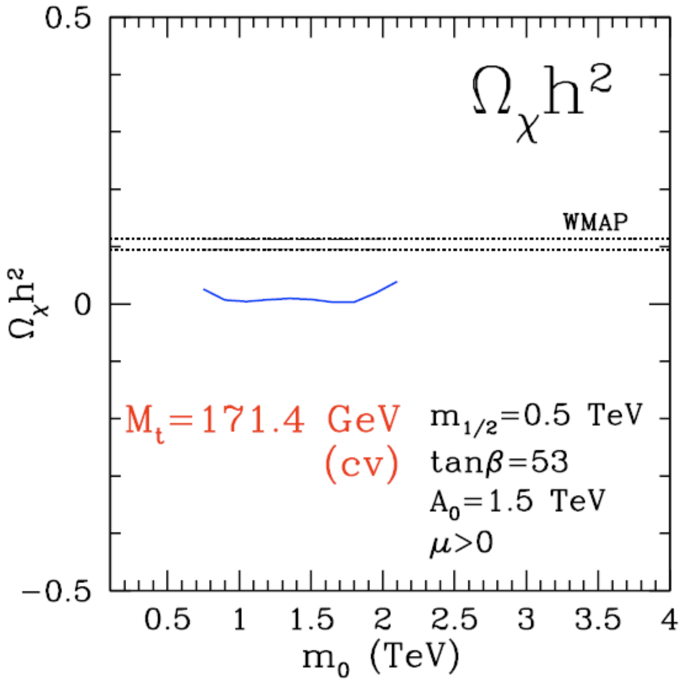

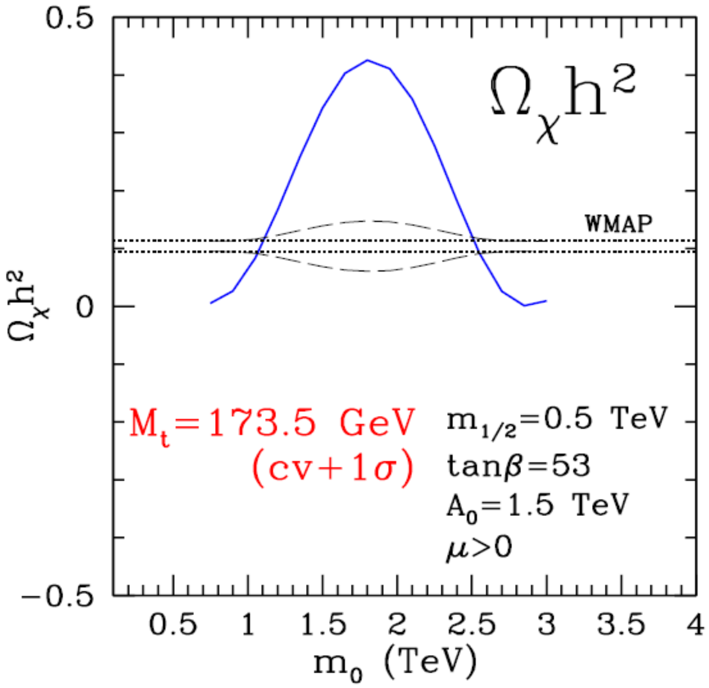

Roszkowki et al (2007)

Random scans

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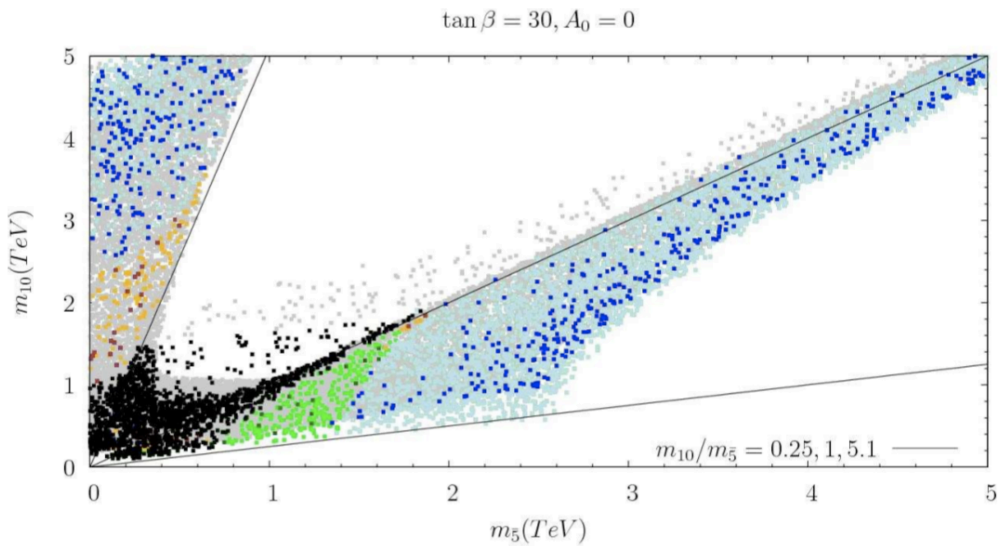

Gogoladze et al (2008)

- Points accepted/rejected in a in/out fashion (e.g., 2-sigma cuts)
- No statistical measure attached to density of points
- No probabilistic interpretation of results possible
- Inefficient/Unfeasible in high dimensional parameters spaces (N>3)

Bayesian parameter inference

The Bayesian approach

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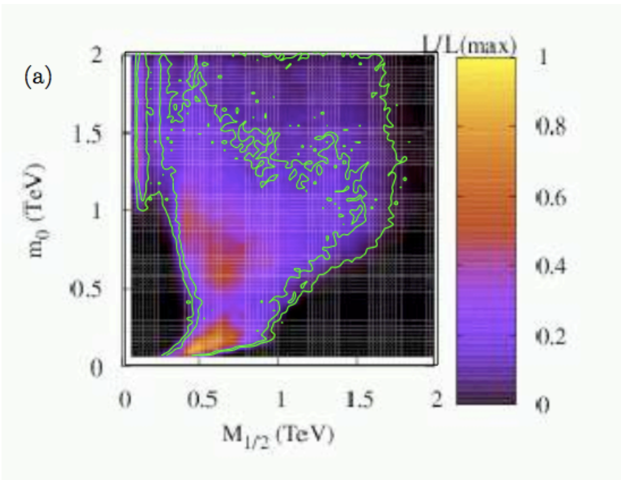
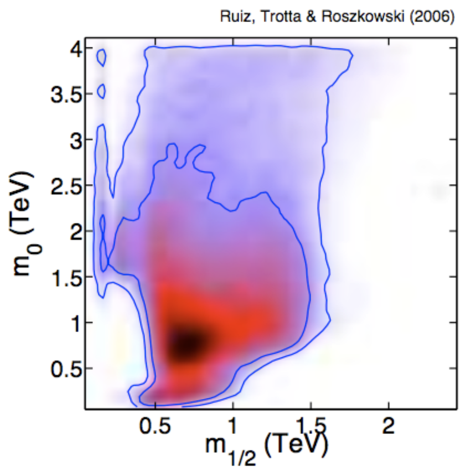
- Bayesian approach led by two groups (early work by Baltz & Gondolo, 2004):
- Ben Allanach (DAMPT) et al (Allanach & Lester, 2006 onwards, Cranmer, and others)
- Ruiz de Austri, Roszkowski & RT (2006 onwards)
  SuperBayeS public code (available from: superbayes.org)
  + Feroz & Hobson (MultiNest), + Silk (indirect detection), + Strigari (direct detection), + Martinez et al (dwarfs), + de los Heros (IceCube)



Allanach & Lester (2006)



Ruiz de Austri, Roszkowski & RT (2006)

See also, e.g.: Ellis et al (2004, 2005, 2006), Baltz & Gondolo (2004), Buchmuller et al (2008)

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Key advantages

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- **Efficiency:** computational effort scales ~ N rather than k^N as in grid-scanning methods. Orders of magnitude improvement over previously used techniques.
- **Marginalisation:** integration over hidden dimensions comes for free.
- **Inclusion of nuisance parameters:** simply include them in the scan and marginalise over them. **Notice:** nuisance parameters in this context must be well constrained using independent data.
- **Derived quantities**: probabilities distributions can be derived for any function of the input variables (crucial for DD/ID/LHC predictions)

Bayesian pdf for an 8D model

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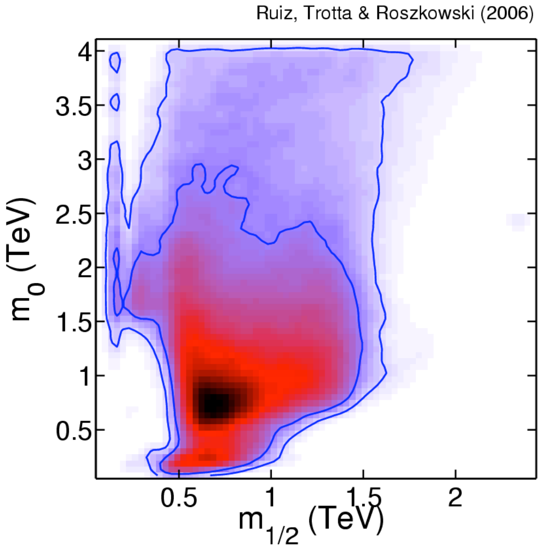
Experimental value: BR(b →sγ) = $(3.55 \pm 0.26)\times 10^{-4}$
Theoretical value SM: BR(b →sγ) = $(3.12 \pm 0.21)\times 10^{-4}$ (Misiak et al , 2006)
Previous SM value: BR(b →sγ) = $(3.60 \pm 0.30)\times 10^{-4}$

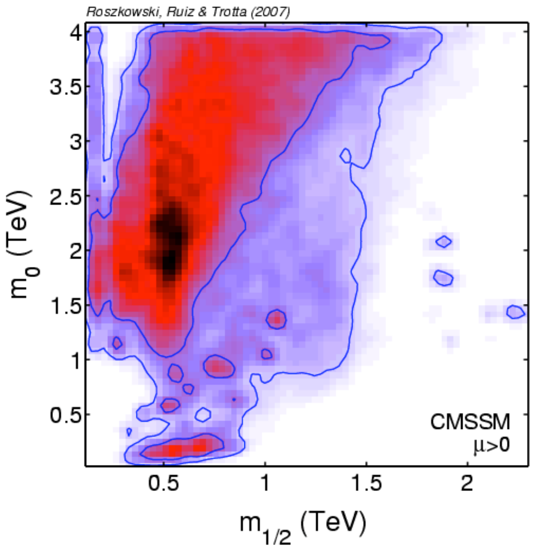
2006

(Ruiz de Austri et al 2006)

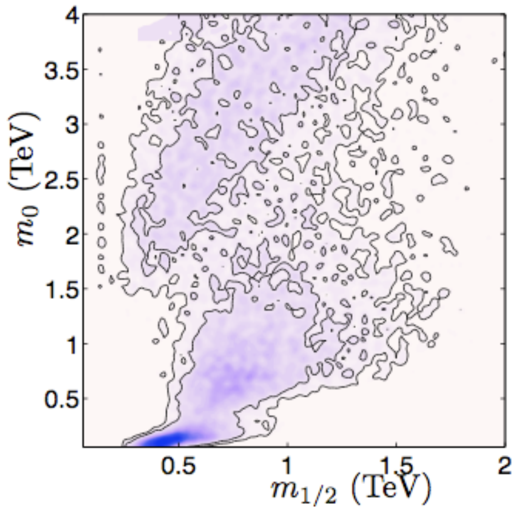


2007

(Roszkowski et al 2007)



(Feroz et al 2008)



including new SM BR(b →sγ)

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Bayes' theorem

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posterior

likelihood

prior

$$P(H|d,I) = \frac{P(d|H,I)P(H|I)}{P(d|I)}$$

evidence

H: hypothesis
d: data
I: external information

- **Prior**: what we know about H (given information I) before seeing the data
- **Likelihood**: the probability of obtaining data d if hypothesis H is true
- **Posterior:** our state of knowledge about H after we have seen data d
- **Evidence**: normalization constant (independent of H), crucial for model comparison

Continuous parameters

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$$P(\theta|d,I) = \frac{P(d|\theta,I)P(\theta|I)}{P(d|I)}$$

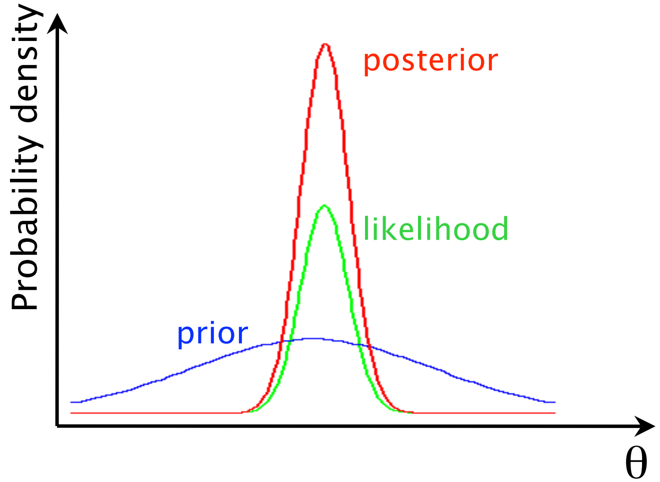
Bayesian evidence: average of the likelihood over the prior

$$P(d|I) = \int d\theta P(d|\theta,I)P(\theta|I)$$

For parameter inference it is sufficient to consider

$$P(\theta|d,I) \propto P(d|\theta,I)P(\theta|I)$$

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$



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Prior dependence

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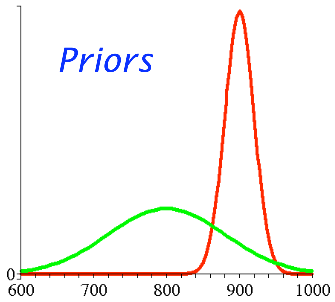
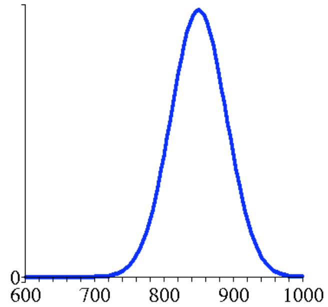
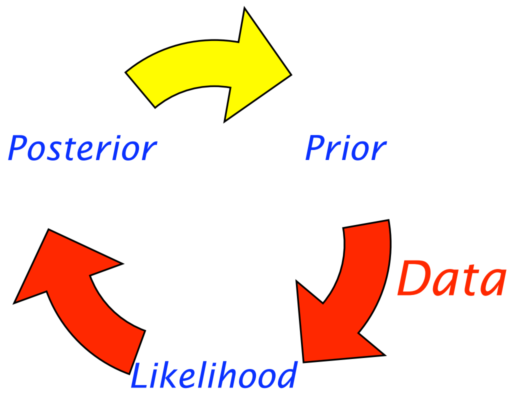
- In parameter inference, prior dependence will **in principle** vanish for strongly constraining data.
  THIS IS CURRENTLY NOT THE CASE EVEN FOR THE CMSSM!

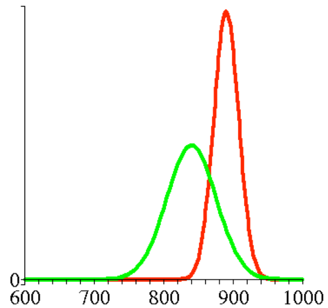


Likelihood (1 datum)

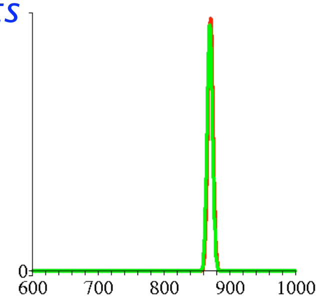




Posterior after 1 datum



Posterior after 100 data points



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The SuperBayeS package (superbayes.org)

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- Supersymmetry Parameters Extraction Routines for Bayesian Statistics
- Implements the CMSSM, but can be easily extended to the general MSSM
- Currently linked to SoftSusy 2.0.18, DarkSusy 4.1, MICROMEGAS 2.2, FeynHiggs 2.5.1, Hdecay 3.102. **New release (v 1.36) upcoming!**
- Includes up-to-date constraints from all observables
- Fully parallelized, MPI-ready, user-friendly interface à la cosmomc (thanks Sarah Bridle & Antony Lewis)
- Bayesian MCMC or grid scan mode, plotting routines.
  NEW: MULTI-MODAL NESTED SAMPLING (Feroz & Hobson 2008), efficiency increased by a factor 200. **A full 8D scan now takes 3 days on a single CPU** (previously: 6 weeks on 10 CPUs)

MCMC estimation

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$$P(\theta|d, I) \propto P(d|\theta, I)P(\theta|I)$$

- A Markov Chain is a list of samples θ_1, θ_2, θ_3,... whose density reflects the (unnormalized) value of the posterior

- A MC is a sequence of random variables whose *(n+1)*-th elements only depends on the value of the *n*-th element

- **Crucial property:** a Markov Chain converges to a stationary distribution, i.e. one that does not change with time. In our case, the posterior.

- From the chain, expectation values wrt the posterior are obtained very simply:

$$\langle\theta\rangle = \int d\theta P(\theta|d)\theta \approx \frac{1}{N}\sum_i \theta_i$$

$$\langle f(\theta)\rangle = \int d\theta P(\theta|d)f(\theta) \approx \frac{1}{N}\sum_i f(\theta_i)$$

MCMC estimation

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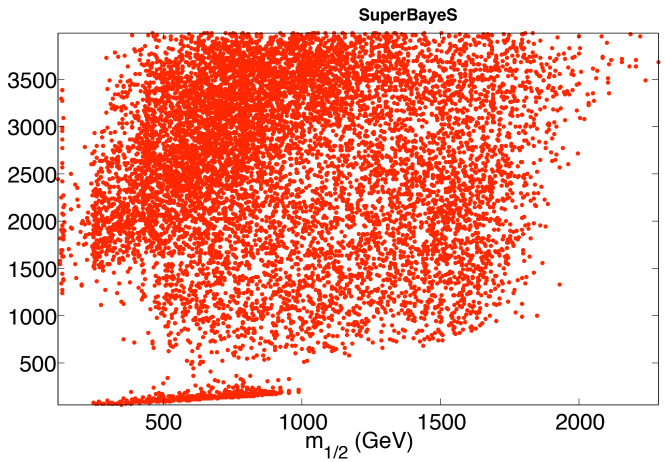
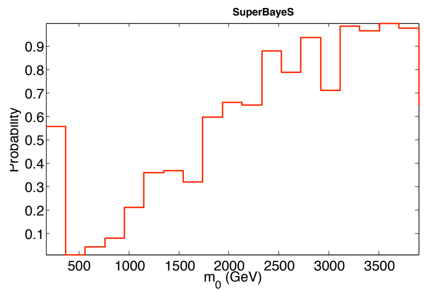
- **Marginalisation becomes trivial:** create bins along the dimension of interest and simply count samples falling within each bins ignoring all other coordinates

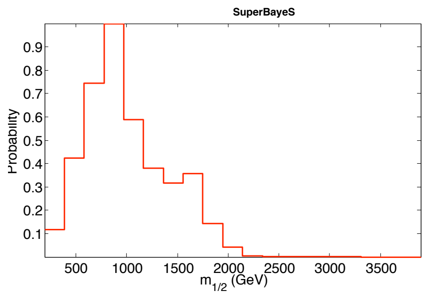
- Examples (from **superbayes.org**) :

2D distribution of samples from joint posterior





1D marginalised posterior (along y)



1D marginalised posterior (along x)

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Fancier stuff

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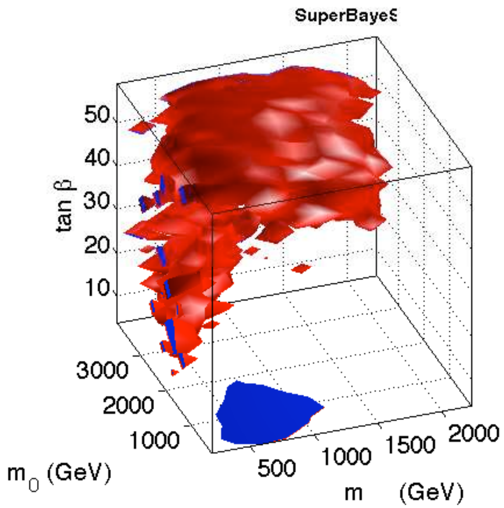

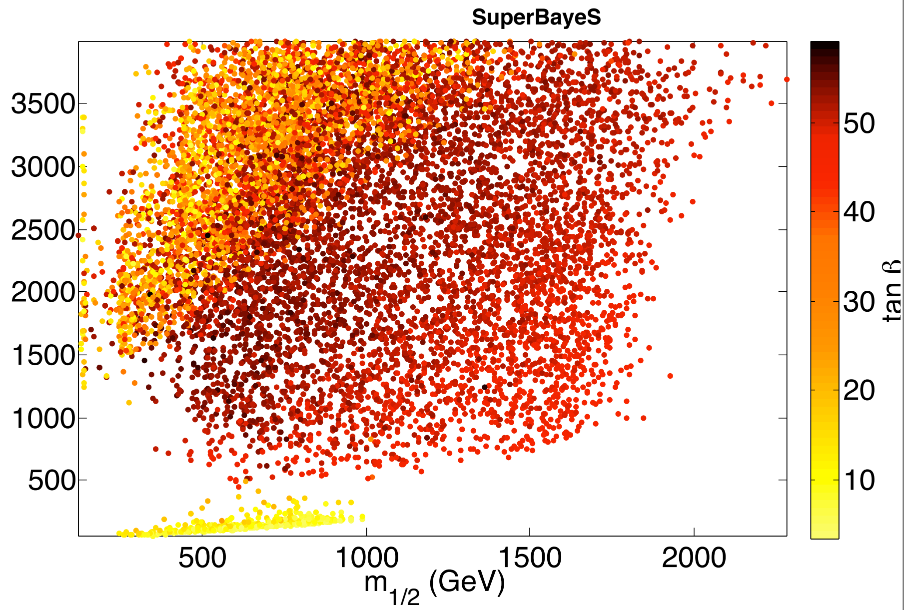

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Nested sampling

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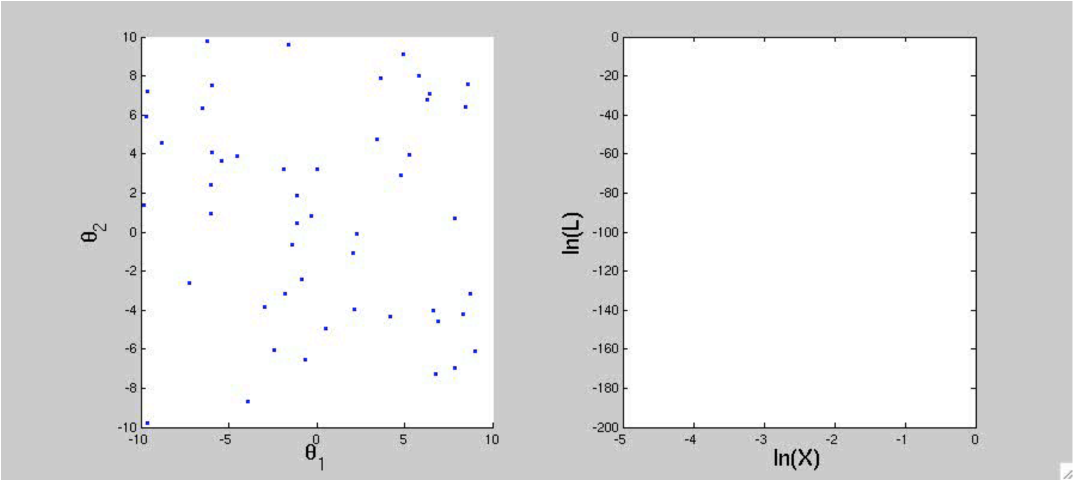
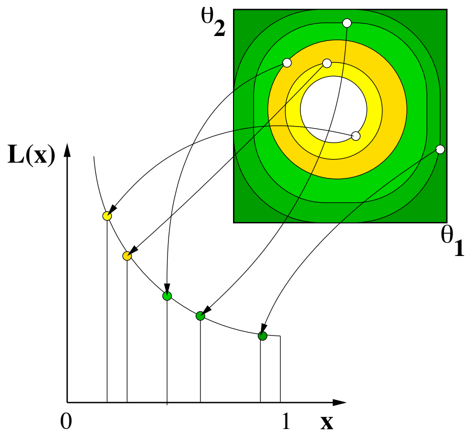


(animation courtesy of David Parkinson)



An algorithm originally aimed primarily at the Bayesian evidence computation (Skilling, 2006):

$$X(\lambda) = \int_{\mathcal{L}(\theta)>\lambda} P(\theta) d\theta$$

$$P(d) = \int d\theta \mathcal{L}(\theta) P(\theta) = \int_0^1 X(\lambda) d\lambda$$

Feroz et al (2008), *arxiv: 0807.4512*, Trotta et al (2008), *arxiv: 0809.3792*

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The MultiNest algorithm

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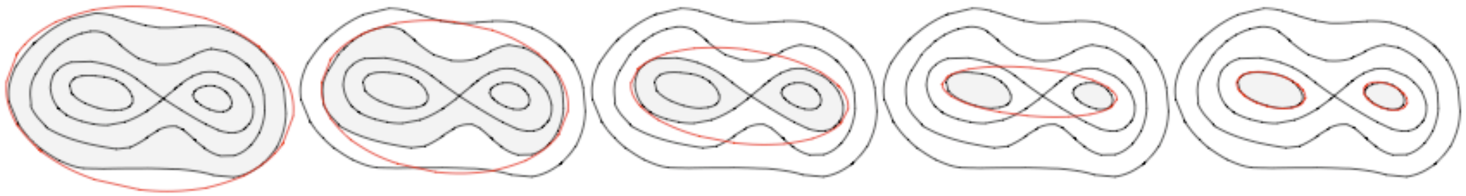
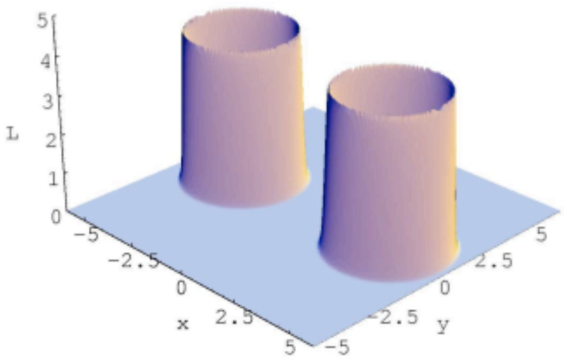
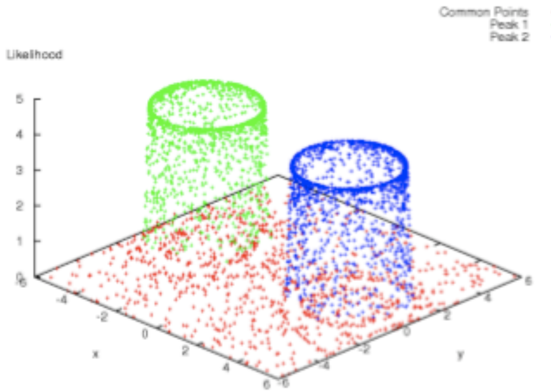
- MultiNest: Also an extremely efficient sampler for multi-modal likelihoods!
  Feroz & Hobson (2007), RT et al (2008), Feroz et al (2008)





Target Likelihood



Sampled Likelihood

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Global CMSSM constraints

See also recent works by Ellis et al (2004, 2005, 2006), Baltz & Gondolo (2004), Buchmuller et al (2008), Allanach & collaborators (2006, 2007,2008)

Priors

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- There is a vast literature on priors: Jeffreys', conjugate, non-informative, ignorance, reference, ...
- In simple problems, "good" priors are dictated by symmetry properties
- "Flat priors" (i.e., uniform in the model's parameters) are often uncritically adopted as default by cosmologists/physicists: **they do not necessarily reflect indifference/ ignorance.** Beware: in large dimensions, most of the volume of a sphere is near its surface!
- For the SM parameters we adopt flat priors (with cutoff well beyond the region where the likelihood is non-zero). This is largely unproblematic as the nuisance parameters are directly constrained by the likelihood hence the posterior is dominated by the likelihood
- Priors for the CMSSM parameters: this is a difficult issue

Parameter inference (all data included)

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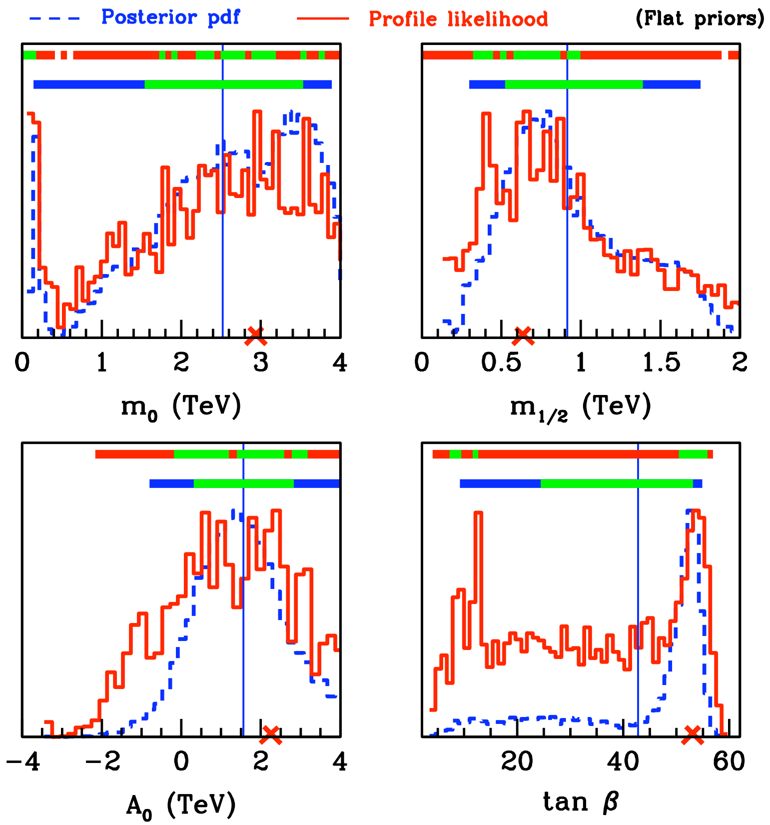
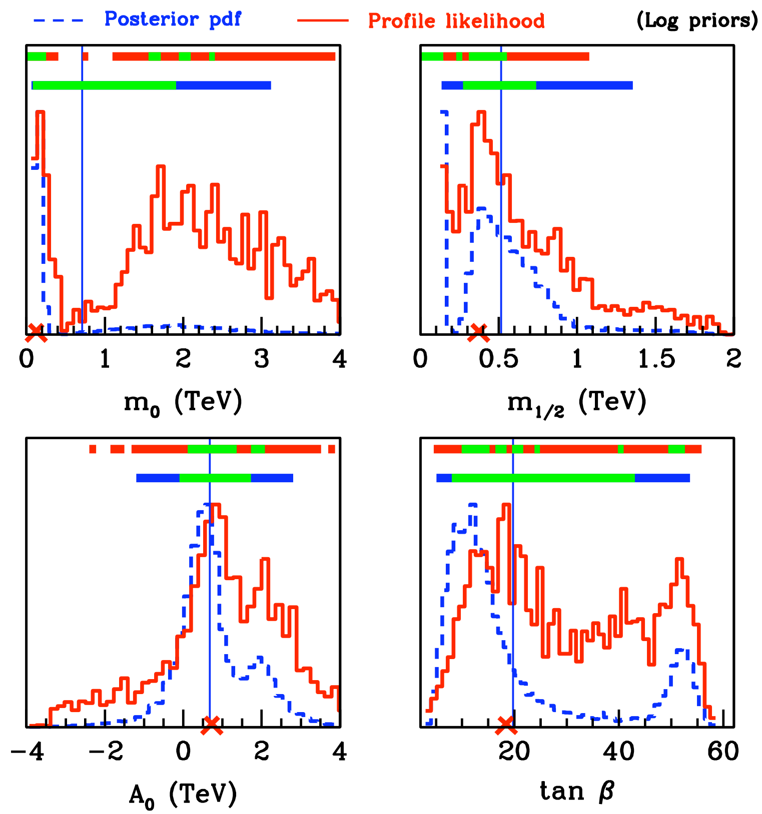
Flat priors



Log priors



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2D posterior vs profile likelihood

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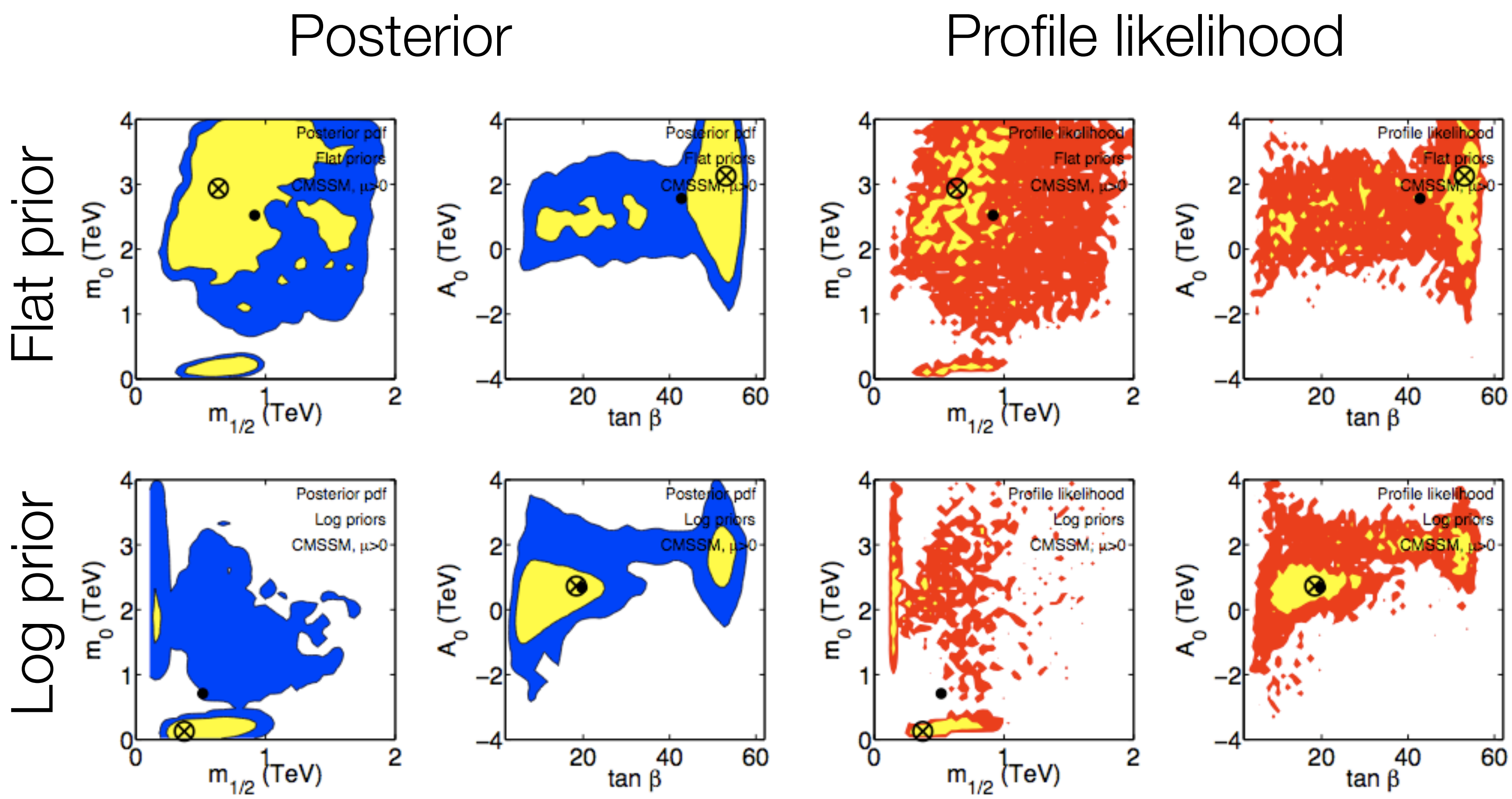

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Comparison with other work

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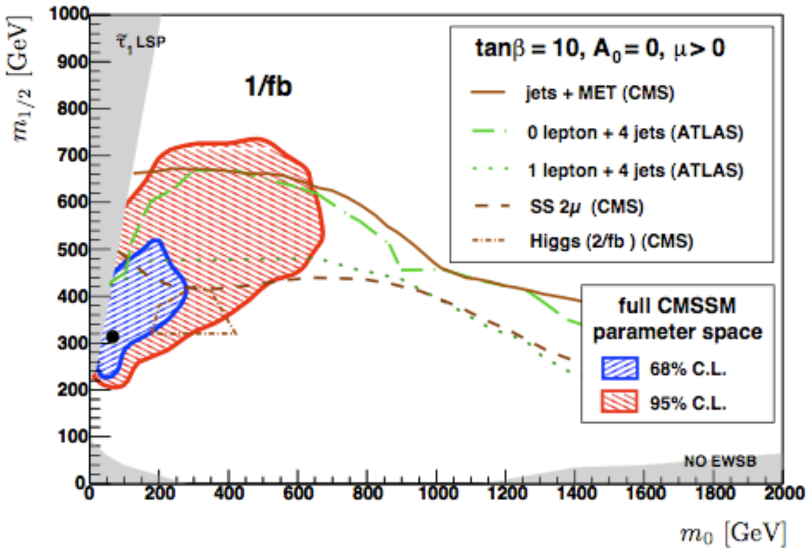
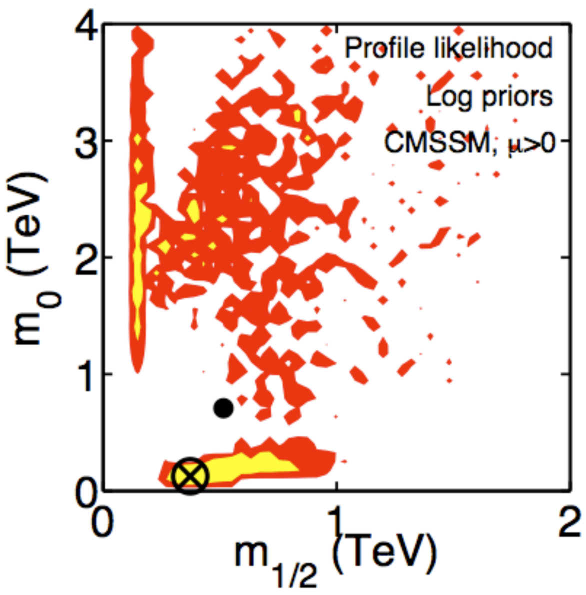
Buchmuller et al (2008)



Trotta et al (2008)



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Posterior for some observables

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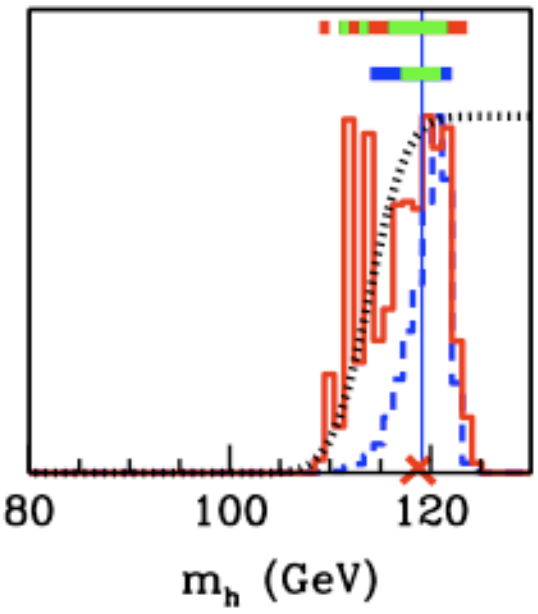
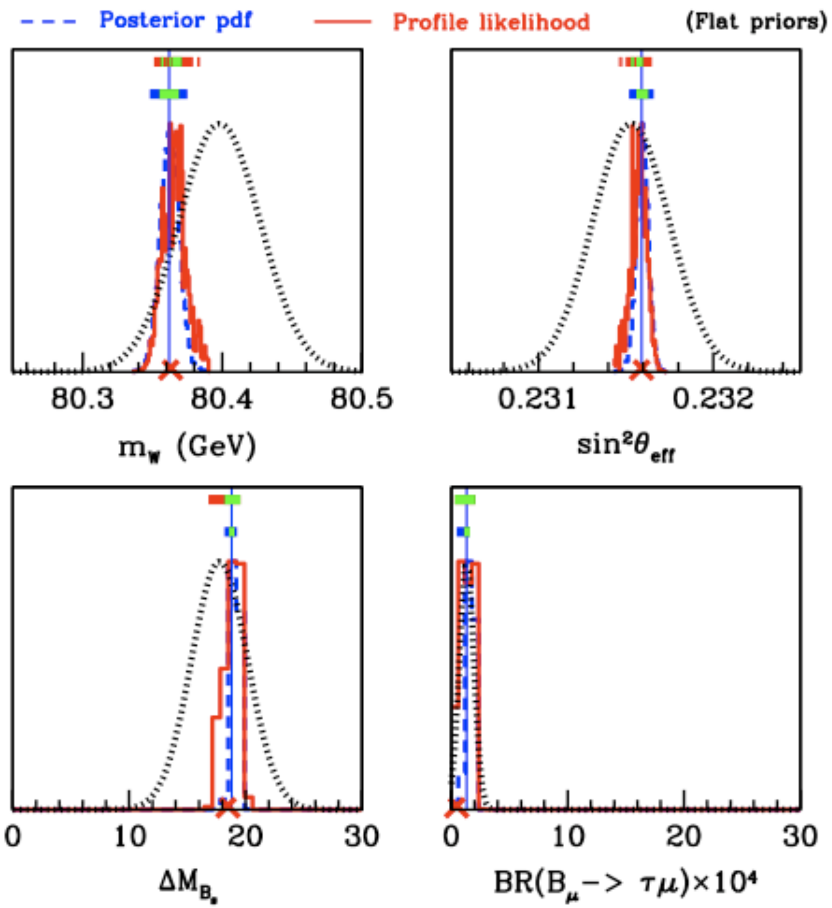
Higgs prospects





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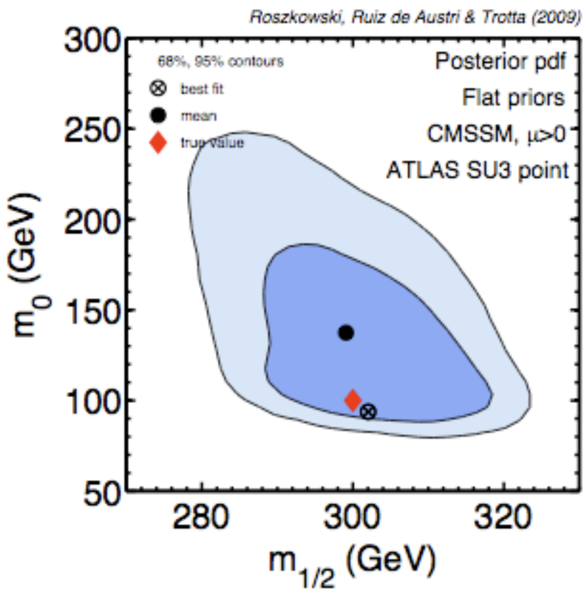
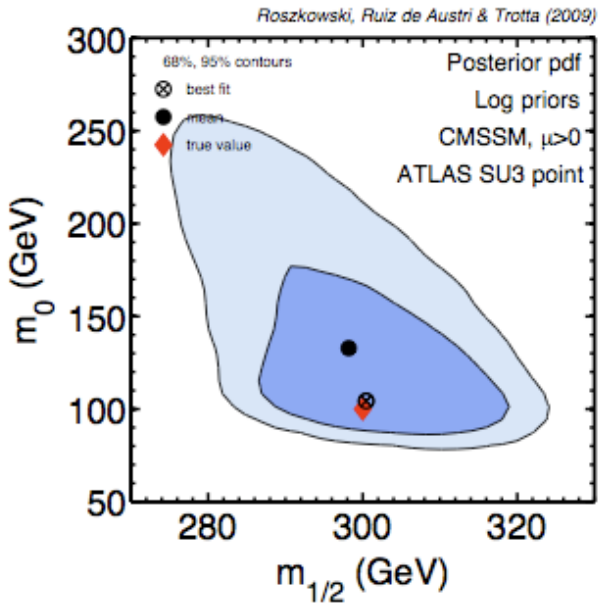
ATLAS will solve the prior dependency

- Projected constraints from ATLAS, (dilepton and lepton+jets edges, 1 fb^{-1} luminosity)

Flat prior



Log prior



Roszkowski, Ruiz & RT (2009, 0907.0594)

Residual dependency on the statistics

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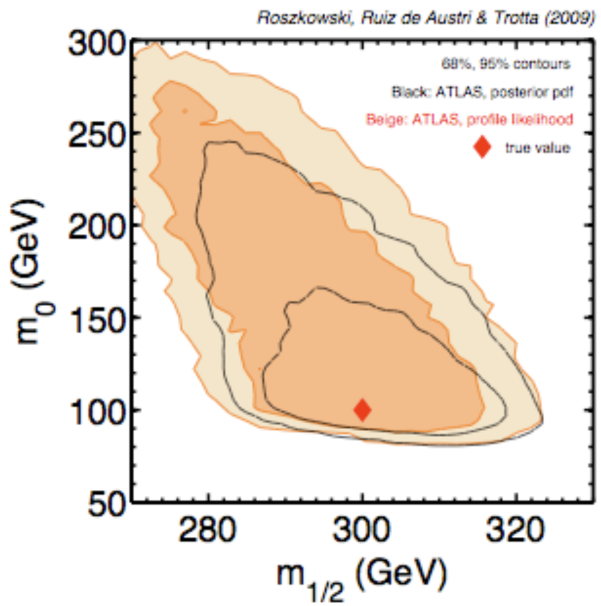
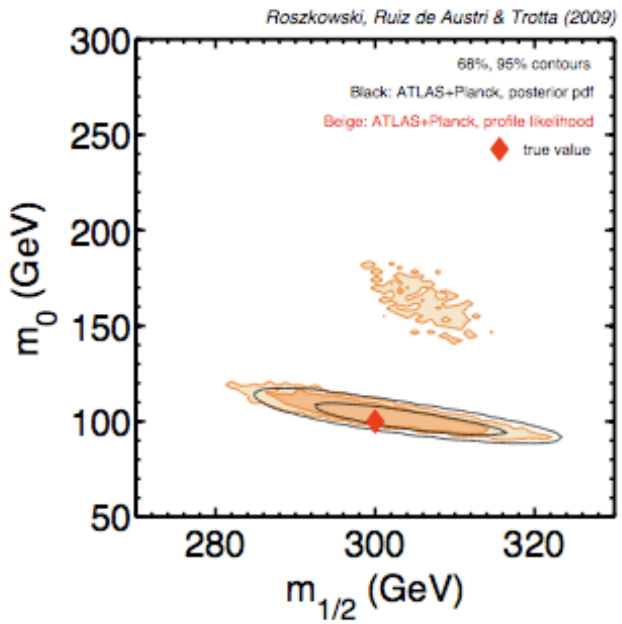
- Marginal posterior and profile likelihood will remain somewhat discrepant using ATLAS alone. Much better agreement from ATLAS+Planck CDM determination.

ATLAS alone



ATLAS+Planck



Direct and indirect detection prospects

Astrophysical probes

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- Direct detection: underground detectors looking for nuclear recoils from WIMP scattering. **It is fundamental to account for the uncertainty in the local WIMP distribution.**

- Indirect detection: detection of annihilation products from WIMP-WIMP annihilation.

  - **Gamma ray** (galactic centre, galactic halo, diffuse extragalactic sources, nearby dwarf galaxies)

  - **Antimatter** (positrons, anti-proton) from local clumps

  - **Neutrinos** from the center of the Sun/Earth.

  - In all cases: it is fundamental to include a **modeling of background sources**. For gamma ray and neutrinos the unknown branching ratios have to be estimated simultaneously **(bias!).**

Direct detection prospects

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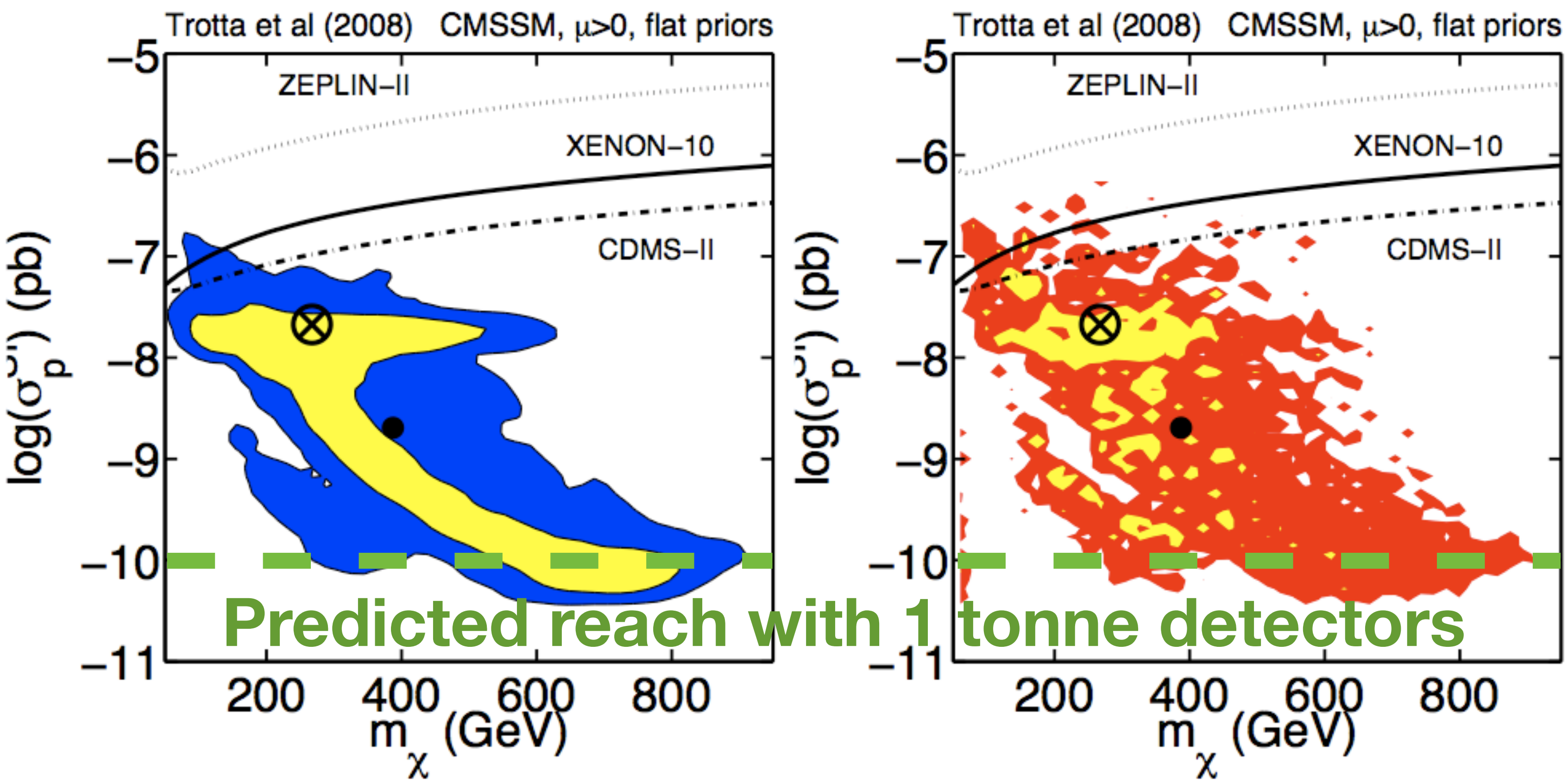
R. Trotta, F. Feroz, M.P. Hobson, R. Ruiz de Austri and L. Roszkowski, 0809.3792

Bayesian pdf | Profile likelihood



Predicting the gamma ray flux

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Differential flux: $\frac{d\Phi_\gamma}{dE_\gamma} \propto \sum_i \frac{\langle \sigma_i v \rangle}{m_\chi^2} \frac{dN_\gamma^i}{dE_\gamma} \int \rho_\chi^2 dl$

particle physics — astrophysics

DM density profile:

$$\rho_\chi(r) = \rho_0 \frac{(r/r_0)^{-\gamma}}{[1+(r/a)^\alpha]^{\frac{\beta-\gamma}{\alpha}}} [1 + (r/a)^\alpha]^{\frac{\beta-\gamma}{\alpha}}$$

$$J(\Psi) = \int_{\rm los} dl \rho_\chi^2(r(l,\Psi)) \qquad \bar{J} = \frac{1}{\Delta\Omega} \int_{\Delta\Omega} J(\Psi) d\Omega$$

Halo model	a (kpc)	α	β	γ	$\bar{J}(10^{-3}\text{sr})$	$\bar{J}(10^{-5}\text{sr})$
isothermal cored	3.5	2	2	0	30.35	30.40
NFW	20.0	1	3	1	1.21×10^3	1.26×10^4
NFW+ac	20.0	0.8	2.7	1.45	1.25×10^5	1.02×10^7
Moore	28.0	1.5	3	1.5	1.05×10^5	9.68×10^6
Moore+ac	28.0	0.8	2.7	1.65	1.59×10^6	3.12×10^8

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Predictions for Fermi in the CMSSM

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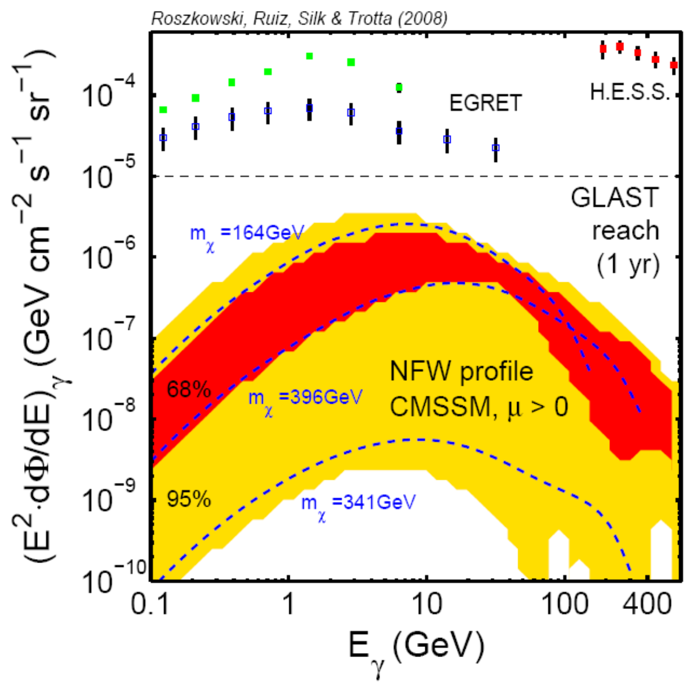
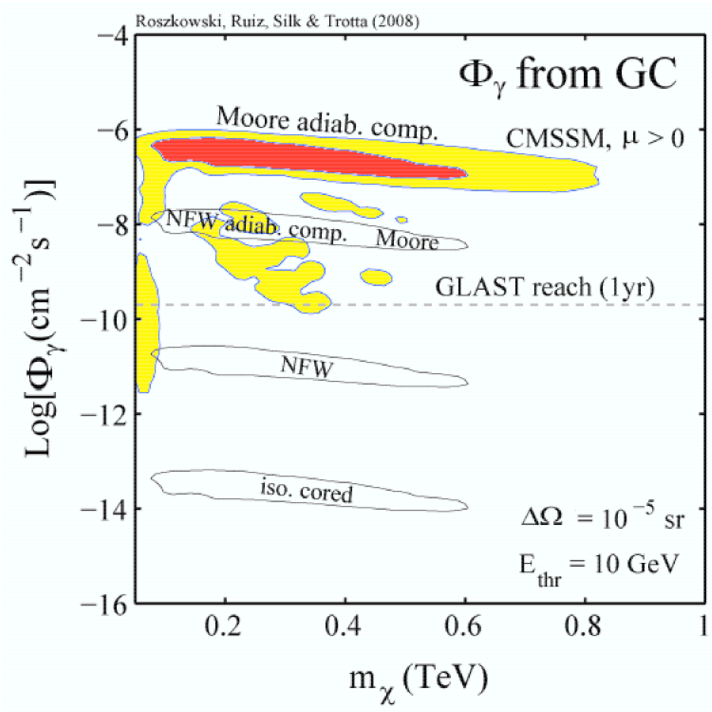
Predicted gamma-ray spectrum probability distribution from the galactic center at Fermi resolution



Roszkowski, Ruiz, Silk & RT (2008)

Predicted gamma-ray flux above 10 GeV at Fermi resolution



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Predictions for the positrons spectrum

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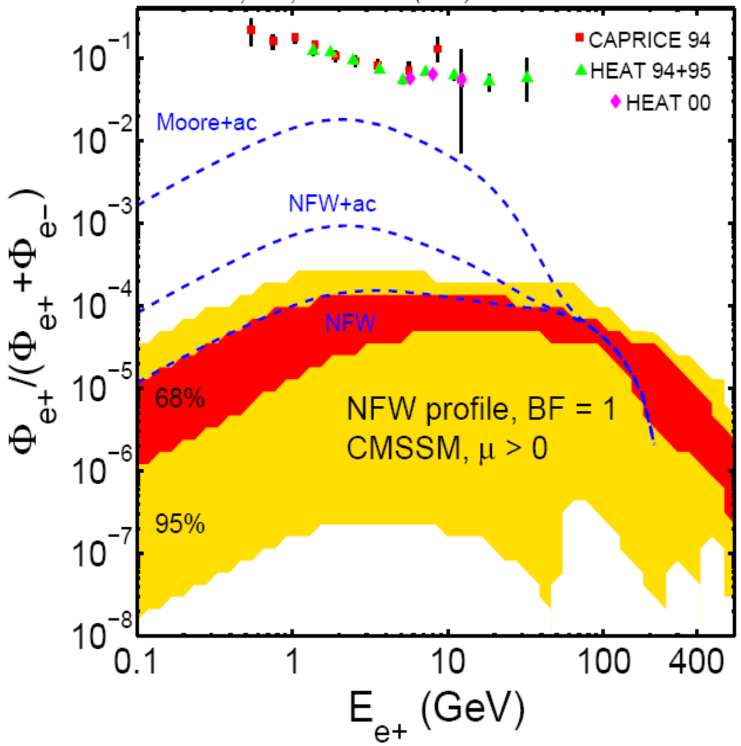
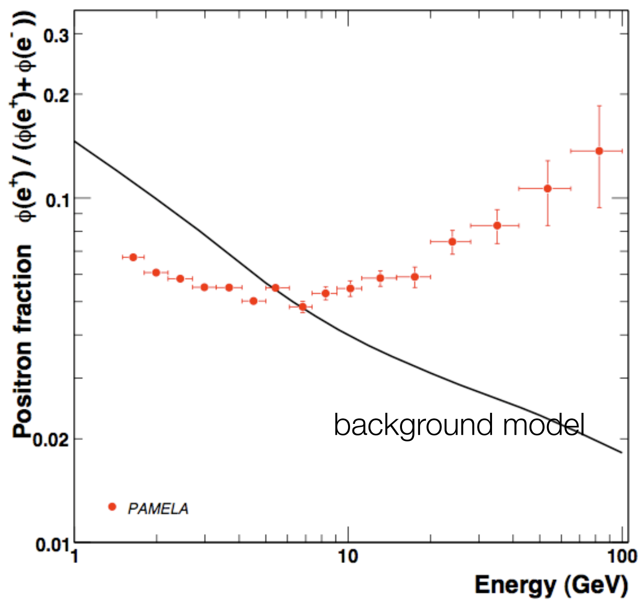
Notice: this is for a fixed choice of propagation parameters!

Roszkowski, Ruiz, Silk & Trotta (2008)



PAMELA data

(Adriani et al 2008)



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Neutrinos from WIMP annihilations in the Sun

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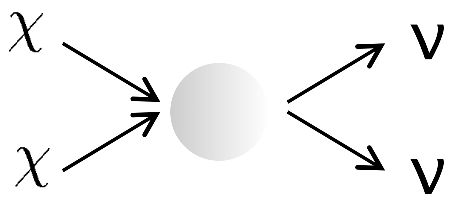

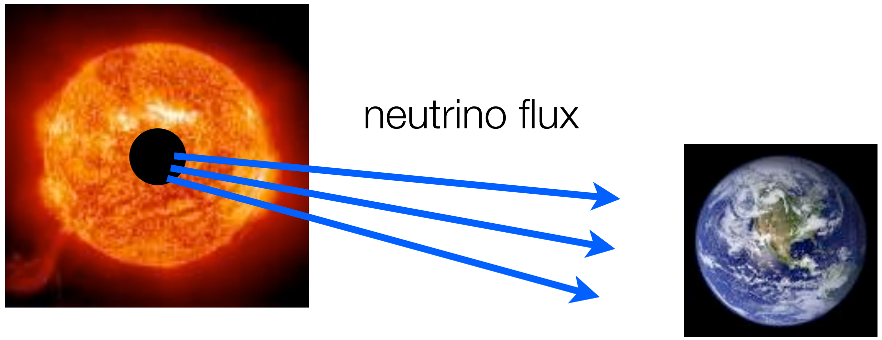

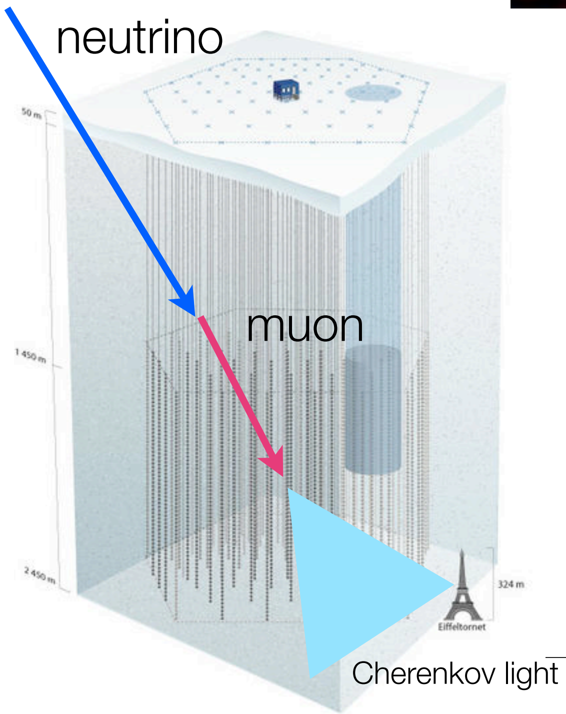

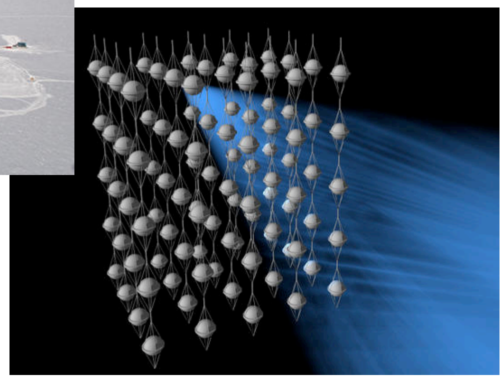

Detection prospects with IceCube

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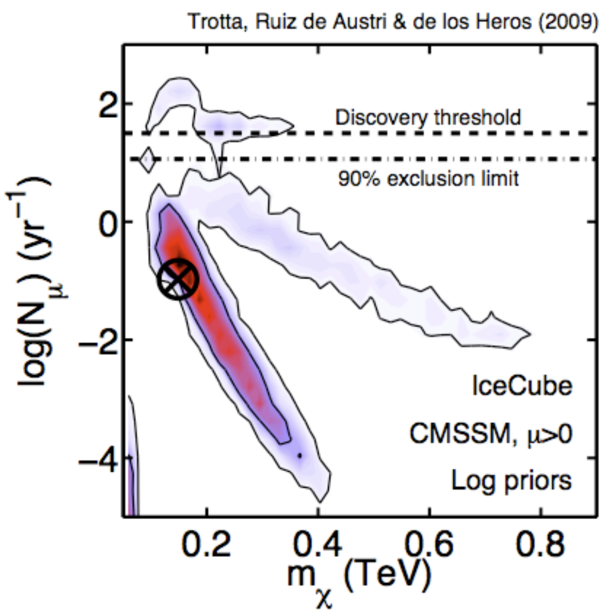
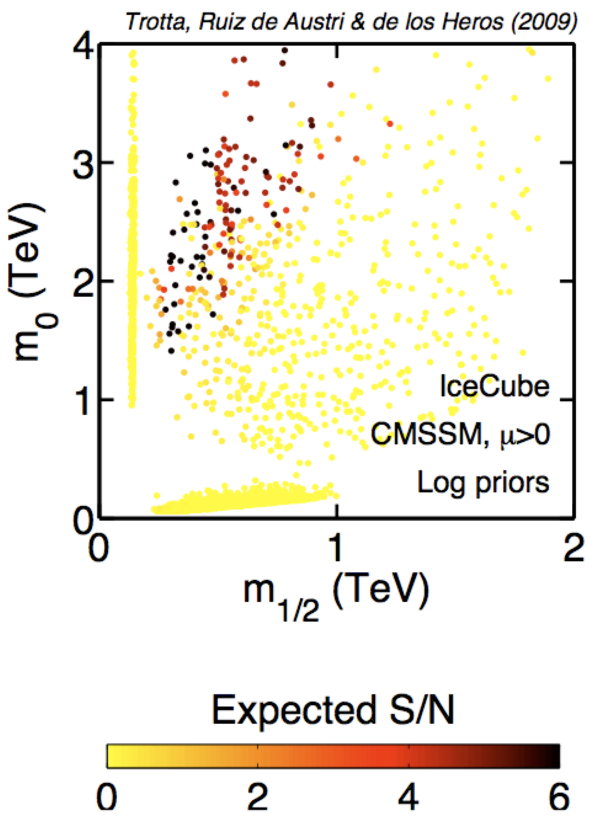
- In the context of the CMSSM, the final configuration of IceCube (with 80 strings) has between 2% and 12% probability of achieving a 5-sigma detection





Trotta, Ruiz de Austri & de los Heros (0906.0366)

otta

Bias from assuming the wrong final state

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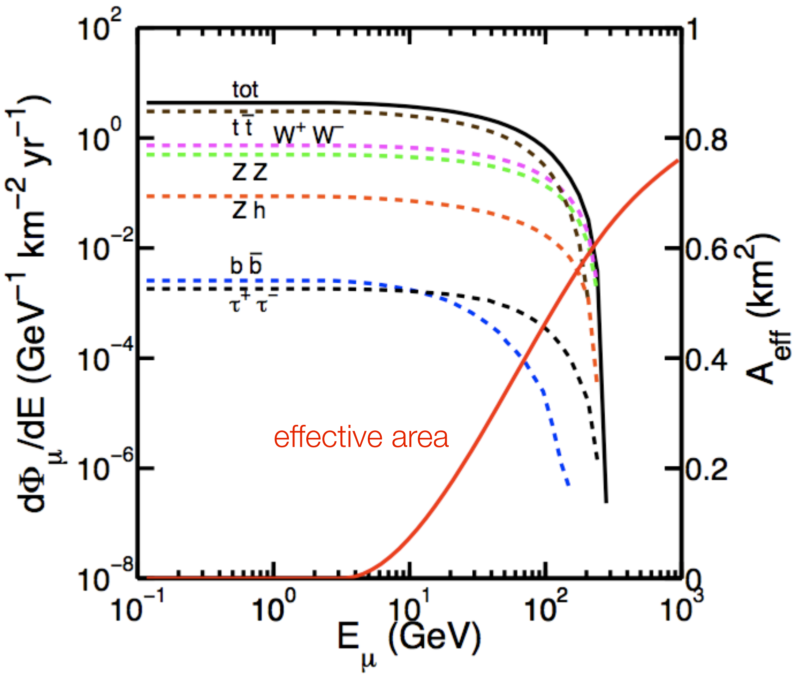
Muon events rate = $N_\mu = \int_{E_\mu^{\rm th}}^{+\infty} dE_\mu A_\mu^{\rm eff}(E_\mu) \frac{d\Phi_\mu}{dE_\mu}$

effective area

total spectrum - sum over final states



Total events = 34 1/yr

BR's for dominant channels		Events/yr assuming only 1 channel
$\mathrm{BR}(\chi\chi \to b\bar{b})$	2.1×10^{-3}	
$\mathrm{BR}(\chi\chi \to t\bar{t})$	0.717	
$\mathrm{BR}(\chi\chi \to W^+W^-)$	0.162	
$\mathrm{BR}(\chi\chi \to ZZ)$	0.094	
Muon events fraction for IceCube		
$N_{b\bar{b}}/N_\mu$	5×10^{-5}	0.8
$N_{t\bar{t}}/N_\mu$	0.46	
$N_{W^+W^-}/N_\mu$	0.30	64
N_{ZZ}/N_μ	0.21	

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Bias from assuming the wrong final state

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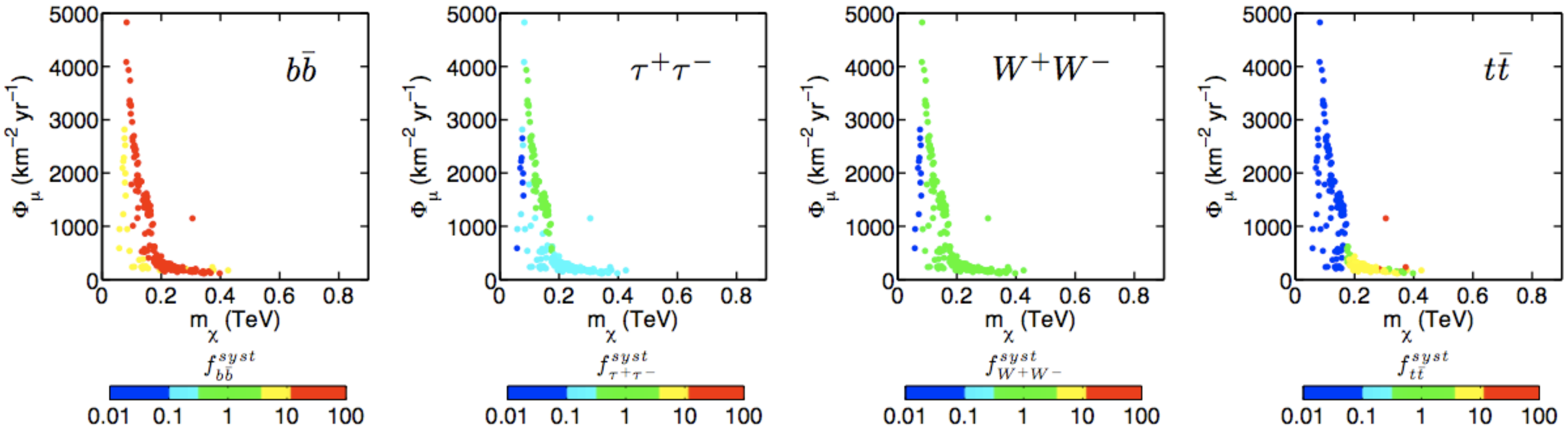
- In general, the systematic error from assuming only 1 dominating channel is given by

$$f_i^{\rm syst} = \frac{{\rm BR}(\chi\chi \to i)}{N_i/N_\mu}$$

Systematic bias from assuming single-channel domination (IceCube)



Trotta, Ruiz de Austri & de los Heros (0906.0366)

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Some uses for the Bayesian evidence

Model comparison

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- The Bayesian evidence is the prime tool for Bayesian model comparison. It automatically includes the notion of "Occam's razor" (see RT, 0803.4089)

$$P(d|M) = \int_\Omega d\theta P(d|\theta, M)P(\theta|M)$$

Model's posterior probability : $$P(M|d) = \frac{P(d|M)P(M)}{P(d)}$$

Two models (e.g. sgn(μ)=±1): $$\frac{P(M_0|d)}{P(M_1|d)} = \frac{P(d|M_0)}{P(d|M_1)}\frac{P(M_0)}{P(M_1)}$$

Posterior odds = Bayes factor × prior odds

Feroz et al (2008)

Prior	"2 TeV"		"4 TeV"	
	flat	log	flat	log
$\log \Delta E$ (our determination)	2.7 ± 0.1	4.1 ± 0.1	1.8 ± 0.1	3.2 ± 0.1
P_+/P_- (our determination)	15.6 ± 1.1	61.6 ± 1.1	5.9 ± 1.1	24.0 ± 1.1

sgn(μ)=+1 between 6 and 60 times more probable than sgn(μ)=-1

Information content of observables

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- Which observable is the most constraining for the CMSSM?

- The information content (i.e., constraining power) of each observable with respect to the model and prior can be quantified using the Kullback-Leibler divergence between prior and posterior:

$$D_{KL} = \int d\theta P(\theta|d) \ln \frac{P(\theta|d)}{P(\theta)} = -\ln P(d) - \langle \chi^2/2 \rangle$$

Information content = -ln(Bayesian evidence) - average chi-square/2

RT et al (2008)

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Constraining power of observables

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Constraints	Data points	Flat priors χ^2_{min}	Flat priors $\langle\chi^2\rangle$	Flat priors D_{KL}	Log priors χ^2_{min}	Log priors $\langle\chi^2\rangle$	Log priors D_{KL}
PHYS+NUIS	4	0.06	3.89	1.00	0.02	3.88	1.00
+CDM	5	0.05	4.36	3.22	0.10	4.32	2.59
+BSG	5	0.31	6.48	1.11	0.10	5.48	1.21
+GM2	5	0.27	11.55	1.35	0.13	6.38	1.20
+COLL+CDM	5+	0.28	4.60	3.20	0.15	5.04	2.98
+COLL+BSG	5+	0.99	6.82	1.11	0.45	6.54	1.24
+COLL+GM2	5+	1.79	13.43	1.10	0.17	9.92	1.49
+COLL+CDM+BSG	6+	0.75	7.15	3.36	0.68	7.72	3.29
+COLL+CDM+GM2	6+	0.62	9.24	2.90	0.43	7.49	3.23
+COLL+CDM+BSG+GM2	7+	6.27	15.83	3.48	4.67	14.89	3.39
ALL but GM2	10+	3.51	9.45	3.42	3.22	9.51	3.28
ALL but CDM	10+	12.17	18.86	1.10	4.14	18.30	1.24
ALL	11+	13.51	19.29	3.38	11.90	18.41	3.26

Cosmology provides 80% for flat priors (95% for log priors) of the total constraining power on the CMSSM

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Data consistency

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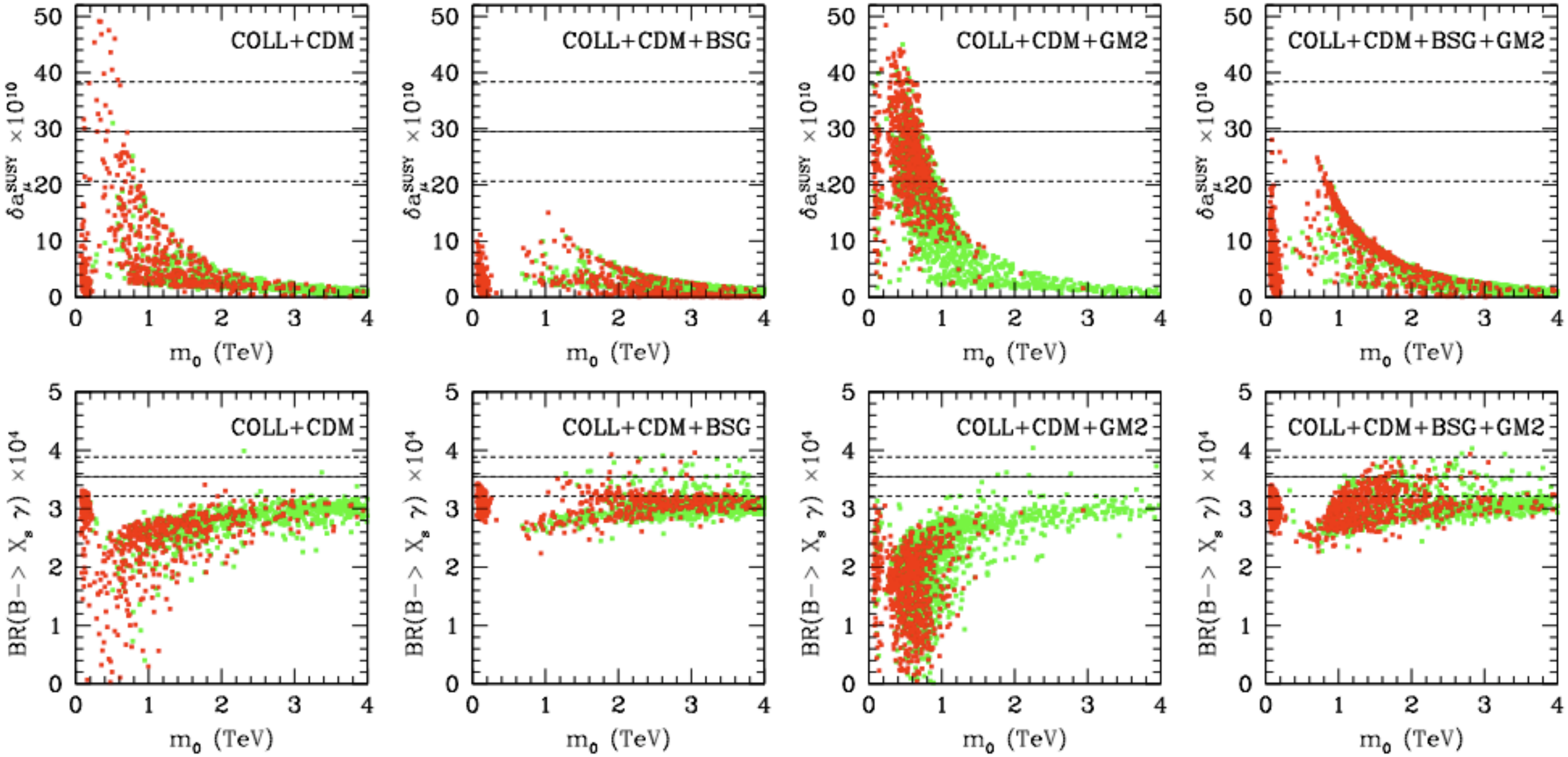
Tension between the anomalous magnetic moment and b -> sγ



RT et al (2008)

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Testing for data consistency

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- Evaluate the probability of “systematic inconsistency” between g-2 and and b → sγ within the CMSSM:

- Baysian model comparison:
  H_0: (g-2, b → sγ) compatible and described by a unique set of CMSSM parameters
  vs
  H_1: (g-2, b → sγ) systematically incompatible

$$R = \frac{\Pr(\mathbf{D}|H_1)}{\Pr(\mathbf{D}|H_0)} = \frac{\Pr(\mathbf{D}|H_1)}{\prod_i \Pr(D_i|H_0)}.$$

Feroz et al (2008) find that $|\ln R| < 1.0$, hence no significant evidence for tension (but no conclusive evidence for H_0, either)

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Predictive data distributions

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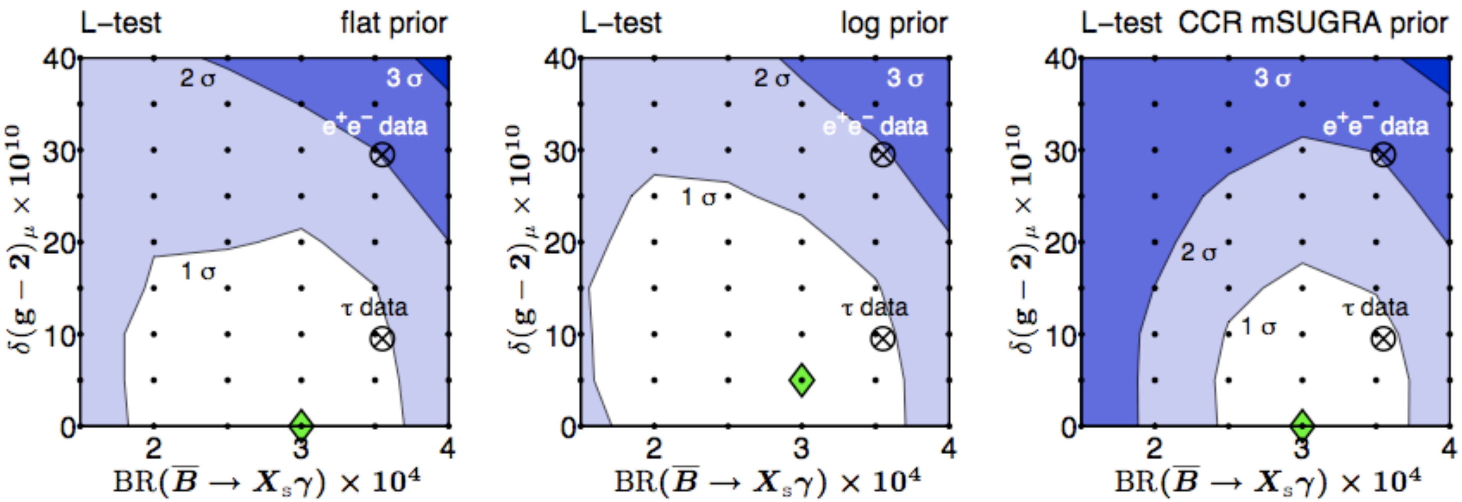
- What is the probability of observing (g-2, b → sγ) given the CMSSM and all other constraints? Compute the predictive data distribution (i.e., conditional evidence) for D^{obs} given the other observations D

$$\mathscr{L}(\mathscr{D}^{\rm obs}|D) \equiv \frac{p(\mathscr{D}^{\rm obs}|D)}{p(\mathscr{D}^{\rm max}|D)} = \frac{p(\mathscr{D}^{\rm obs}, D)}{p(\mathscr{D}^{\rm max}, D)}$$



Feroz, Hobson, Ruiz, Roszkowski & RT, 2009, 0903.2487

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Conclusions

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- SUSY phenomenology provides a timely and challenging problem for parameter inference and model selection. A considerably harder problem than cosmological parameter extraction!
- Bayesian advantages: higher efficiency, inclusion of nuisance parameters, predictions for derived quantities, model comparison
- CMSSM only a case study. There are several models around (NUHM, pMSSM, ... -> more free parameters) that will need to be analyzed as soon as new data flow in
- Currently, even the CMSSM is somewhat underconstrained: ATLAS+Planck will take us to “statistics nirvana” (Bob Cousins)
- The Bayesian evidence can be used for model selection, data consistency checks and prediction. Also useful to quantify constraining power of the data.