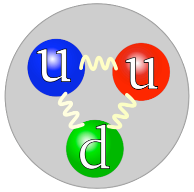
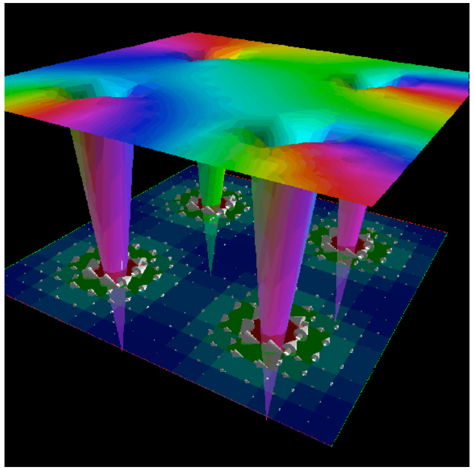
Soliton physics in holographic models: From Baryons to superconductor vortices





Alex Pomarol (Univ. Autonoma Barcelona)

AdS/CFT correspondence:

Maldacena 97

Strongly coupled 4D theories in the large-N limit	$\xleftrightarrow{\text{duality}}$	Weakly coupled string theories in 10D
At very large g^2N	$\longleftrightarrow$	10D gravity theories

New tool to tackle strongly interacting physical systems:

- QCD-like theories: Low-energy spectrum, properties of the Quark-gluon plasma
- Condensed matter systems: Cold atoms, superfluidity, superconductors,...

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New tool to tackle strongly interacting physical systems:

- QCD-like theories: Low-energy spectrum, properties of the Quark-gluon plasma

  → Solitons: Baryons

- Condensed matter systems: Cold atoms, superfluidity, superconductors,...

  → Solitons: Abrikosov Vortices

I. Gravity duals of QCD-like theories
(Holographic QCD)

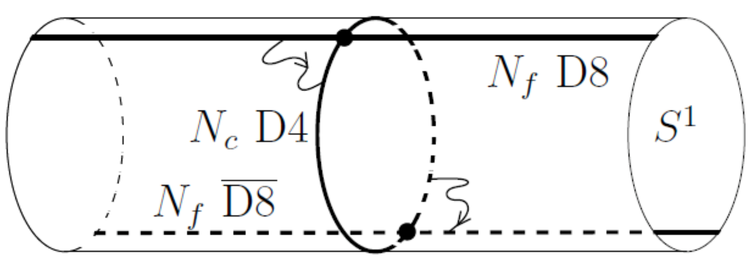
The closest example to QCD

Sakai-Sugimoto Model (D4/D8 system)

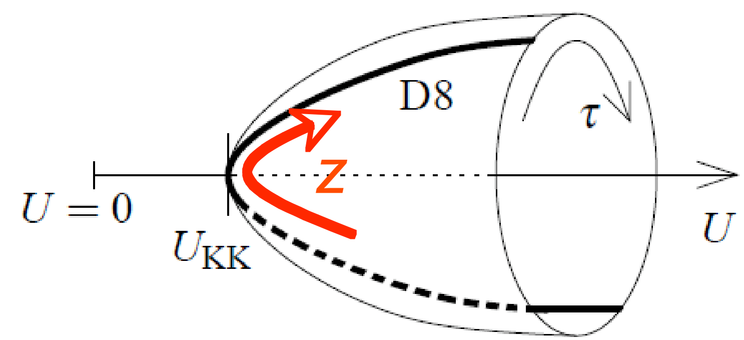
Massless spectrum: Gluons and quarks

But not true QCD since
extra massive states do not decouple

Nice feature: Dual realization of Chiral symmetry breaking:

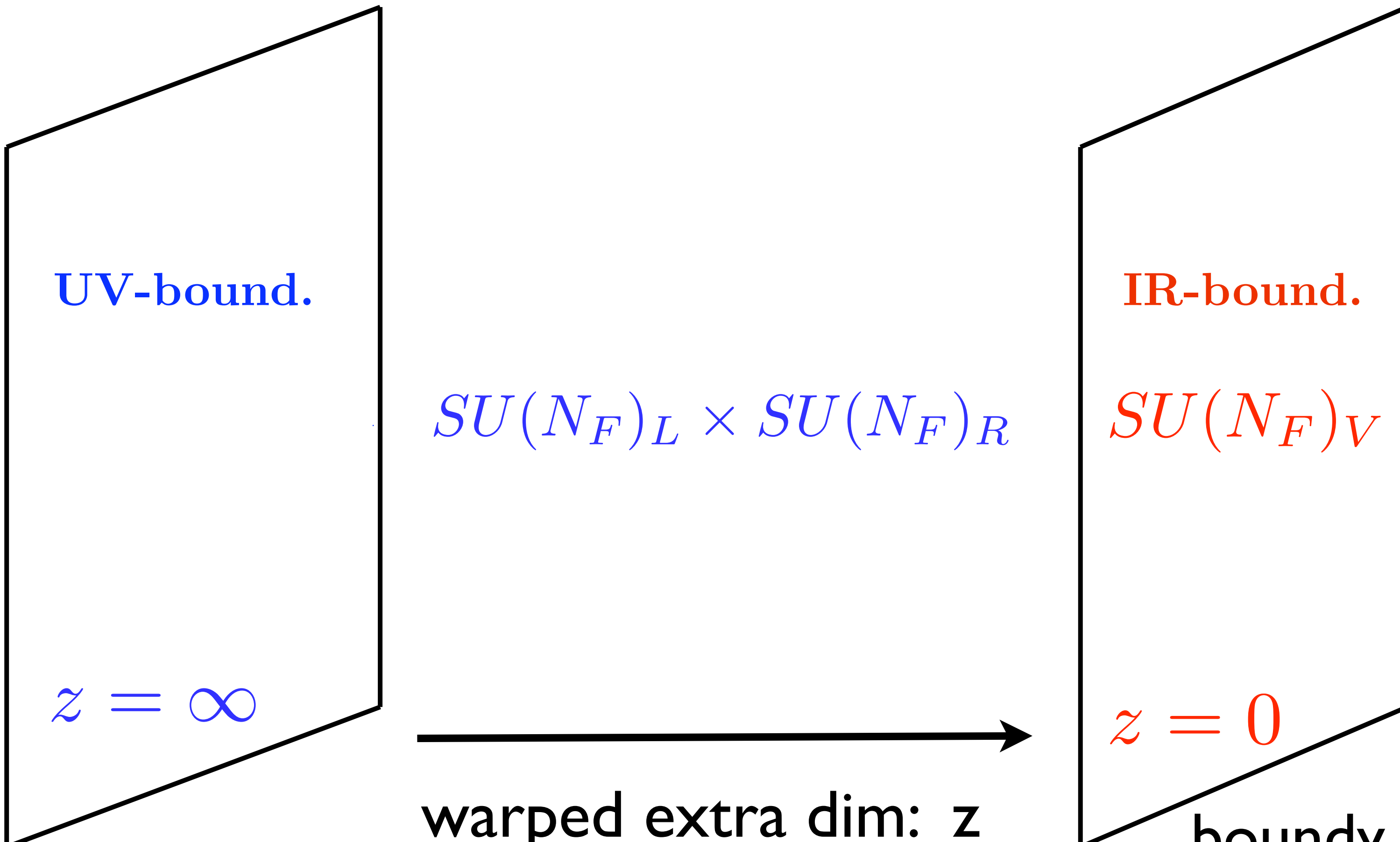


large N



$$SU(N_F)_L \times SU(N_F)_R \longrightarrow SU(N_F)_V$$

At large N and large ‘tHooft coupling (g^2N in the 4D gauge theory), the string dual theory is, at low-energies, a weakly-coupled gauge theory in 5D with chiral symmetry breaking on the z=0 boundary



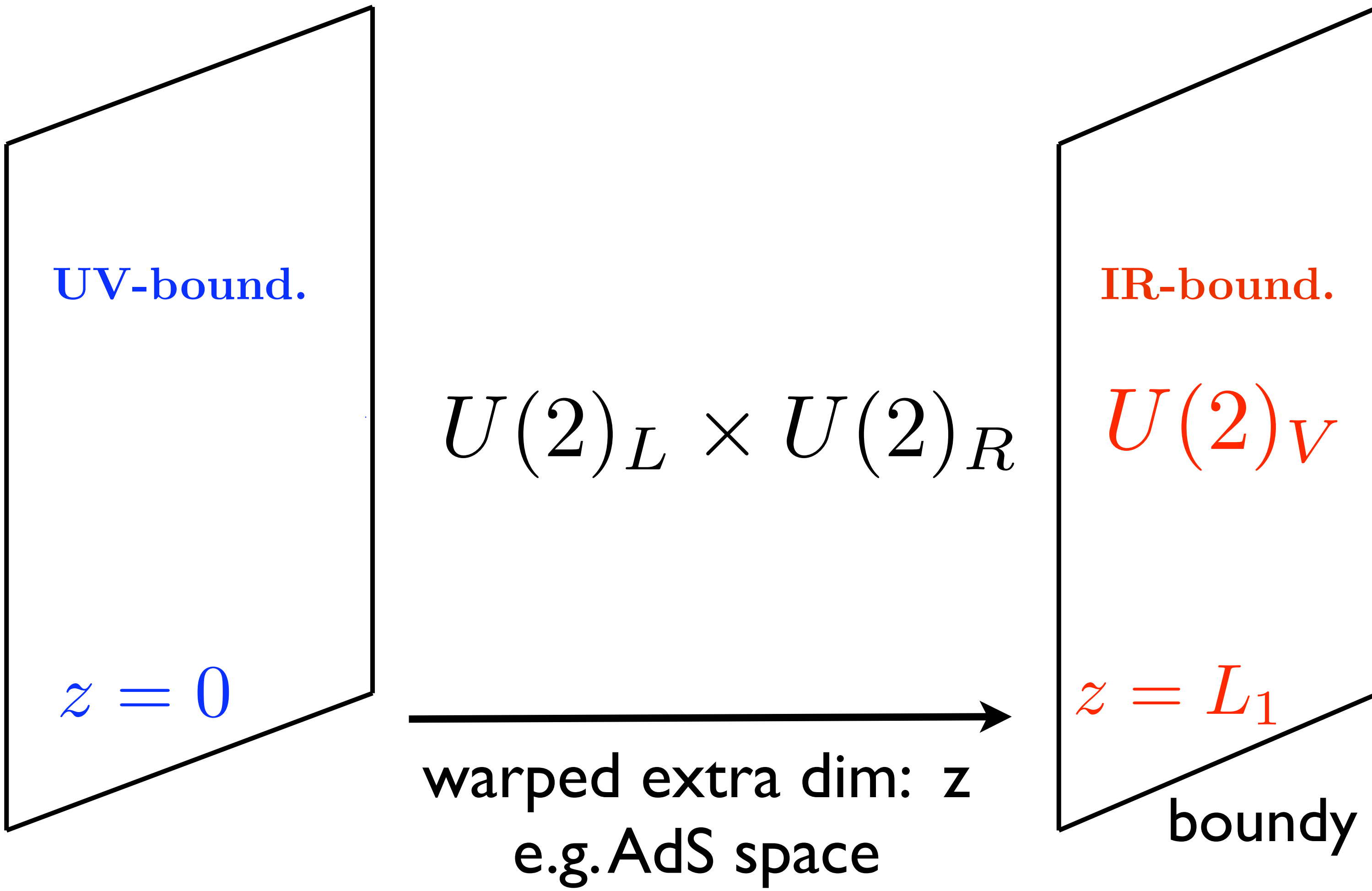
boundy conditions:

Dirichlet: $A_L^\mu - A_R^\mu = 0$

Neumann: $F_L^{\mu 5} + F_R^{\mu 5} = 0$

Proposed Holographic QCD model of two massless quarks

Erlich,Katz,Son,Stephanov
Da Rold,A.P.; Hirn,Sanz



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Dirichlet: $A^\mu_L - A^\mu_R = 0$

Neumann: $F^{\mu 5}_L + F^{\mu 5}_R = 0$

5D Lagrangian:

$$\boxed{ds^2 = a(z)^2[dx^2 + dz^2]}$$

$$-\int a(z)\frac{M_5}{2}\left[Tr[L_{MN}L^{MN}] + \frac{\alpha^2}{2}\hat{L}_{MN}\hat{L}^{MN} + \{L \to R\}\right]$$

$$+\frac{N_c}{64\pi^2}\int \epsilon^{MNOPQ}\widehat{L}_M\, Tr\left[L_{NO}L_{PQ}\right] - \{L \to R\} + \ldots$$

Chern-Simons term:

Needed to reproduce the U(1)-anomaly in QCD

Coefficient fixed $\longrightarrow$ no extra parameter!

5D lagrangian:

$$-\int a(z)\frac{M_5}{2}\left[Tr[L_{MN}L^{MN}]+\frac{\alpha^2}{2}\hat{L}_{MN}\hat{L}^{MN}+\{L\to R\}\right]$$

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Theory of 3 parameters: $M_5\ ,\ L_1\ ,\ \alpha$

Compactification scale: Mass gap

Kaluza-Klein states → QCD-like mesons ($q\bar{q}$-states)

Predictions very close to QCD meson sector:

A.P., Wulzer 08

Table 1.1. Global fit to mesonic physical quantities. Masses, decay constants and widths are given in MeV. Physical masses have been used in the kinematic factors of the partial decay widths.

	Experiment	AdS_5	Deviation
m_ρ	775	824	+6%
m_{a_1}	1230	1347	+10%
m_ω	782	824	+5%
F_ρ	153	169	+11%
F_ω/F_ρ	0.88	0.94	+7%
F_π	87	88	+1%
$g_{\rho\pi\pi}$	6.0	5.4	−10%
L_9	$6.9 \cdot 10^{-3}$	$6.2 \cdot 10^{-3}$	−10%
L_{10}	$-5.2 \cdot 10^{-3}$	$-6.2 \cdot 10^{-3}$	−12%
$\Gamma(\omega \to \pi\gamma)$	0.75	0.81	+8%
$\Gamma(\omega \to 3\pi)$	7.5	6.7	−11%
$\Gamma(\rho \to \pi\gamma)$	0.068	0.077	+13%
$\Gamma(\omega \to \pi\mu\mu)$	$8.2 \cdot 10^{-4}$	$7.3 \cdot 10^{-4}$	−10%
$\Gamma(\omega \to \pi ee)$	$6.5 \cdot 10^{-3}$	$7.3 \cdot 10^{-3}$	+12%

Baryon sector?

Baryon sector?

In the large-N_c limit:

Witten 77

$$M_B \propto N_c = \frac{1}{1/N_c}$$

Baryon are solitons!

Original motivation for Baryons as Skyrmions:

solitons of the chiral lagrangian

Skyrme 61, Adkins+Nappi+Witten 83

If only the F²-term is considered...

Baryons: Solitons of the 5D theory $\approx$ instanton in 4D ($t_E \rightarrow z$)

Atiyah,Manton89
Son,Stephanov04

Topological charge:

$$Q = \frac{1}{16\pi^2} \int d^3x \int_0^{L_1} dz \, Tr \left[L^{\hat{\mu}\hat{\nu}} \widetilde{L}_{\hat{\mu}\hat{\nu}} - R^{\hat{\mu}\hat{\nu}} \widetilde{R}^{\hat{\mu}\hat{\nu}} \right] = N_{inst}(L) - N_{inst}(R)$$

$$\hat{\mu} = 1, 2, 3, 4 \ (L_0 = R_0 = 0)$$

reduces to
4D Skyrmion charge

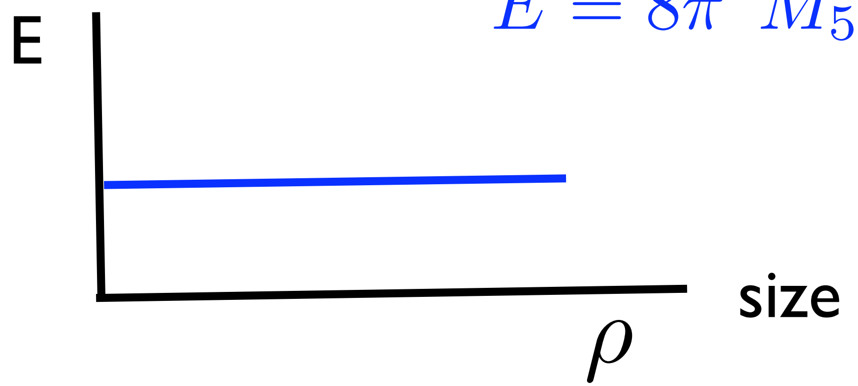
$$Q = \frac{1}{24\pi^2} \int d^3x \, \epsilon^{ijk} \mathrm{Tr} \left[U\partial_i U^\dagger \, U\partial_j U^\dagger \, U\partial_k U^\dagger \right] \in \mathbb{Z}$$

U(x): pion field

Is the Baryon given by the 4D instanton configuraton?

Yes, in an infinite flat space. Energy independent of size:

$$E = 8\pi^2 M_5$$



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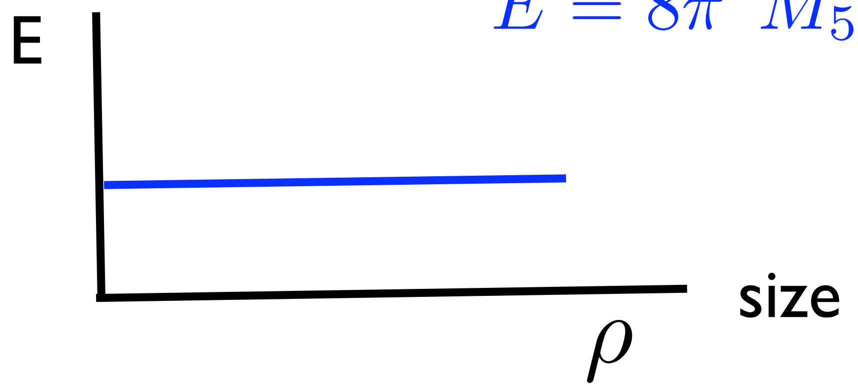
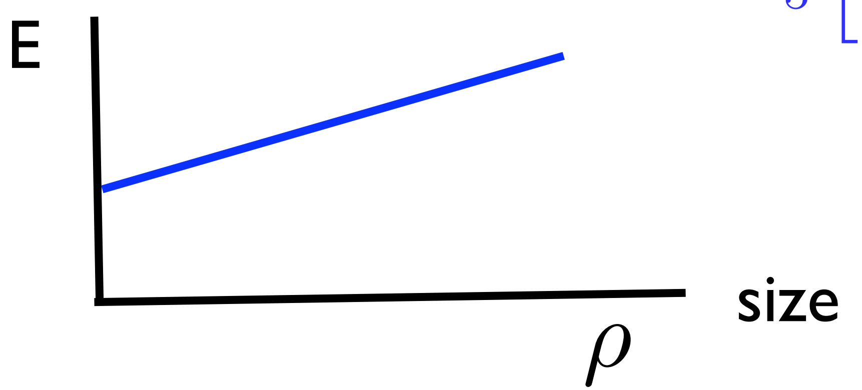


But curvature and compactification breaks the scale invariance of the instanton:

$$E = 8\pi^2 M_5 \left[1 + \frac{\rho}{2R}\right]$$



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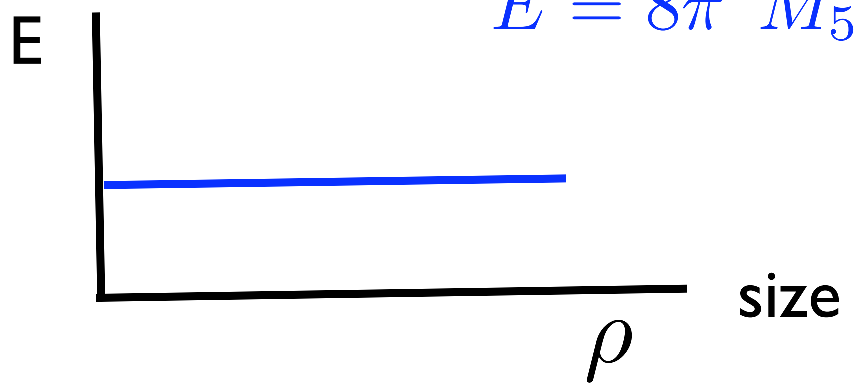

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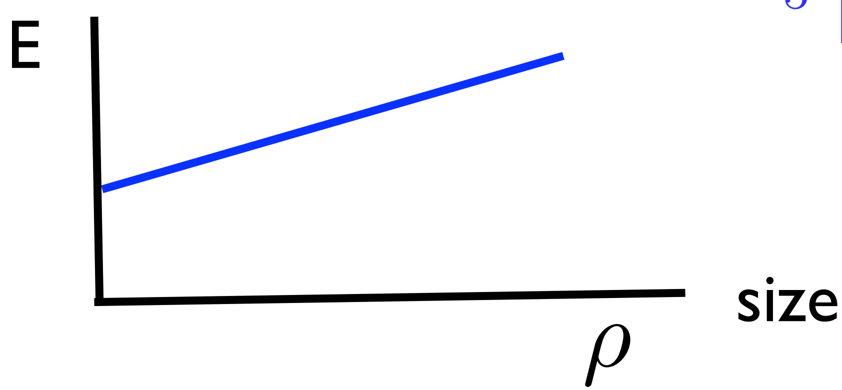

It shrinks to zero size!

Its stability is UV-sensitive:
Depends on the higher-dimensional operators of the theory

Lowest higher-dimensional operator:

Dimension five-operator: Chern-Simons term

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Dimension five-operator: Chern-Simons term

Stability and consistency of the model:

5D model: $$\mathcal{L}_5 \propto F^2 + \frac{1}{\Lambda_5} AFF + \frac{1}{\Lambda_5^4} F^4 + \ldots$$

Soliton energy $$E(\rho) \sim \rho M_{KK} + \frac{1}{\rho^2 \Lambda_5^2} \Rightarrow \rho \sim \frac{1}{M_{KK}^{1/3} \Lambda_5^{2/3}} >> \frac{1}{\Lambda_5}$$

No sensitive to higher-dim operators

Different from the 4D Skyrmion model:

4D Skyrmion: $$\mathcal{L}_\chi \propto (D_\mu U)^2 + \frac{1}{m_\rho^2} (D_\mu U)^4 + \ldots$$

$$\rho \sim \frac{1}{m_\rho}$$

Sensitive to higher-dim operators

Mainly two approaches towards Holographic Baryons:

a) Treat baryons as approximated instanton configurations

H.Hata,T.Sakai,S.Sugimoto,S.Yamato; D.K.Hong,T.Inami,H.U.Yee; D.K.Hong, M.Rho,H.U. Yee,P.YH.Hata,M.Murata,S.Yamato; K.Hashimoto,T.Sakai,S.Sugimoto

b) Find the new 5D soliton configuration including the curvature of the space and the CS-term

A.P., Wulzer; Wulzer, Panico

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A.P., Wulzer; Wulzer, Panico

Several problems:

- Not fully consistent (CS-term is sizable)
- Does not reproduce large-N expectations

Wulzer,Panico; A.Cherman,T.D. Cohen,M.Nielsen

Solution of the SU(2) part:

Ansatz (Witten 77):

"cylindrical symmetry" invariance under the combine SU(2) gauge and rotation + Parity:

$$L_j^a = -\frac{1+\phi_2^L(r,z)}{r^2}\,\epsilon_{jak}x_k + \frac{\phi_1^L(r,z)}{r^3}\left(r^2\delta_{ja} - x_j x_a\right) + \frac{A_1^L(r,z)}{r^2}x_j x_a$$

$$L_5^a = \frac{A_2^L(r,z)}{r}x^a$$

$$R_j^a(x,z) = -L_j^a(-x,z)$$

$$R_5^a(x,z) = L_5^a(-x,z)$$

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4 fields: combine in a
2D gauge boson + a complex scalar

2D Abelian Higgs models:

$$E = 16\pi \int_0^\infty dr \int_{z_{\rm UV}}^{z_{\rm IR}} dz\, M_5\, a(z)\left[\frac{1}{2}|D_{\bar\mu}\phi|^2 + \frac{1}{8}r^2 F_{\bar\mu\bar\nu}^2 + \frac{1}{4r^2}\left(1-|\phi|^2\right)^2\right]$$

Solution of the U(1) part:

Ansatz: $$\hat{L}_0 = \hat{R}_0 = \frac{1}{\alpha}\frac{s(r,z)}{r}$$

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scalar coupled to the topological charge density of the 2D Abelian model

$$E = 8\pi M_5 \int_0^\infty dr \int_0^{L_1} dz \left[a(z)\frac{1}{2}(\partial_\mu s)^2 - \pi\gamma\,\frac{s}{r}\rho_{topo} \right]$$

Creates a $1/r^2$ potential that prevents the shrinking of the soliton

2D Abelian Higgs model + Scalar

Total: 5 fields

System of 5 non-linear PDEs in 2D

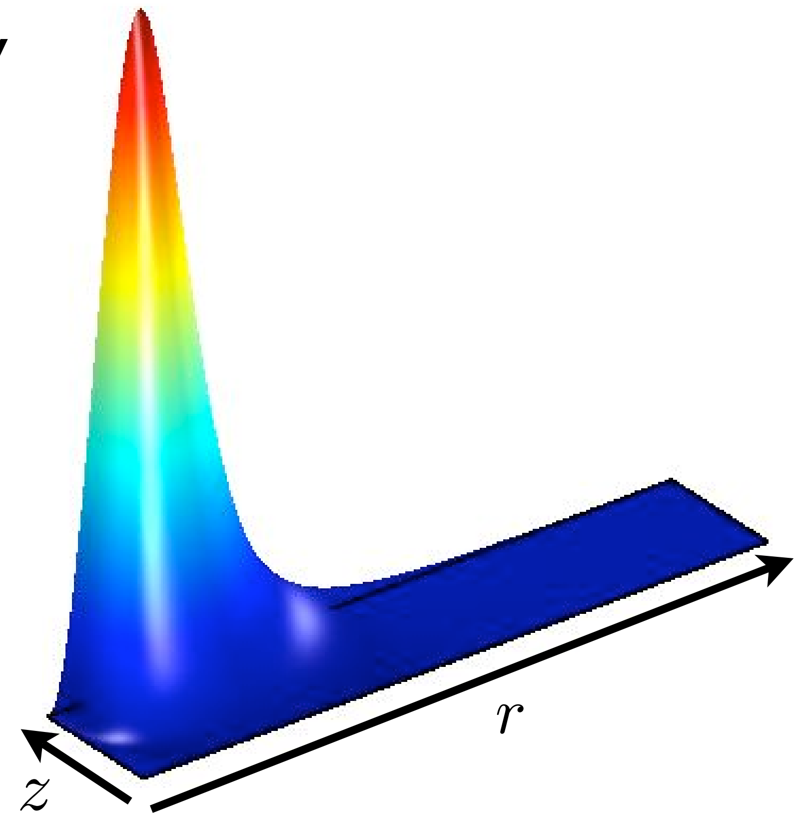
+ suitable b.c. to ensure Q=1

Solution must be found numerically

We rely on FEMLAB (COMSOL) package

used by engineers in many physical systems

Soliton energy density



$$E = \int dr dz\, \rho_E = f(M_5, L_1, \alpha) \simeq 1140 \text{ MeV}$$

taking $F_\pi,\ m_\rho,\ F_\omega/F_\rho$ to fix the 3 parameters

a instanton-like baryon, without the CS, will have 1/2 of the energy

Identification of the the proton and neutron:

Standard Procedure:

Adkins+Nappi+Witten 83

1) Identify the time-dependent fluctuations of the rotational zero-modes:

SU(2)-rotation of the soliton doesn't change its energy

$$U = a_0 + i\vec{\sigma}\cdot\vec{a}_i \;\; \in SU(2)$$

$a_0 \, , \, a_i$: collective coordinates (quantum mechanical variables)

2) Calculate H

3) Calculate the spin and isospin operator

4) Eigenstates of spin and isospin 1/2

$$|p\uparrow\rangle = \frac{1}{\pi}(a_1 + ia_2) \, , \qquad |p\downarrow\rangle = -\frac{i}{\pi}(a_0 - ia_3) \, ,$$

$$|n\uparrow\rangle = \frac{i}{\pi}(a_0 + ia_3) \, , \qquad |n\downarrow\rangle = -\frac{1}{\pi}(a_1 - ia_2) \, ,$$

Extra difficulty:

After turning on $a_{0,i}(t)$ we must assure that the EOM are satisfied:
Fields that were zero in our Ansatz turn on

Static properties of the baryon

Baryon couplings to external sources:

$$\langle B|J_\mu|B\rangle = J_\mu^{\text{boundary}}\Big|_{\text{soliton}}$$

$$J_\mu^{\text{boundary}} = \frac{\delta\mathcal{L}_{5\text{D}}}{\delta A_\mu^{\text{non-norma}}} = F_{5\mu}\Big|_{\text{boundary}}$$

Axial coupling, magnetic and electric form factors (and moments) can be calculated

Tree-level:

$$\begin{aligned}
G_E^S &= -\frac{N_c}{6\pi\gamma L}\int dr\, r\, j_0(qr)\left(a(z)\partial_z s\right)_{UV}\\
G_E^V &= \frac{4\pi M_5}{3\lambda}\int dr\, r^2\, j_0(qr)\left[a(z)\left(\partial_z v - 2\,(D_z\chi)_{(2)}\right)\right]_{UV}\\
G_M^S &= \frac{8\pi M_N M_5\alpha}{3\lambda}\int dr\, r^3\,\frac{j_1(qr)}{qr}\left(a(z)\partial_z Q\right)_{UV}\\
G_M^V &= \frac{M_N N_c}{3\pi L\gamma\alpha}\int dr\, r^2\,\frac{j_1(qr)}{qr}\left(a(z)\,(D_z\phi)_{(2)}\right)_{UV}\\
G_A &= \frac{M_N}{E}\frac{N_c}{3\pi\alpha\gamma L}\int dr\, r\left[a(z)\frac{j_1(qr)}{qr}\left((D_z\phi)_{(1)} - r\,A_{zr}\right) - a(z)\,(D_z\phi)_{(1)}\, j_0(qr)\right]_{UV}
\end{aligned}$$

Results:

A.P.,Wulzer; Wulzer,Panico

	Experiment	AdS_5	Deviation
M_N	940 MeV	1130 MeV	20%
μ_S	0.44	0.34	30%
μ_V	2.35	1.79	31%
g_A	1.25	0.70	79%
$\sqrt{\langle r^2_{E,S}\rangle}$	0.79 fm	0.88 fm	11%
$\sqrt{\langle r^2_{E,V}\rangle}$	0.93 fm	∞	
$\sqrt{\langle r^2_{M,S}\rangle}$	0.82 fm	0.92 fm	12%
$\sqrt{\langle r^2_{M,V}\rangle}$	0.87 fm	∞	
$\sqrt{\langle r^2_{A}\rangle}$	0.68 fm	0.76 fm	12%
μ_p/μ_n	-1.461	-1.459	0.1%

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Large deviations: Possible due to have a massless pion

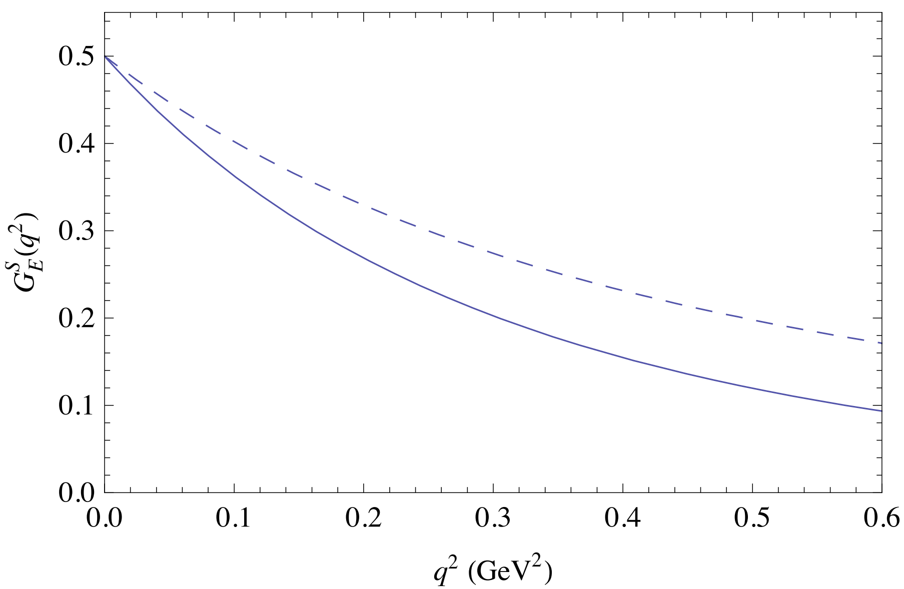
Due to have a massless pion

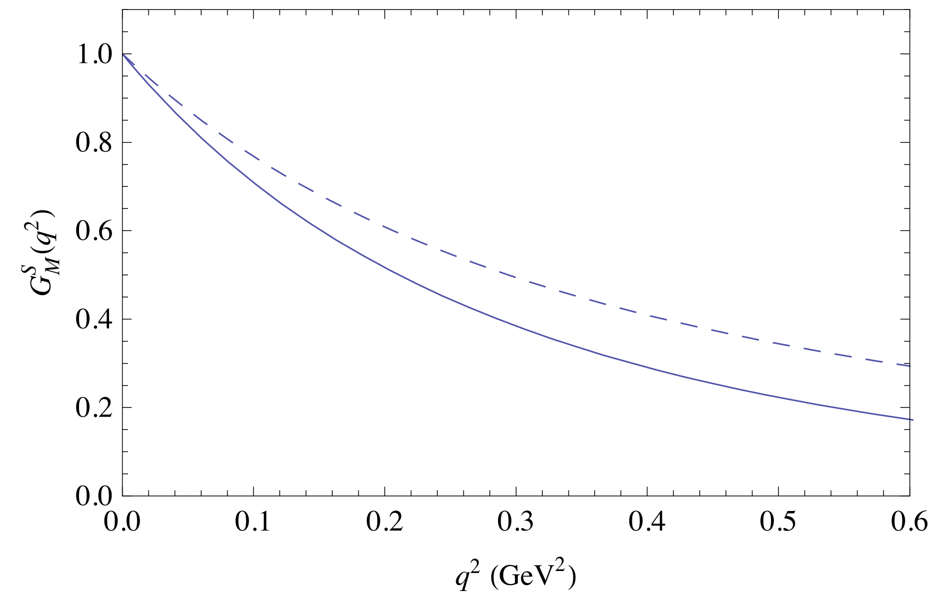
Very small deviation:
Prediction including subleading large-N corrections

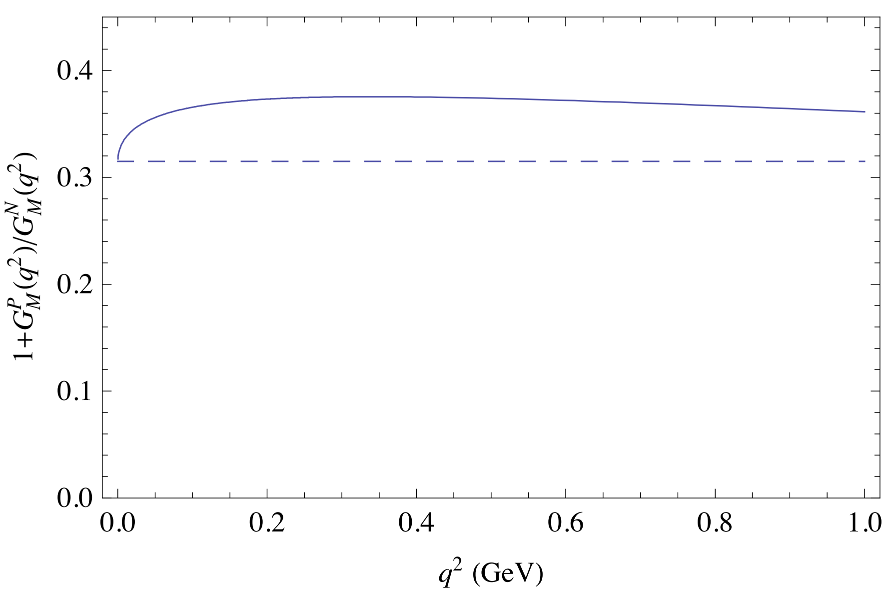
$$\mu_p/\mu_n = -(\mu_V+\mu_S)/(\mu_V-\mu_S) \simeq -1 - 2\mu_S/\mu_V$$

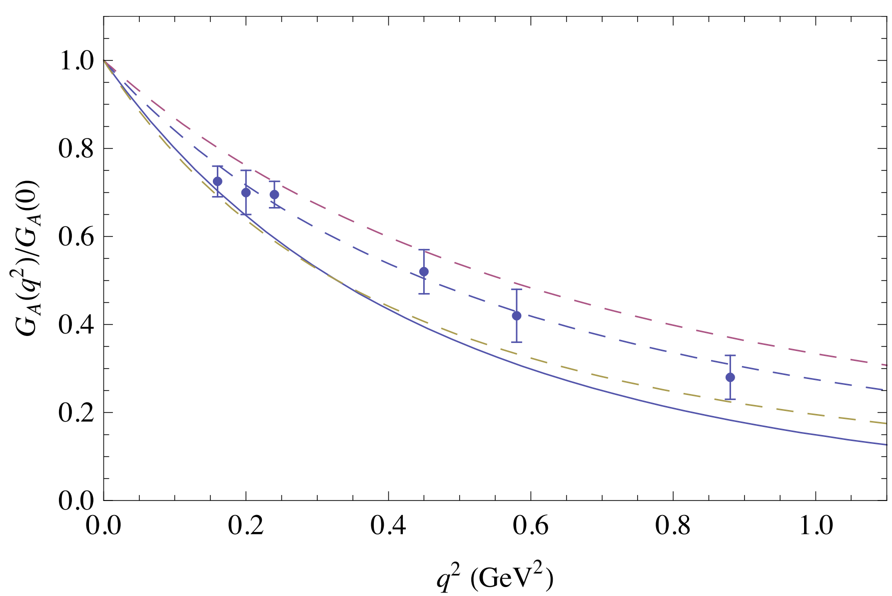
Form factors: Dashed line: Empirical dipole fit

Wulzer,Panico









II. Gravity duals of superconductors
(Holographic superconductors)

Superconductor:

Material inside which the EM U(1) is spontaneously broken at certain $T<T_c$

("Higgs mechanism")

Order parameter $<O>$ (e.g. condensation of cooper-pairs) turns on at low-temperature

Proposed model for 3D holographic superconductor

Harnoll,Herzog,Horowitz 09

4D gravity theory: $$S = \int d^4x \sqrt{-G} \left\{ \frac{1}{16\pi G_N} (R + \Lambda) - \frac{1}{g^2} \mathcal{L} \right\}$$

$$\mathcal{L} = \frac{1}{4} F^2 + \frac{1}{L^2} |D_\mu \Psi|^2 + \frac{m^2}{L^4} |\Psi|^2$$

Metric: AdS-Schwarzschild Black Hole:

$$ds^2 = \frac{L^2}{z^2} \left(-f(z) dt^2 + dr^2 + r^2 d\phi^2 \right) + \frac{L^2}{z^2 f(z)} dz^2$$

$$f(z) = 1 - (z/z_h)^3$$

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Metric: AdS-Schwarzschil

$$ds^2 = \frac{L^2}{z^2} \left(-f(z) dt^2 + dr^2 + r \right.$$

$$f(z) = 1 - (z/z$$

AdS ↔ CFT

Gauge field: A_μ ↔ Conserved current

Charged Scalar: Ψ ↔ Order parameter: <O>

BH ↔ Finite Temperature

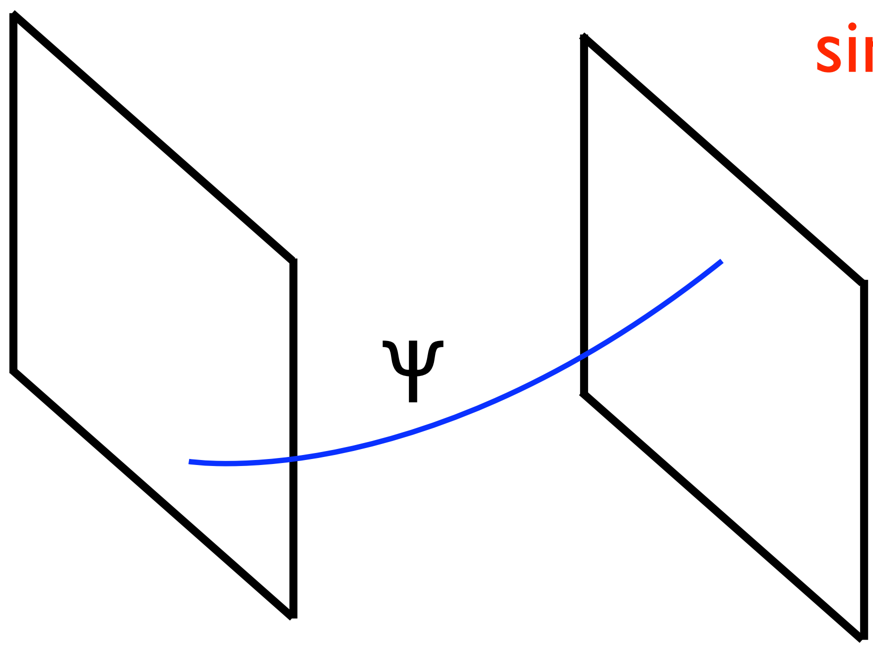
$$T = 3/(4\pi z_h)$$

Charge density of the superconductor → $A_0 \neq 0$ in the bulk

(sets the scale)

Below certain T, the scalar turns on towards the BH

since $m_{eff}^2 \sim m^2 - A_0^2$



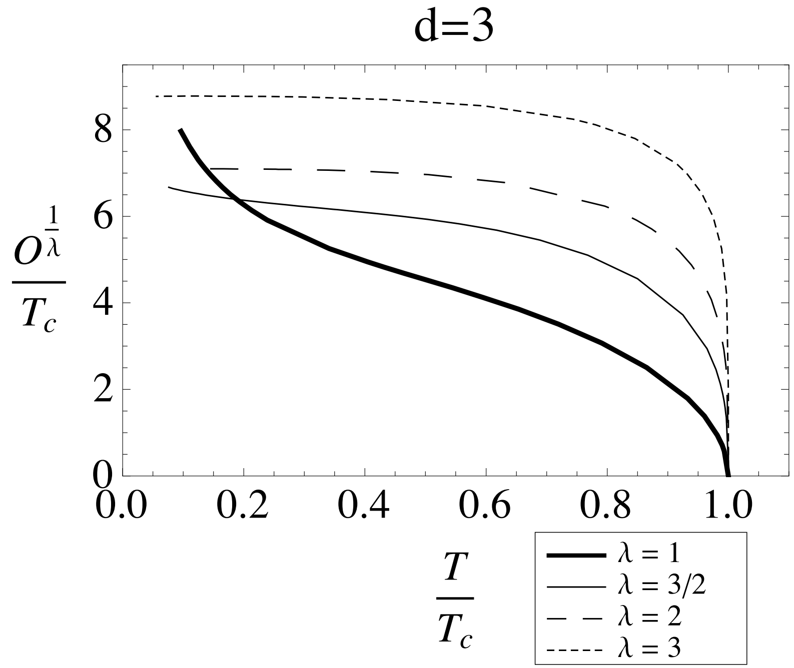
BH horizon

New phase:

Superconducting phase: $\langle \Psi \rangle \neq 0$ → Nonzero "photon mass"

☛ Order parameter is nonzero for $T<T_c$:

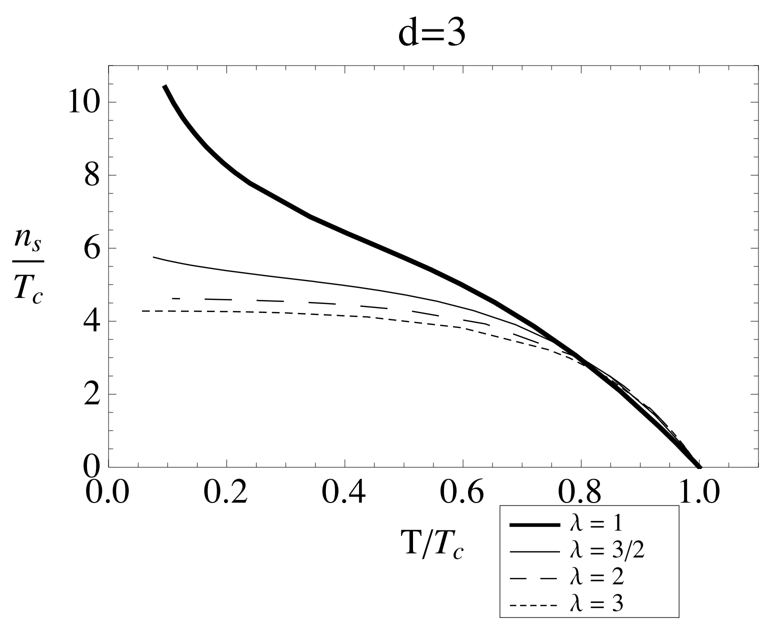
Horowitz,Roberts



λ=Dim[O]

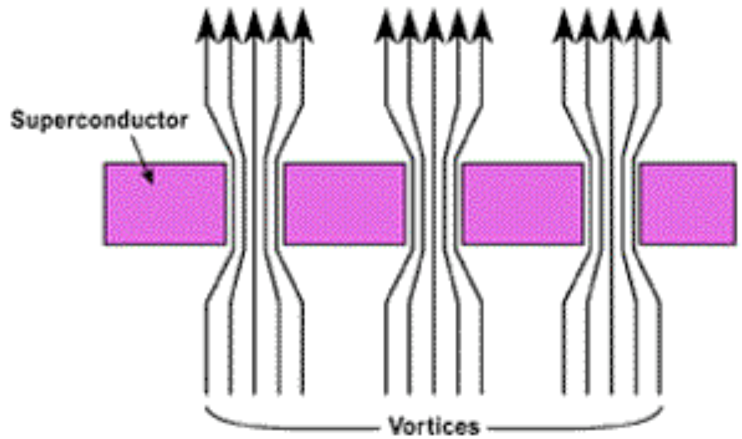
☛ "Superconducting density" (<JJ> correlator at zero ω):

("photon mass")



Turning on magnetic fields:

Expected: Abrikosov Vortices (if it is a Type II superconductor)



Do these type of vortices exist in holographic superconductors?

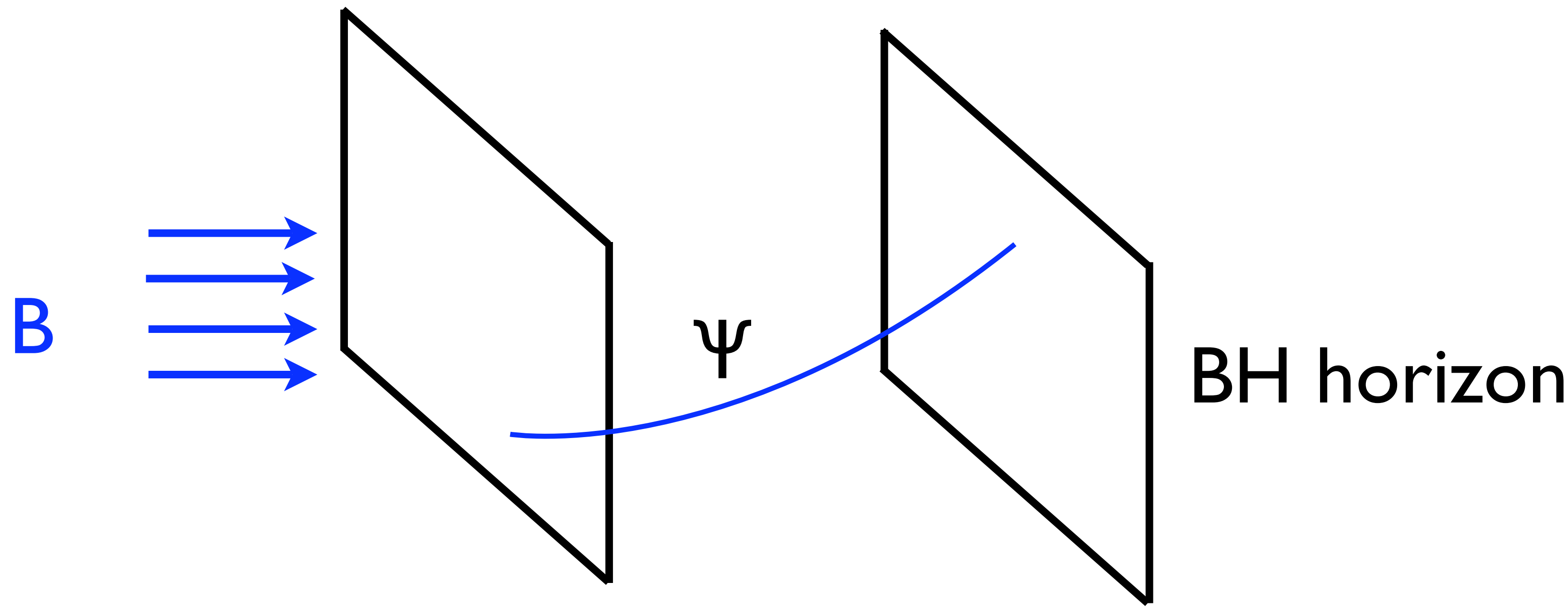
Yes, and they can be energetically favorable for certain B

Montull,A.P.,Silva 09

(see also Albash,Johnson 09)

Turning on magnetic fields:

Gravity theory:



Vortex Ansatz:

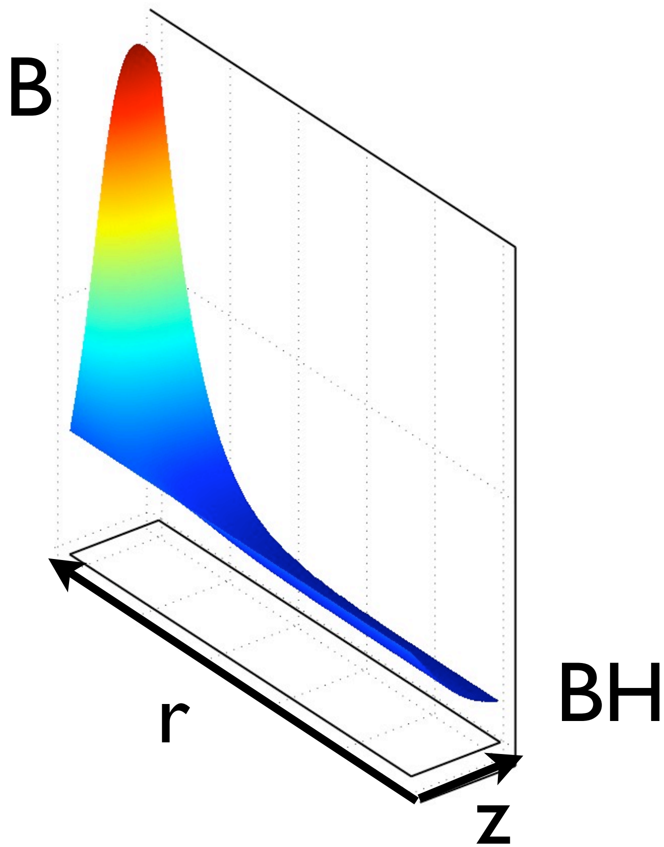
$$\Psi = \psi(r,z)\, e^{in\phi} \ , \quad A_0 = A_0(r,z) \ , \quad A_\phi = A_\phi(r,z)$$

working in the probe limit: g →Infty

→ configurations must be found numerically

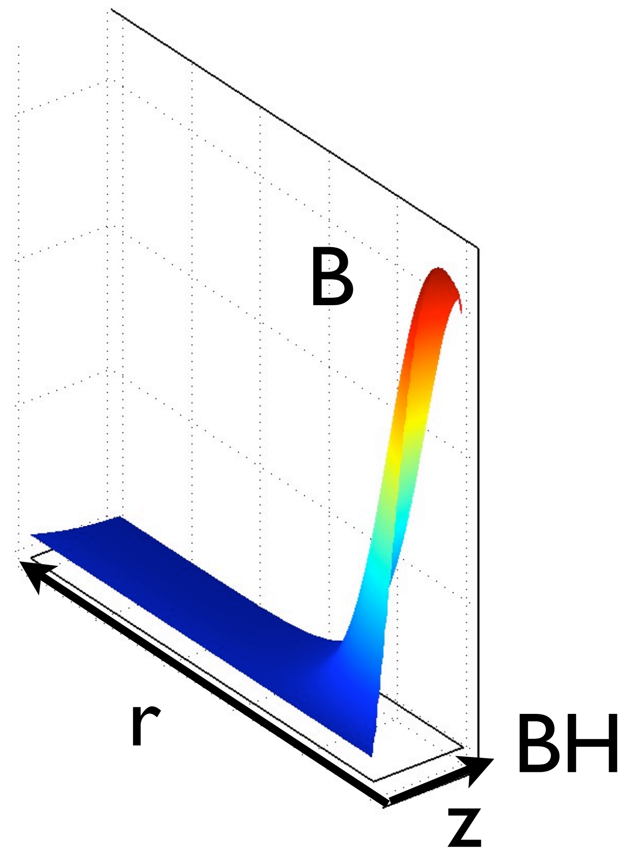
Numerical solutions of B in the bulk:

1) Small B:



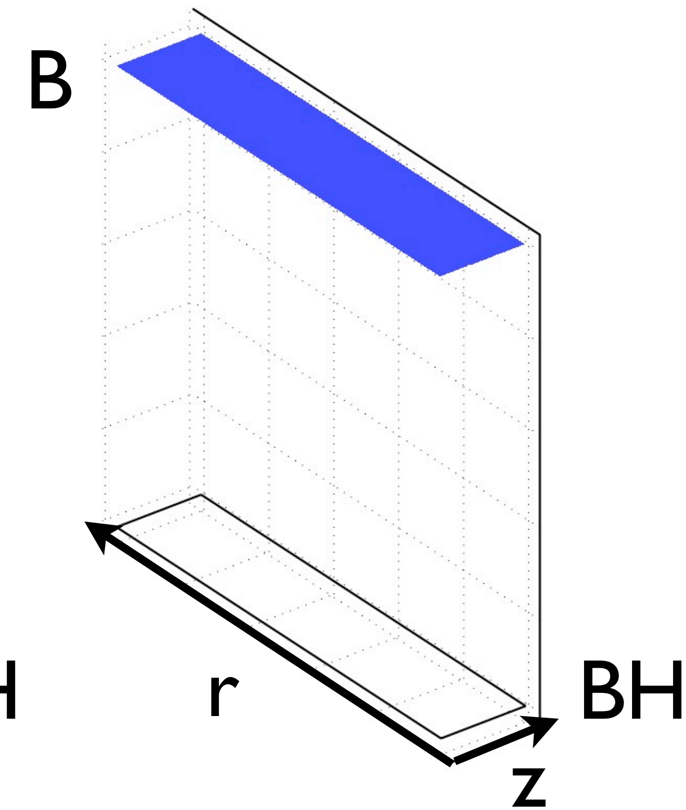
Magnetic field expelled:
bulk “Meissner effect”

2) Intermediate B:



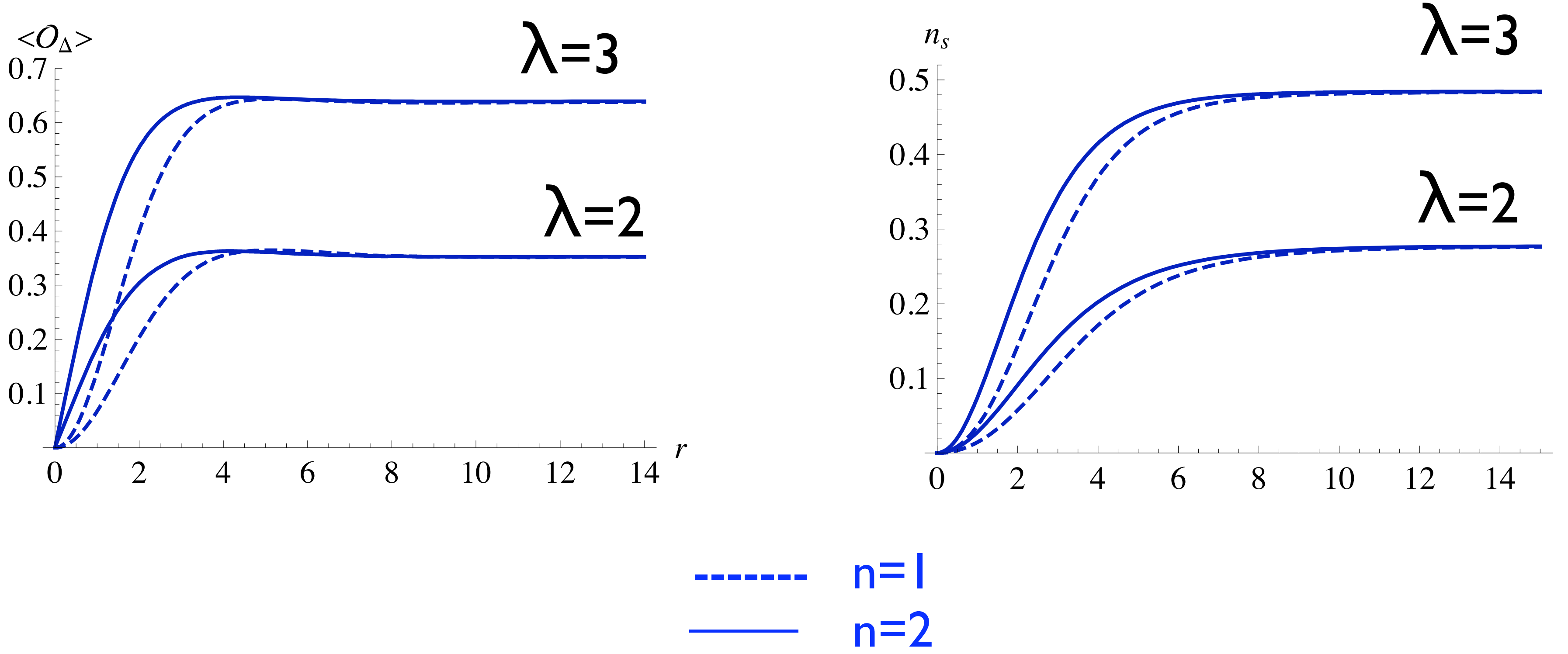
Magnetic field passes
through a vortex

3) Large B:



Magnetic field not expelled:
normal phase

Numerical solution shows order parameter and "superconducting density" goes to zero for small r → Core of the vortex



Conclusions

- Solitonic physics in holographic models give interesting phenomena associated with either

- ➦ Baryons of a 5D QCD-like model: We have found the exact solution (numerically) and calculated their properties (masses, couplings, ...) → behave like real baryons

- ➦ Vortices in holographic superconductors: Energetically favorable for certain B-fields

- Still a lot of questions to answer: e.g. Lattice of baryons/vortices