Inhomogeneities and Cosmological Expansion

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September 5, 2009

Outline

- Effect of inhomogeneities on the perceived acceleration
- A similar problem in brane cosmology
- Large-scale structures in quintessence cosmology
- Prospects

Standard framework

- Basic assumptions: Homogeneity and isotropy

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

$$\left(\frac{\dot{a}}{a_D}\right)^2 = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

- Inhomogeneities can be treated as small perturbations of this background
- Indications for the acceleration of the cosmological expansion
  1. Distant supernovae
  2. Power spectrum of the galaxy distribution
  3. Cosmic microwave background
- For acceleration: $p < -\rho/3$

Question

- Could the acceleration of the cosmological expansion be related to the appearance of inhomogeneities in a pressureless cosmological fluid (dark matter)?

Various attempts

- Kolb, Matarrese, Notari, Riotto (2005)

$$ds^2 = -dt^2 + a^2(t)e^{-2\Psi(\vec{x},t)}\delta_{ij}dx^i dx^j$$

  Acceleration from super-horizon perturbations
  Does not work!

- Holz, Wald (1998)

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1-2\Psi(\vec{x},t))\delta_{ij}dx^i dx^j$$

  Ψ: Newtonian potential
  No acceleration

- Buchert(2000), Rasanen (2004)

$$ds^2 = -dt^2 + g_{ij} dx^i dx^j$$
$$\langle \Phi \rangle_D(t) = \frac{1}{V_D} \int_D \Phi(t, x^i) \sqrt{\det g} d^3x$$

Average scale factor: $a_D \sim V_D^{1/3}$

$$\left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G}{3} \langle \rho \rangle_D - \frac{1}{6} \langle R \rangle_D - \frac{1}{6} Q_D$$
$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G}{3} \langle \rho \rangle_D + \frac{1}{3} Q_D$$
$$Q_D = \frac{2}{3} \left(\langle \theta^2 \rangle_D - \langle \theta \rangle_D^2 \right) - \langle \sigma_{ij} \sigma^{ij} \rangle_D$$
$$\left(a_D^6 Q_D \right)^{\cdot} + a^4 \left(a_D^2 \langle R \rangle_D \right)^{\cdot} = 0.$$

There are several unclear points in this approach.

- Apostolopoulos, Brouzakis, N.T., Tzavara (2006)
  In inhomogeneous backgrounds the perceived acceleration depends on the direction of observation

$$ds^2 = -dt^2 + R'^2 dr^2 + R^2(t,r)(d\theta^2 + \sin^2\theta \, d\phi^2)$$

Expansion rate

$$H = \frac{2}{3}H_\theta + \frac{1}{3}H_r$$

$$H_r = \frac{\dot{R}'}{R'}$$

$$H_\theta = \frac{\dot{R}}{R}$$

The expansion may be accelerating in one direction and decelerating in the other.

Our approach

- All the information about the expansion of the Universe is obtained through light signals.
- Study light propagation in an exact background that mimics a Universe with structure.
- Calculate observables: Luminosity distance of a light source a function of its redshift.
- P. Apostolopoulos, N. Brouzakis, N. T., E. Tzavara
  astro-ph/0603234, JCAP 0606:009, 2006
  N. Brouzakis, N. T., E. Tzavara
  astro-ph/0612179, JCAP 0702:013, 2007
  astro-ph/0703586, JCAP 0804:008, 2008
  N. Brouzakis, N. T.
  arXiv:0802.0859 [astro-ph], Phys.Lett.B665:344-348,2008

The background

- A variation of the Swiss-cheese model, applicable to length scales larger than $\sim \mathcal{O}(10)\, h^{-1}$ Mpc.
- Cheese: Homogeneous and isotropic (Friedmann-Robertson-Walker metric)
- Hole: Spherically symmetric, but inhomogeneous (Lemaitre-Tolman-Bondi metric):

$$ds^2 = -dt^2 + \frac{R'^2(t,r)}{1+f(r)}dr^2 + R^2(t,r)d\Omega^2,$$

  with $f(r)$ arbitrary.
- Einstein equations:

$$\dot{R}^2(t,r) = \frac{1}{8\pi M^2}\frac{\mathcal{M}(r)}{R} + f(r)$$
$$\mathcal{M}'(r) = 4\pi R^2 \rho R'.$$

  The mass function $\mathcal{M}(r)$ is arbitrary.

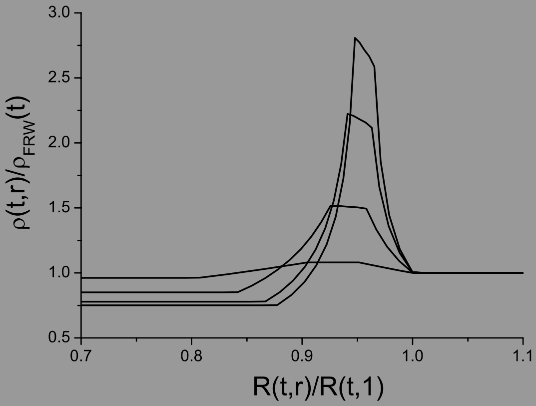


Figure: *The evolution of the density profile for a central underdensity surrounded by an overdensity.*

Photon paths

- Photons follow geodesics in this background.
- The cross-section area A of a light beam changes along its trajectory. The expansion of the beam is (λ: affine parameter)

$$\theta = \frac{1}{2A}\frac{dA}{d\lambda} \tag{1}$$

- The symmetric and traceless shear tensor

$$\sigma_{ab} = \begin{pmatrix} \sigma_1 & \sigma_2 \\ \sigma_2 & -\sigma_1 \end{pmatrix} \tag{2}$$

describes deformations of the beam.
- Sachs optical equations ($\sigma^2 = \sigma_1^2 + \sigma_2^2$):

$$\frac{d\theta}{d\lambda} = -\frac{1}{4M^2}\rho\left(k^0\right)^2 - \theta^2 - \sigma^2$$

$$\frac{d\sigma}{d\lambda} + 2\theta\sigma = \frac{\left(k^3\right)^2 R^2}{4M^2}\left(\rho - \frac{3\mathcal{M}(r)}{4\pi R^3}\right).$$

Luminosity distance and redshift

- Consider photons emitted within a solid angle Ω_s by an isotropic source with luminosity L. These photons are detected by an observer for whom the light beam has a cross-section A_o.
- The redshift factor is

$$1+z=\frac{\omega_s}{\omega_o}=\frac{k_s^0}{k_o^0},$$

- The energy flux f_o measured by the observer is

$$f_o=\frac{L}{4\pi D_L^2}=\frac{L}{4\pi}\frac{\Omega_s}{(1+z)^2A_o}.$$

- Integrating the optical equations allows the determination of the luminosity distance D_L as a function of the redshift z.

Observer at the center of a large hole

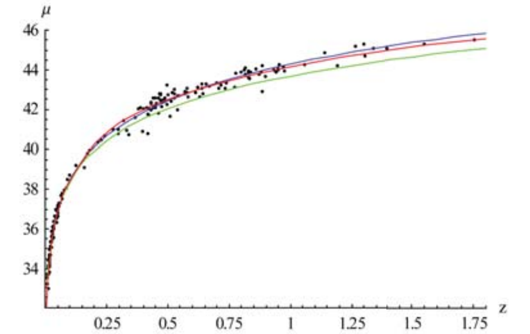


Figure: *The distance modulus* $\mu = m - M = 5\log(D_L/\mathrm{Mpc}) + 25$ *as a function of redshift* z*.*
a) Green line: FRW cosmology with $\Omega_m = 1$, $\Omega_\Lambda = 0$.
b) Blue line: FRW cosmology with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$.
c) Red line: LTB cosmology with the observer at the center of an underdense region of present size ~ 800 *Mpc.*

Observer and source at random positions

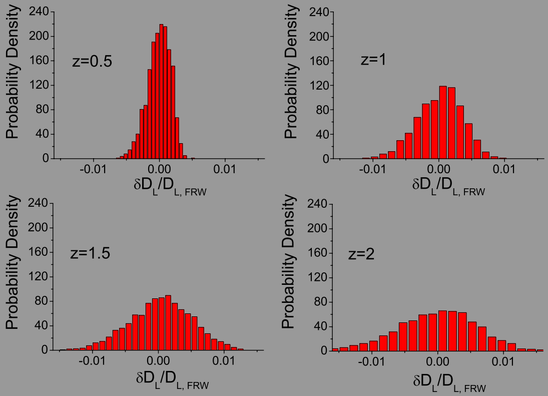


Figure: *The distribution of luminosity distances for various redshifts in the LTB Swiss-cheese model for inhomogeneities with length scale $40\,h^{-1}$ Mpc.*

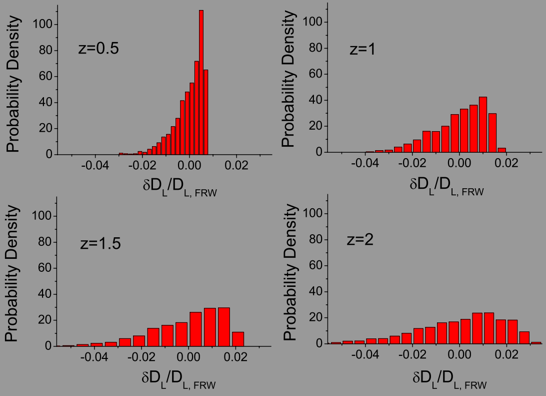


Figure: *Same as before for a characteristic scale of* $400\,h^{-1}$ *Mpc.*

The effect on the determination of $w = p/\rho$

- The presence of inhomogeneities induces a statistical error in the value of w deduced from astrophysical data, as well as a shift of its average value if the sample is small.
- For inhomogeneities with a characteristic scale of $40\,h^{-1}$ Mpc the error is $\delta w \simeq 0.015$ for all z between 0.5 and 2, while the average value for a small sample is $\bar{w} \lesssim -0.003$.
- For inhomogeneities with a characteristic scale of $400\,h^{-1}$ Mpc the error increases from 0.03 to 0.05, while the average is $\bar{w} \simeq -0.015$.

Analytical estimate

- $\bar{H} = \frac{\text{size of the inhomogeneity}}{\text{horizon distance}} = \frac{r_0}{1/H} = r_0 H < 1$
- Central observer
  - $\delta z / z_{FRW} = \mathcal{O}(\bar{H}^2)$
  - $\delta A / A_{FRW} = \mathcal{O}(\bar{H}^2)$
- Random observer
  - $\delta z / z_{FRW} = \mathcal{O}(\bar{H}^3)$
  - $\delta A / A_{FRW} = \mathcal{O}(\bar{H}^2)$
  - $(\delta A / A_{FRW})_{\text{average}} = \mathcal{O}(\bar{H}^3)$
- Consistent with flux conservation.

A similar problem in brane cosmology

- Identify the Universe with a hypersurface (brane) in five-dimensional space-time. Low-energy gravity is localized near the brane (Randall, Sundrum (1999)).
- Assume an inhomogeneous energy distribution along the fourth spatial dimension. Is accelerated expansion possible along the brane?
- For an arbitrary energy distribution, accelerated expansion requires negative pressure either on the brane or in the bulk.
- This holds even when corrections, such as an induced gravity term on the brane, or a Gauss-Bonnet term in the bulk, are taken into account.
- P. Apostolopoulos, N. T.
  astro-ph/0604014, Phys. Rev. D 74 (2006) 064021
  P. Apostolopoulos, N. Brouzakis, N. T., E. Tzavara
  arXiv:0708.0469, Phys. Rev. D 76 (2007) 084029

Quintessence cosmology

- It seem unlikely that the acceleration of the cosmological expansion can be attributed to the growth of inhomogeneities.
- Negative pressure is needed.
- The simplest scenario assumes the presence of a cosmological constant.
- The quintessence scenario (Wetterich (1988)) can provide a dynamical explanation for the smallness of the present value of the vacuum energy.
- We shall discuss coupled quintessence (Wetterich (1994)): a quintessence field coupled with dark matter (or neutrinos).
- What kind of new structures can appear in such cosmologies?
- Are they observable?

Basic relations

- Action

$$\mathcal{S} = \int d^4x \sqrt{-g}\left(\frac{1}{16\pi G}R - \frac{1}{2}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x_\mu} - U(\phi)\right) - \sum_i \int m(\phi(x_i))d\tau_i,$$

  with $d\tau_i = \sqrt{-g_{\mu\nu}(x_i)dx_i^\mu dx_i^\nu}$ and the second integral taken over particle trajectories.

- Equation of motion

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}\, g^{\mu\nu}\frac{\partial\phi}{\partial x^\nu}\right) = \frac{dU}{d\phi} - \frac{d\ln m(\phi(x))}{d\phi}\ (T_M)^\mu_{\ \mu}.$$

Cosmological evolution

- Homogeneous background

$$\ddot{\phi} + 3H\dot{\phi} = \frac{dU}{d\phi} + \frac{d\ln m(\phi)}{d\phi}(\rho - 3p)$$

$$\dot{\rho} + 3H\rho = -\frac{d\ln m(\phi)}{d\phi}(\rho - 3p)\dot{\phi}$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + U(\phi) + \rho\right)$$

Static spherically symmetric configurations

- Metric:

$$ds^2 = -B(r)dt^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + A(r)dr^2.$$

- Fermi-Dirac distribution at every point in space:

$$f(p) = \left[\exp\left(\frac{\sqrt{p^2 + m^2(\phi(r))} - \mu(r)}{T(r)}\right) + 1\right]^{-1}.$$

- The Einstein equations give:

$$T(r) = T_0/\sqrt{B(r)}, \qquad \mu(r) = \mu_0/\sqrt{B(r)}.$$

- N. T.
  hep-ph/0507288, Phys. Lett. B 632: 463-466, 2006
  N. Brouzakis, N. T.
  astro-ph/0509755, JCAP 0601:004, 2006
  N. T., J.D. Vergados, A. Faessler
  hep-ph/0609078, Phys. Rev. D 75 (2007) 023504

Dark matter in galaxy haloes

- The coupling between DM and the quintessence field generates an attractive force between DM particles.
- The typical DM velocity is larger than in the decoupled case.
- Implications for DM detection.

Compact astrophysical objects made of dark matter

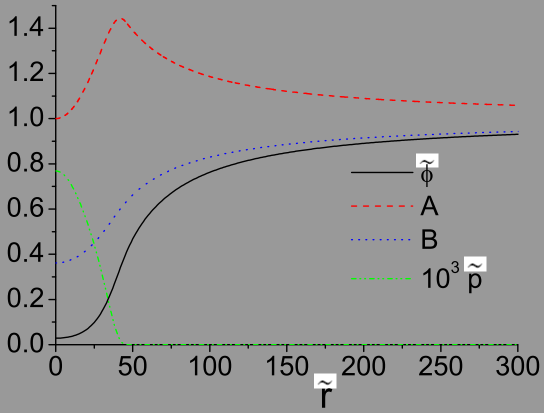


Figure: *A typical configuration*

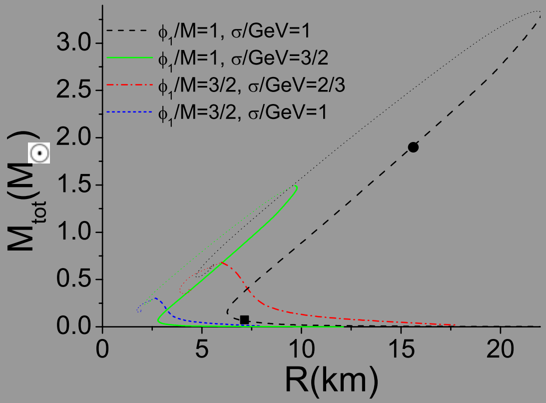
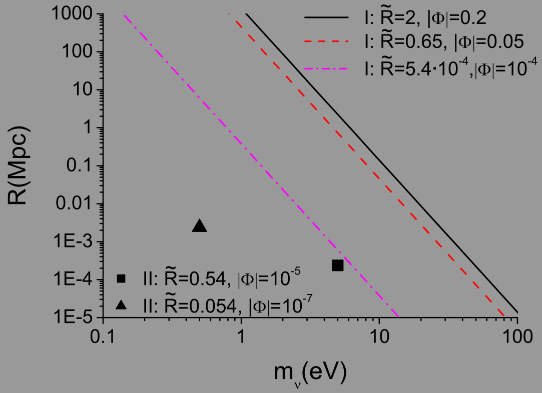


Figure: *The mass to radius relation.*

Astrophysical objects made of neutrinos



N. Brouzakis, N. T., C. Wetterich
e-Print: arXiv:0711.2226 [astro-ph] , Phys. Lett. B 665: 131-134,

Prospects

- Study the formation of such structures.
- The evolution of inhomogeneities depends on the cosmological background.
- The matter spectrum at various redshifts reflects the detailed structure of the cosmological model.
- Comparison with observations of the galaxy distribution can differentiate between models.
- Work in progress: Analytical calculation of the matter spectrum beyond the linear level.