Open Problems in String Cosmology

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Observational evidence supports a very rich, but highly involved, version of hot Big Bang cosmology.

Some of its main features are:

an early period of slow-roll inflation, during which the Universe grew to a large macroscopic size

very high temperature, and symmetry breaking phase transitions

a proportionally large amount of dark matter and dark energy, dominating the late time evolution . . .

This largely phenomenological model, presents some of the greatest challenges to fundamental physics.

Two in particular:

If we extrapolate the cosmological evolution arbitrarily back in time, using the equations of General Relativity and Quantum Field Theory $\rightarrow$

we are driven to an initial singularity, the Big Bang, where these descriptions are breaking down.

Inflation alleviates some of the problems of the standard hot Big Bang model, such as the flatness, the large size and the horizon problems,

$\longrightarrow$ but it is generally believed that it is not past complete . . .

A second is concerned with the nature of the dark energy, the simplest explanation for it being a positive, however unnaturally small, cosmological constant $\longrightarrow$

many orders of magnitude smaller than the Planck scale . . .

Moreover, if the dark energy persists arbitrarily long, it would imply that
the Universe approaches de Sitter space far in the future $\longrightarrow$

and so portions of space will remain out of causal reach of a single observer . . .

Within the context of General Relativity and the Standard Model we lack a coherent framework to analyze the cosmology of our Universe, from beginning to end.

If string theory is a complete theory of quantum gravity, it should eventually provide us with a coherent framework for studying cosmology.

The hope is that by incorporating fundamental duality symmetries and new degrees of freedom of string theory in time-dependent, cosmological settings $\rightarrow$

some of the greatest cosmological puzzles will find a natural resolution with important implications and new tools for cosmological model building.

Indeed, string dualities give us profound insights into the nature of *Spacetime*,

with many surprising phenomena arising, just when we try to probe features of spacetime and geometry at short distances, distances of order the string scale l_s or the Planck length l_p.

These lead to important properties and consequences such as

- UV finiteness
- The stringy spacetime uncertainty principle: $\Delta x \Delta t \sim l_s^2$
- T-duality
- Resolution of orbifold and conifold singularities
- Smooth topology changing transitions ...

illustrating how String Theory can provide concrete answers to many of the puzzles one has to face in trying to quantize Einstein's theory of general relativity.

It could be, for example, that the initial singularity is actually a transition between a contracting big crunch phase and an expanding big bang phase

as in pre big bang models (Gasperini, Veneziano)

with a non-singular eternal cosmology emerging in string theory.

Then many puzzles of the standard hot big bang theory, such as the horizon and entropy problems are absent . . .

A period of inflation may still be needed to wash out probable inhomogeneities produced at the bounce.

Exact, non-singular cosmological solutions to classical superstring theory already exist $\longrightarrow$

described by a two-dimensional worldsheet CFT of the form $SL(2,R)_{-|k|}/U(1) \ \times \ K$ (C. Kounnas, D. Lust)

K is an internal, compact CFT.

The exact CFT description allows us to resolve singularities $\longrightarrow$

that arise in the sigma-model approximation to these backgrounds.

Often the resulting global description is a non-geometrical one, in terms of T-folds,

where patches of space are glued together using T-duality (C. Kounnas, NT, J. Troost).

In the rest of this talk, I will be interested in non-trivial string theory cosmological solutions, which in a large region of moduli space are characterized by an underlying "no scale" structure. (Catelin-Jullien, Kounnas, Partouche, NT)

Many of the problems we need to address within the broader context of Stringy Cosmology can be illustrated using this class of examples.

Start with a weakly coupled supersymmetric string theory, on an initially flat background:

$$R^4 \times T^6$$

We then introduce sources of supersymmetry breaking, by utilizing geometrical fluxes, threading some cycles of the internal toroidal manifold.

These fluxes can be easily adapted at the full string level in the framework of Freely Acting Orbifolds,

a generalization in string theory of Scherk-Schwarz compactification.

(Notice that by using (non-perturbative) string dualities, we can also map this system into a dual one, where the sources of supersymmetry breaking are due to wrapped branes and other non-geometrical fluxes)

Provided that the radii of moduli participating in the SUSY breaking mechanism are large enough, as compared to the string scale, these models are free of tachyonic instabilities.

At low energies, we get an effective "no-scale" supergravity theory with spontaneously broken SUSY.

Namely:
At tree level, the moduli participating in the SUSY breaking mechanism are flat directions, (while many other moduli get a soft breaking mass proportional to the gravitino mass scale.)

Non-trivial time dependence arises when we take into account the thermal and quantum corrections.

To analyze it, we first identify a regime of computational control:

$$T\,,\;\; m_{3/2} \ll M_s$$

In this regime,

- The thermal effective potential is calculable, at the full string level, and it is free of UV and IR ambiguities
- When the VEVs of moduli that are not participating in the SUSY breaking mechanism are of order unity, they give exponentially suppressed contributions to the thermal effective potential
- Complex structure moduli of the form R_x/R_y, (the corresponding radii are involved in the SUSY breaking mechanism), are stabilized (by geometrical fluxes)

The gravitino mass scale is set by a single running modulus, the no-scale modulus.

Thermal Effective Potential

$$P \sim T^4 F\left(\frac{m_{3/2}}{T}\right),$$

The function F can be expressed neatly in terms of Eisenstein series.

Notice that we do not include exponentially suppressed terms of the form e^{-S}, e^{-T} ...
but we keep all corrections involving the ratio of the two SUSY breaking scales, $m_{3/2}$, T.

(Adding estimated exponentially suppressed terms randomly, destroys the no-scale structure)

Incorporating the backreaction on the initially flat background, we obtain in several cases a cosmological solution that follows the critical trajectory:

$$m_{3/2} = uT = \frac{1}{\gamma a},$$

a is the scale factor of the universe, u, γ are model dependent constants (see also the talk by Herve Partouche in this workshop).

The T, a relation, is characteristic of a radiation dominated cosmological evolution.

This phase persists, but it is eventually interrupted at both ends of the temperature scale$\longrightarrow$

1.) by a symmetry breaking phase transition at lower temperatures, such as the electroweak breaking phase transition

2.) by the onset of Hagedorn instabilities at higher temperatures, temperatures of order the string scale, *before the "Big Bang"* .

It is important to look for rich enough models$\longrightarrow$
to provide a mechanism that stabilizes the SUSY breaking no-scale modulus (and other relevant moduli), *at least* just after the elecroweak symmetry breaking scale.

At around the Hagedorn temperature, new string theoretic degrees of freedom, oscillators and string winding states, become relevant.

Clearly, to understand the the very early history of these cosmologies, we need to be able to handle the instabilities of string theory at high temperature.

(obtaining a concrete realization of the String gas cosmological scenario (Brandenberger, Vafa)).

I will conclude by describing some new ideas towards this direction (Angelantonj, Kounnas, Partouche, NT).

Hagedorn divergences of the canonical ensemble: due to an exponential growth in the density of single string particle states as a function of the mass.

$\rightarrow$ phase transition at the Hagedorn temperature.

$$T_H \sim \frac{1}{l_s}$$

The partition function can be computed via a Euclidean path integral on $S^1 \times \mathcal{M}$ (S^1 is the Euclidean time circle with period $\beta = 1/(2\pi T)$).

At $T > T_H$ certain stringy winding modes ($n \neq 0$) become tachyonic. $\rightarrow$ divergence can be thought of as an IR instability and the phase transition is driven by tachyon condensation.

The IR instability can be removed by deforming appropriately the underlying Euclidean background.

It has been argued $\rightarrow$ that in the context of $N = 4$ Heterotic strings, the exact tachyon potential has a stable minimum (Antoniadis, Derendinger, Kounnas).

The potential was derived using properties of $N = 4$ gauged supergravity.

A stable high temperature phase exists, with the Free energy getting a genus zero contribution.

This phase is in fact characterized by *thermal duality symmetry:* $\beta \rightarrow 1/\beta$.

But there is another way to stabilize the high temperature phase (AKPT):

The winding tachyons are charged under the graviphoton, and axial vector gauge field, the latter being associated to the $B_{\mu\nu}$ background field of string theory.

We can lift the tachyonic instabilities by turning on appropriate geometrical fluxes associated with these gauge fields.

The analogue to keep in mind is the motion of a charged particle on a plane, under an inverted harmonic oscillator potential:

A large enough magnetic field can stabilize the motion of the particle.

The gravito-magnetic fluxes correspond in fact to gauge field condensates, of zero field strength, but with a non-zero value of the Wilson line

$$U = P\exp(i\int_0^\beta A_0 dX^0)$$

At finite temperature, (abelian) vacuum potentials in the range $0 \leq \frac{A_0}{T} \leq \pi$ are gauge inequivalent.

These Wilson lines *refine* the canonical ensemble, and it turns out that they are also described by thermal duality symmetry.

The Model

Consider type IIB on $T^2 \times T^8$.

*T^8 is a very large eight-torus

*T^2 is a rectangular torus $S^1_T \times S^1$, the first circle is the Euclidean time circle of radius R_0.

Initially the model is supersymmetric

$$\mathcal{Z} = \frac{1}{4} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \frac{1}{(\eta\bar{\eta})^{12}} \Gamma_{(1,1)}(R_0) \Gamma_{(1,1)}(R_1) \Gamma_{(8,8)}$$

$$\times \sum_{a,b=0,1} (-1)^{a+b+ab} \, \theta^4 \begin{bmatrix} a \\ b \end{bmatrix} \sum_{\bar{a},\bar{b}=0,1} (-1)^{\bar{a}+\bar{b}+\bar{a}\bar{b}} \, \bar{\theta}^4 \begin{bmatrix} \bar{a} \\ \bar{b} \end{bmatrix}$$

Or in terms of the $SO(8)$ characters

$$O_8 = \frac{\theta_3^4 + \theta_4^4}{2\,\eta^4}\,, \qquad V_8 = \frac{\theta_3^4 - \theta_4^4}{2\,\eta^4}\,,$$

$$S_8 = \frac{\theta_2^4 - \theta_1^4}{2\,\eta^4}\,, \qquad C_8 = \frac{\theta_2^4 + \theta_1^4}{2\,\eta^4}\,,$$

$$\mathcal{Z} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \frac{1}{(\eta\bar{\eta})^8}\,|V_8 - S_8|^2\,\Gamma_{(1,1)}(R_0)\,\Gamma_{(1,1)}(R_1)\,\Gamma_{(8,8)}$$

- All oscillators along the X^0, X^1 directions can be gauged away
- When we decompactify the T^8 torus, we get an $SO(8)$ symmetry

Spacetime fermion number receives contributions from both the left and right worldsheet movers

$F = F_L + F_R$

Under $(-1)^{F_L}$ the left moving R sector changes sign, similarly for F_R.

Conventional thermal deformation: Insert the phase

$$(-1)^{\tilde{m}_0(a+\bar{a})+n_0(b+\bar{b})}$$

Instead we consider the asymmetric deformation:

$$\frac{R_0}{\sqrt{\tau_2}}\sum_{\tilde{m}_0,n_0} e^{-\frac{\pi R_0^2}{\tau_2}|\tilde{m}_0+n_0\tau|^2}(-)^{\tilde{m}_0 a+n_0 b+\tilde{m}_0 n_0}$$

$$\frac{R_1}{\sqrt{\tau_2}}\sum_{\tilde{m}_1,n_1} e^{-\frac{\pi R_1^2}{\tau_2}|\tilde{m}_1+n_1\tau|^2}(-)^{\tilde{m}_1 \bar{a}+n_1 \bar{b}+\tilde{m}_1 n_1}$$

In this way, the X^0 lattice is "thermally" coupled to the left-moving world-sheet degrees of freedom, while the X^1 lattice is "thermally" coupled to the right-moving world-sheet degrees of freedom.

In the n_0 (n_1) odd winding sector the left (right) GSO projection is reversed

The string partition function takes the form

$$\mathcal{Z} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \frac{\Gamma_{(8,8)}}{(\eta\bar{\eta})^8}$$

$$\times \sum_{m_0,n_0} \left(V_8\, \Gamma_{m_0,2n_0} + O_8\, \Gamma_{m_0+\frac{1}{2},2n_0+1} - S_8\, \Gamma_{m_0+\frac{1}{2},2n_0} - C_8\, \Gamma_{m_0,2n_0+1} \right)$$

$$\times \sum_{m_1,n_1} \left(\bar{V}_8\, \Gamma_{m_1,2n_1} + \bar{O}_8\, \Gamma_{m_1+\frac{1}{2},2n_1+1} - \bar{S}_8\, \Gamma_{m_1+\frac{1}{2},2n_1} - \bar{C}_8\, \Gamma_{m_1,2n_1+1} \right) .$$

→ The $O\bar{O}$ sector appears in the spectrum, which typically becomes tachyonic in some regions of moduli space.

Here, however it carries non-zero momentum and winding charges and so

$$2\, m^2_{O\bar{O}} = \left(\frac{1}{\sqrt{2}R_0} - \sqrt{2}R_0\right)^2 + \left(\frac{1}{\sqrt{2}R_1} - \sqrt{2}R_1\right)^2$$

It is never tachyonic. Massless when $R_0 = R_1 = \frac{1}{\sqrt{2}}$, dual fermionic point.

Spectrum

Only the $V\bar{V}$ ($G_{\mu\nu}$, $B_{\mu\nu}$, Φ) sector is massless.

Fermions

$$2\,m^2_{V\bar{S}} = \frac{1}{(\sqrt{2}R_1)^2}\,, \qquad 2\,m^2_{S\bar{V}} = \frac{1}{(\sqrt{2}R_0)^2}$$

and from the odd winding sectors

$$2\,m^2_{V\bar{C}} = (\sqrt{2}R_1)^2\,, \qquad 2\,m^2_{C\bar{V}} = (\sqrt{2}R_0)^2$$

- At large radii the spinors S, $\bar{S}$ are light.
- At small radii the conjugate spinors C, $\bar{C}$ are light.

All RR fields are massive since these are charged under $(-1)^{F_L}$ and $(-1)^{F_R}$.

In fact the spectrum is T-duality invariant under

$$R_0,\ R_1 \to \frac{1}{2R_0},\ \frac{1}{2R_1}$$

$$S,\ \bar{S} \to C, \bar{C}$$

Supersymmetry is restored at both ends of moduli space.
At large radii: chiral IIB model
At small radii: the equivalent chiral IIB' model.

At the self-dual point $R_0 = R_1 = \frac{1}{\sqrt{2}}$, we get additional massless states from the $O\bar{O}$, $V\bar{O}$ and $O\bar{V}$ sectors $\to$

$SU(2)_L \times SU(2)_R$ enhanced gauge symmetry

Observe also that when

$$R_0 >> 1, \qquad R_1 \sim 1$$

the light states arise in the $V\bar{V}$ and $S\bar{V}$ sectors with masses

$$m^2_{V\bar{V}} = 0\,, \qquad 2\,m^2_{S\bar{V}} = \frac{1}{(\sqrt{2}R_0)^2}$$

→ the light spectrum is precisely thermal.

Thermal Interpretation

Shift

$$\tilde{m}_1 \to \tilde{m}_1 + \tilde{m}_0 \,, \qquad n_1 \to n_1 + n_0$$

We obtain a thermally coupled $\Gamma_{(2,2)}$ torus lattice, where there is a non-trivial B field background

$$B_{01} = -B_{10} = \frac{1}{2}$$

and a non-diagonal metric

$$ds^2 = R_0^2 (dx^0)^2 + R_1^2 (dx^1 + G\, dx^0)^2$$

$$\frac{G_{01}}{R_1^2} = G = 1$$

$$\frac{R_0\, R_1}{\tau_2} \sum_{\tilde{m},n} e^{-\frac{\pi}{\tau_2}\left[R_0^2|\tilde{m}_0+\tau n_0|^2+R_1^2|\tilde{m}_1+G\tilde{m}_0+\tau(n_1+Gn_0)|^2\right]}$$

$$\times e^{2i\pi B(\tilde{m}_1 n_0-\tilde{m}_0 n_1)}$$

$$\times(-1)^{\tilde{m}_0(a+\bar{a})+n_0(b+\bar{b})}\,(-1)^{\tilde{m}_1\bar{a}+n_1\bar{b}+\tilde{m}_1 n_1}$$

X^0-cycle: the deformation acts as a standard thermal deformation.

X^1-cycle: couples only to the right-moving fermion number F_R.

Special point: $2B = G = 1$

We think of the model as follows:

Start with the $10D$ type IIB theory and compactify the X^1 direction on a circle.

Coupling this circle to F_R, breaks the initial $(4,4)$ susy to $(4,0)$.

In addition we get two $U(1)$ gauge fields:

- The graviphoton field: $A_\mu = G_{1\mu}$
- The axial gauge field: $\tilde{A}_\mu = B_{1\mu}$

We then heat the system, giving vevs to $A_0 \to G = 1$ and to $\tilde{A}_0 \to B = 1/2$

Or turn on Polyakov loops for these gauge fields.

The 1-loop partition function is finite and is characterized by a "thermal duality" symmetry: $R_0 \to 1/2R_0$.

In terms of the T-dual variables, the system at small R_0 is effectively cold.

The line in moduli space, $R = R_0 = R_1$, is interesting.

As we decrease R, the system contracts and heats up, until we reach the fermionic point. Then it expands and cools.

It would be interesting to see if the Euclidean state, at self-dual radii, can define an initial, non-singular state, or a bounce, for a Lorentzian cosmological evolution.

The Euclidean system is characterized by both $a \to 1/a$ and $\beta \to 1/\beta$ dualities.

We need to obtain a Lorentzian dilaton/gravity action, incorporating the thermal effective potential, with these symmetries manifest.

Hopefully, the $T \sim 1/a$ relation of the radiation dominated era gets replaced by a duality symmetric relation, restricting the values of the scale factor to the fundamental domain, and bounding the curvature of the cosmology.

It would be interesting to see if we can achieve this in the context of weak coupling . . .

One can also find models, where at points in moduli space susy is broken but the massive spectrum is characterized by a boson/fermion degeneracy symmetry. (Kounnas; Kounnas, Florakis)
(see the talk by Ioannis Florakis to this workshop)

It would be interesting to see if we can use these models to generate non-singular cosmologies.