A Minimal S_3-Invariant Extension of the Standard Model

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Contents

Flavour permutational symmetry

- Prior to the introduction of the Higgs boson and mass terms, the Langrangian of the Standard Model is chiral and invariant with respect to any permutation of the left and right quark and lepton fields. $G_F \sim S_{3L} \otimes S_{3R}$
- Charged currents J_μ are invariant under G_F if the d and $u-$type fields are transformed with the same family group matrix

$$J_\mu = -i\bar{u}_L \gamma_\mu d_L + h.c. \Rightarrow \ G_F \sim S_3 \subset S_{3L} \otimes S_{3R}$$

- When $< 0|\Phi_H|0 > \neq 0$, the Yukawa couplings give mass to quarks and leptons, if we assume that the S_3 permutational symmetry is not broken

$$M_q = m_{3q} \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

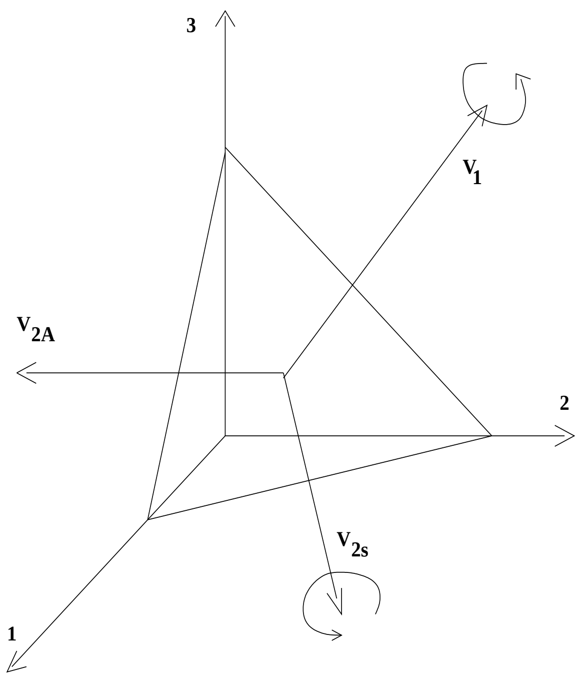
$$m_t \neq 0, \ m_e = m_\mu = 0; \ m_b \neq 0, m_s = m_d = 0$$
$$m_\tau \neq 0, m_\mu = m_e = 0; m_{\nu_\tau} = m_{\nu_\mu} = m_{\nu_e} = 0$$

$$\mathbf{V} = \mathbf{1}$$

There is no mixing nor CP- violation.

The Group S_3

The group S_3 of permutations of three objects



Permutations		Rotations
$\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}$	$\Longleftrightarrow$	a $120^\circ-$ rotation around the invariant vector $\mathbf{V}_1$
$\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix}$	$\Longleftrightarrow$	a 180°rotation around the invariant vector $\mathbf{V}_{2s}$

Symmetry adapted basis

$$|v_{2A}>=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix},\ |v_{2s}>=\frac{1}{\sqrt{6}}\begin{pmatrix}1\\1\\-2\end{pmatrix},\ |v_1>=\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

Irreducible representations of S_3

The group S_3 has two one-dimensional irreps (singlets) and one two-dimensional irrep (doublet)

- one dimensional: $\mathbf{1}_A$ antisymmetric singlet, $\mathbf{1}_s$ symmetric singlet
- Two - dimensional: 2 doublet

Direct product of irreps of S_3

$$\mathbf{1}_s \otimes \mathbf{1}_s = \mathbf{1}_s, \quad \mathbf{1}_s \otimes \mathbf{1}_A = \mathbf{1}_A, \quad \mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_s, \quad \mathbf{1}_s \otimes \mathbf{2} = \mathbf{2}, \quad \mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}$$

$$\boxed{\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_s \oplus \mathbf{1}_A \oplus \mathbf{2}}$$

the direct (tensor) product of two doublets

$$\mathbf{p_D} = \begin{pmatrix} p_{D1} \\ p_{D2} \end{pmatrix} \quad \text{and} \quad \mathbf{q}_D = \begin{pmatrix} q_{D1} \\ q_{D2} \end{pmatrix}$$

has two singlets, r_s and r_A, and one doublet r_D^T

$$r_s = p_{D1}q_{D1} + p_{D2}q_{D2} \quad \text{is invariant}, \quad r_A = p_{D1}q_{D2} - p_{D2}q_{D1} \quad \text{is not invariant}$$

$$r_D^T = \begin{pmatrix} p_{D1}q_{D2} + p_{D2}q_{D1} \\ p_{D1}q_{D1} - p_{D2}q_{D2} \end{pmatrix}$$

A Minimal, S_3 invariant extension of the SM

The Higgs sector is modified,

$$\Phi \rightarrow H \quad = \quad (\Phi_1, \Phi_2, \Phi_3)^T$$

H is a reducible $\mathbf{1_s} \oplus 2$ rep. of S_3

$$H_s \quad = \quad \frac{1}{\sqrt{3}}\Big(\Phi_1 + \Phi_2 + \Phi_3\Big)$$

$$H_D \quad = \quad \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_1 - \Phi_2) \\ \\ \frac{1}{\sqrt{6}}(\Phi_1 + \Phi_2 - 2\Phi_3) \end{pmatrix}$$

Quark, lepton and Higgs fields are

$$Q^T = (u_L, d_L), u_R, d_R, \qquad L^\dagger = (\nu_L, e_L), e_R, \nu_R, \qquad H$$

All these fields have three species (flavours) and belong to a reducible $\mathbf{1} \oplus 2$ rep. of S_3

The most general renormalizable Yukawa interactions

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_u} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}$$

Quarks

$$\begin{aligned}
\mathcal{L}_{Y_D} &= -Y_1^d \overline{Q}_I H_S d_{IR} - Y_3^d \overline{Q}_3 H_S d_{3R} \\
&- Y_2^d [\, \overline{Q}_I \kappa_{IJ} H_1 d_{JR} + \overline{Q}_I \eta_{IJ} H_2 d_{JR} \,] \\
&- Y_4^d \overline{Q}_3 H_I d_{IR} - Y_5^d \overline{Q}_I H_I d_{3R} + h.c
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{Y_U} &= -Y_1^u \overline{Q}_I (i\sigma_2) H_S^* u_{IR} - Y_3^u \overline{Q}_3 (i\sigma_2) H_S^* u_{3R} \\
&- Y_2^u [\, \overline{Q}_I \kappa_{IJ} (i\sigma_2) H_1^* u_{JR} + \eta \overline{Q}_I \eta_{IJ} (i\sigma_2) H_2^* u_{JR} \,] \\
&- Y_4^u \overline{Q}_3 (i\sigma_2) H_I^* u_{IR} - Y_5^u \overline{Q}_I (i\sigma_2) H_I^* u_{3R} + h.c.,
\end{aligned}$$

Doublets carry indices $I, J = 1, 2$

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{y} \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Singlets carry the index s or 3

Leptons' Yukawa interactions

Leptons

$$\begin{aligned}\mathcal{L}_{Y_E} &= -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} - Y_2^e [\, \overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR} \,] \\ &- Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + h.c.,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{Y_\nu} &= -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} \\ &- Y_2^\nu [\, \overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} \,] \\ &- Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + h.c.\end{aligned}$$

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad I, J = 1, 2$$

Furthermore, the Majorana mass terms for the right handed neutrinos are

$$\mathcal{L}_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R},$$

C is the charge conjugation matrix.

The Higgs potential

$$V = V_1 + V_2$$

$$\begin{aligned} V_1 &= \mu^2\Big[(\bar{H}_{D1}H_{D1}) + (\bar{H}_{D2}H_{D2}) + (\bar{H}_3H_3)\Big] \\ &+ \frac{1}{2}\lambda_1\Big[(\bar{H}_{D1}H_{D1}) + (\bar{H}_{D2}H_{D2}) + (\bar{H}_3H_3)\Big]^2 \end{aligned}$$

$$V_2 = \eta_1(\bar{H}_3H_3)\Big[(\bar{H}_{D1}H_{D1} + \bar{H}_{D2}H_{D2})\Big]$$

where

$$H_{D1} = \frac{1}{\sqrt{2}}(\Phi_1 - \Phi_2), \qquad H_{D2} = \frac{1}{\sqrt{6}}(\Phi_1 + \Phi_2 - 2\Phi_3)$$

$$H_3 = \frac{1}{\sqrt{3}}(\Phi_1 + \Phi_2 + \Phi_3)$$

Mass matrices

We will assume that

$$< H_{D1} > \;=\; < H_{D2} > \;\neq 0 \qquad \text{and} \qquad < H_3 > \neq\; 0$$

and

$$< H_3 >^2 + < H_{D1} >^2 + < H_{D2} >^2 \;\approx\; \left(\frac{246}{2} GeV\right)^2$$

Then, the Yukawa interactions yield mass matrices of the general form

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}$$

The Majorana masses for ν_L are obtained from the see-saw mechanism

$$M_\nu = M_{\nu D} \tilde{\mathsf{M}}^{-1} (M_{\nu D})^T \qquad \text{with} \qquad \tilde{\mathsf{M}} = \mathrm{diag}(M_1, M_1, M_3)$$

Mixing matrices

The mass matrices are diagonalized by unitary matrices

$$U^{\dagger}_{d(u,e)L}\mathbf{M}_{d(u,e)}\mathbf{U}_{d(u,e)R} \quad = \quad \mathrm{diag}\Big(m_{d(u,e)}m_{s(c,\mu)}m_{b(t,\tau)}\Big)$$

and

$$U^{T}_{\nu}M_{\nu}U_{\nu} \quad = \quad \mathrm{diag}\Big(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\Big)$$

The masses can be complex, and so, U_{eL} is such that

$$U^{\dagger}_{eL}M_{e}M^{\dagger}_{e}U_{eL} \quad = \quad \mathrm{diag}\Big(|m_e|^2, |m_\mu|^2, |m_\tau|^2\Big), \qquad etc.$$

The quark mixing matrix is

$$V_{CKM} \quad = \quad U^{\dagger}_{uL}U_{dL}$$

and, the neutrino mixing matrix is

$$\mathbf{V}_{MNS} \quad = \quad U^{\dagger}_{eL}U_{\nu}$$

Masses and mixings in the quark sector

The mass matrices for the quark sector take the general form

$$\mathbf{M}_{u(d)} = \begin{pmatrix} \mu_1^{u(d)} + \mu_2^{u(d)} & \mu_2^{u(d)} & \mu_5^{u(d)} \\ \mu_2^{u(d)} & \mu_1^{u(d)} - \mu_2^{u(d)} & \mu_5^{u(d)} \\ \mu_4^{u(d)} & \mu_4^{u(d)} & \mu_3^{u(d)} \end{pmatrix}$$

$$\begin{aligned} U^\dagger_{u(d)L} M_{u(d)} M^\dagger_{u(d)} U_{u(d)L} &= \text{diag}\Big(|m_{u(d)}|^2, |m_{c(s)}|^2, |m_{t(b)}|^2\Big), \\ V_{CKM} &= U^\dagger_{uL} U_{dL} \end{aligned}$$

The set of dimensionless parameters

$$\begin{aligned} \mu_1^u/\mu_0^u &= -0.000293, \quad \mu_2^u/\mu_0^u = -0.00028, \quad \mu_3^u/\mu_0^u = 1, \\ \mu_4^u/\mu_0^u &= 0.031, \quad \mu_5^u/\mu_0^u = 0.0386, \\ \mu_1^d/\mu_0^d &= 0.0004, \quad \mu_2^d/\mu_0^d = 0.00275, \quad \mu_3^d/\mu_0^d = 1 + 1.2I, \\ \mu_4^d/\mu_0^d &= 0.283, \quad \mu_5^d/\mu_0^d = 0.058, \end{aligned}$$

The quark mixing matrix

Yields the mass hierarchy and the mixing matrix

$$m_u/m_t = 2.5469 \times 10^{-5}\ , \quad m_c/m_t = 3.9918 \times 10^{-3},$$
$$m_d/m_b = 1.5261 \times 10^{-3}\ , \quad m_s/m_b = 3.2319 \times 10^{-2},$$

The computed mixing matrix is

$$\mathbf{V}_{CKM} = \begin{pmatrix} 0.968 + 0.117I & 0.198 + 0.0974I & -0.00253 - 0.00354I \\ -0.198 + 0.0969I & 0.968 - 0.115I & -0.0222 - 0.0376I \\ 0.00211 + 0.00648I & 0.0179 - 0.0395I & 0.999 - 0.00206I \end{pmatrix}$$

$$|\mathbf{V}^{th}_{CKM}| = \begin{pmatrix} 0.975 & 0.221 & 0.00435 \\ 0.221 & 0.974 & 0.0437 \\ 0.00682 & 0.0434 & 0.999 \end{pmatrix}$$

which should be compared with

$$|V^{\rm exp}_{\rm CKM}| \quad = \quad \begin{pmatrix} 0.97383 \pm 0.00024 & 0.2272 \pm 0.0010 & (3.96 \pm 0.09) \times 10^{-3} \\ 0.2271 \pm 0.010 & 0.97296 \pm 0.00024 & (42.21 \pm 0.45 \pm 0.09) \times 10^{-3} \\ (8.14 \pm 0.5) \times 10^{-3} & (41.61 \pm 0.12) \times 10^{-3} & 0.9991 \pm 0.000034 \end{pmatrix}$$

The Jarlskog invariant is

$$J = \mathrm{Im}\ [(V_{\rm CKM})_{11}(V_{\rm CKM})_{22}(V^*_{\rm CKM})_{12}(V^*_{\rm CKM})_{21}] = 2.9 \times 10^{-5} \qquad J^{exp} = (3.0 \pm 0.3) \times 10^{-5}$$

The leptonic sector

To achieve a further reduction of the number of parameters, in the leptonic sector, we introduce an additional discrete Z_2 symmetry

$-$	$+$
$H_I,\ \nu_{3R}$	$H_S,\ L_3,\ L_I,\ e_{3R},\ e_{IR},\ \nu_{IR}$

then,

$$Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0$$

Hence, the leptonic mass matrices are

$$M_e = \begin{pmatrix} \mu_2^e & \mu_2^e & \mu_5^e \\ \mu_2^e & -\mu_2^e & \mu_5^e \\ \mu_4^e & \mu_4^e & 0 \end{pmatrix} \qquad \text{and} \qquad M_{\nu D} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}$$

The Mass Matrix of the charged leptons as function of its eigenvalues

The mass matrix of the charged leptons is

$$M_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 \end{pmatrix}.$$

$x = m_e/m_\mu$, $\tilde{m}_\mu = m_\mu/m_\tau$ and $\tilde{m}_e = m_e/m_\tau$ This expression is accurate to order 10^{-9} in units of the τ mass

There are no free parameters in $\mathbf{M}_e$ other than the Dirac Phase δ!!

The Unitary Matrix U_{eL}

The unitary matrix U_{eL} is calculated from

$$U^\dagger_{eL} M_e M^\dagger_{eL} U_{eL} = diag\big(m_e^2, m_\mu^2, m_\tau^2\big)$$

We find

$$U_{eL} = \Phi_{eL} O_{eL}, \qquad \Phi_{eL} = diag\big[1, 1, e^{i\delta_D}\big]$$

and

$$\mathbf{O}_{eL} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} x \frac{(1+2\tilde{m}_\mu^2+4x^2+\tilde{m}_\mu^4+2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -\frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} x \frac{(1+4x^2-\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & \frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}(1+\tilde{m}_\mu^2+x^2-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -x \frac{(1+x^2-\tilde{m}_\mu^2-2\tilde{m}_e^2)\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{\sqrt{1+x^2}\tilde{m}_e\tilde{m}_\mu}{\sqrt{1+x^2-\tilde{m}_\mu^2}} \end{pmatrix},$$

$x = m_e/m_\mu$, $\tilde{m}_\mu = m_\mu/m_\tau$ and $\tilde{m}_e = m_e/m_\tau$

The neutrino mass matrix I

The neutrino mass matrix is obtained from the see-saw mechanism

$$\mathbf{M}_\nu \quad = \quad \mathbf{M}_{\nu_\mathbf{D}} \tilde{\mathbf{M}}^{-1} (\mathbf{M}_{\nu_\mathbf{D}})^T = \begin{pmatrix} 2(\rho_2^\nu)^2 & 0 & 2\rho_2^\nu \rho_4^\nu \\ 0 & 2(\rho_2^\nu)^2 & 0 \\ 2\rho_2^\nu \rho_4^\nu & 0 & 2(\rho_4^\nu)^2 + (\rho_3^\nu)^2 \end{pmatrix}$$

M_ν is reparametrized in terms of its eigenvalues

$$M_\nu = \begin{pmatrix} m_{\nu_3} & 0 & \sqrt{(m_{\nu_3} - m_{\nu_1})(m_{\nu_2} - m_{\nu_3})} e^{-i\delta_\nu} \\ 0 & m_{\nu_3} & 0 \\ \sqrt{(m_{\nu_3} - m_{\nu_1})(m_{\nu_2} - m_{\nu_3})} e^{-i\delta_\nu} & 0 & (m_{\nu_1} + m_{\nu_2} - m_{\nu_3}) e^{-2i\delta_\nu} \end{pmatrix}.$$

The Unitary Matrix U_ν

The complex symmetric matrix M_ν is diagonalized as

$$\mathbf{U}_\nu^T \mathbf{M}_\nu \mathbf{U}_\nu = \begin{pmatrix} |m_{\nu_1}|e^{i\phi_1 - i\phi_\nu} & 0 & 0 \\ 0 & |m_{\nu_2}|e^{i\phi_2 - i\phi_\nu} & 0 \\ 0 & 0 & |m_{\nu_3}| \end{pmatrix}$$

where

$$U_\nu = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_\nu} \end{array} \right) \left(\begin{array}{ccc} \sqrt{\dfrac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_2} - m_{\nu_1}}} & \sqrt{\dfrac{m_{\nu_3} - m_{\nu_1}}{m_{\nu_2} - m_{\nu_1}}} & 0 \\ 0 & 0 & 1 \\ -\sqrt{\dfrac{m_{\nu_3} - m_{\nu_1}}{m_{\nu_2} - m_{\nu_1}}} & \sqrt{\dfrac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_2} - m_{\nu_1}}} & 0 \end{array} \right).$$

the mass eigenvalues, m_{ν_1}, m_{ν_2} and m_{ν_3} are, in general, complex numbers

Unitarity of U_ν

All the phases in M_ν except for one, ϕ_ν, can be absorbed in a rephasing of the fields

The phases ϕ_1 and ϕ_2 are fixed by the unitarity condition on U_ν

$$|m_{\nu_3}| \sin\phi_\nu = |m_{\nu_2}| \sin\phi_2 = |m_{\nu_1}| \sin\phi_1$$

therefore

$$\frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_3} - m_{\nu_1}} = \frac{(\Delta m_{12}^2 + \Delta m_{13}^2 + |m_{\nu_3}|^2 \cos^2\phi_\nu)^{1/2} - |m_{\nu_3}||\cos\phi_\nu|}{(\Delta m_{13}^2 + |m_{\nu_3}|^2 \cos^2\phi_\nu)^{1/2} + |m_{\nu_3}||\cos\phi_\nu|}.$$

The neutrino mixing matrix I

$$V^{th}_{PMNS} \;=\; U^{\dagger}_{eL} U_{\nu}$$

The theoretical mixing matrix V^{th}_{PMNS} is

$$V^{th}_{PMNS} \;=\; \begin{pmatrix} O_{11}\cos\eta + O_{31}\sin\eta e^{i\delta} & O_{11}\sin\eta - O_{31}\cos\eta e^{i\delta} & -O_{21} \\ -O_{12}\cos\eta + O_{32}\sin\eta e^{i\delta} & -O_{12}\sin\eta - O_{32}\cos\eta e^{i\delta} & O_{22} \\ O_{13}\cos\eta - O_{33}\sin\eta e^{i\delta} & O_{13}\sin\eta + O_{33}\cos\eta e^{i\delta} & O_{23} \end{pmatrix}$$

$$\times \; \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

where O_{ij} are the absolute values of the elements of $\mathbf{O}_e$

$$\mathbf{V}^{PDG}_{PMNS} \;=\; \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

Neutrino Mixing Angles

From a comparison of $\mathbf{V}^{th}_{PMNS}$ with $\mathbf{V}^{exp}_{PMNS}$, we obtain the neutrino mixing angles as functions of the lepton masses

The mixing angles θ_{13} and θ_{23} depend only on the charged lepton masses

$$\sin\theta_{13} \approx \frac{1}{\sqrt{2}} x \frac{(1+4x^2-\tilde{m}_\mu^4)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4}},$$

$$\sin\theta_{23} \approx \frac{1}{\sqrt{2}} \frac{1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4}}.$$

$x = m_e/m_\mu$, $\tilde{m}_\mu = m_\mu/m_\tau$

The solar angle θ_{12} is strongly dependent on the neutrino masses but depends only very weakly on the charged lepton masses

$$\tan\theta_{12}^2 = \frac{(\Delta m_{12}^2 + \Delta m_{13}^2 + |m_{\nu_3}|^2 \cos^2\phi_\nu)^{1/2} - |m_{\nu_3}||\cos\phi_\nu|}{(\Delta m_{13}^2 + |m_{\nu_3}|^2 \cos^2\phi_\nu)^{1/2} + |m_{\nu_3}||\cos\phi_\nu|}.$$

Majorana Phases

The Majorana phases are

$$\sin 2\alpha = \sin(\phi_1 - \phi_2) = \frac{|m_{\nu_3}| \sin \phi_\nu}{|m_{\nu_1}||m_{\nu_2}|} \times$$

$$\left(\sqrt{|m_{\nu_2}|^2 - |m_{\nu_3}|^2 \sin^2 \phi_\nu} + \sqrt{|m_{\nu_1}|^2 - |m_{\nu_3}|^2 \sin^2 \phi_\nu}\right)$$

$$\sin 2\beta = \sin(\phi_1 - \phi_\nu) =$$

$$\frac{\sin \phi_\nu}{|m_{\nu_1}|} \left(|m_{\nu_3}|\sqrt{1 - \sin^2 \phi_\nu} + \sqrt{|m_{\nu_1}|^2 - |m_{\nu_3}|^2 \sin^2 \phi_\nu}\right).$$

The mixing angles θ_{13} and θ_{23} depend only on the charged lepton masses and are in excellent agreement with the experimental values

Experimental	Theoretical
$(\sin^2 \theta_{13})^{exp} \leq 0.025$	$\sin^2 \theta_{13} = 1.1 \times 10^{-5}$
$(\sin \theta_{23})^{exp} = 0.5^{+0.08}_{-0.07}$	$\sin^2 \theta_{23} = 0.5$

The neutrino mass spectrum I

In the present model, the experimental restriction

$$|\Delta m^2_{21}| < |\Delta m^2_{23}|$$

implies an inverted neutrino mass spectrum $\quad m_{\nu_3} \; < \; m_{\nu_1}, m_{\nu_2}$

From our previous expressions for $\tan\theta_{12}$

$$|m_{\nu_3}| = \frac{\sqrt{\Delta m^2_{13}}}{2\cos\phi_\nu \tan\theta_{12}} \frac{1 - \tan^4\theta_{12} + r^2}{\sqrt{1+\tan^2\theta_{12}}\sqrt{1+\tan^2\theta_{12}+r^2}},$$

where $\quad r = \Delta m^2_{21}/\Delta m^2_{23}$.

The mass $|m_{\nu_3}|$ assumes its minimal value when $\sin\phi_\nu = 0$,

$$|m_{\nu_3}| \approx \frac{1}{2}\frac{\sqrt{\Delta m^2_{13}}}{\tan\theta_{12}}(1 - \tan^2\theta_{12})$$

Neutrino mass spectrum II

- We wrote the neutrino mass differences, $m_{\nu_i} - m_{\nu_j}$, in terms of the differences of the squared masses $\Delta^2_{ij} = m^2_{\nu_i} - m^2_{\nu_j}$ and one of the neutrino masses, say m_{ν_3}.
- The mass m_{ν_2} was taken as a free parameter in the fitting of our formula for $\tan\theta_{12}$ to the experimental value
- with

$$\Delta m^2_{21} = 7.6 \times 10^{-5} eV^2 \qquad \Delta m^2_{13} = 2.4 \times 10^{-3} eV^2$$

and

$$\tan\theta_{12} = 0.696$$

we get

$$|m_{\nu_3}| \approx 0.019\ eV \implies |m_{\nu_2}| \approx 0.053\ eV \quad \text{and} \quad |m_{\nu_1}| \approx 0.052\ eV$$

- The neutrino mass spectrum has an inverted hierarchy of masses

S_3 symmetry and tribimaximal form of the mixing matrix I

WE WILL ASSUME $\delta = \delta_\nu - \delta_e = \pi/2$!!!!

$$\mathbf{V}_{PMNS}^{th} = \mathbf{V}_{PMNS}^{tri} + \mathbf{\Delta V}_{PMNS}^{tri}$$

where

$$\mathbf{V}_{PMNS}^{tri} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

$$\mathbf{\Delta V}_{PMNS}^{tri} = \mathbf{\Delta V_e} + \delta t_{12} \frac{(\sqrt{2} + \delta t_{12})}{1 + \frac{2}{3}\delta_{t12}(\sqrt{2} + \delta t_{12})} \widetilde{\mathbf{\Delta V}}_\nu$$

$$\delta t_{12} = \frac{\sqrt{2}}{2} - t_{12}$$

S_3 symmetry and tribimaximal form of the mixing matrix II

$$\widetilde{\Delta \mathbf{V}}_\nu = \begin{pmatrix} -\left(\frac{2}{3}\right)^{3/2} \dfrac{1-\frac{s_{13}^2}{1+c_{13}}}{1+\sqrt{1-\frac{2}{3}\frac{\delta_\theta(\sqrt{2}+\delta_\theta)}{g(\delta_\theta)}}} & \left(\frac{1}{3}\right)^{3/2} \dfrac{1-\frac{s_{13}^2}{1+c_{13}}}{1+\sqrt{1+\frac{1}{3}\frac{\delta_\theta(\sqrt{2}+\delta_\theta)}{g(\delta_\theta)}}} & 0 \\ \left(\frac{2}{3}\right)^{3/2} \dfrac{1-2x^2}{\sqrt{1+\frac{5}{2}x^2}\left(1+\sqrt{1+\frac{4}{3}\frac{\delta_\theta(\sqrt{2}+\delta_\theta)}{g(\delta_\theta)}\frac{1-2x^2}{1+\frac{5}{2}x^2}}\right)} & -\frac{2}{3\sqrt{3}} \dfrac{1-2x^2}{\sqrt{1+\frac{1}{4}x^2}\left(1+\sqrt{1-\frac{2}{3}\frac{\delta_\theta(\sqrt{2}+\delta_\theta)}{g(\delta_\theta)}\frac{1-2x^2}{1+\frac{1}{4}x^2}}\right)} & 0 \\ \left(\frac{2}{3}\right)^{3/2} \dfrac{1+\frac{1}{4}x^2}{\sqrt{1+x^2}\left(1+\sqrt{1+\frac{4}{3}\frac{\delta_\theta(\sqrt{2}+\delta_\theta)}{g(\delta_\theta)}\frac{1+\frac{1}{4}x^2}{1+x^2}}\right)} & -\frac{2}{3\sqrt{3}} \dfrac{1+x^2}{\sqrt{1+\frac{1}{4}x^2}\left(1+\sqrt{1+\frac{2}{3}\frac{\delta_\theta(\sqrt{2}+\delta_\theta)}{g(\delta_\theta)}\frac{1+x^2}{1+\frac{1}{4}x^2}}\right)} & 0 \end{pmatrix}$$

$$g(\delta_\theta) = 1 + \frac{2}{3}\delta_\theta(\sqrt{2} + \delta_\theta)$$

$$\mathbf{\Delta V}_e \approx \begin{pmatrix} -\frac{2}{3}\frac{s_{13}^2}{1+c_{13}} & -\frac{1}{3}\frac{s_{13}^2}{1+c_{13}} & s_{13} \\ \frac{5}{2\sqrt{6}}\frac{x^2}{1+\sqrt{1+\frac{5}{2}x^2}} & \frac{1}{4}\sqrt{\frac{1}{3}}\frac{x^2}{1+\sqrt{1+\frac{1}{4}x^2}} & -\frac{1}{2\sqrt{2}}\frac{x^2}{\sqrt{1-4\tilde{m}_\mu^2+x^2}} \\ \sqrt{\frac{1}{6}}\frac{x^2}{1+\sqrt{1+x^2}} & \frac{1}{4}\sqrt{\frac{1}{3}}\frac{x^2}{1+\sqrt{1+\frac{1}{4}x^2}} & 0 \end{pmatrix}$$

$$s_{13} \approx \frac{1}{\sqrt{2}} x \frac{(1+4x^2-\tilde{m}_\mu^4)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4}}$$

$$\Downarrow$$

$$\mathbf{\Delta V_e} \sim m_e/m_\mu$$

FCNC I

In the Standard Model the FCNC at tree level are suppressed by the GIM mechanism.

Models with more than one Higgs $SU(2)$ doublet have tree level FCNC due to the exchange of scalar fields. The mass matrix written in terms of the Yukawa couplings is

$$\mathcal{M}_Y^e = Y_w^{E1} H_1^0 + Y_w^{E2} H_2^0,$$

FCNC processes:

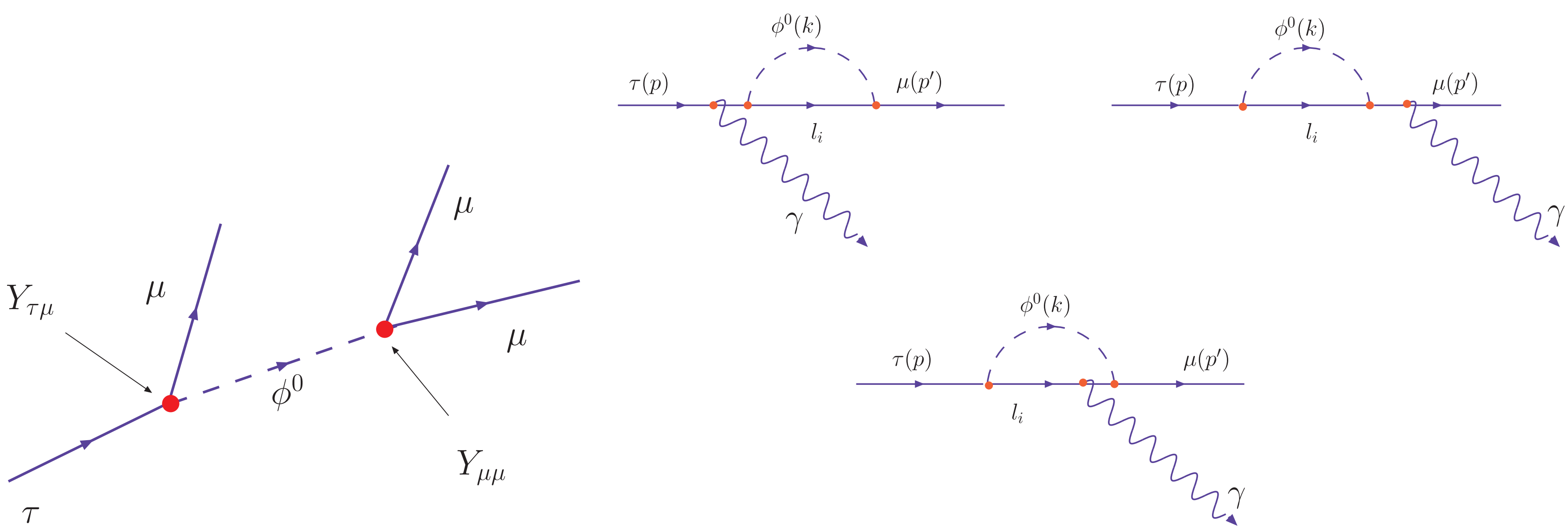


Figure 1: The diagram in the left contributes to the process $\tau^- \longrightarrow 3\mu$. The three diagrams in the right contribute to the process $\tau \longrightarrow \mu\gamma$.

The Yukawa matrices

The Yukawa matrices in the weak basis are

$$Y_w^{E1} = \frac{m_\tau}{v_1} \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & 0 & 0 \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 & 0 \end{pmatrix}$$

and

$$Y_w^{E2} = \frac{m_\tau}{v_2} \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ 0 & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 \end{pmatrix}.$$

Yukawa matrices in the mass representation

The Yukawa matrices in the mass basis defined by

$$\tilde{Y}_m^{EI} = U_{eL}^{\dagger} Y_w^{EI} U_{eR}$$

$$\tilde{Y}_m^{E1} \approx \frac{m_\tau}{v_1} \begin{pmatrix} 2\tilde{m}_e & -\frac{1}{2}\tilde{m}_e & \frac{1}{2}x \\ -\tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & -\frac{1}{2} \\ \frac{1}{2}\tilde{m}_\mu x^2 & -\frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m ,$$

and

$$\tilde{Y}_m^{E2} \approx \frac{m_\tau}{v_2} \begin{pmatrix} -\tilde{m}_e & \frac{1}{2}\tilde{m}_e & -\frac{1}{2}x \\ \tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \\ -\frac{1}{2}\tilde{m}_\mu x^2 & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m ,$$

all off diagonal terms give rise to FCNC processes!!

Branching ratios

We define the partial branching ratio (only leptonic decays)

$$Br(\tau \to \mu e^+ e^-) = \frac{\Gamma(\tau \to \mu e^+ e^-)}{\Gamma(\tau \to e\nu\bar{\nu}) + \Gamma(\tau \to \mu\nu\bar{\nu})}, \quad \Gamma(\tau \to \mu e^+ e^-) \approx \frac{m_\tau^5}{3 \times 2^{10}\pi^3} \frac{(Y_{\tau\mu}^{1,2} Y_{ee'}^{1,2})^2}{M_{H_{1,2}}^4}$$

thus

$$Br(\tau \to \mu e^+ e^-) \approx \frac{9}{4} \left(\frac{m_e m_\mu}{m_\tau^2}\right)^2 \left(\frac{m_\tau}{M_{H_{1,2}}}\right)^4,$$

Similar computations lead to

$$Br(\tau \to e\gamma) \approx \frac{3\alpha}{8\pi} \left(\frac{m_\mu}{M_H}\right)^4,$$

$$Br(\tau \to \mu\gamma) \approx \frac{3\alpha}{128\pi} \left(\frac{m_\mu}{m_\tau}\right)^2 \left(\frac{m_\tau}{M_H}\right)^4,$$

$$Br(\tau \to 3\mu) \approx \frac{9}{64} \left(\frac{m_\mu}{M_H}\right)^4,$$

$$Br(\mu \to 3e) \approx 18 \left(\frac{m_e m_\mu}{m_\tau^2}\right)^2 \left(\frac{m_\tau}{M_H}\right)^4,$$

$$Br(\mu \to e\gamma) \approx \frac{27\alpha}{64\pi} \left(\frac{m_e}{m_\mu}\right)^4 \left(\frac{m_\tau}{M_H}\right)^4.$$

Numerical results

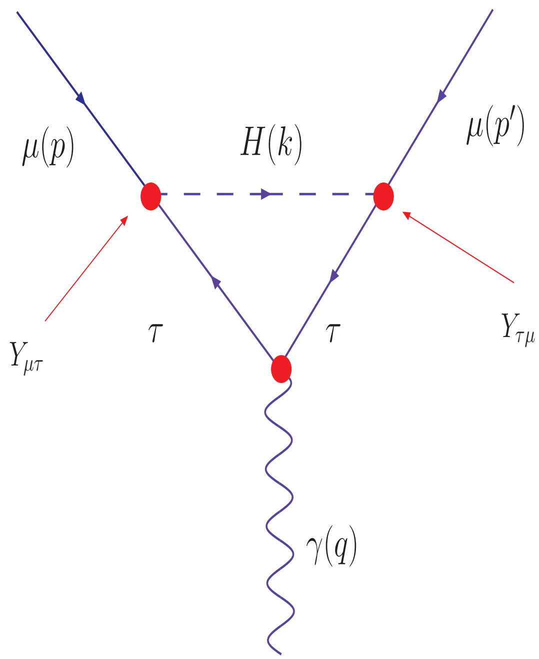
Table 1: Leptonic processes via FCNC

FCNC processes	Theoretical BR	Experimental upper bound BR	References
$\tau \longrightarrow 3\mu$	8.43×10^{-14}	5.3×10^{-8}	B. Aubert *et. al.* (2007)
$\tau \longrightarrow \mu e^+ e^-$	3.15×10^{-17}	8×10^{-8}	B. Aubert *et. al.* (2007)
$\tau \longrightarrow \mu\gamma$	9.24×10^{-15}	6.8×10^{-8}	B. Aubert *et. al.*(2005)
$\tau \longrightarrow e\gamma$	5.22×10^{-16}	1.1×10^{-11}	B. Aubert *et. al.* (2006)
$\mu \longrightarrow 3e$	2.53×10^{-16}	1×10^{-12}	U. Bellgardt *et al.* (1998)
$\mu \longrightarrow e\gamma$	2.42×10^{-20}	1.2×10^{-11}	M. L. Brooks *et al.* (1999)

Small FCNC processes mediating non-standard quark-neutrino interactions could be important in the theoretical description of the **gravitational core collapse and shock generation** in the explosion stage of a supernova

Muon Anomalous Magnetic Moment

The anomalous magnetic moment of the muon is related to the gyroscopic ratio by



$$a_\mu = \frac{\mu_\mu}{\mu_B} - 1 = \frac{1}{2}(g_\mu - 2)$$

In models with more than one Higgs $SU(2)$ doublet, the exchange of flavour changing neutral scalars also contribute to the anomalous magnetic moment of the muon

$$\delta a_\mu^{(H)} = \frac{Y_{\mu\tau} Y_{\tau\mu}}{16\pi^2} \frac{m_\mu m_\tau}{M_H^2} \left(log\left(\frac{M_H^2}{m_\tau^2}\right) - \frac{3}{2} \right)$$

From our results: $Y_{\mu\tau} Y_{\tau\mu} = \frac{m_\mu m_\tau}{4 v_1 v_2}$

$$\delta a_\mu^{(H)} = \frac{m_\tau^2}{(246\ GeV)^2} \frac{(2 + \tan^2\beta)}{32\pi^2} \frac{m_\mu^2}{M_H^2} \left(log\left(\frac{M_H^2}{m_\tau^2}\right) - \frac{3}{2} \right), \quad \tan\beta = \frac{v_s}{v_1}$$

From the experimental upper bound on $(\mu \to 3e)$, we get $\tan\beta \leq 14$, Hence

$$\delta a_\mu = 1.7 \times 10^{-10}$$

Contribution to the anomaly of the muon's magnetic moment

The difference between the experimental value and the Standard Model prediction for the anomaly is

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (28.7 \pm 9.1) \times 10^{-10}$$

$\Delta a_\mu \sim 3\sigma$ (three standard deviations) !!

But, the uncertainty in the computation of higher order hadronic effects is large

$$\delta a_\mu^{LBL}(3, had) \approx 1.59 \times 10^{-9}; \quad \delta a_\mu^{VP}(3, had) \approx -1.82 \times 10^{-9}$$

$$\frac{\delta a_\mu^{(H)}}{\Delta a_\mu} \approx \frac{1.7}{28} \approx 6\% \quad \text{and} \quad \delta a_\mu^{(H)} < \delta a_\mu(3, had)$$

The contribution of the exchange of flavour changing scalars to the anomaly of the muon's magnetic moment, $\delta a_\mu^{(H)}$, is small but not negligible, and it is compatible with the best, state of the art, measurements and theoretical predictions.

Summary

- By introducing three $SU(2)_L$ Higgs doublet fields, in the theory, we extended the concept of flavour and generations to the Higgs sector and formulated a minimal S_3-invariant Extension of the Standard Model
- A further reduction of free parameters is achieved in the leptonic sector by introducing a Z_2 symmetry
- The magnitudes of the three mixing angles θ_{12}, θ_{23} and θ_{13}, are determined by an interplay of the $S_3 \times Z_2$ symmetry, the see-saw mechanism and the lepton mass hierarchy
- The mixing angles, θ_{23} and θ_{13}, depend only on the masses of the charged leptons and are in excellent agreement with the best experimental values
- The solar mixing angle, θ_{12}, fixes the scale and origin of the neutrino mass spectrum which has an inverted mass hierarchy with values

$$|m_{\nu_2}| \approx 0.056eV, \qquad |m_{\nu_1}| \approx 0.055eV, \qquad |m_{\nu_3}| \approx 0.022eV$$

- The branching ratios of all flavour changing neutral processes in the leptonic sector are strongly suppressed by the $S_3 \times Z_2$ symmetry and powers of the small mass ratios m_e/m_τ, m_μ/m_τ, and $\left(m_\tau/M_{H_{1,2}}\right)^4$, but could be important in astrophysical processes
- The anomalous magnetic moment of the muon gets a small but non-negligible contribution from the exchange of flavour changing scalar fields

Conclusions

- The Minimal S_3-Invariant Extension of the Standard Model describes succesfully masses and mixings in the quark and leptonic sectors with a small number of free parameters (8).
- It predicts the numerical values of θ_{13} and θ_{23} neutrino mixing angles, as well as, all flavour changing neutral current processes in the leptonic sector in excellent agreement with experiment.
- It gives a small but non-negligible contribution to the magnetic moment of the muon.