M.B., Haack, Kang ‘09

Corfu, Sep 12, 2009

# Thrice twisted open strings

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# One-loop Kähler metric of D-brane scalars from open strings in the N=1 sector of toroidal orientifolds

Co

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# Why orientifolds?

A few reasons:

- Supersymmetry reduced (e.g. Type IIB to Type I)
- In compact models: consistency conditions (D-branes)
- As orbifolds: wide range (toroidal, Calabi-Yau, F, ...)

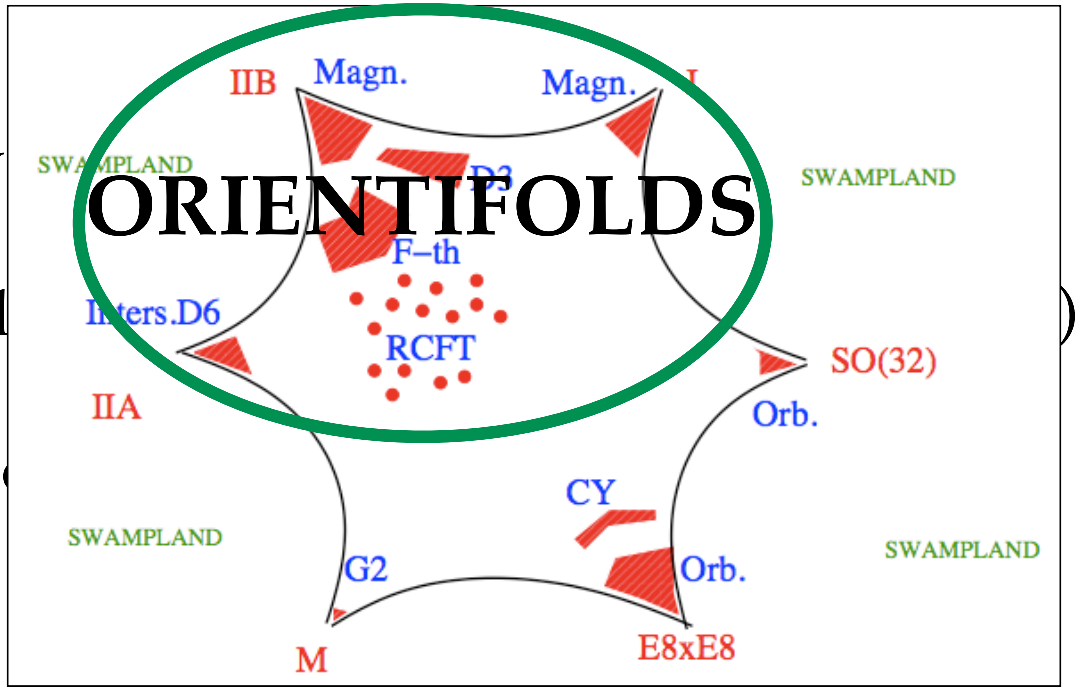
free CFT

# Why orientifolds?

Ibanez, Strings ’08: “state of string model building”

A few reasons:

- Supersymmetry
- In compact mod )
- As orbifolds: wi



# General D=4, N=1 effective theory

open and closed string moduli

$$\mathcal{L}_{\text{eff}} = \frac{1}{2\kappa^2}R - K_{\Phi_i\bar{\Phi}_{\bar{\jmath}}}\partial_\mu\Phi^i\partial^\mu\bar{\Phi}^{\bar{\jmath}} - \frac{1}{g^2(\Phi)}\text{tr}_a F^2$$

$$-V(\Phi) + \text{corrections}$$

$\alpha'$ and $g_s$ corrections

# General D=4, N=1 effective theory

open and closed string moduli

$$\mathcal{L}_{\rm eff} = \frac{1}{2\kappa^2}R - K_{\Phi_i\bar{\Phi}_{\bar{\jmath}}}\partial_\mu\Phi^i\partial^\mu\bar{\Phi}^{\bar{\jmath}} - \frac{1}{g^2(\Phi)}{\rm tr}_a F^2$$

$$-V(\Phi) + \text{corrections}$$

*Can't always think "negligible" even when numerically small at a point in moduli space (i.e. at fixed $\Phi^i$).*

# General D=4, N=1 effective theory

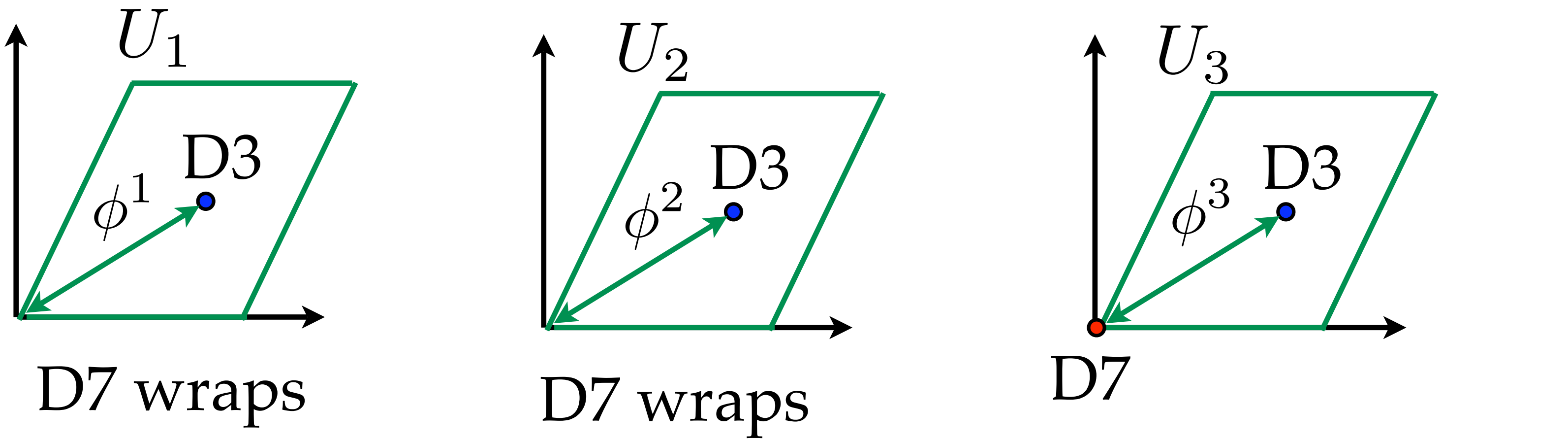
To know vacua,

would like to know the complete moduli-dependence of the effective action to one-loop order.

# A first look at corrections: f

sample $\mathcal{N}=1$ orientifolds: $\mathbb{T}^6/\mathbb{Z}_6'$, $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$

$\Theta Z^1 = e^{2\pi i v_1} Z^1$ $\quad$ $\Theta Z^2 = e^{2\pi i v_2} Z^2$ $\quad$ $\Theta Z^3 = e^{2\pi i v_3} Z^3$




$$\mathbb{Z}_6' : (v_1, v_2, v_3) = \left(\frac{1}{6}, -\frac{1}{2}, \frac{1}{3}\right)$$

$\Theta^2$

"N=2 sector"

"partially twisted"

"twice twisted"

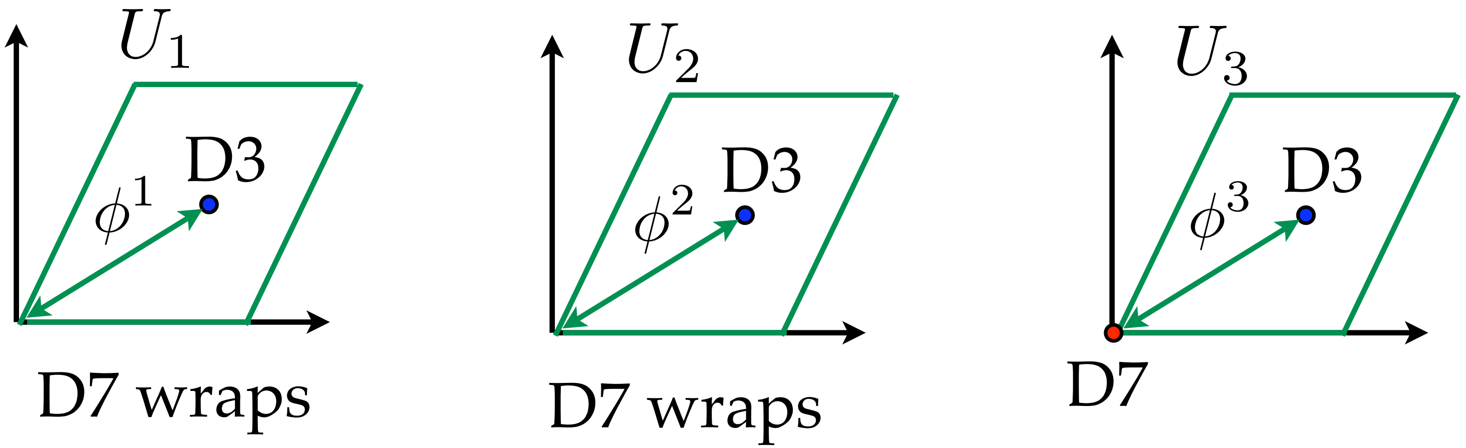
$\Theta$

"N=1 sector"

"completely twisted"

"thrice twisted"

$$\Theta Z^1 = e^{2\pi i v_1} Z^1 \qquad \Theta Z^2 = e^{2\pi i v_2} Z^2 \qquad \Theta Z^3 = e^{2\pi i v_3} Z^3$$

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# A first look at corrections: f

sample $\mathcal{N}=1$ orientifolds: $\mathbb{T}^6/\mathbb{Z}_6'$, $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$

threshold corrections

$$\left(\frac{1}{g^2(\phi^i)}\right)^{1-\mathrm{loop}} = \beta\,\ln\frac{M_{\mathrm{string}}}{\mu} + \Delta(\phi,U)$$

correct

$$K_{\phi_i\bar{\phi}_{\bar{j}}} = \partial_{\phi^i}\partial_{\bar{\phi}^{\bar{j}}}K \quad \text{and} \quad \frac{1}{g^2(\phi)} = \mathrm{Re}f(\phi)$$

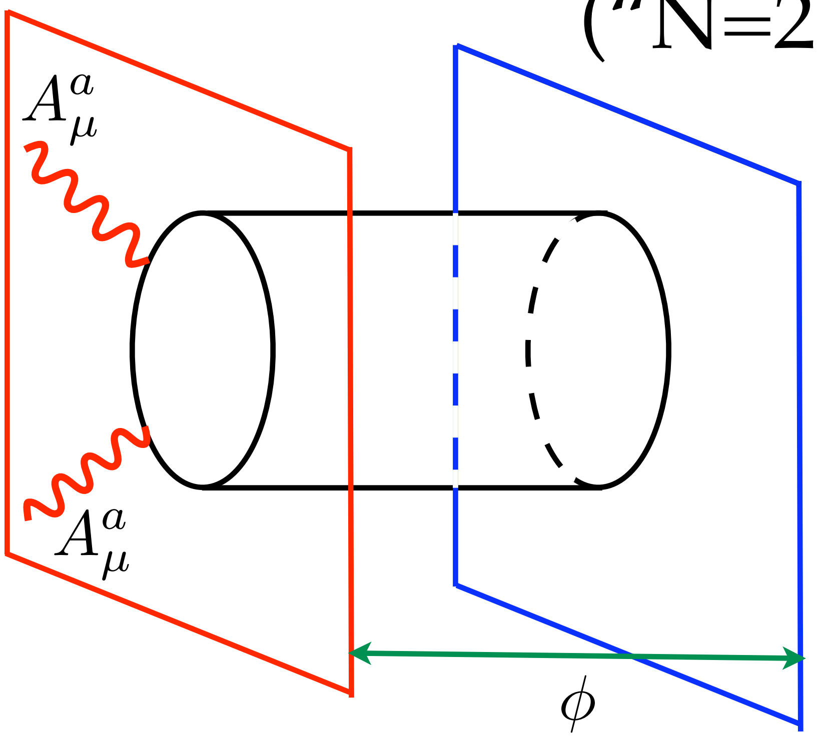
# f: twice twisted strings

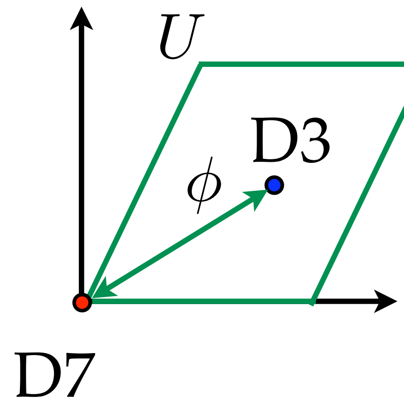
("N=2 sectors")

Dixon, Kaplunovsky, Louis '91

...

M.B., Haack, Körs '04




$$\Delta(\phi, U) = -\frac{1}{2} \ln \left| \frac{\vartheta_1(\phi/2\pi, U)}{\eta(U)} \right|^2 + \frac{(\mathrm{Im}\phi)^2}{4\pi\, \mathrm{Im}U}$$

$$f^{1-\mathrm{loop}} = -2 \ln \vartheta_1(\phi/2\pi, U) + \ldots$$

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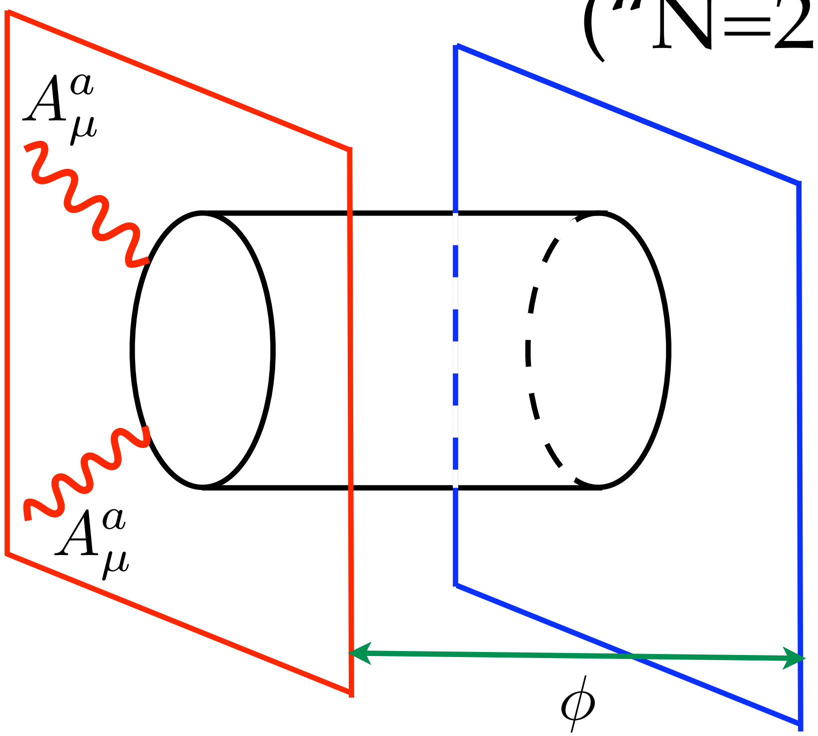
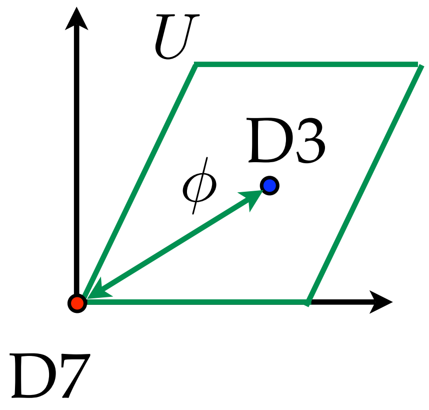


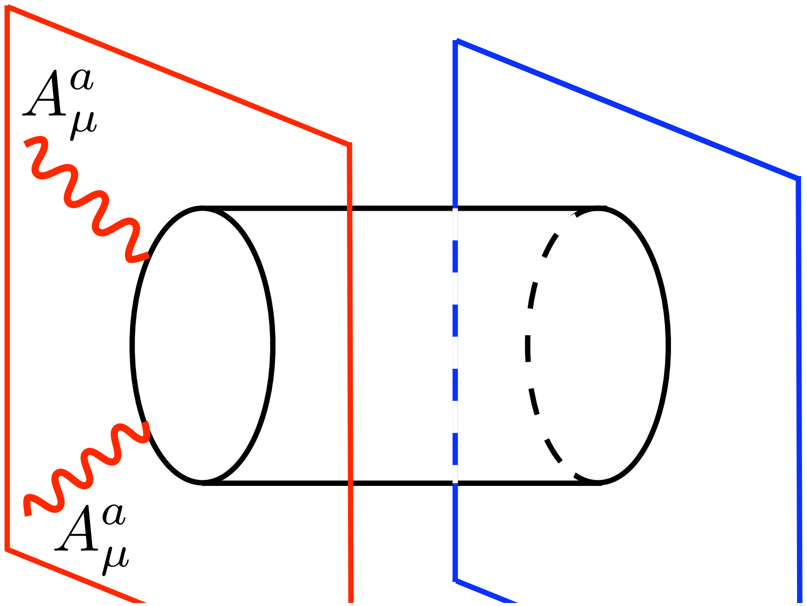



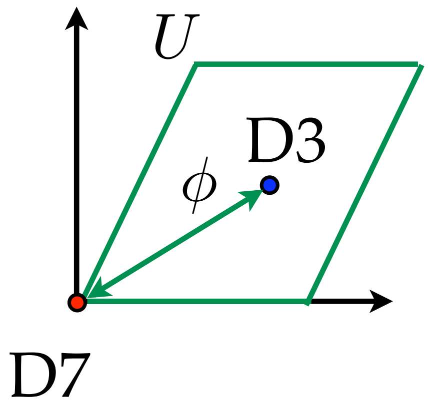

$$\ln|z|^2 = \ln z + \ln \bar{z} = \tfrac{1}{2}\mathrm{Re}\ln z$$

$$\Delta(\phi, U) = -\frac{1}{2}\ln\left|\frac{\vartheta_1(\phi/2\pi, U)}{\eta(U)}\right|^2 + \frac{(\mathrm{Im}\phi)^2}{4\pi\,\mathrm{Im}U}$$

$$f^{1-\mathrm{loop}} = -2\ln\vartheta_1(\phi/2\pi, U) + \ldots$$

# A first look at corrections: f




where could this play a role in, say, KKLT?

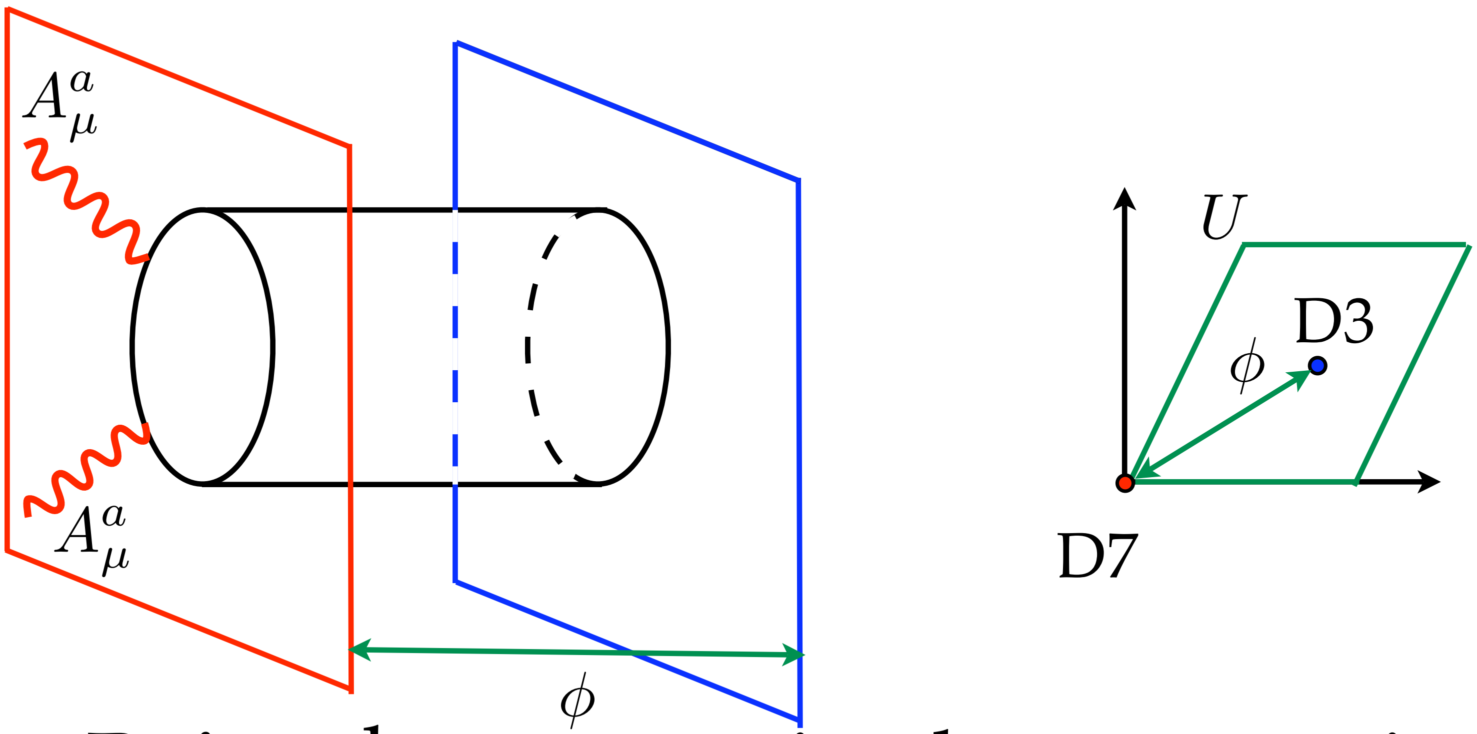
M.B., Haack, Körs '04

$$W_{\rm np} = Ae^{-af} = Ae^{-a(f^{\rm tree}+f^{\rm 1-loop}+\ldots)}$$

$$= \underbrace{A \cdot \left(\vartheta_1(\phi/2\pi, U)^{2a}\cdots\right)}_{\tilde{A}(\phi,U)} e^{-a(f^{\rm tree}+\ldots)}$$

$$a \sim 1/N_{\rm D7}$$

# A first look at corrections: f




***Point***: these are string loop corrections, but without them, $W_{\rm np}$ doesn't depend on the D3-brane scalar $\phi$ at all. So they are not "negligible" in any real sense.

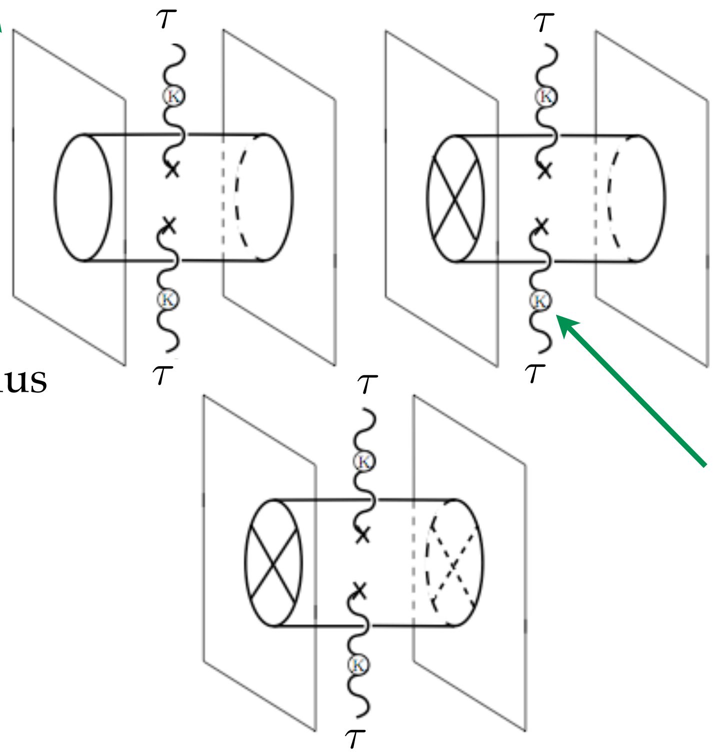
# K: twice twisted strings

M.B., Haack, Körs, '05

brane at arbitrary position $\phi$

$\tau = \mathrm{Re}\, T$

$\langle \tau \, \tau \rangle =$



Annulus

Möbius strip

Klein bottle

"Kähler adapted
vertex operators"

# K: twice twisted strings

M.B., Haack, Körs, '05

"integrate" one-loop corrected Kähler metric to get one-loop corrected Kähler potential:

$$K = -\ln\left((S+\bar{S})(T+\bar{T})(U+\bar{U})\right)$$
$$-\ln\left(1 - \frac{1}{8\pi}\sum_i \frac{N_i(\phi_i+\bar{\phi}_i)^2}{(T+\bar{T})(U+\bar{U})} - \frac{1}{128\pi^6}\sum_i \frac{\mathcal{E}_2(\phi_i,U)}{(S+\bar{S})(T+\bar{T})}\right)$$

sum over images of $E_2(\phi_i, U)$, i.e.

$$E_2(\phi,U) = \sum_{(n,m)=(0,0)} \frac{\mathrm{Re}(U)^2}{|n+mU|^4} \exp\left(2\pi i \frac{\phi(n+m\bar{U})+\bar{\phi}(n+mU)}{U+\bar{U}}\right)$$

# twice twisted strings: summary

Efforts by many people:

- moduli-dependent gauge coupling: one-loop order

$$g_{\rm ph}(\Phi, \bar{\Phi}) = g_{\rm ph}(S, \bar{S}, T, \bar{T}, U, \bar{U}, \phi, \bar{\phi})$$

- Kähler potential: one-loop order

$$K(\Phi, \bar{\Phi}) = K(S, \bar{S}, T, \bar{T}, U, \bar{U}, \phi, \bar{\phi})$$

# What about thrice twisted?

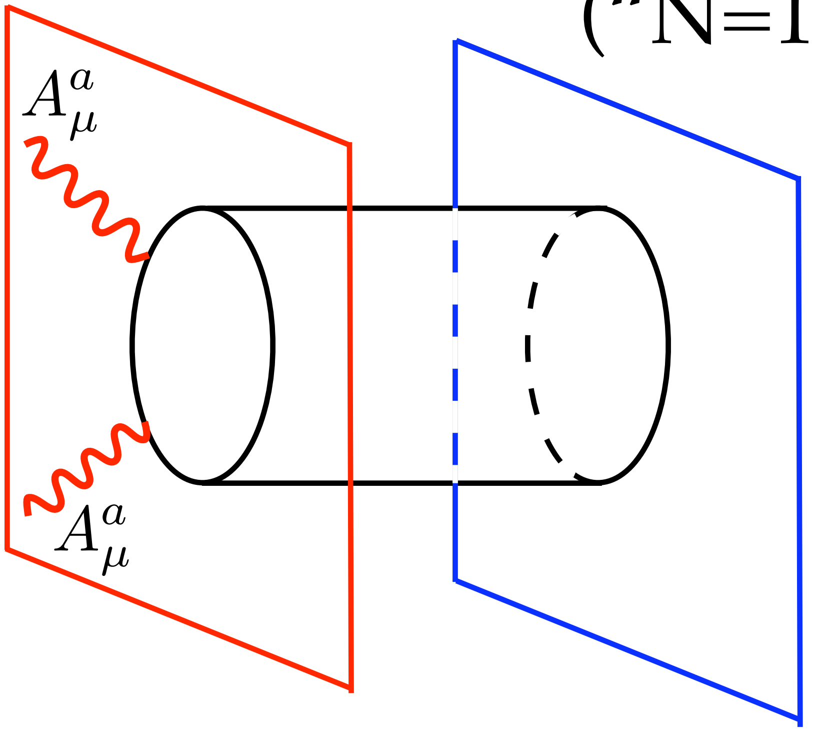
Less is known. Are they not interesting?

“[thrice twisted strings] effectively force the fields in the path integral to lie near some fixed point. The path integral is therefore insensitive to the shape of the *spacetime* torus and so is independent of the untwisted moduli. If on the other hand [they are only twice twisted], then the amplitude can depend on the moduli.”

Polchinski, Vol. 2, p. 299

# f: thrice twisted strings

("N=1 sectors")




Branes at angles: Completely twisted strings give moduli dependence too

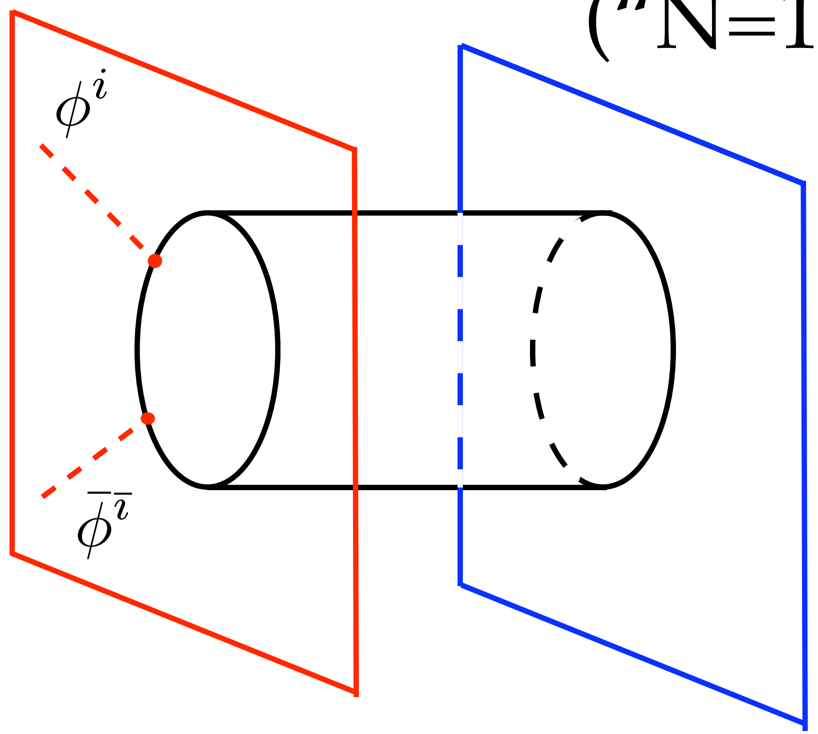
Lust, Stieberger '03

$$\int_0^\infty d\ell \, \frac{\vartheta_1'(\gamma)}{\vartheta_1(\gamma)} = -\frac{\pi}{2} \log \left[ e^{-2i\gamma_E \gamma} \frac{\Gamma(1-\gamma)}{\Gamma(1+\gamma)} \right]$$

(some fine print here)

# K: thrice twisted strings

("N=1 sectors")




Kähler metric
of D-brane scalars

$$K_{\phi\bar\phi}$$

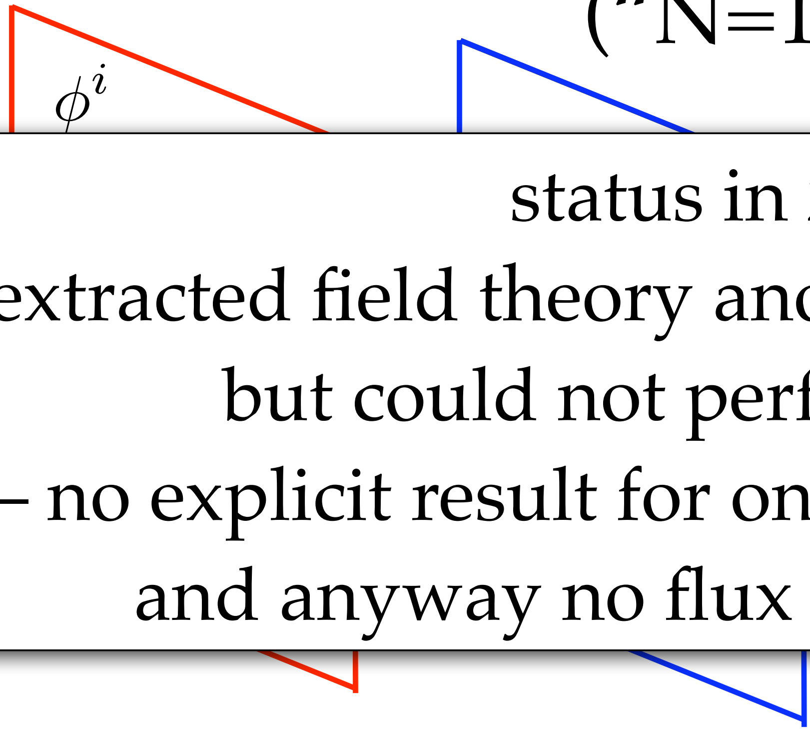
Bain, M.B. '00

$$\mathcal{A}_{99}^{k=1,5}(\phi^3,\bar\phi^{\bar 3}) \quad = \quad \frac{\delta e^3 \bar e^{\bar 3}}{96\pi^2}\,\mathrm{tr}(\gamma_9^k\lambda_1\lambda_2^\dagger)\mathrm{tr}(\gamma_9^k)\prod_{i=1}^{3}(-2\sin\pi k v_j)$$

$$\times\int_0^\infty \frac{dt}{t^2}\,\frac{\eta(it)^{3(1-\delta)}}{\vartheta_1(a,it)}\int_0^t d\nu\; e^{-\pi\delta\nu^2/t}\,\frac{\vartheta_1(i\nu+a,it)}{\vartheta_1(i\nu,it)^{1-\delta}}$$

# K: thrice twisted strings

("N=1 sectors")




status in 2000:
extracted field theory anomalous dimensions,
but could not perform integrals
– no explicit result for one-loop Kähler metric
and anyway no flux (so just constant)

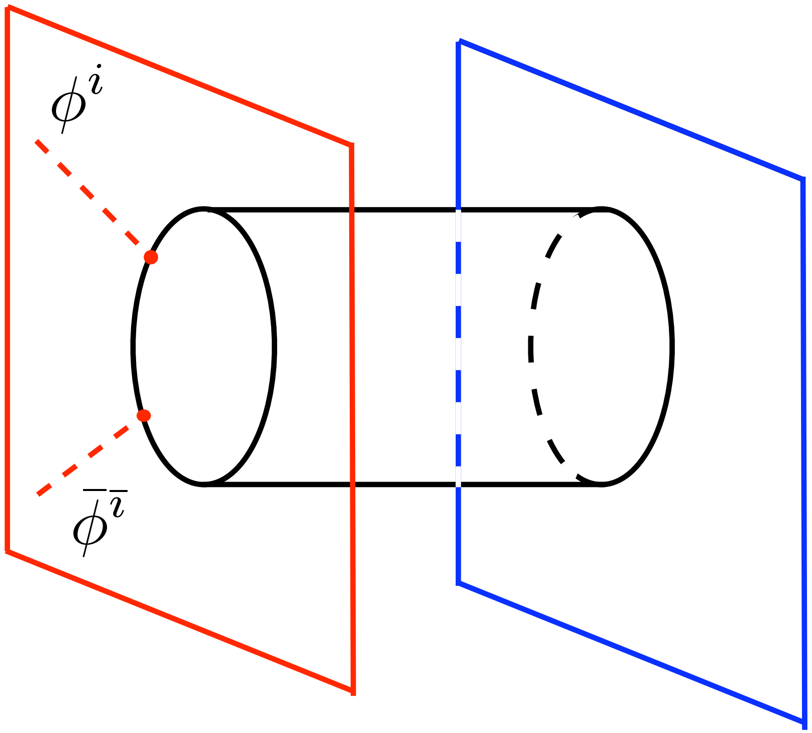
Bain, M.B. '00

$$\mathcal{A}_{99}^{k=1,5}(\phi^3,\bar{\phi}^{\bar{3}}) = \frac{\delta e^3 \bar{e}^{\bar{3}}}{96\pi^2}\,\mathrm{tr}(\gamma_9^k\lambda_1\lambda_2^\dagger)\mathrm{tr}(\gamma_9^k)\prod_{i=1}^{3}(-2\sin\pi k v_j)$$

$$\times\int_0^\infty \frac{dt}{t^2}\frac{\eta(it)^{3(1-\delta)}}{\vartheta_1(a,it)}\int_0^t d\nu\; e^{-\pi\delta\nu^2/t}\frac{\vartheta_1(i\nu+a,it)}{\vartheta_1(i\nu,it)^{1-\delta}}$$

# K: thrice twisted strings

M.B., Haack, Kang '09



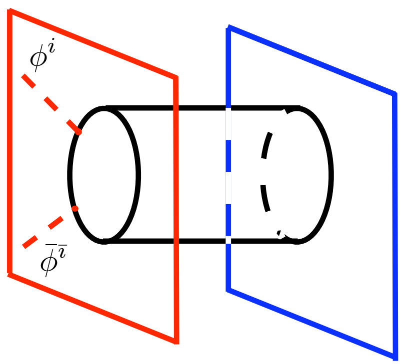

$$S = \sum_{\vec{\alpha}} \int d^2\tau \underbrace{Z^{\vec{\alpha}}}_{\text{dep. on } \tau} \int d^2\nu_i \underbrace{\langle \mathcal{V}_\phi \mathcal{V}_{\bar{\phi}^{\bar{\imath}}} \rangle^{\vec{\alpha}}}_{\text{dep. on } \nu_i \text{ and } \tau}$$

# K: thrice twisted strings

$$\sum_{\vec{\alpha}} Z^{\vec{\alpha}} \langle \mathcal{V}_{\phi} \mathcal{V}_{\bar{\phi}^{\bar{\imath}}} \rangle^{\vec{\alpha}} = e^{-\delta \cdot G_B(\nu,\tau)} G_F\left[ {}^{1/2}_{1/2+\gamma} \right](\nu,\tau)$$

$$e^{-\delta \cdot G_B(\nu,\tau)} G_F\left[ {}^{1/2}_{1/2+\gamma} \right](\nu,\tau) = e^{-\delta \cdot G_B(\nu,\tau)} \left( G_F\left[ {}^{1/2}_{1/2+\gamma} \right](\nu,\tau) - \frac{1}{\nu} \right) + e^{-\delta \cdot G_B(\nu,\tau)} \frac{1}{\nu}$$

$\delta = p_1 \cdot p_2$



split off divergence

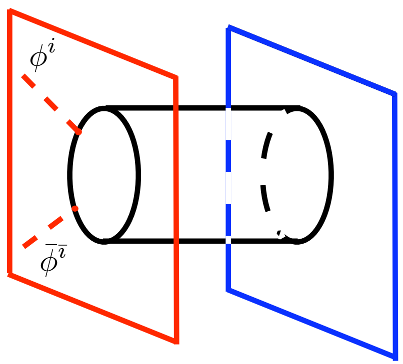
# K: thrice twisted strings

$$\sum_{\vec{\alpha}} Z^{\vec{\alpha}} \langle \mathcal{V}_\phi \mathcal{V}_{\bar{\phi}^{\bar{\imath}}} \rangle^{\vec{\alpha}} = e^{-\delta \cdot G_B(\nu,\tau)} G_F[{}^{1/2}_{1/2+\gamma}](\nu,\tau)$$

$$e^{-\delta \cdot G_B(\nu,\tau)} G_F[{}^{1/2}_{1/2+\gamma}](\nu,\tau) \; = \; e^{-\delta \cdot G_B(\nu,\tau)} \left( G_F[{}^{1/2}_{1/2+\gamma}](\nu,\tau) - \frac{1}{\nu} \right)$$

$$+ e^{-\delta \cdot G_B(\nu,\tau)} \frac{1}{\nu}$$

$$\delta = p_1 \cdot p_2$$



# K: thrice twisted strings

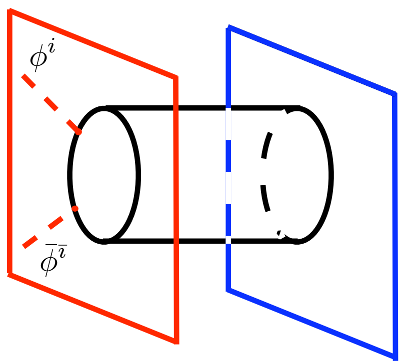
$$\sum_{\vec{\alpha}} Z^{\vec{\alpha}} \langle \mathcal{V}_\phi \mathcal{V}_{\bar{\phi}^{\bar{\imath}}} \rangle^{\vec{\alpha}} = e^{-\delta \cdot G_B(\nu,\tau)} G_F[{}^{1/2}_{1/2+\gamma}](\nu,\tau)$$

set $\delta = 0$

$$e^{-\delta \cdot G_B(\nu,\tau)} G_F[{}^{1/2}_{1/2+\gamma}](\nu,\tau) \;=\; e^{-\delta \cdot G_B(\nu,\tau)} \left( G_F[{}^{1/2}_{1/2+\gamma}](\nu,\tau) - \frac{1}{\nu} \right)$$

$$+e^{-\delta \cdot G_B(\nu,\tau)} \frac{1}{\nu}$$

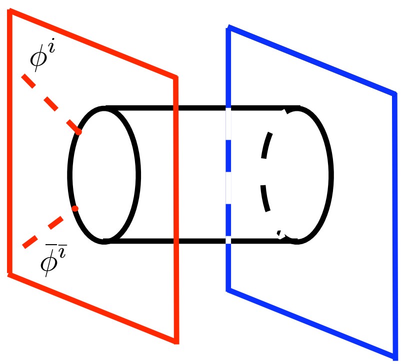
$\delta = p_1 \cdot p_2$



# K: thrice twisted strings

$$\int_0^\infty d\ell \int_0^1 d\nu\, e^{-2\pi i\gamma\nu} \left( G_F\left[{}^{\;\;1/2}_{1/2+f-i\gamma\ell}\right](\nu, i\ell) - \frac{\pi e^{\pi i\nu}}{\sin \pi\nu} \right)$$

$$G_F\left[{}^{\;\;1/2}_{1/2+z}\right](\nu,\tau) \quad = \quad \frac{\vartheta_1(\nu+z,\tau)\vartheta_1'(0,\tau)}{\vartheta_1(z,\tau)\vartheta_1(\nu,\tau)}$$

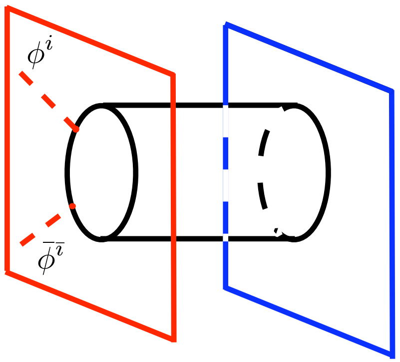


# K: thrice twisted strings

The twisted fermionic propagator $G_F$ can be represented as

a) theta functions (previous slide)
b) $q$-series
c) Lambert series
d) twisted Weierstrass function e.g. Gaberdiel, Keller '09

...



11. By considering $\int \frac{\vartheta_4'(z)}{\vartheta_4(z)} e^{2niz}\, dz$ taken along the contour formed by the parallelogram whose corners are $-\frac{1}{2}\pi, \frac{1}{2}\pi, \frac{1}{2}\pi + \pi\tau, -\frac{1}{2}\pi + \pi\tau$, shew that, when $n$ is a positive integer,

$$(1-q^{2n}) \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{\vartheta_4'(z)}{\vartheta_4(z)} e^{2niz}\, dz = 2\pi i q^n,$$

and deduce that, when $|I(z)| < \frac{1}{2} I(\pi\tau)$,

$$\frac{\vartheta_4'(z)}{\vartheta_4(z)} = 4 \sum_{n=1}^{\infty} \frac{q^n \sin 2nz}{1-q^{2n}}.$$

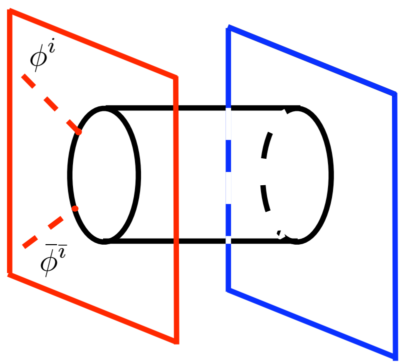
Whittaker & Watson exercises 21.11, 21.13

b) *q*-series

c) Lambert series

d) twisted Weierstrass function e.g. Gaberdiel, Keller '09

...

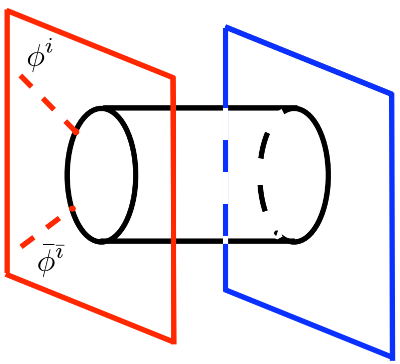


# K: thrice twisted strings

$$G_F\left[\begin{smallmatrix} 1/2 \\ 1/2+z \end{smallmatrix}\right](\nu,\tau) = \frac{\vartheta_1(\nu+z,\tau)\vartheta_1'(0,\tau)}{\vartheta_1(z,\tau)\vartheta(\nu,\tau)}$$

$$= \pi\cot\pi\nu + \pi\cot\pi z$$

$$+4\pi\sum_{m,n=1}^{\infty} q^{mn}\sin(2\pi m\nu + 2\pi n z)$$




$$(q = e^{2\pi i\tau})$$

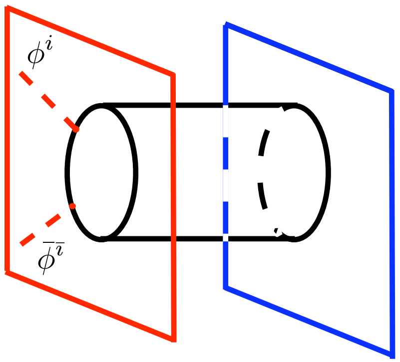
$q$-series representation

# K: thrice twisted strings

$$G_F\left[{}^{1/2}_{1/2+z}\right](\nu,\tau) \quad = \quad \frac{\vartheta_1(\nu+z,\tau)\vartheta_1'(0,\tau)}{\vartheta_1(z,\tau)\vartheta(\nu,\tau)}$$

$$= \pi\cot\pi\nu + \pi\cot\pi z$$

$$+4\pi\sum_{m,n=1}^{\infty} q^{mn}\sin(2\pi m\nu + 2\pi n z)$$



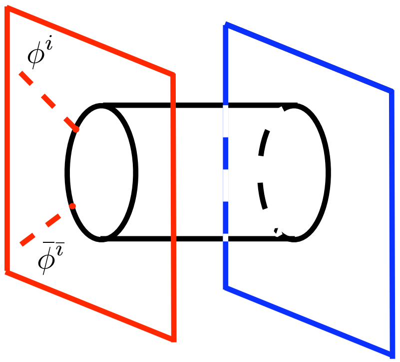

skip to final result for amplitude

# K: thrice twisted strings

$$\int_{\mu/M_s}^{\Lambda/M_s} d\ell \int_0^1 d\nu\, e^{-2\pi i\gamma\nu} \left( G_F\left[{}^{\;\;1/2}_{1/2-i\gamma\ell}\right](\nu, i\ell) - \frac{\pi e^{\pi i\nu}}{\sin \pi\nu} \right)$$

$$= \frac{\pi e^{-\pi i\gamma}}{\sin \pi\gamma} \left( \log \frac{M_s}{\mu} - \log(2\pi) \right) +$$

$$\frac{1}{\pi} e^{-\pi i\gamma} \sin \pi\gamma \left(\zeta'(2, 1-\gamma) + \zeta'(2, \gamma)\right)$$




Hurwitz zeta

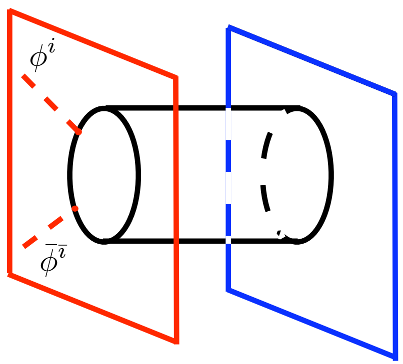
$$\zeta(s, \gamma) = \sum_{n=0}^{\infty} \frac{1}{(n+\gamma)^s}$$

# K: thrice twisted strings

$$\int_{\mu/M_s}^{\Lambda/M_s} d\ell \int_0^1 d\nu\, e^{-2\pi i\gamma\nu} \left( G_F\left[{}^{\;\;1/2}_{1/2-i\gamma\ell}\right](\nu, i\ell) - \frac{\pi e^{\pi i\nu}}{\sin \pi\nu} \right)$$

$$= \frac{\pi e^{-\pi i\gamma}}{\sin \pi\gamma} \left( \log \frac{M_s}{\mu} - \log(2\pi) \right) +$$

$$\frac{1}{\pi} e^{-\pi i\gamma} \sin \pi\gamma \left(\zeta'(2, 1-\gamma) + \zeta'(2,\gamma)\right)$$

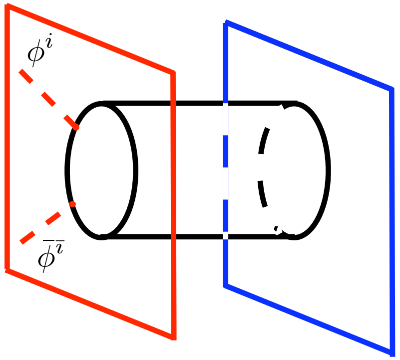



reproduces old result on anomalous dimensions

# K: thrice twisted strings

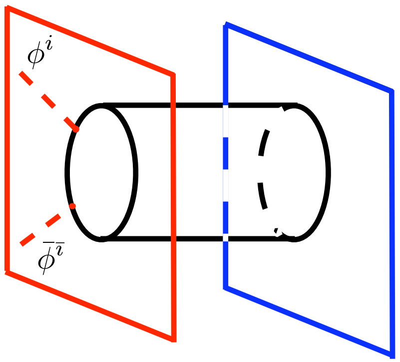
• computed twisted 2-point functions of D-brane scalars with worldvolume flux in N=1 toroidal orientifolds

• in paper: extract the moduli-dependent one-loop Kähler metric for D-brane scalar



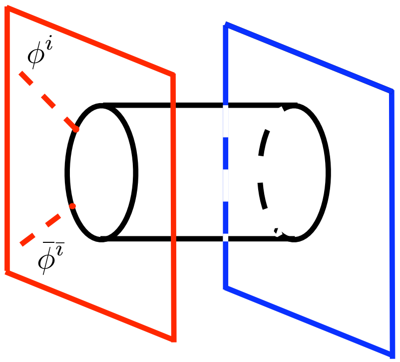
# K: thrice twisted strings

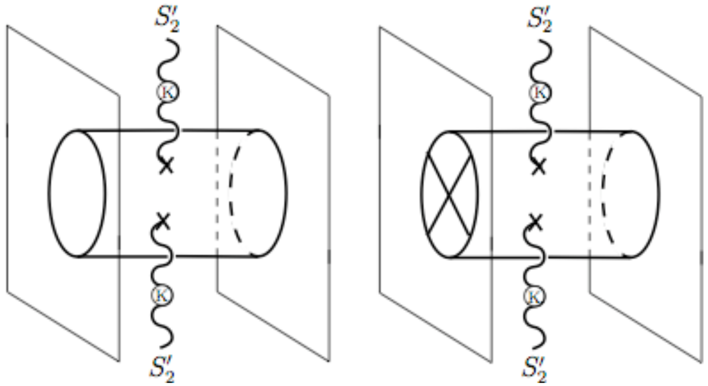
- currently resolving last (?) issues (using the Kiritsis textbook!)



# K: thrice twisted strings

- work in progress: the analogous closed string one-loop amplitudes.
Using Kähler adapted vertex operators, we should be able to “integrate” to find the Kähler potential $K(S, \bar{S}, T, \bar{T}, U, \bar{U}, \phi, \bar{\phi})$



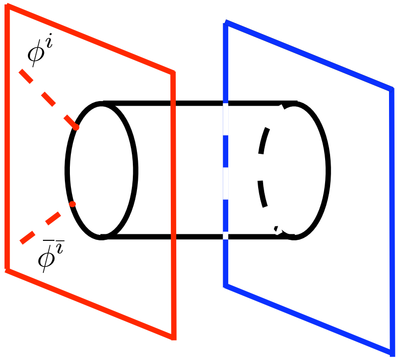



# Outlook

• I didn't discuss it here, but in the twice twisted (N=2) case,
one finds interesting relations between corrections to gauge couplings and Kähler metric

eq. (C.32) in M.B., Haack, Körs '05

$$\partial_\phi \partial_{\bar{\phi}} E_2(\phi, U) = \frac{-2\pi^2 i}{U - \bar{U}} E_1(\phi, U)$$

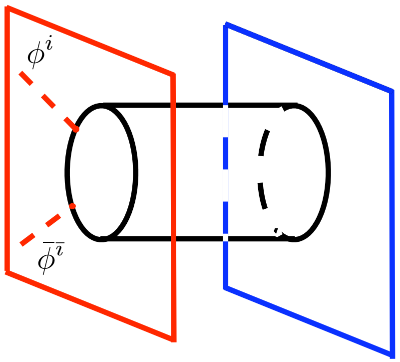


# Outlook

Giddings, Maharana '05
Baumann, Dymarsky, Klebanov, , Maldacena, McAllister, Murugan '06
...

- These kinds of arguments can lead to better understanding of corrections in much more general backgrounds

$$\partial_\phi \partial_{\bar{\phi}} E_2(\phi, U) = \frac{-2\pi^2 i}{U - \bar{U}} E_1(\phi, U)$$



# Outlook

- Why does there seem to be some relation between thrice twisted (N=1) corrections to gauge coupling and Kähler metric corrections ?

$$\frac{d}{ds}\zeta(s,\gamma)\Big|_{s=0} = \log\Gamma(\gamma) - \frac{1}{2}\log(2\pi)$$