Electroweak corrections to tri-boson production at the ILC

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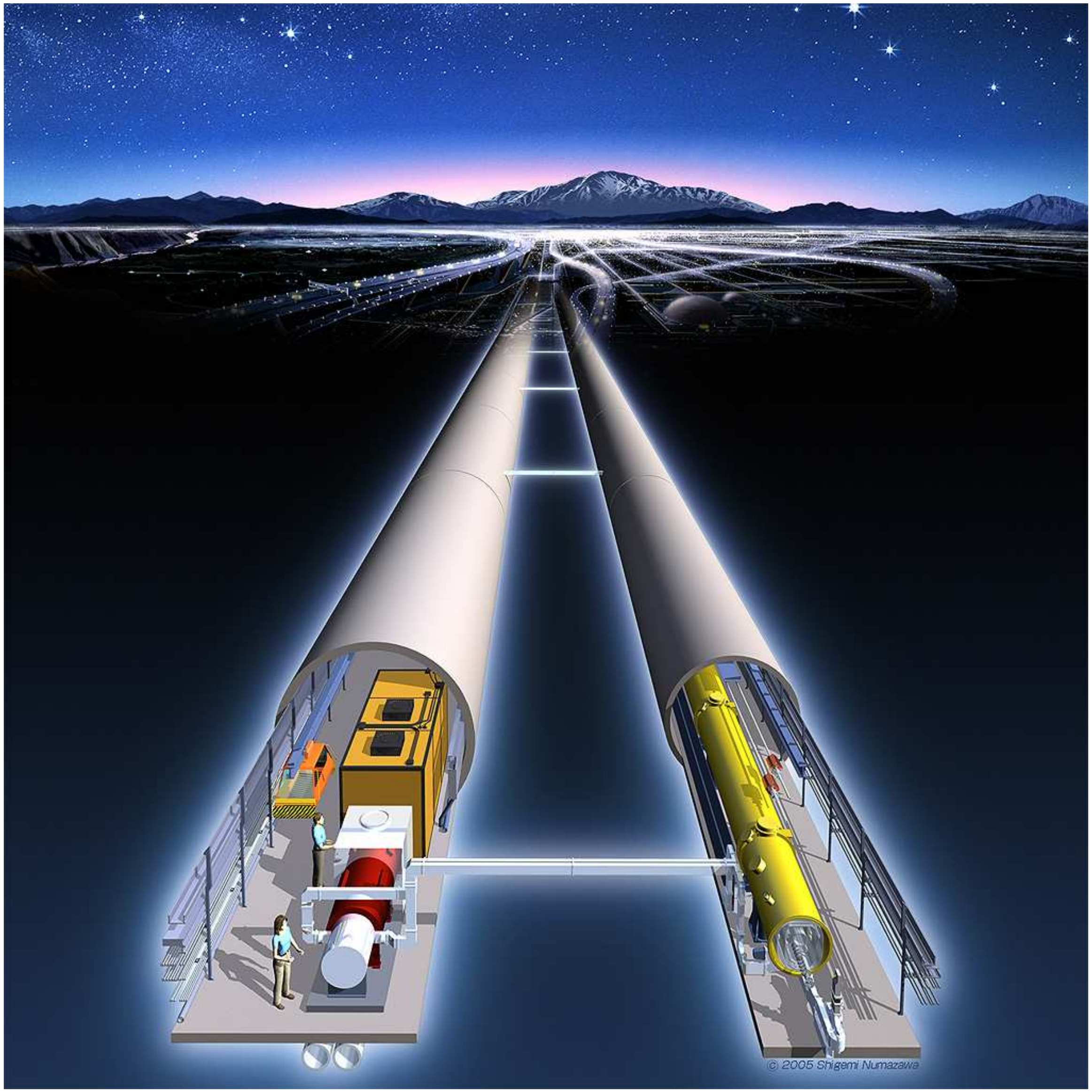
Outline

- Motivation
- Calculations: $e^+e^- \to ZZZ, WWZ$
- Preliminary results
- Conclusions

The LCs



The International Linear Collider (ILC, see Behnke's lecture):

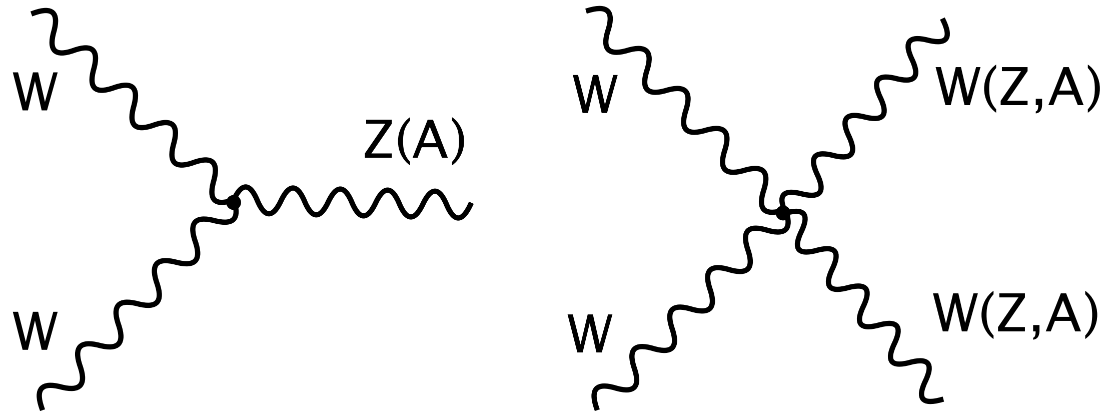
- CM energy: $500 \div 1000$GeV.
- Luminosity: of the order 10^{34}cm^{-2}s^{-1}.

The Compact LInear Collider (CLIC, see Tsesmelis's talk):

- CM energy: $0.5 \div 3$TeV.
- Luminosity: also of the order 10^{34}cm^{-2}s^{-1}.

e^+e^- colliders are high precision machines.

SM trilinear and quartic gauge couplings



- Trilinear couplings: checking the non-abelian gauge structure.

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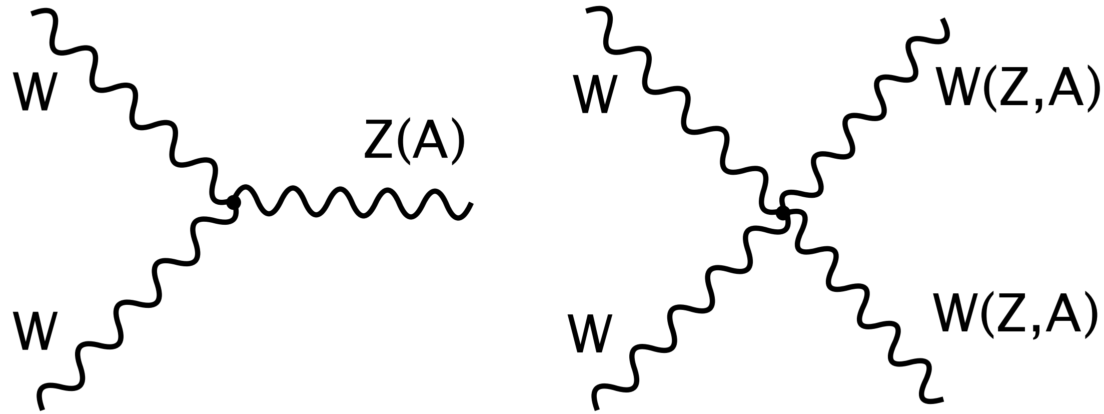

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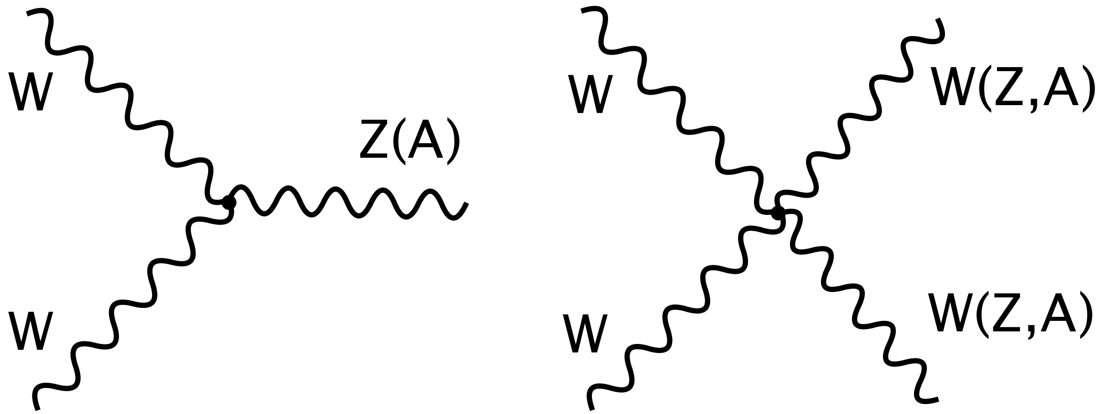
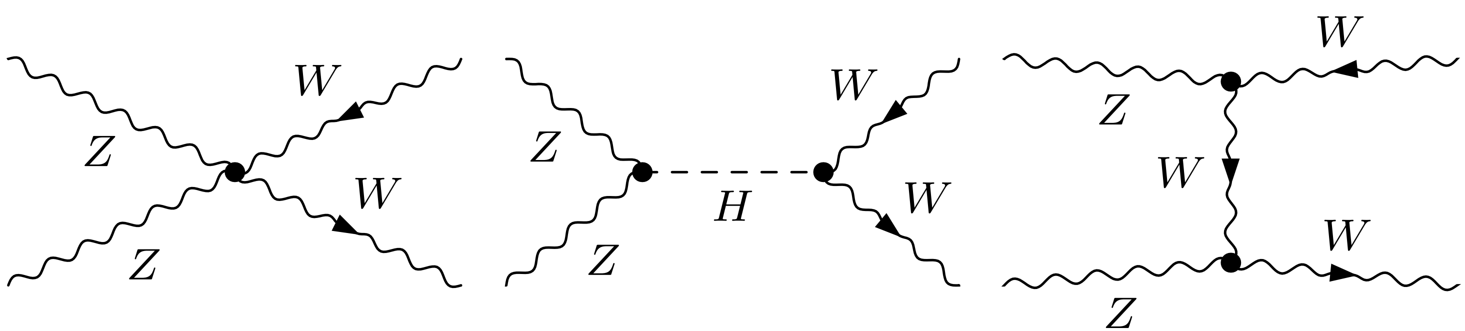
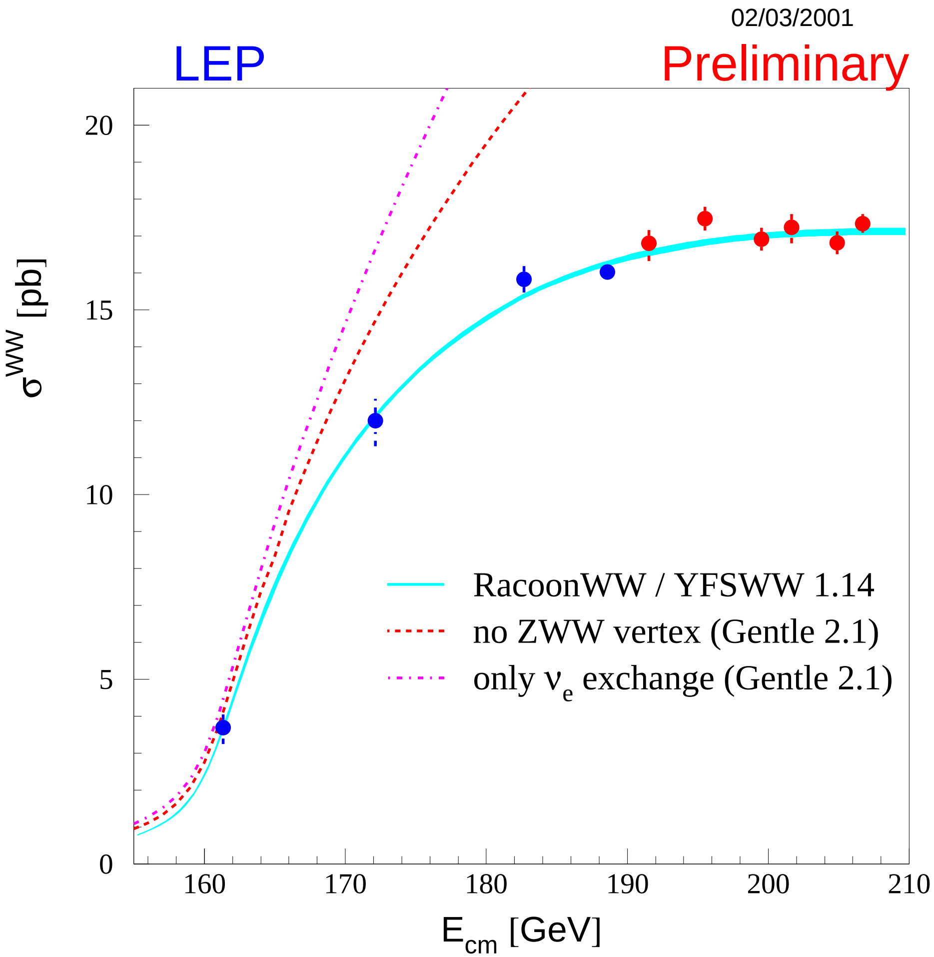


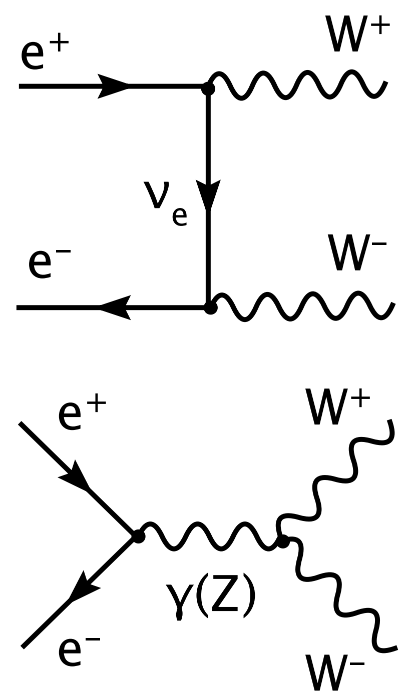
- Trilinear couplings: checking the non-abelian gauge structure.
- Quartic couplings: also give a window on the spontaneous symmetry breaking (SSB) mechanism.
- SM: unitarity constraints and the perturbative condition (*and* experimental facts) indicate the existence of a small-mass Higgs and new physics at TeV scale.



$\Longrightarrow$ this suggests some connection between the Higgs, new physics and quartic gauge couplings (contact interactions).

WW production at LEP





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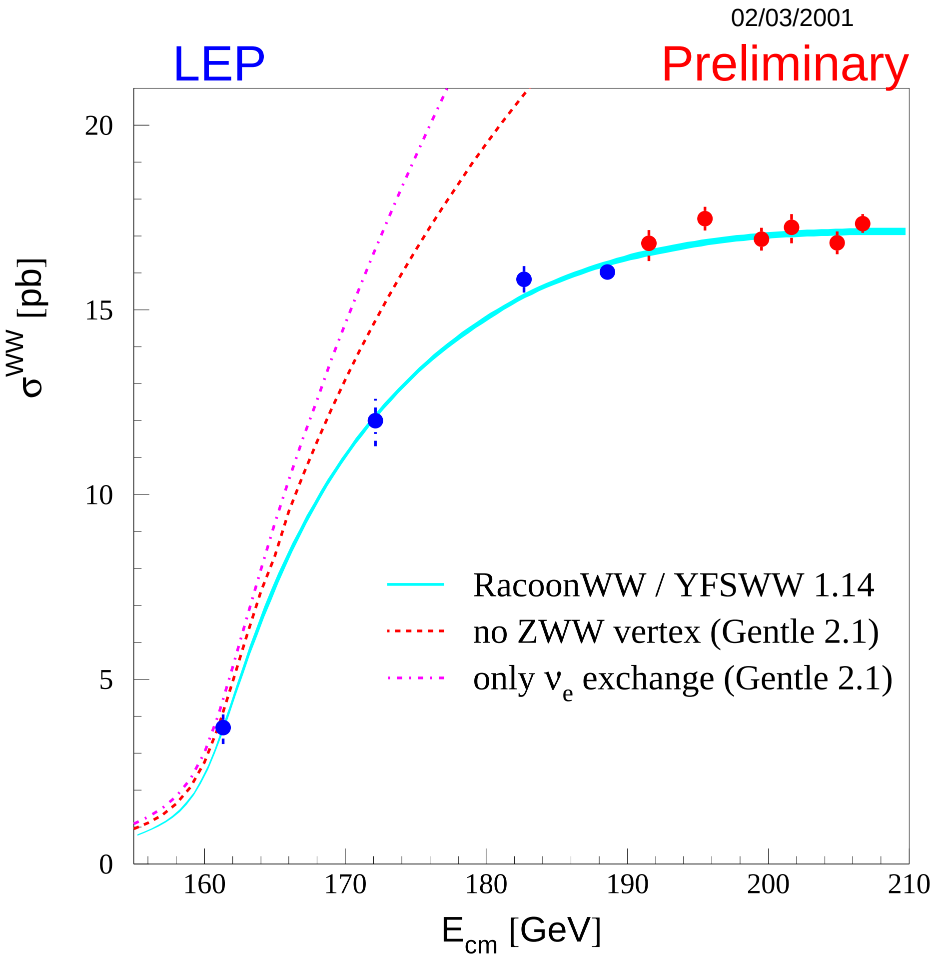
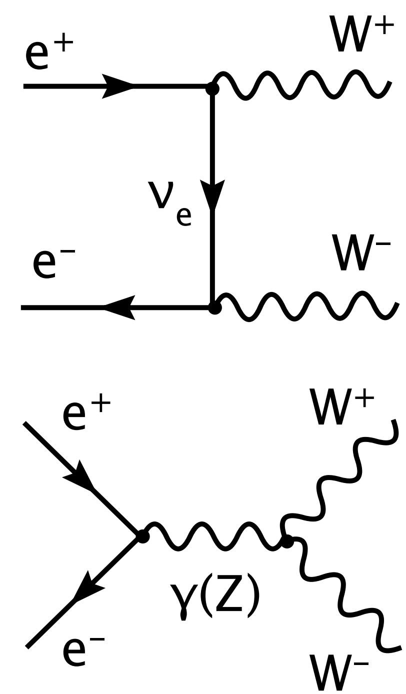
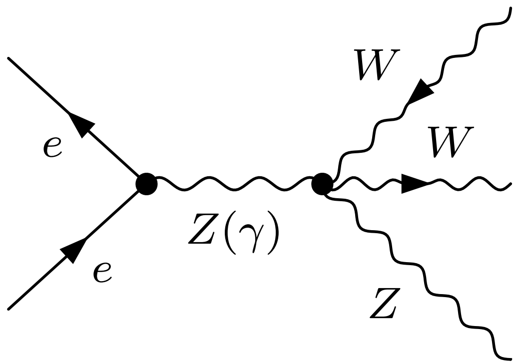




- SM trilinear couplings: well tested at LEP.
- What about the quartic gauge couplings? Not well tested.

Quartic couplings and Higgs mechanism?



- The threshold for WWZ is about 250GeV ($\sqrt{s}_{LEP} = 209$GeV). We need the LCs (LHC upgrade ???).

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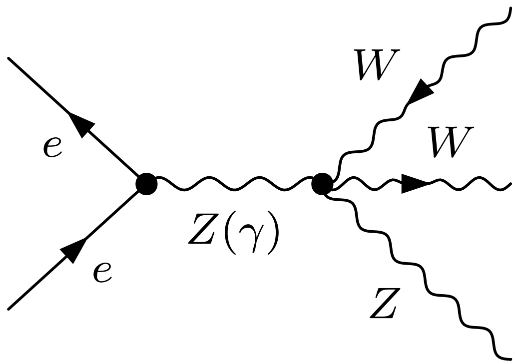

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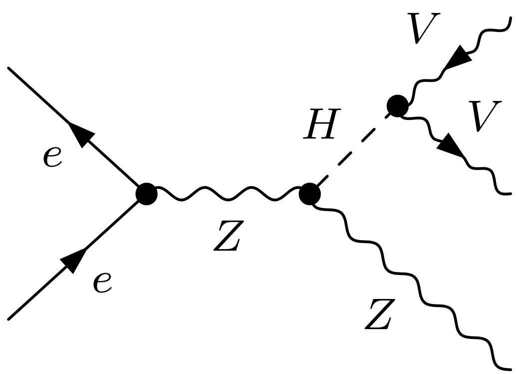

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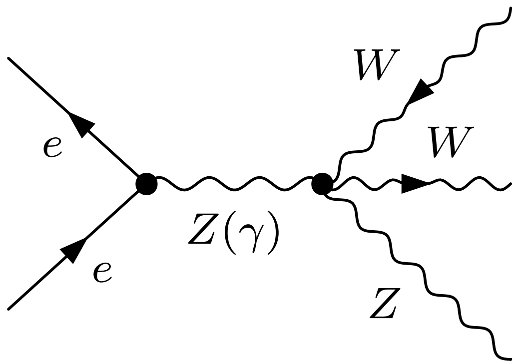

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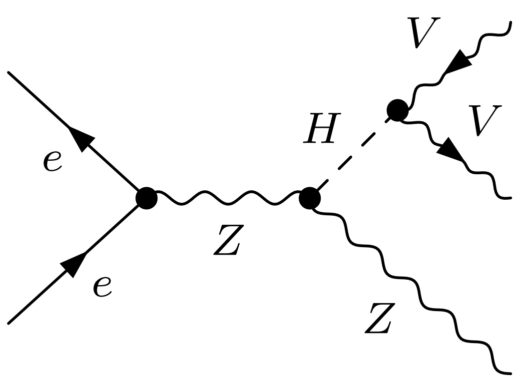

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- At the LCs the radiative EW corrections must be taken into account (obvious from LEP's results: the famous Higgs blue band, top mass prediction, ...).

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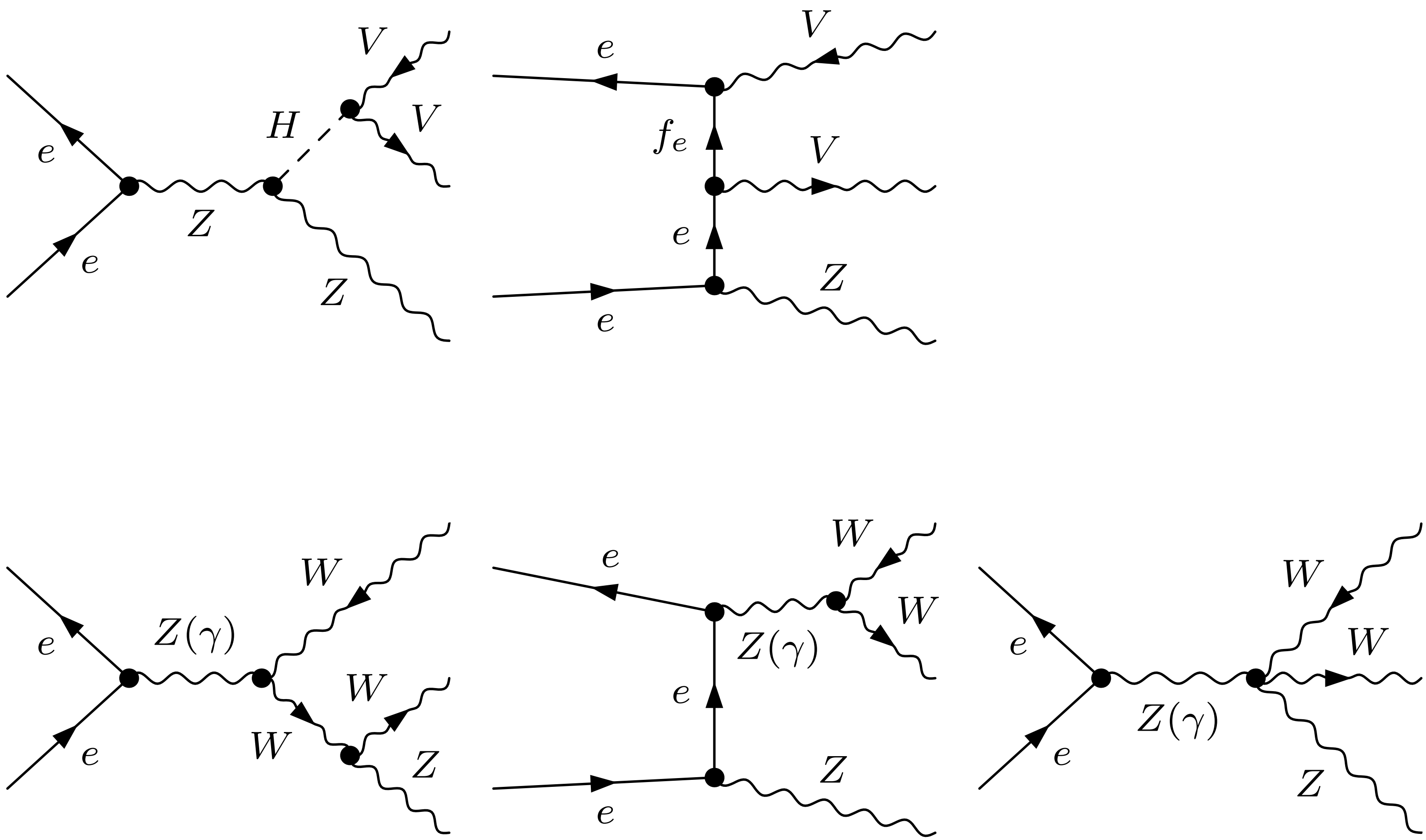
- Among the long list, gauge boson production is an important topic and attracts a lot of interest:
  - 1-loop QCD correction to $pp \to VVV$ at the LHC: Binoth, Ossola, Papadopoulos and Pittau (2008); Hankele and Zeppenfeld (2008); Lazopoulos, Melnikov and Petriello (2007) ...
  - 1-loop EW correction to $e^+e^- \to ZZZ$: Su, Ma, Zhang, Wang and Guo (2008).
  - $e^+e^- \to W^+W^-Z$: more interesting but also much more difficult. This has not been calculated.

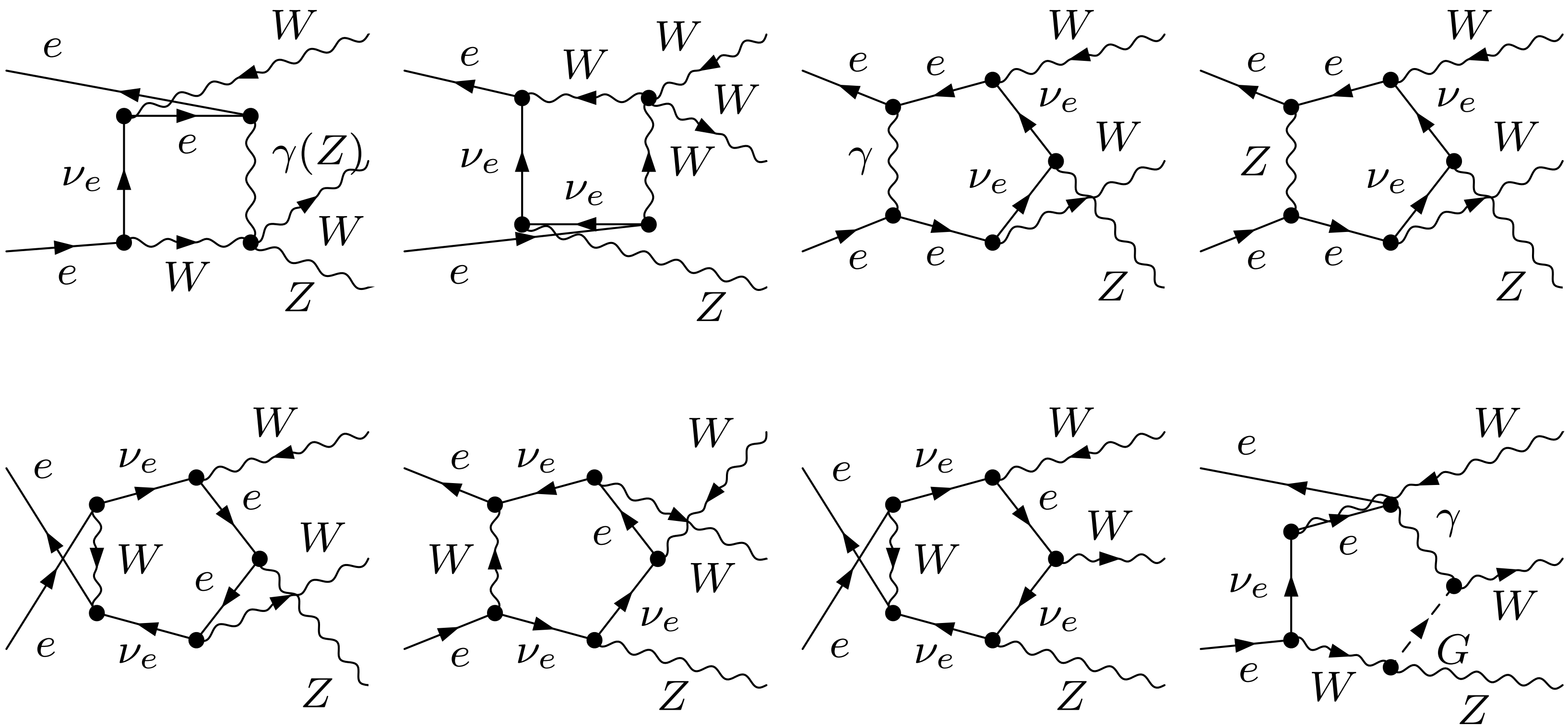
$e^+e^- \to VVZ$: tree diagrams

- ZZZ: 9 diagrams, no trilinear and quartic couplings in SM
- WWZ: 20 diagrams, trilinear and quartic couplings contribute in SM



$e^+e^- \to W^+W^-Z$: one-loop diagrams

't Hooft-Feynman guage, neglecting $< eeS >$ couplings:



Topology	$ZZZ(1767)$	$WWZ(2736)$
Loop Amp. (`FormCalc-6.0`)	6.4MB	6.9MB
4-point	384	396
5-point	64	109

calculation framework

$$d\sigma_{1-loop}^{e^+e^-\to VVZ} = d\sigma_{virt}^{e^+e^-\to VVZ} + d\sigma_{real}^{e^+e^-\to VVZ\gamma}$$

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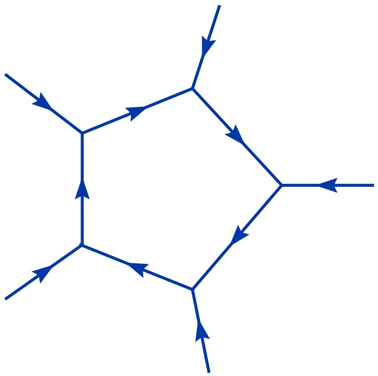
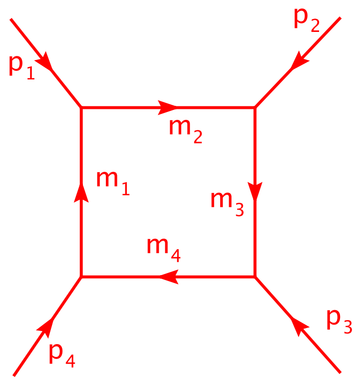
- LoopTools(FF+ Tensor reduction + ...) to calculate all one-loop integrals. [Hahn, van Oldenborgh and Vermaseren]
- BASES(Kawabata) and VEGAS to do phase space integration and to get distributions.

Loop integrals and numerical instabilities





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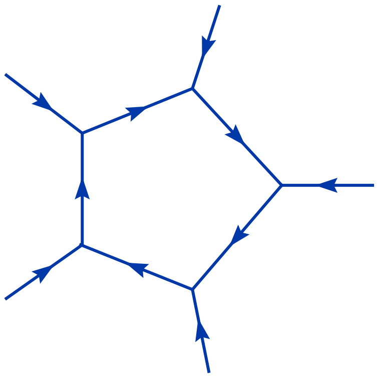

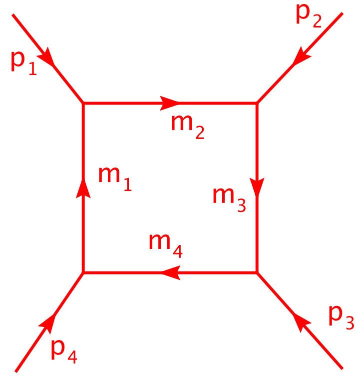

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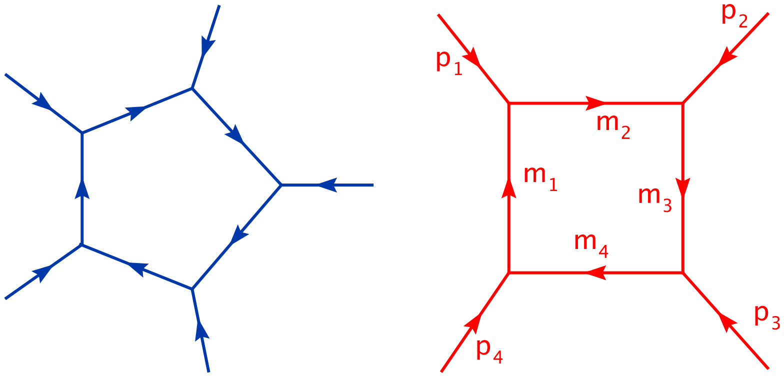

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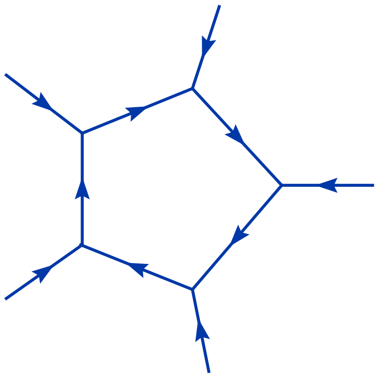

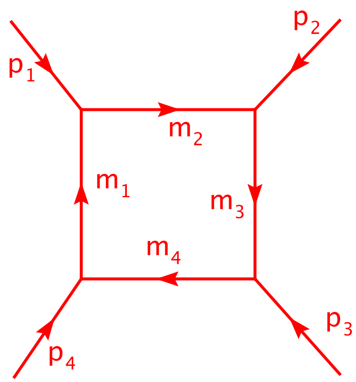

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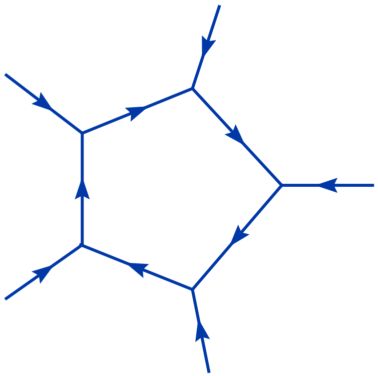
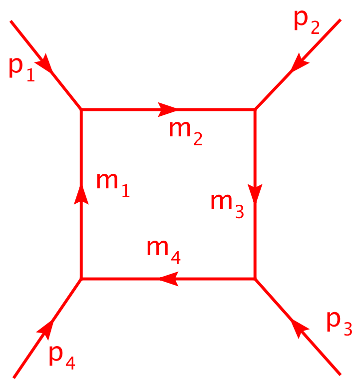
Loop integrals and numerical instabilities





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- Solutions: small DetG expansion or using quadruple precision (loop library only, the results become stable, 5 times slower).

Real correction (I)

1) Two cutoff phase space slicing approach: easy to implement but time consuming

$$d\sigma_{real}^{e^+e^-\to W^+W^-Z\gamma} = d\sigma_{soft}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s) + d\sigma_{hard}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s),$$
$$d\sigma_{hard}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s) = d\sigma_{coll}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s,\delta_c) + d\sigma_{fin}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s,\delta_c)$$

Soft part: $E_\gamma < \delta_s\sqrt{s}/2 = \Delta E$,

$$d\sigma_{soft} = -d\sigma_{Born}\frac{\alpha}{2\pi^2}\sum_{i,j=1}^{4}\int_{|\mathbf{k}|<\Delta E}\frac{d^3k}{2\omega_k}\frac{\pm p_i p_j Q_i Q_j}{(p_i.k)(p_j.k)}.$$

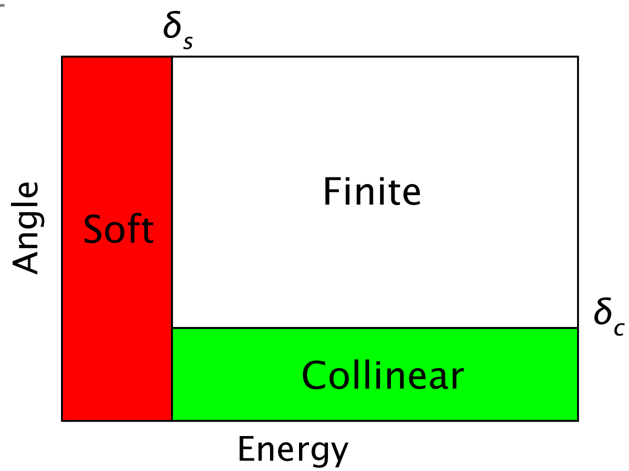
Collinear part: $\{E_\gamma \geq \Delta E, \cos\theta_{\gamma f} > 1-\delta_c\}$, $\hat{s} = xs$,

$$d\sigma_{coll} = \sum_{i=1}^{2}\frac{\alpha}{2\pi}Q_i^2\int_0^{1-\delta_s} dx d\sigma_{Born}(\hat{s})\left[\frac{1+x^2}{1-x}\ln\frac{\hat{s}\delta_c}{2m_i^2 x} - \frac{2x}{1-x}\right]$$

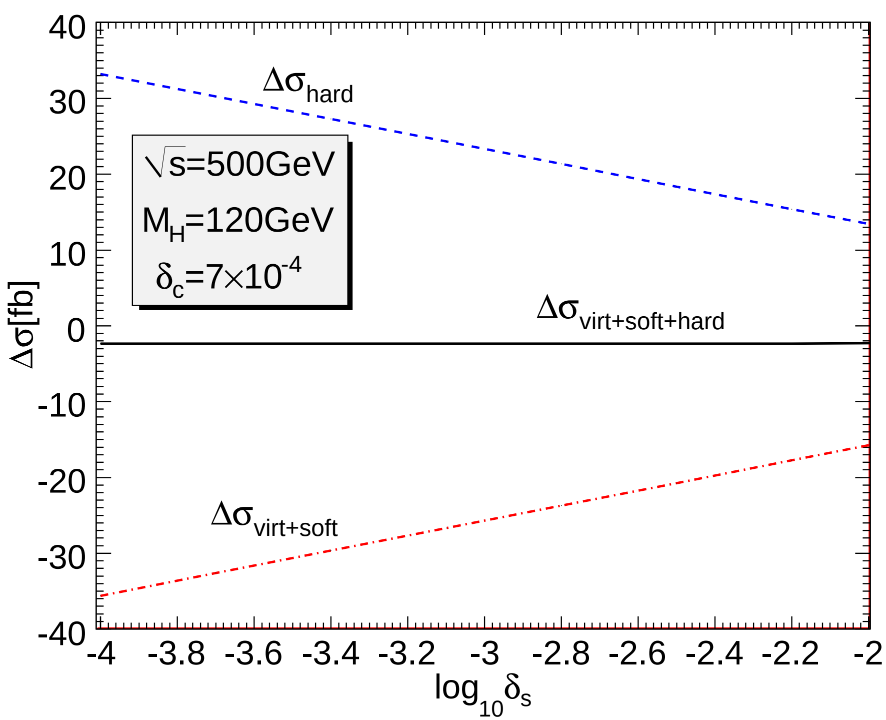
Finite part: $\{E_\gamma \geq \Delta E, \cos\theta_{\gamma f} \leq 1-\delta_c\}$, numerical integration using Monte Carlo BASES.

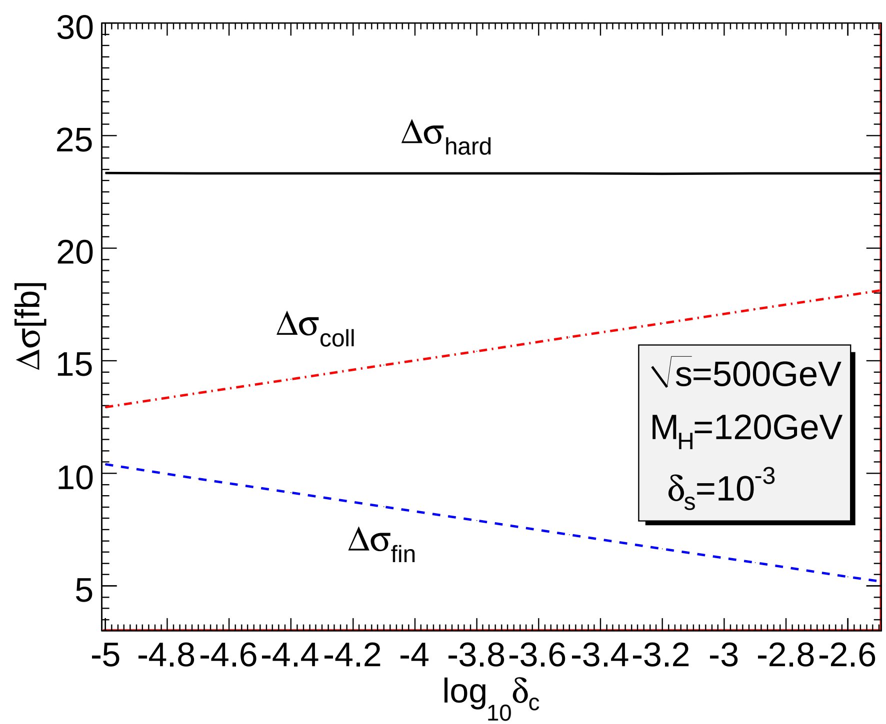
2) Dipole subtraction approach: to cross check, in progress.

Real correction (II)



- Real correction is cutoff-independent.
- Factorization condition: δ_s and δ_c are sufficiently small. And $\delta_c \gg 2m_e^2/s$ to use the collinear integration formula.





Genuine weak correction

The universal QED contribution from ISR:

$$\delta_{V+S}^{QED} = \frac{2\alpha}{\pi}\left[(L_e - 1)\ln\delta_s + \frac{3}{4}L_e + \frac{\pi^2}{6} - 1\right] \ , \ L_e = \ln(s/m_e^2) \ .$$

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- Large QED corrections require higher order treatment. The above procedure paves the way to resummation.

Non-Linear gauge check

General Non-Linear gauge (NLG) fixing Lagrangian (Boudjema, Chopin 1995):

$$\mathcal{L}_{GF} = -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu+} + \xi_W\frac{g}{2}(v+\tilde{\delta}H + i\tilde{\kappa}\chi_3)\chi^+|^2$$
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Non-Linear gauge check

General Non-Linear gauge (NLG) fixing Lagrangian (Boudjema, Chopin 1995):

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- Squared amplitude: independent of those parameters (gauge invariant).

NLG Check and numerical instability

$(\tilde{\alpha}, \tilde{\beta})$	ZZZ	WWZ(1)	WWZ(2)
(0,0)	-7.8077709362570481E-4	-6.3768793214220439E-2	5.5880925111126470478198203067272 17E-2
(1,0)	-7.8077709362570731E-4	-6.3767676883630841E-2	5.5880925111110349911426963080135 26E-2
(0,1)	-7.8077709361534624E-4	-6.3772289648961160E-2	5.5880925111146084510166610529723 81E-2

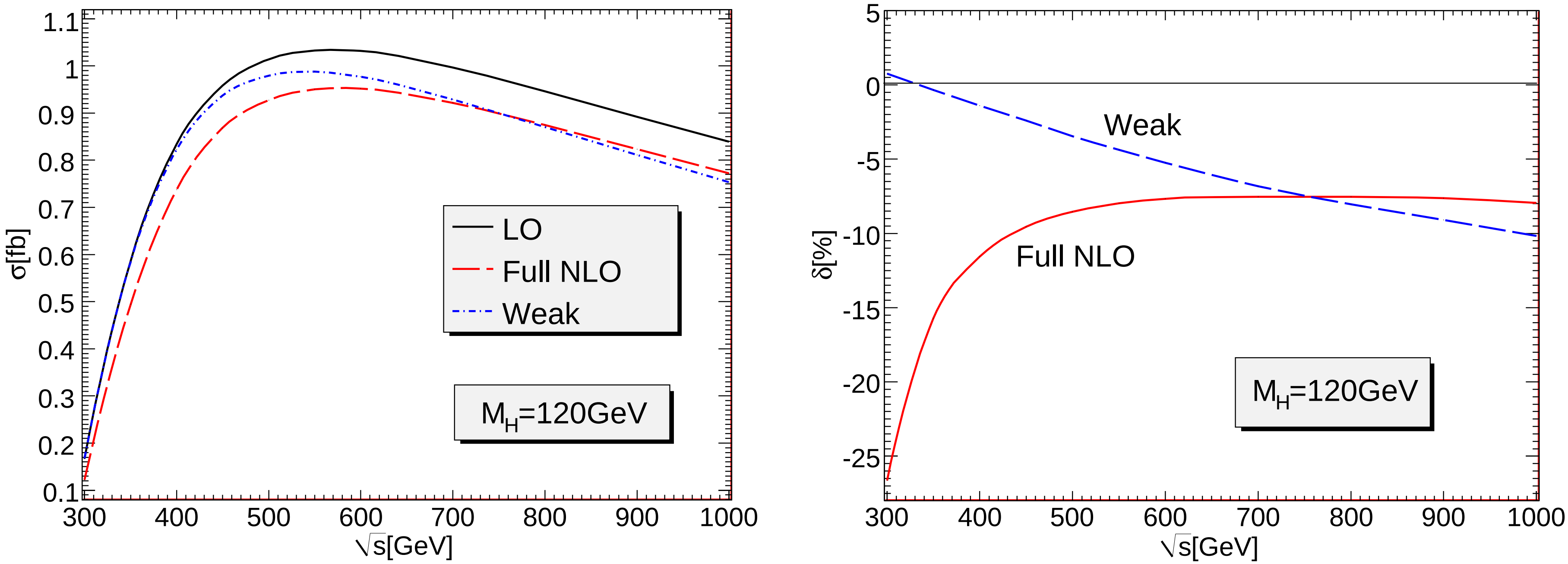
- ZZZ: at least 10 digit agreement at a random point with double precision.
- WWZ: for a DP random point, got only 4 digit agreement. By using QP, got 12 digits. Gauge invariance check is much worse for WWZ. This is an indication of numerical instability.

Checks on the results

- gauge invariance check: tree and one-loop squared amplitude level.
- UV and IR finiteness: one-loop squared amplitude level and for the virtual + soft corrections.
- Two independent calculations: different loop integral libraries, different integrators (BASES and VEGAS). This final check is in progress.

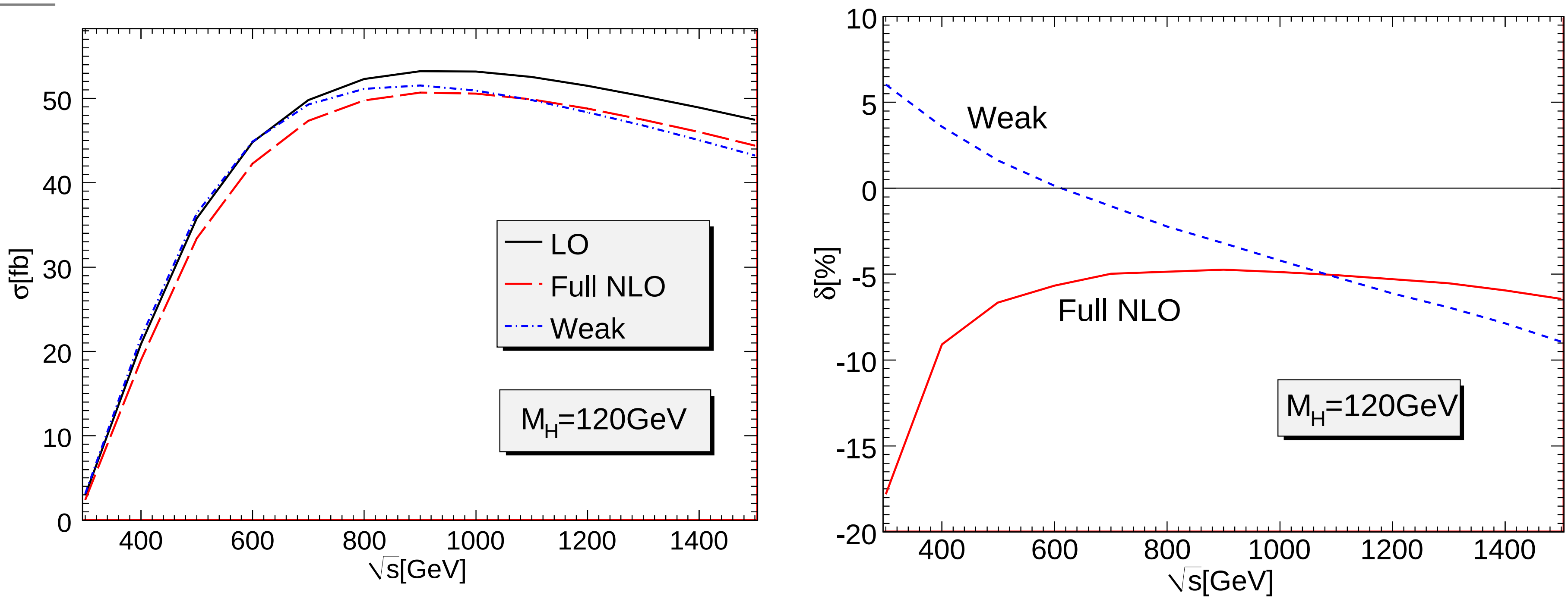
All results are **preliminary**

$e^+e^- \to ZZZ$: Total Xsection



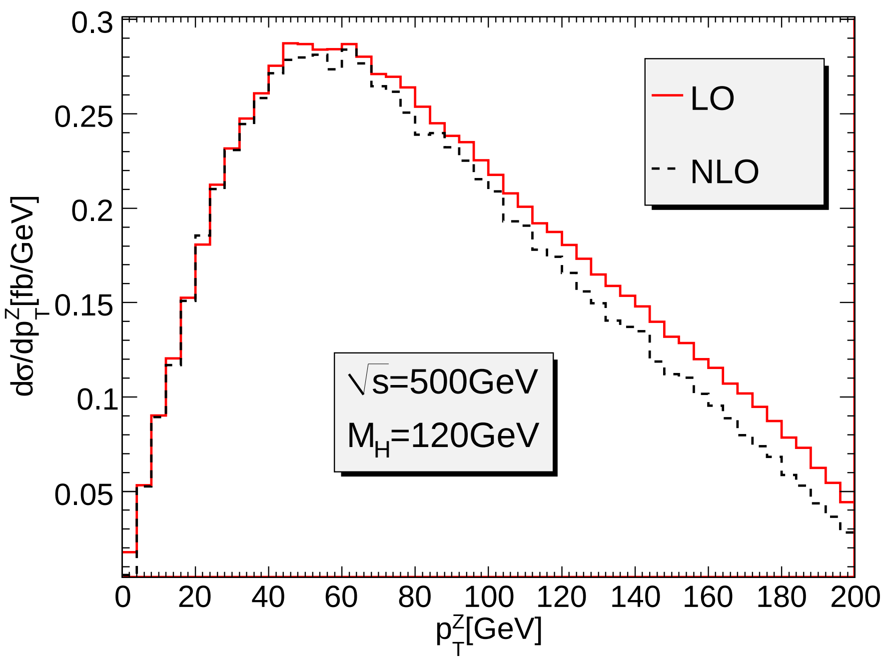
- Total Xsection peak about 1fb is at $\sqrt{s} \approx 550$GeV.
- The weak correction goes from -3.5% to -10% when $\sqrt{s}$ increases from 500GeV to 1TeV.
- Comparisons with *Su et al. arXiv:0807.0669*: Low energies ($350, 500$GeV) quite good agreement (less than 0.2%), at 1TeV about 0.5%.

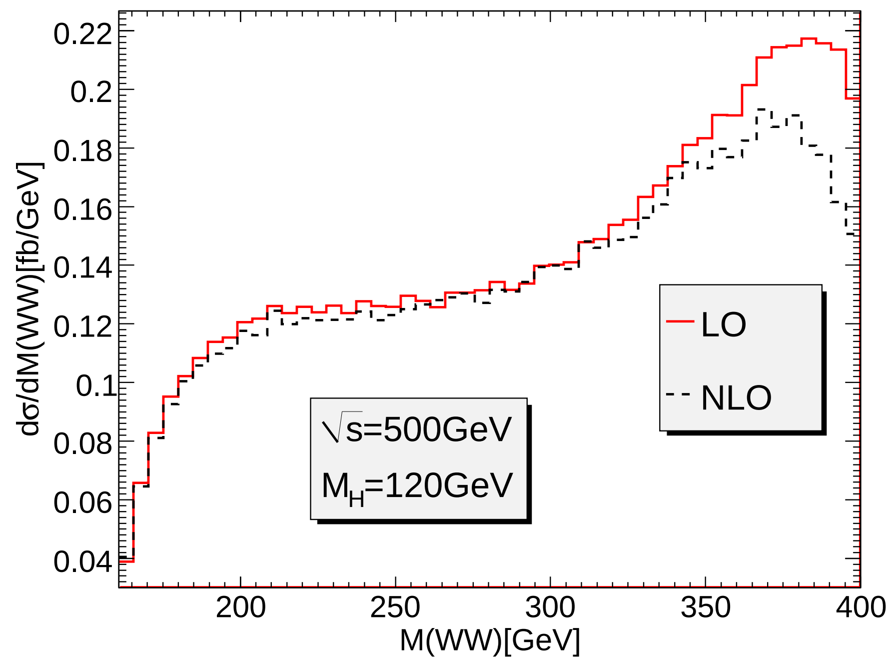
$e^+e^- \to W^+W^-Z$: Total Xsection



- Total Xsection peak about 50fb (50 times larger than σ_{ZZZ}) is at $\sqrt{s} \approx 900$GeV.
- The weak correction goes from 1.6% to -8.9% when $\sqrt{s}$ increases from 500GeV to 1.5TeV.

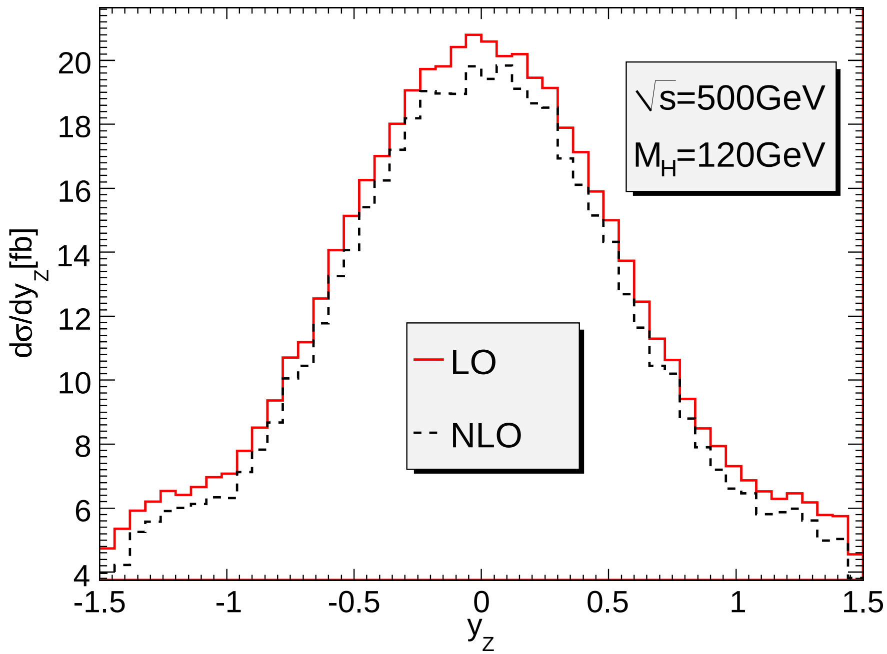
$e^+e^- \to W^+W^-Z$: Distributions (I)

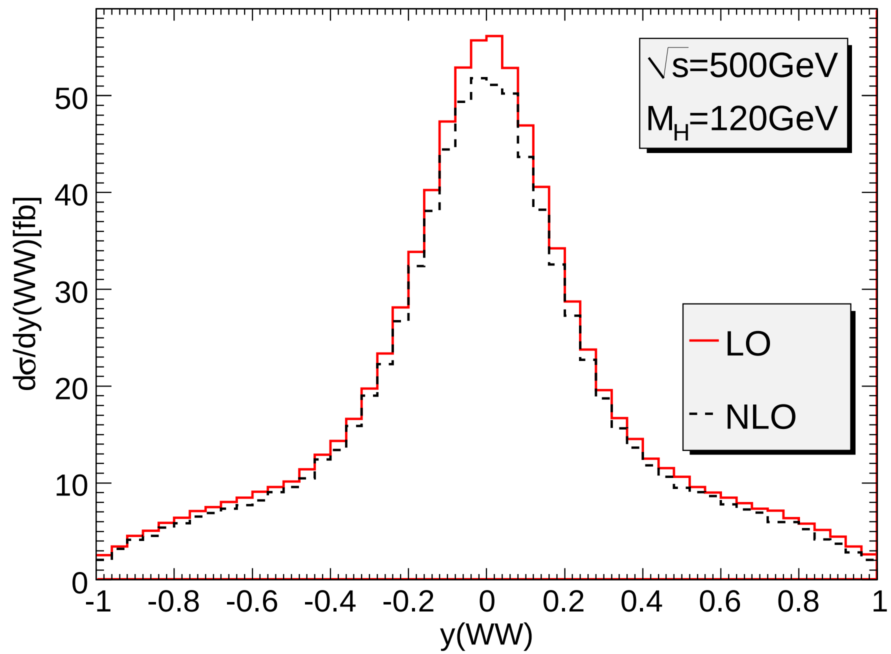




- Quite small corrections (less than -5%) at small GeV. At large GeV, large corrections (-30%) due to the hard photon effect [dominant contribution comes from the low-energy photon region (see the δ_s-plot) which corresponds to large p_T^Z and large M_{WW}.]

$e^+e^- \to W^+W^-Z$: Distributions (II)





- small corrections ($-4 \div -10\%$), shape unchanged.

Conclusions

- Tri-boson production (ZZZ and WWZ) at the ILC is a very important process to test the quartic gauge couplings and the Higss mechanism. This is the first step towards the understanding of SSB mechanism if the LHC cannot find the Higgs.
- The preliminary results of our calculation indicate that EW corrections are significant and have to be taken into account when doing analysis.
- Final results will be published soon.