Primordial perturbations from inflation

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Astroparticules
et Cosmologie

Cosmological evolution

- Homogeneous and isotropic Universe

$$ds^2 = -dt^2 + a^2(t)\overbrace{\gamma_{ij}dx^i dx^j}^{\gamma_{ij} = \delta_{ij} \quad (\kappa = 0)}$$

- Einstein's equations $\quad G_{\mu\nu} = 8\pi G \; T_{\mu\nu}$

$\Rightarrow$ Friedmann equations

$$\begin{cases} H^2 \equiv \left(\dfrac{\dot{a}}{a}\right)^2 = \dfrac{8\pi G}{3}\rho - \dfrac{\kappa}{a^2} \\ \dfrac{\ddot{a}}{a} = -\dfrac{4\pi G}{3}(\rho + 3P)\,. \end{cases}$$

The Universe in the Past

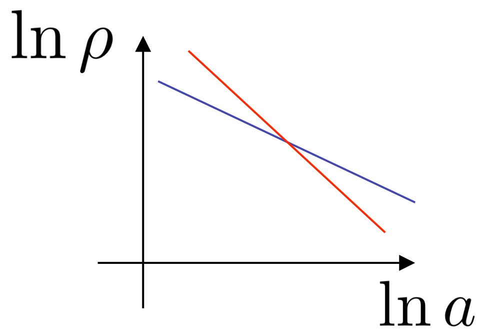
The energy densities dilute at various rates:

- pressureless matter

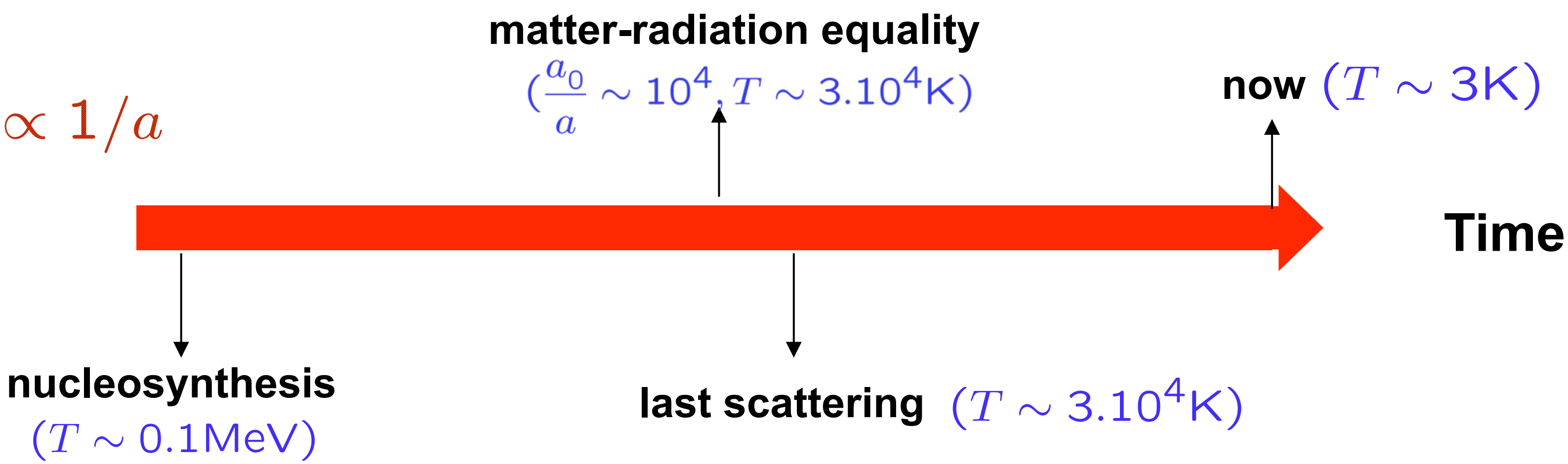
$$P_m \simeq 0 \quad \Longrightarrow \quad \rho_m \propto \frac{1}{a^3}$$

- radiation

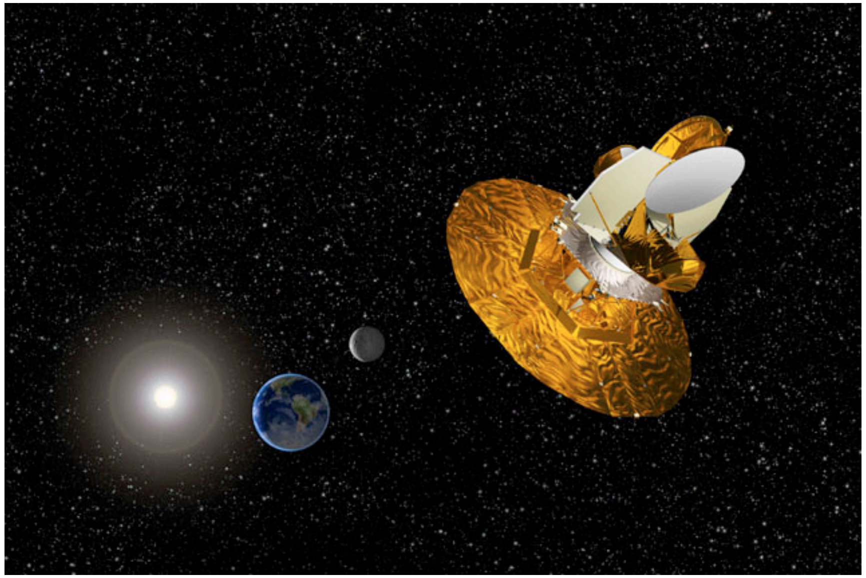
$$P_r \simeq \frac{1}{3}\rho_r \quad \Longrightarrow \quad \rho_r \propto \frac{1}{a^4}$$

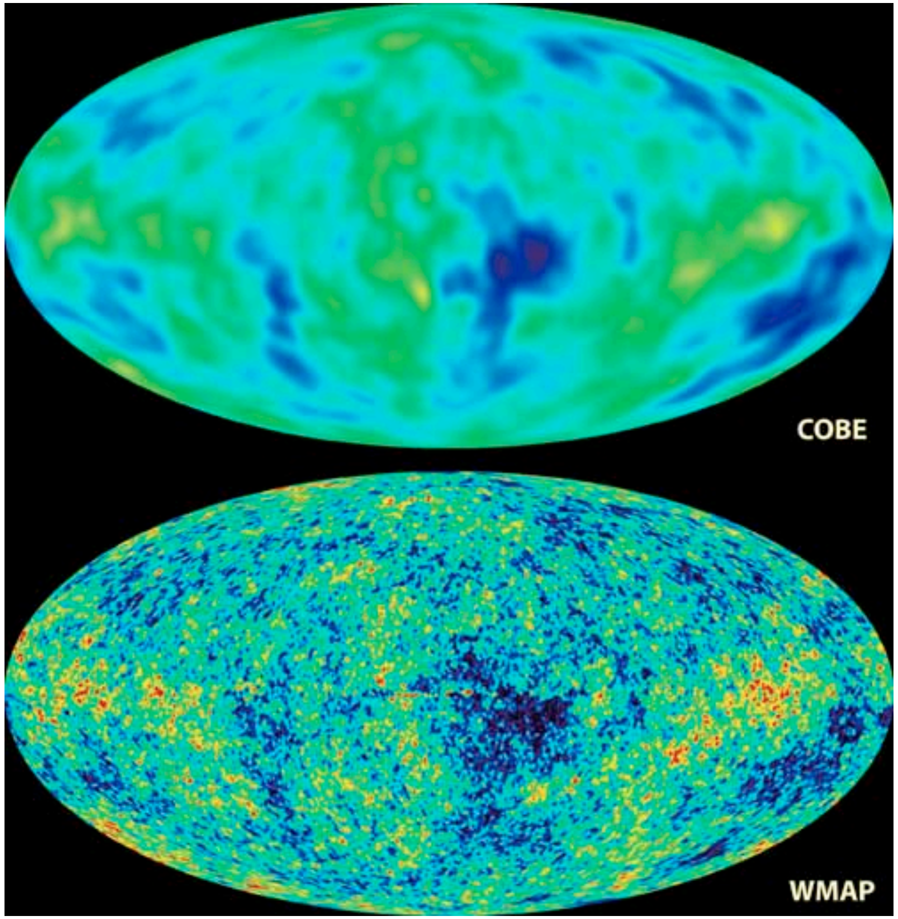


$T \propto 1/a$



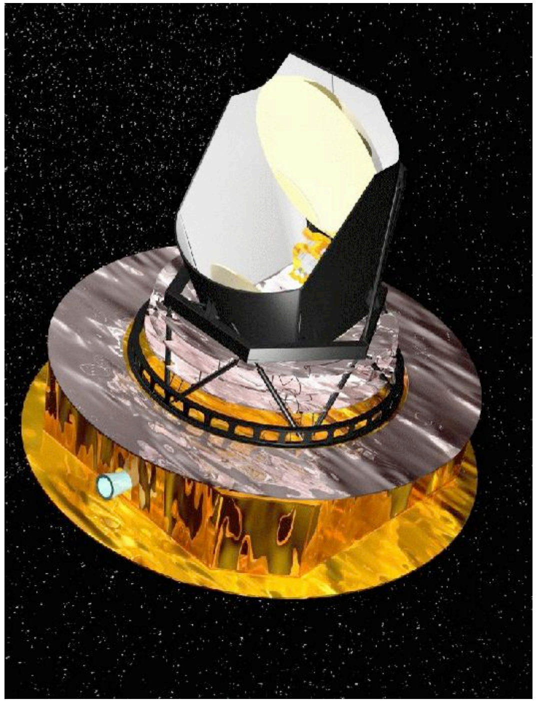
CMB seen by WMAP

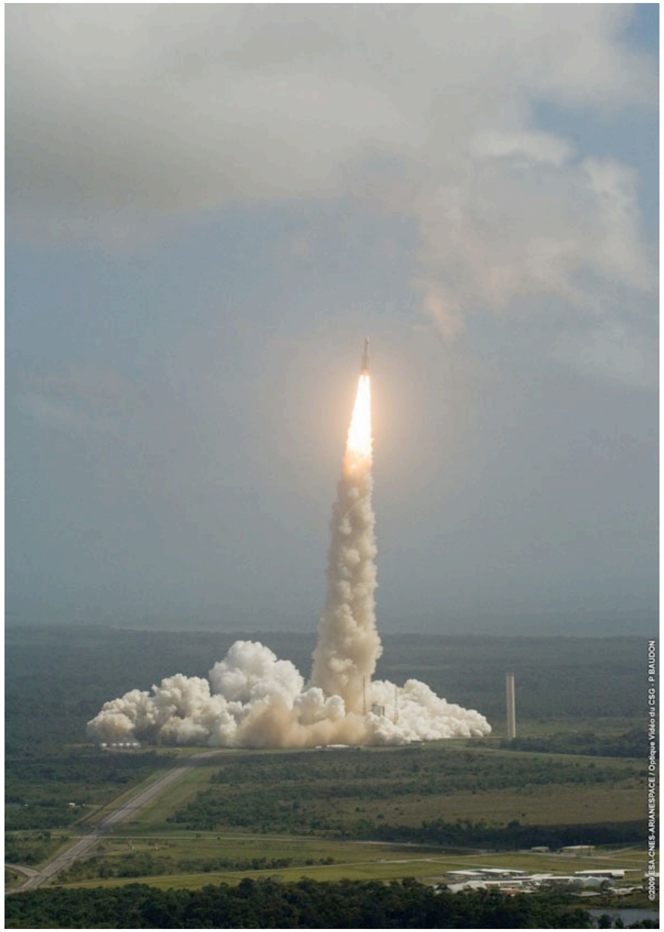




$$\frac{\delta T}{T} \sim 10^{-5}$$

CMB seen by Planck ?





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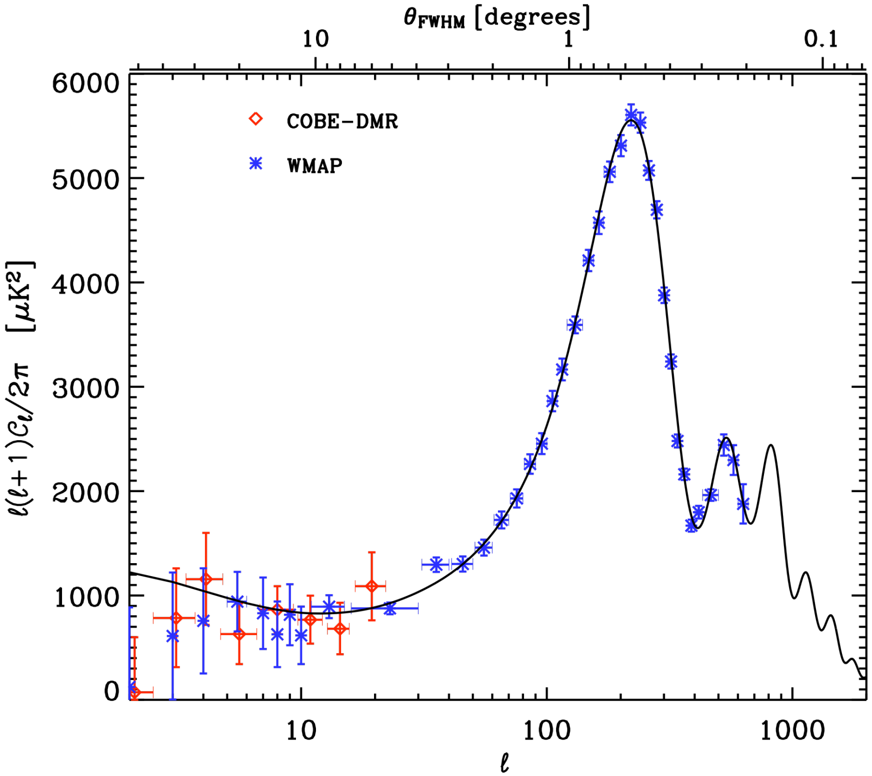
CMB seen by … theorists



$$\frac{\delta T}{T}(\theta,\phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\phi)$$

$$C_l = \langle |a_{lm}|^2 \rangle$$

Oscillations = sound waves

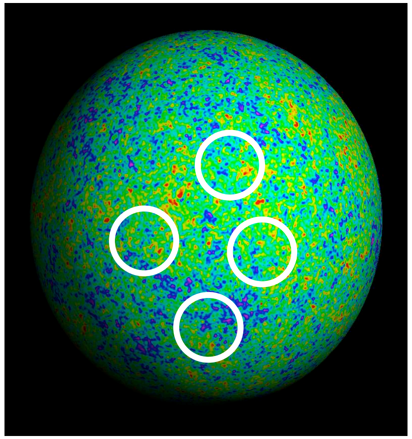
Primordial spectrum + cosmological parameters ➡ CMB predictions

What is the origin of the primordial fluctuations ?

Inflation

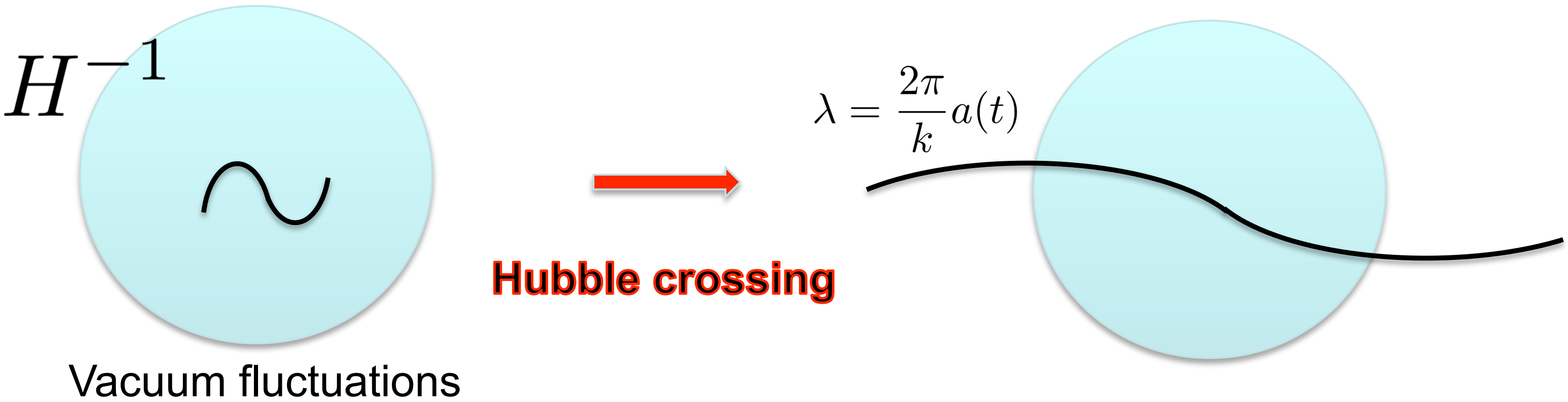
- A period of acceleration in the early Universe

$$\ddot{a} > 0$$



- Solves the horizon and flatness problems

- Origin of the primordial perturbations



Scalar field inflation

- How to get inflation ? $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)\,.$$
- Scalar field $$S_\phi = \int d^4x\sqrt{-g}\left(-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right)$$
- Homogeneous equations

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

$$3H^2 = 8\pi G\,\rho \qquad \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- Slow-roll motion $$\frac{1}{2}\dot{\phi}^2 \ll V(\phi), \quad \ddot{\phi} \ll 3H\dot{\phi}$$

$$P \simeq -\rho$$

Perturbations during inflation

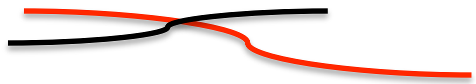
- Scalar field fluctuations: $\phi(t,\vec{x}) = \bar{\phi}(t) + \delta\phi(t,\vec{x})$

- Metric fluctuations (of scalar type)

$$ds^2 = -(1+2A)dt^2 + 2a(t)\partial_i B\, dx^i dt + a^2(t)\left[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\right] dx^i dx^j$$

- Freedom in the choice of coordinates

$$Q = \delta\phi + \frac{\dot{\phi}}{H}\psi = \frac{\dot{\phi}}{H}\mathcal{R}$$



Single scalar degree of freedom

- Its dynamics is governed by a Lagrangian of the form

$$S_{(2)} = \frac{1}{2}\int dt\, d^3x\, a^3\left[\dot{Q}^2 - \frac{1}{a^2}\partial_i Q\partial^i Q - \mathcal{M}^2(t)Q^2\right]$$

Quantization

- Using the conformal time $d\tau = dt/a$ and $u = a\,Q$

$$S_u = \frac{1}{2}\int d\tau\, d^3x \left[u'^2 + \partial_i u \partial^i u + \frac{z''}{z}u^2\right] \qquad z \equiv a\frac{\dot{\phi}}{H}$$

- In the slow-regime, quasi-de Sitter spacetime

$$a(\tau) = -\frac{1}{H\tau} \qquad \frac{z''}{z} \simeq \frac{a''}{a} = \frac{2}{\tau^2}$$

- **Quantization**

$$\hat{u}(\tau,\vec{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3k \left\{\hat{a}_{\vec{k}} u_k(\tau) e^{i\vec{k}.\vec{x}} + \hat{a}^\dagger_{\vec{k}} u_k^*(\tau) e^{-i\vec{k}.\vec{x}}\right\}$$

$$\left[\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}'}\right] = \left[\hat{a}^\dagger_{\vec{k}}, \hat{a}^\dagger_{\vec{k}'}\right] = 0, \qquad \left[\hat{a}_{\vec{k}}, \hat{a}^\dagger_{\vec{k}'}\right] = \delta(\vec{k} - \vec{k}')$$

- Minkowski-like vacuum on small scales $k\tau \gg 1$

$$u_k = \sqrt{\frac{\hbar}{2k}} e^{-ik\tau}\left(1 - \frac{i}{k\tau}\right)$$

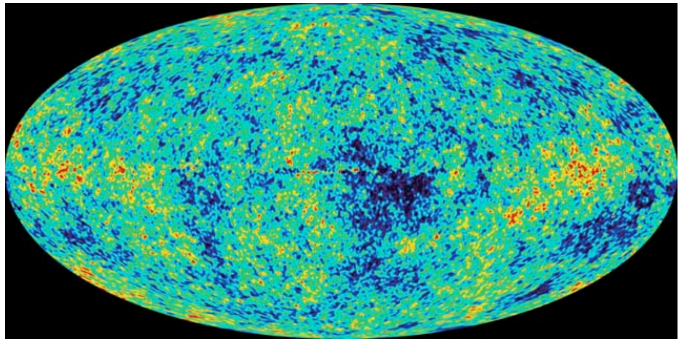
Power spectra

- 2-point function $$\langle 0|\hat{Q}(\vec{x}_1)\hat{Q}(\vec{x}_2)|0\rangle = \int d^3k\ e^{i\vec{k}.(\vec{x}_1-\vec{x}_2)}\frac{\mathcal{P}_Q(k)}{4\pi k^3}$$
- Power spectrum $$\mathcal{P}_Q(k) = \frac{k^3}{2\pi^2}\frac{|u_k|^2}{a^2} \simeq \hbar\left(\frac{H}{2\pi}\right)^2 \qquad (k \ll aH)$$
- Curvature perturbation
$$Q = \delta\phi + \frac{\dot{\phi}}{H}\psi = \frac{\dot{\phi}}{H}\mathcal{R}$$
$$\mathcal{P}_\mathcal{R} \simeq \frac{\hbar}{4\pi^2}\left(\frac{H^4}{\dot{\phi}^2}\right)_{k=aH}$$
- Quasi-scale-invariant spectrum $$n_s - 1 = \frac{d\ln\mathcal{P}_\mathcal{R}}{d\ln k}$$
- Gravitational waves
$$\mathcal{P}_T = 2\left(\frac{2}{M_P}\right)^2\left(\frac{H}{2\pi}\right)^2 = \frac{2H^2}{\pi^2 M_P^2}$$

From inflation to present cosmology

Early Universe
(inflation)

conservation law

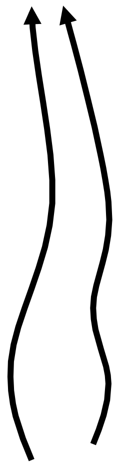


- For a cosmological perfect fluid, with

$$T_{ab} = (\rho + P)u_a u_b + P g_{ab}$$

one can define:

- Expansion: $\Theta = \nabla_a u^a$
- **Integrated expansion**: $\alpha = \frac{1}{3}\int d\tau\, \Theta$



u^a

Local scale factor $S = e^{\alpha}$

Conservation law

- The conservation law $\nabla_a T^a_{\ b} = 0$ implies that the **covector**

$$\zeta_a \equiv \nabla_a \alpha - \frac{\dot{\alpha}}{\dot{\rho}} \nabla_a \rho$$

satisfies the identity

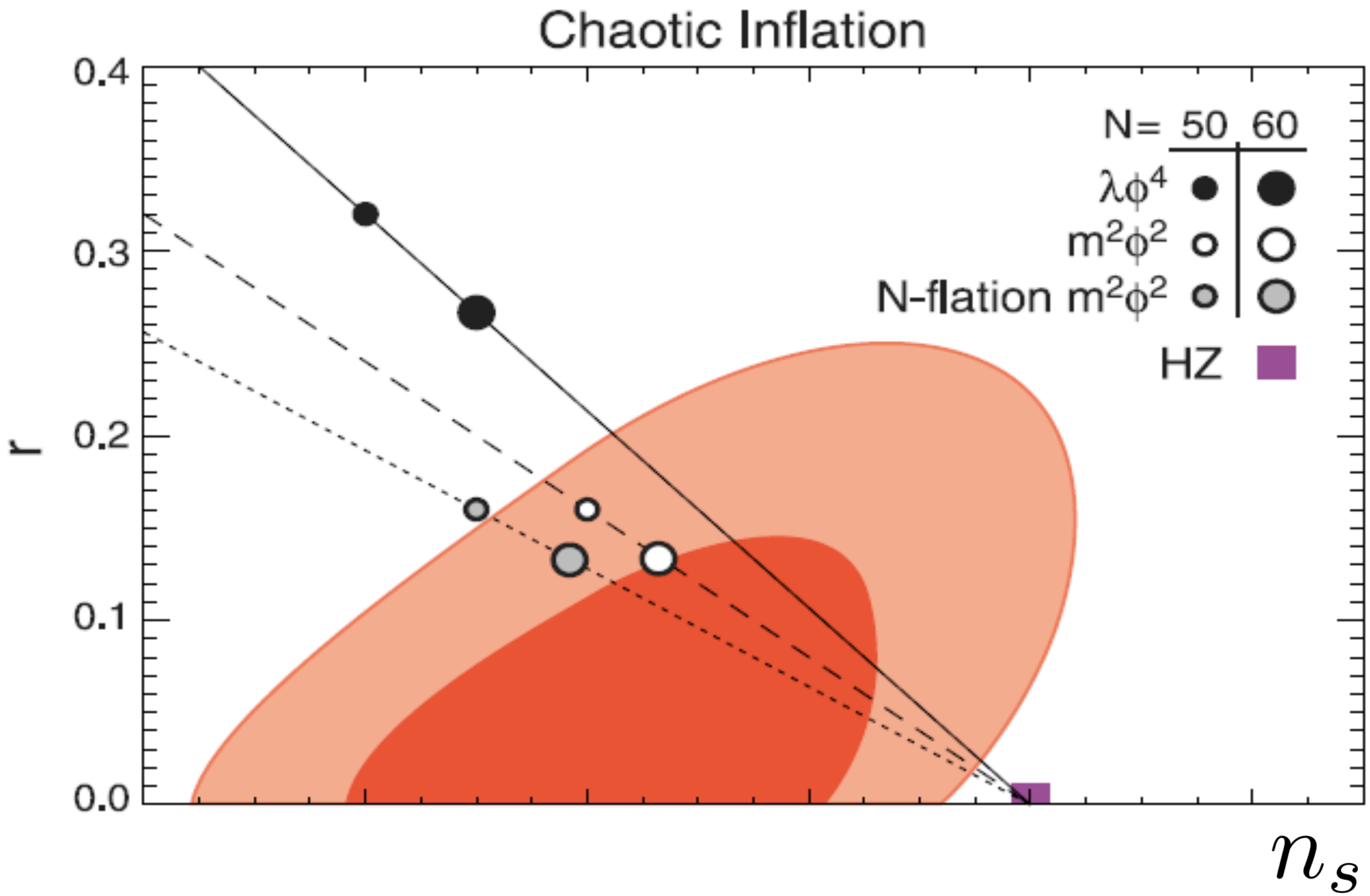
[DL & Vernizzi '05]

$$\dot{\zeta}_a \equiv \mathcal{L}_u \zeta_a = -\frac{\Theta}{3(\rho + P)} \left(\nabla_a P - \frac{\dot{P}}{\dot{\rho}} \nabla_a \rho \right)$$

- For linear perturbations, this implies that $\zeta \equiv -\psi - \frac{H}{\dot{\rho}} \delta\rho$ is conserved on large scales for adiabatic perturbations, i.e. such that $\delta p - (\dot{p}/\dot{\rho})\delta\rho = 0$

- Moreover, $-\zeta$ coincides with $\mathcal{R} = \psi - \frac{H}{\rho + p} \delta q$ on large scales

Observational constraints



On large scales

$$\frac{\delta T}{T} = \frac{1}{5}\mathcal{R}$$

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\mathcal{R}}$$

WMAP5+BAO+SN

[From WMAP5: Komatsu et al.]

Beyond the simplest models

- In high energy physics models, one usually finds many scalar fields.

  $\Rightarrow$ **multi-field inflation** !

- The kinetic terms can also be non-standard

- **General multi-field Lagrangians** [DL, Renaux-Petel, Steer & Tanaka ‘08]

$$S = \int d^4x \sqrt{-g}\, P(X^{IJ}, \phi^K), \qquad X^{IJ} \equiv -\frac{1}{2}\partial_\mu \phi^I \partial^\mu \phi^J$$

- **Linear perturbations**

$$S_{(2)} = \tfrac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\, a^3 \quad \Big[\Big(P_{<IJ>} + P_{<MJ>,<IK>}\dot{\phi}^M \dot{\phi}^K\Big)\dot{Q}^I \dot{Q}^J$$

$$-P_{<IJ>} h^{ij} \partial_i Q^I \partial_j Q^J - \mathcal{M}_{KL} Q^K Q^L + 2\,\Omega_{KI} Q^K \dot{Q}^I\Big]$$

$$P_{<IJ>} \equiv \frac{\partial P}{\partial X^{(IJ)}}$$

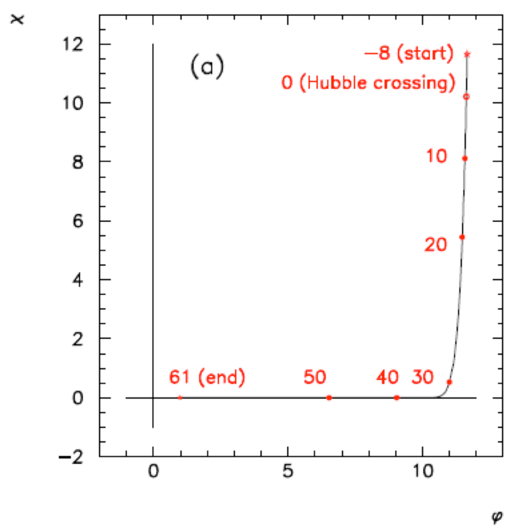
Multi-field inflation

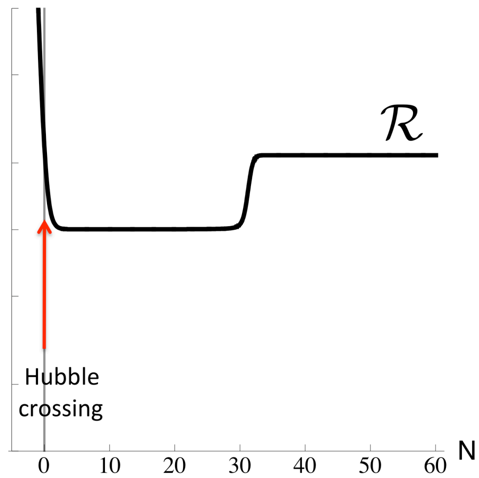
- $\mathcal{R}$ **no longer conserved on large scales**

  [Starobinsky, Yokayama '95]

- Example

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$$

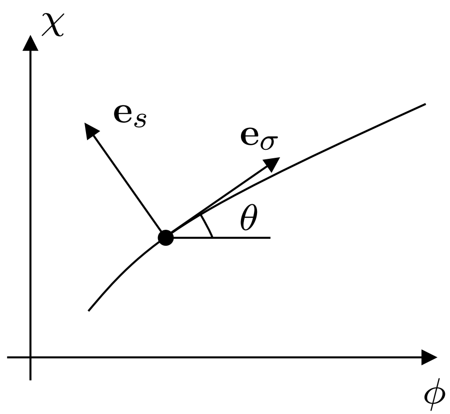




- **Transfer** from the **entropy** mode(s) into the **adiabatic** mode.

$$\begin{aligned} \delta\sigma &= \cos\theta\ \delta\phi + \sin\theta\ \delta\chi, \\ \delta s &= -\sin\theta\ \delta\phi + \cos\theta\ \delta\chi \end{aligned}$$

$$\dot\sigma = \sqrt{\dot\phi^2 + \dot\chi^2}$$



$$\dot{\mathcal{R}} = \frac{2H}{\dot\sigma}\dot\theta\delta s + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

Gordon et al. '00, Groot Nibbelink & Van Tent '00

Non-Gaussianities

- **Bispectrum**

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \equiv (2\pi)^3 \delta^{(3)}(\sum_i \mathbf{k}_i) B_\zeta(k_1, k_2, k_3).$$

One also uses the f_{NL} parameter

$$B_\zeta(k_1, k_2, k_3) \equiv \frac{6}{5} f_{\rm NL}(k_1, k_2, k_3) \left[P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perms} \right]$$

$$\left[\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \, P(k_1) \right]$$

- **Link with inflation**

  Using the δN-formalism [Lyth & Rodriguez '05]

$$\zeta \simeq \sum_I N_{,I} \delta\varphi_*^I + \frac{1}{2} \sum_{IJ} N_{,IJ} \delta\varphi_*^I \delta\varphi_*^J$$

Non-Gaussianities

$$\begin{aligned}\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle \;&=\; \sum_{IJK} N_{,I}N_{,J}N_{,K}\langle \delta\varphi^I_{k_1}\delta\varphi^J_{k_2}\delta\varphi^K_{k_3}\rangle + \\ &\quad \frac{1}{2}\sum_{IJKL} N_{,I}N_{,J}N_{,KL}\langle \delta\varphi^I_{k_1}\delta\varphi^J_{k_2}(\delta\varphi^K \star \delta\varphi^L)_{k_3}\rangle + \text{perms},\end{aligned}$$

- If the scalar field perturbations are quasi-Gaussian, **local NG**

$$\frac{6}{5}f_{\rm NL} = \frac{N_I N_J N^{IJ}}{(N_K N^K)^2}$$

- If the scalar field three-point function is significant, like in models with non standard kinetic terms, **equilateral NG**

- **Observational constraints** $\qquad -9 < f_{NL}^{(\rm local)} < 111 \quad (95\%\ \rm CL)$

  [WMAP5: Komatsu et al '08] $\qquad -151 < f_{NL}^{(\rm equil)} < 253 \quad (95\%\ \rm CL)$

Example: multi-field DBI inflation

[DL, Renaux-Petel, Steer & Tanaka '08]

- Inflation: motion of a D3-brane, described by the Lagrangian

$$P = -\frac{1}{f(\phi^I)}\left[\sqrt{\det(\delta^\mu_\nu + f\, G_{IJ}\partial^\mu\phi^I\partial_\nu\phi^J)} - 1\right] - V(\phi^I)$$

- Power spectra

$$\mathcal{P}_{Q_\sigma} = \left(\frac{H}{2\pi}\right)^2 \qquad \mathcal{P}_{Q_s} = \left(\frac{H}{2\pi c_s}\right)^2 \qquad \mathcal{P}_{\mathcal{R}} = \frac{H^4}{4\pi^2\dot\sigma^2}\left(1 + T_{\mathcal{RS}}^2\right)$$

- Expansion of the action up to third order

$$S^{(3)} = \int dt\, d^3x \left\{\frac{a^3}{2c_s^5\dot\sigma}\left[(\dot Q_\sigma)^3 + c_s^2\dot Q_\sigma(\dot Q_s)^2\right] - \frac{a}{2c_s^3\dot\sigma}\left[\dot Q_\sigma(\partial Q_\sigma)^2 - c_s^2\dot Q_\sigma(\partial Q_s)^2 + 2c_s^2\dot Q_s\partial Q_\sigma\partial Q_s\right]\right\}$$

- Equilateral non-Gaussianity

$$f_{NL}^{(\mathrm{equi})} = -\frac{35}{108}\,\frac{1}{c_s^2}\,\frac{1}{1+T_{\mathcal{RS}}^2}$$

Conclusions

- After nearly 30 years of existence, inflation has been so far successful to account for observational data.

- However, the nature of the inflaton(s) remains an open question.

- More sophisticated models involve multiple fields and/or non-standard kinetic terms.

- Hope that future cosmological data will enable us to discriminate between various types of models.

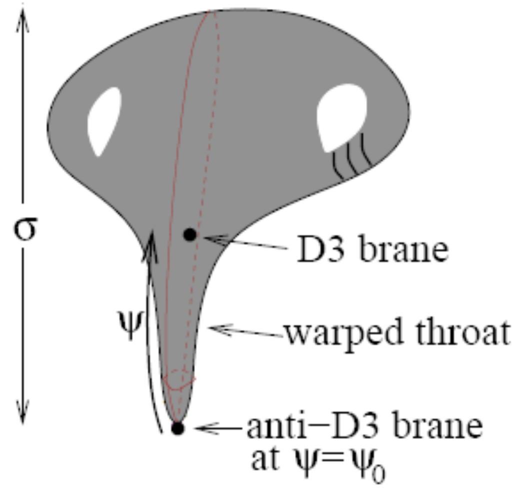
  Signatures: non-Gaussianities, gravitational waves, entropy modes.

DBI inflation

– Brane inflation: inflaton as the position of a brane

⇨ effective 4D scalar

– Moving D3-brane in a higher-dimensional background



$$ds^2 = h^{-1/2}(y^K) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y^K) G_{IJ}(y^K) dy^I dy^J$$

Its dynamics is governed by a Dirac-Born-Infeld action

$$L_{DBI} = -\frac{1}{f}\sqrt{-\det\left(g_{\mu\nu} + f\, G_{IJ}\,\partial_\mu\phi^I \partial_\nu\phi^J\right)}$$

DBI inflation

- One dimensional effective motion (radial motion)

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{f} \sqrt{1 + f\, \partial_\mu \phi \partial^\mu \phi} - V(\phi) \right] = \int d^4x \sqrt{-g}\, P(X, \phi)$$

- In the homogeneous case,

$$S = \int dt\, a^3 \left[-\frac{1}{f} \sqrt{1 - f\, \dot{\phi}^2} - V(\phi) \right]$$

1. Slow-roll regime: $f\, \dot{\phi}^2 \ll 1$ [KKLMMT]

2. “Relativistic” regime:

$$1 - f\, \dot{\phi}^2 \ll 1 \Rightarrow |\dot{\phi}| \simeq 1/\sqrt{f}$$

[Silverstein, Tong ’04; Alishahiha, Silverstein, Tong’04]

Multi-field DBI inflation

- Take into account the other internal coordinates

  ⇨ **multi-field effective description !**

[Easson, Gregory, Tasinato & Zavala '07; Huang, Shiu & Underwood '07]

$$P(X^{IJ}, \phi^K) = \tilde{P}(\tilde{X}, \phi^K) = -\frac{1}{f}\sqrt{1-2f\tilde{X}} - V$$

[DL, Renaux-Petel, Steer & Tanaka, PRL '08]

$$\det\left(\delta^\mu_\nu + f\, G_{IJ}\, \partial^\mu \phi^I \partial_\nu \phi^J\right) \;=\; 1 - 2f\tilde{X}$$

$$\tilde{X} \;\equiv\; G_{IJ}X^{IJ} - 2f X^{[I}_I X^{J]}_J + 4f^2 X^{[I}_I X^J_J X^{K]}_K - 8f^3 X^{[I}_I X^J_J X^K_K X^{L]}_L$$

$$X^{IJ} \equiv -\frac{1}{2}\partial_\mu \phi^I \partial^\mu \phi^J, \qquad X^I_K = X^{IJ} G_{JK}$$

- Homogeneous case

$$S = \int dt\, a^3 \left[-\frac{1}{f}\sqrt{1 - f\, G_{IJ}\, \dot{\phi}^I \dot{\phi}^J} - V(\phi^K)\right]$$

Including bulk forms

[DL, Renaux-Petel, Steer, 0902.2941]

- One can include the NS-NS and R-R bulk forms

$$S_{\mathrm{DBI}} = -T_3 \int \mathrm{d}^4x \, e^{-\Phi} \sqrt{-\det\left(\hat{\gamma}_{\mu\nu} + \hat{B}_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right)}$$

$$S_{\mathrm{WZ}} = -T_3 \int_{\mathrm{brane}} \sum_{n=0,2,4} \hat{C}_n \wedge e^{\left(\hat{B}_2 + 2\pi\alpha' F_2\right)}\Big|_{4-\mathrm{form}}$$

- Variation w.r.t A_0 yields the constraint

$$2\pi\alpha' h^{1/2} A_0 - b_{IJ}\dot{\phi}^I Q^J = 0$$

⇒ all new scalar terms cancel in the 2nd and 3rd order actions !

- Vector degrees of freedom