Bound states within AdS/CFT and their stability behavior

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- Based on works with:
  - S. Avramis and K. Sfetsos:
    Nucl.Phys. **B769** (2007) 44-78, hep-th/0612139.
    Nucl.Phys.**B793** (2008) 1-33, arXiv:0706.2655 [hep-th].
  - K. Sfetsos:
    Nucl. Phys. **B797** (2008) 268-294, arXiv:0710.3162 [hep-th].
    JHEP **0808** (2008) 071, arXiv:0807.0236 [hep-th].

Motivation-Conclusions

- The AdS/CFT correspondence is a tool to extract information for gauge theories at strong coupling from gravity. Bound states of quarks are dual to classical string probe solutions.
- Not all probe solutions are suitable for such a description. Apart from identifying a correct asymptotic behavior, a stability analysis of the solution proves to be essential.
- Discrepancies arise in many examples between field theory /experimental expectations and their gravitational description, ie multivalued potentials, confinement in $\mathcal{N} = 4$ SYM.
- Discrepancies are resolved as these solutions prove to be unstable. Findings from the dual gravity side coincide with gauge theory expectations.

Plan of the talk:

- Construction of bound states within the gravity/gauge theory duality.
- Calculation of binding energy.
- Applications: heavy mesons, baryons and dyons (comment on them).
- Stability analysis:
  - Based on general statements concerning the perturbative stability of such string solutions (transcendental equation), rather than (heavy) numerics.
  - Applications and resolutions of the discrepancies.

Quark-antiquark potential within AdS/CFT

- Heavy quark-antiquark potential $E(L)$ is extracted from Wilson loop expectation values $\langle W(C)\rangle$.
- Within AdS/CFT, the interaction potential energy of the quark-antiquark static pair is given by

$$e^{-\mathrm{i}ET} = \langle W(C)\rangle \simeq \exp\left(\mathrm{i}S[C]\right) \ ,$$

where $S[C]$ is the Nambu–Goto action for a string extending in the bulk, with its endpoints on the contour C at the boundary.
Note: $\mathcal{N} = 4$ SYM is massless. Quarks are external.

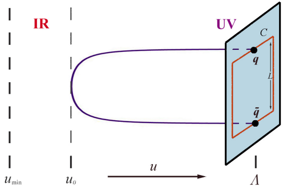


Figure: The string's turning point is u_0. Its minimum is $u_{\min}$.

The Nambu–Goto action is given in terms of the induced metric on the worldsheet, namely $\gamma_{\alpha\beta}$ as

$$S[C] = -\int d\tau d\sigma \sqrt{-\det \gamma_{\alpha\beta}} \; ; \qquad \gamma_{\alpha\beta} = G_{\mu\nu}(u,\theta)\partial_\alpha x^\mu \partial_\beta x^\nu \; .$$

- At first, for simplicity we consider diagonal metrics of the form

$$d\ell^2 = G_{tt}dt^2 + G_{uu}du^2 + G_{yy}dy^2 + G_{xx}dx^2 + G_{\theta\theta}d\theta^2 + \dots \; ,$$

where the different type of coordinates are:
u: radial AdS direction, x: generic cyclic coordinate,
y: direction of $q\bar{q}$ axis, θ: non-cyclic angular coordinate.

- Gauge choice

$$\alpha \equiv (\tau, \sigma) \; , \qquad \tau = t \; , \qquad \sigma = u \; .$$

- Ansatz for the classical solution

$$y = y_{\rm cl}(u) \; , \quad x = 0 \; , \quad \theta = \theta_0 = \text{const.} \; , \quad \text{rest} = \text{const.} \; .$$

- Impose the boundary condition

$$y(u \to \infty) = \pm L/2 \; ,$$

where L is the separation length of the $q\bar{q}$ pair.

The potential: Integration of the e.o.m. and the b.c. gives for the length

$$L(u_0) = 2f_{y0}^{1/2} \int_{u_0}^{\infty} du \sqrt{\frac{g}{f_y(f_y - f_{y0})}} \ ,$$

and for the binding energy (heavy quarks)

$$E(u_0) = \int_{u_0}^{\infty} du \left(\sqrt{\frac{g f_y}{f_y - f_{y0}}} - \sqrt{g} \right) - \int_{u_{\min}}^{u_0} du \sqrt{g},$$

where we have substracted the the divergent contribution of disconnected worldsheets; $g \equiv -G_{tt} G_{uu}$, $f_y \equiv -G_{tt} G_{yy}$. Eliminating u_0 determines the $q\bar{q}$ potential $E(L)$.

- The potential must satisfy the [Baumgartner *et al* 85, Bachas 86].

concavity condition : $\frac{dE}{dL} > 0 \ , \qquad \frac{d^2 E}{dL^2} \leqslant 0 \ .$

- In conformal case [Maldacena 98, Rey-Yee 98] Coulomb behavior

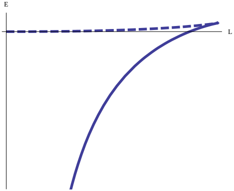
$$E(L) \sim -\sqrt{g_{\rm YM}^2 N}/L \ .$$

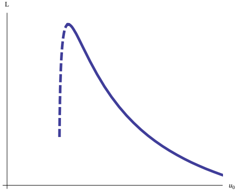
- In non-conformal (non-extremal, multicenter), complicated...

Examples-Discrepancies

We shall next present two generic examples and consider applications of the first one.

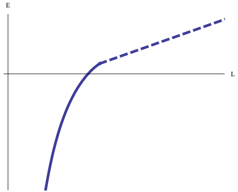
I. Double-valued potential, maximal length.





- **Applications**:
  $\mathcal{N} = 4$ at finite temperature using black D3-branes: Static $q\bar{q}$ [Rey et al, Brandhuber et al 98].
  Moving: Applications of AdS/CFT to Physics at RHIC.
  [Liu-Rajagopal-Wiedemann, Friess-Gubser-Michalogiorgakis-Pufu 06]
  Rindler space, similar behavior with generic black holes near the horizon [Avramis-Sfetsos-KS 07].
- Upper branch is energetically unfavorable and it violates concavity. Stability analysis discards it.

II. No critical points, Coulomb/Confining potential



- Coulomb branch of $\mathcal{N} = 4$, multicenter supersymmetric D3-brane solutions [Brandhuber-Sfetsos,99]. Confining behavior is not expected for $\mathcal{N} = 4$ SYM.[Seiberg 1988]
  Stability analysis discards the confining behavior.
- Less supersymmetric backgrounds.
  - Confining behavior can exist in less SUSY backgrounds. β-deformed backgrounds $\Leftrightarrow \mathcal{N} = 1$ SCFTs [Hernández-Sfetsos-Zoakos, 05].
  - Indeed this proves to be stable [Avramis-Sfetsos-KS, 07].

Stability analysis

Objective: A stability analysis will determine the parametric region of u_0 and the parameters of the problem for which fluctuations about the classical configurations become unstable-unphysical.

Small fluctuations-Generalities

- Three types of fluctuations:

Transverse δx	:	cyclic coordinate transverse to $q\bar{q}$ axis
Longitudinal δy	:	cyclic coordinate along $q\bar{q}$ axis
Angular $\delta\theta$	:	non-cyclic angular coordinate

- Perturbation of the embedding

$$x = \delta x(t,u) \ , \qquad y = y_{\rm cl}(u) + \delta y(t,u) \ , \qquad \theta = \theta_0 + \delta\theta(t,u) \ .$$

- Due to the worldsheet diffeomorphism invariance, we keep the gauge choice $(\tau = t, \sigma = u)$ fixed.

Perturbation

We expand the action around the classical solution. Writing down the equations of motion for the quadratic part of the action, using independence of the various functions from t, we set

$$\delta x_\mu(t,u) = \Phi_\mu(u) e^{-i\omega t} , \qquad \mu = x, y, \theta$$

and we obtain 3 decoupled differential equations of the general Sturm–Liouville type for the Φ_μ's defined in the infinite interval $u \in [u_0, \infty)$.

Boundary Conditions

- We keep the endpoints at the boundary fixed

$$u \to \infty : \quad \Phi_\mu = 0 , \quad \text{for } \mu = x, y, \theta .$$

- To have a well defined variational problem we obtain the following conditions at the turning point $u = u_0$ of the string,

$$\Phi_y + 2(u-u_0)\Phi'_y = 0 , \qquad (u-u_0)^{1/2}\Phi'_{x,\theta} = 0 .$$

The zero mode problem

- In most of the cases, it is impossible to exactly determine the spectrum of the Sturm-Liuville problem(for these b.c. $\omega^2 \in \mathbb{R}$). It turns out that we can obtain useful information by studying the zero-mode problem(simpler).
- The spectrum may become negative if dimensionless parameters can be formed from those in the supergravity background and u_0. For example in cases with temperature, angular momentum, velocity, deformation parameter,... there are instabilities.
- If there are instabilities then, as we lower u_0, we must cross a point u_{0c} where the differential equation has a zero mode.
- **Note**: For asymptotically AdS spaces and for sufficiently large u_0 we always have positive spectrum (stable).

Zero Modes: Transverse

Transverse fluctuations are stable.

Zero Modes: Longitudinal

The zero mode solution for the longitudinal which vanishes at the boundary reads

$$\Phi_y \sim \int_u^\infty du \frac{\sqrt{g f_y}}{(f_y - f_{y0})^{3/2}} ,$$

expanding this solution around $u = u_0$ we obtain

$$\Phi_y \sim \mathrm{const.}(u - u_0)^{-1/2} + 2 \int_{u_0}^\infty \frac{du}{\sqrt{f_y - f_{y0}}} \partial_u \left(\frac{\sqrt{g f_y}}{f_y'} \right) + \cdots .$$

- This solution should satisfy the boundary condition for the longitudinal fluctuations at u_0. Satisfying this condition leads that the constant term in the above expansion of the should be zero. So, by just solving at most a transcendental equation we get an indication of an instability.

- On the other hand we have that the derivative of the length with respect to u_0 reads

$$L'(u_0) = 2\frac{f'_{y0}}{\sqrt{f_{y0}}}\int_{u_0}^{\infty}\frac{du}{\sqrt{f_y - f_{y0}}}\partial_u\left(\frac{\sqrt{gf_y}}{f'_y}\right) \ .$$

- **Theorem**: Longitudinal zero modes are "1-1" with the (potential) extrema of $L(u_0)$.
- Existence of a zero mode is an indication of an instability. We have to perturb the diff. eqn. around $u_{0\mathrm{c}}$, so to examine its stability behavior. Doing it so, we find

$$\omega_0^2 = A(u_0 - u_{0\mathrm{c}}) + \dots \ , \quad u_{0\mathrm{c}} = u_{0\mathrm{m}} \ ,$$

where $A = \mathrm{const.}$ depends on the parameters of the problem.

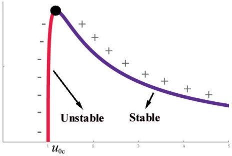


Figure: A>0 $\rightarrow$ The upper branch in Ex. I is unstable, ie $u_0 < u_{0m}$.

Analytic Example:
Non-extremal D3-branes/ Dual to $\mathcal{N}=4$ SYM at finite temperature.

Metric for stack of non-extremal D3-branes
(in units of πT_H, $R=1$).

$$d\ell^2 = u^2\left[-\left(1-\frac{1}{u^4}\right)dt^2 + d\vec{x}_3^2\right] + \frac{u^2}{u^4-1}\,du^2 + d\Omega_5^2\ .$$

- Meson separation length and energy in terms of u_0

$$L \sim \frac{\sqrt{u_0^4-1}}{u_0^3}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4},\frac{5}{4};\frac{1}{u_0^4}\right)\ ,\quad E \sim -u_0\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4},\frac{1}{4};\frac{1}{u_0^4}\right) + \text{const.}.$$

  The plots of $L=L(u_0)$ and $E=E(L)$, were given in example A. Maximum of the length yields to a longitudinal instability. Numerically: $u_{0c}\simeq 1.177$, $L_m \simeq 0.869$.

- Similar results for mesons moving in hot plasma and for Rindler space (soap film). [Friess-Gubser-Michalogiorgakis-Pufu 06, Avramis-Sfetsos-KS 07]

Summary-extensions

- Built up: We constructed configurations dual to bound states. Gravity side predictions may contradict with gauge theory /experimental expectations. A stability analysis of the classical solution is essential.
- Solved puzzles: In line with gauge theory expectations, eg concavity, no-confinement in $\mathcal{N} = 4$ SYM...
- Simple extensions: Similar methods as the ones described here could be applied to other classical solutions involving D-strings, extended branes.
  Overwhelming expectations-dyons: Part(s) energetically unfavorable are pertubatively stable.
  Baryons: Colorless states.