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Theory - Cosmology - Phenomenology

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Lectures in Superstring Cosmology

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1. Introduction

Is superstring theory able to describe the basic features of our Universe?

It is necessary to develop a string theoretic framework for studying cosmology. Still very little is known about the dynamics of strings with branes and fluxes in time-dependent, cosmological settings.

Our aim is to provide, in superstring theory,

A new class of non-trivial, Quantum and
Thermal cosmological solutions.

The classical stringy solutions, are unreliable for exact cosmological solutions. Most of the time the classical ground states correspond to stationary Anti-de Sitter or flat backgrounds but not to cosmological ones. The same uncomfortable situation appears to be true in the effective supergravity theories.

Naively, these results lead to the conclusion that it is unlikely to find cosmological ground states in superstring theory.

The recent years became fashionable the study of the stringy effective "no - scale" supergravities where the SUSY is spontaneously broken by branes and fluxes.

From our viewpoint the "no-scale" setup is the reliable starting point:

• Mainly because ALL classically defined consistent strings in flat space-time provide at the classical level a well define " no - scale " supergravity with spontaneously broken SUSY.

• All moduli participating in the SUSY breaking are flat directions while the most of the others are stabilized having a mass proportional to the gravitino scale $m_{3/2}$. (soft breaking terms).

The second step is to determine the Quantum and Thermal Stringy corrections, if possible at the string level.

• I had to stress here that our approach differs from others. Namely, we will classify the Thermal and Quantum corrections according to their magnitude respect to the ordering of these corrections as dictated by exact calculations at the string level.

• We do not add for instance the exponentially suppressed terms like e^{-T}, e^{-S}, e^{-R} compared to the perturbative corrections.

• We keep however ALL corrections of the type $e^{-\frac{R_1}{R_2}}$ when R_1 and R_2 are comparable.

• As we will show, the exponentially suppressed terms play crucial role only when the moduli R_I participate at the supersymmetry breaking and are close to the string scale (Hagedorn like T_H, R_H^I transitions).

• Adding estimated exponentially suppressed terms destroy randomly the "String no-scale structure" with an apparent moduli stabilization in AdS vacuum.

• Although this study looks to be hopeless and out of any systematic control in field theory, it turns out that in certain "physically" interesting cases both the Quantum and Thermal corrections are under control thanks to the "no-scale structure" of the String Effective SUGRA's in the spontaneously broken SUSY phase.

In this talk I will present two typical string examples. The one in the Heterotic and the other in a very special Type IIB orientifold.

In both cases the Thermal Fluctuations and the Quantum corrections produce a non zero Free Energy Density which is calculable at the string level.

The back reaction on the i) space-time metric, $g_{\mu\nu}$ and ii) moduli fields, Φ_I gives rise to a specific calculable cosmological evolution.

When T is bellow the Hagedorn temperature T_H, the evolution of the universe is well determined together with the evolution of the mass scales

$$\frac{1}{a(t)},\ T(t),\ m_{3/2}(t)$$

following a critical trajectory:

$$a(t)\ T(t) = const., \quad a(t)\ m_{3/2}(t) = const., \quad \frac{T(t)}{m_{3/2}(t)} = const.$$

$a(t)$ is the scale factor of the universe.

2. Magnetic and Thermal SUSY Breaking

• In both examples we have assumed that the breaking of SUSY is via magnetic geometrical fluxes of graviphotons.
The main reason of this particular choice is that the introduction of these fluxes can be adapted easily in strings in the framework of Freely Acting Orbifolds (generalization in strings of the Scherk-Schwarz compactification).

• The Thermal Corrections are implemented by introducing a thermal coupling of the space-time fermion number Q_F to the string momentum and windings associated to the Euclidean time cycle S^1_T.

• The breaking of SUSY is generated by a similar coupling of an internal R-symmetry charge Q_R to the momentum and windings associated to an internal coordinate, say X_5 cycle S^1_M.

Two very special mass scales appear associated with the breaking of supersymmetry.

The temperature scale $T = 1/(2\pi R_0)$

The SUSY breaking scale $M = 1/(2\pi R_5)$

The initially degenerate mass levels of bosons and fermions are shifted by an amount that is proportional to T and/or M according to the charges Q_F and Q_R.

This mass shifting is the signal of supersymmetry breaking and gives rise to a non-trivial free energy density and effective potential at the quantum level.

3. Thermal and Quantum Corrections in the Heterotic Superstring Backgrounds

The one-loop effective action ,(in string frame)

$$S = \int d^4x \sqrt{|g|} \left(e^{-2\phi} (\frac{1}{2} R + 2\partial_\mu \phi \partial^\mu \phi + \ldots) - \mathcal{V}_{\rm String} \right)$$

ϕ is the $4d$ dilaton field and the ellipses stand for the kinetic terms of other moduli fields.

$\mathcal{V}_{\rm String}$ is the one-loop effective potential, which is obtained from the one-loop Euclidean string partition function

$$\frac{Z}{V_4} = -\mathcal{V}_{\rm String}$$

V_4 denotes the $4d$ Euclidean volume.

At finite temperature, the one-loop Euclidean partition function determines the free energy density and pressure

$$\frac{Z}{V_4} = -\mathcal{F}_{\rm String} = P_{\rm String}$$

$$\rho_{\rm string} = T\frac{\partial}{\partial T}\, P_{\rm string} \; - P_{\rm string}$$

In order to determine the back-reaction of the thermal and quantum corrections, it is convenient to work in the Einstein frame where there is no mixing between the metric and the dilaton kinetic terms.

We define as usual the complex field S

$$S = e^{-2\phi} + ia$$

After the Einstein rescaling of the metric, the one loop effective action becomes:

$$S = \int d^4x\sqrt{|g|}\left[\frac{1}{2}R - g^{\mu\nu}\; K_{I\bar{J}}\; \partial_\mu\Phi_I\partial_\nu\bar{\Phi}_{\bar{J}}\;\right] - \left[\frac{1}{s^2}\;\mathcal{V}_{\rm String}(\Phi_I, \bar{\Phi}_{\bar{I}})\right]$$

$K_{i\bar{j}}$ is the metric of the scalar manifold $\{\Phi_I\}$, which is parameterized by the moduli including the field S.

In the Einstein frame the effective potential, $\mathcal{V}_{\rm String}$, is rescaled by a factor $1/s^2$

$$s = \Re S = e^{-2\phi}$$

Taking this rescaling into account, we have

$$\mathcal{V}_{\rm Ein} = \frac{1}{s^2}\,\mathcal{V}_{\rm String}$$

The above relation turns out to be crucial.

We will always work in gravitational mass units, with $M_G = \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}$ GeV.

In the limit $R_0, R_5 \gg 1$ only the temperature scale R_0 and three of the main moduli fields $\{S, T_1, U_1\}$ appears in $\mathcal{V}_{\rm Ein}$. The contribution of all others moduli is exponentially suppressed.

$$\mathcal{V}_{\rm Ein} \simeq \frac{f\left(\frac{sR_0^2}{sR_5^2}, \dots\right)}{(stu)^2} + \mathcal{O}(e^{-c_0 R_0 - c_5 R_5})$$

Freezing all other moduli, the classical Kälher potential is of a no-scale type as was expected from the effective field theory approach:

$$K = -\log\ (S+\bar{S}) - \log\ (T_1+\bar{T}_1) - \log\ (U_1+\bar{U}_1)$$

$$\equiv -3 \log\ (Z+\bar{Z}) + \dots$$

The classical superpotential is constant so that the gravitino mass is:

$$m_{3/2}^2 = \frac{c}{stu} = \frac{c}{z^3}$$

Freezing further the ImZ and defining the field Φ:

$$e^{2\alpha\Phi} = m_{3/2}^2 = \frac{8c}{(Z+\bar{Z})^3}, \quad g_{\mu\nu}\ 3\frac{\partial_\mu Z \partial_\nu \bar{Z}}{(Z+\bar{Z})^2} = g_{\mu\nu}\ \frac{\alpha^2}{3}\ \partial_\mu \Phi \partial_\nu \Phi\ .$$

$\alpha^2 = 3/2$ normalize canonically the kinetic terms of the no-scale modulus Φ.

$$\mathcal{V}_{\rm Ein} \simeq m_{3/2}^4 \, f\left(\frac{sR_0^2}{sR_5^2}, \dots \right) \simeq m_{3/2}^4 \, f\left(\frac{m_{3/2}^2}{T^2}, \frac{m_Y^2}{T^2} \right)$$

The possible dependance on a new SUSY mass scale m_Y^2 appears when non trivial small Wilson lines are turned on.

4. String Determination of the Heterotic Free Energy

We first consider the case of an exact SUSY background of Heterotic superstring with maximal space-time supersymmetry ($N = 4$).

The Euclidean time and all nine spatial directions are compactified on T^{10}.

At zero temperature and in the absence of SUSY breaking the Euclidean string partition function is zero due to space-time SUSY

At finite temperature and non-vanishing SUSY breaking the result is a well defined finite quantity and is given in terms of the thermal partition function :

$$Z = \oint \frac{d\tau d\bar{\tau}}{4\mathrm{Im}\tau}\ \frac{1}{2}\sum_{a,b}\ \frac{(-)^{a+b+ab}\ \theta\begin{bmatrix}a\\b\end{bmatrix}^4}{\eta(\tau)^{12}\ \bar{\eta}(\bar{\tau})^{24}}\ \Gamma_{(5,21)}(R_I)$$

$$\times\ \Gamma_{(3,3)}(R_x = R_y = R_z)$$

$$\times \sum_{h_0,g_0} \Gamma\begin{bmatrix}h_0\\g_0\end{bmatrix}(R_0)\ (-)^{g_0 a + h_0 b + g_o h_o}$$

$$\times \sum_{h_5,g_5} \Gamma\begin{bmatrix}h_5\\g_5\end{bmatrix}(R_5)\ (-)^{g_5 a + h_5 b + g_5 h_5}$$

The non-vanishing of the partition function is due to the non-trivial coupling of the $\Gamma(R_0)$ and $\Gamma(R_5)$ lattices to the spin structures (a, b).

$a = 0$ for space-time bosons and $a = 1$ for space-time fermions.

$$\Gamma\begin{bmatrix}h\\g\end{bmatrix}(R) = \sum_{m,n} R(\mathrm{Im}\tau)^{-\frac{1}{2}}\ e^{-\pi R^2\ \frac{|2m+g+(2n+h)\tau|^2}{\mathrm{Im}\tau}}$$

In the limit of large $3+1$ dimensions and small SUSY breaking

$$R_x = R_y = R_z \equiv R \gg 1 \quad \text{and} \quad R \gg R_0, R_5 \gg 1.$$

The 3d spatial volume factorizes

$$\Gamma_{(3,3)} \cong R^3 \ \mathrm{Im}\tau^{-\frac{3}{2}} = \frac{V_3}{(2\pi)^3} \ \mathrm{Im}\tau^{-\frac{3}{2}}$$

- The sector $(h_0, g_0) = h_5, g_5) = (0,0)$ gives zero contribution. This is due to the fact that we started with a SUSY background.
- In the odd winding sectors,
  $h_0 = 1$ and/or $h_5 = 1$,
  the partition function diverges when
  R_0 and/or R_5

$$\frac{1}{R_H} < R_A < R_H$$

Hagedorn $R_H = (\sqrt{2}+1)/2$ and its dual $1/R_H$

The divergence is due to the winding states that become tachyonic.

- In the regime $R_0, R_5 \gg R_H$, there is no tachyon
  i) the winding sectors and
  ii) the oscillator states
  are both exponentially suppressed.
- When $R_0, R_5 \gg 1$, the contributions of the internal $\Gamma_{(5,21)}(R_I)$ lattice states are also exponentially suppressed, provided that the moduli R_I are of order unity.

For large R_0, R_5, only the sectors

$$(h_0, g_0) = (0, g_T) \text{ and } (h_5, g_5) = (0, g_T) \text{ with } g_0 + g_5 = 1$$

contributes significantly.

Using the identity:

$$\Gamma(R) = \Gamma\begin{bmatrix}0\\0\end{bmatrix} + \Gamma\begin{bmatrix}0\\1\end{bmatrix} + \Gamma\begin{bmatrix}1\\0\end{bmatrix} + \Gamma\begin{bmatrix}1\\1\end{bmatrix}$$

and neglecting the $h = 1$ sectors, we may replace

$$\Gamma\begin{bmatrix}0\\1\end{bmatrix} \to \Gamma(R) - \Gamma\begin{bmatrix}0\\0\end{bmatrix} = \Gamma(R) - \frac{1}{2}\Gamma(2R)$$

in the integral expression for Z.

We decompose the contribution in modular orbits:

$(m_i, n_i) = (0,0)$ and $(m_i, n_i) \neq (0,0)$.

For $(m_i, n_i) \neq (0,0)$, the integration over the fundamental domain is equivalent with the integration over the strip but with $n_i = 0$.

The $(0,0)$ orbit gives zero contribution due to the initial SUSY.

So we are left with the integration over the whole strip:

$$Z = \frac{V_4}{(2\pi)^4} \int_{||} \frac{d\tau d\bar{\tau}}{4\mathrm{Im}\tau^{\frac{7}{2}}} \sum_{g_0, g_5} \frac{\theta \begin{bmatrix} 1 \\ 1+g_0+g_5 \end{bmatrix}^4 \Gamma_{(5,21)}}{\eta(\tau)^{12}\, \bar{\eta}(\bar{\tau})^{24}}$$

$$\times \sum_{m_0, m_5} e^{-\pi R_0^2 \frac{(2m_0+g_0)^2}{\mathrm{Im}\tau}} \times R_5\, e^{-\pi R_5^2 \frac{(2m_5+g_5)^2}{\mathrm{Im}\tau}}$$

The integral over τ_1 imposes the left-right level matching condition.
The left-moving part contains the ratio

$$\frac{\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix}^4}{\eta^{12}} = 2^4 + \mathcal{O}(e^{-\pi\tau_2}),$$

which implies that the lowest contribution is at the massless level.

Finally, after the integration over τ_1 $(\tau_2 \equiv t)$,

$$Z = \frac{V_4 \left(2^4 \ D_0\right)}{(2\pi)^4} \int_0^\infty \frac{dt}{2t^{\frac{7}{2}}} \sum_{m_i} \sum_{g_0+g_5=1} R_5$$

$$e^{-\pi R_0^2 \frac{(2m_0+g_0)^2}{t} - \pi R_5^2 \frac{(2m_5+g_5)^2}{t}} ,$$

$2^4 \ D_0$ is the multiplicity of the massless level.

Changing the integration variable by setting

$$t = \pi \left(R_0^2 (2m_0 + g_0)^2 + R_5^2 (2m_5 + g_5)^2 \right) \ x$$

and integrating over x, we can write Z in terms of Eisenstein functions of order $k = 5/2$:

$$E_k(U) = \sum_{(m,n)\neq(0,0)} \left(\frac{\mathrm{Im}\ U}{|m + nU|^2} \right)^k$$

Define the function

$$f(u) \equiv \sum_{m_1,m_2} \frac{u^4}{|(2m_1+1)iu+2m_2|^5}$$

$$f(u) = u^{\frac{3}{2}} \left(\frac{1}{2^{\frac{5}{2}}} E_{5/2}\left(\frac{iu}{2}\right) - \frac{1}{2^5} E_{5/2}(iu) \right)$$

with

$$u = \frac{R_0}{R_5} = \frac{M}{T}, \quad M = m_{3/2}$$

Then the pressure in the Einstein frame

$$P \equiv \frac{Z}{V_4} = \; C_T \; T^4 \; f(u) + C_V \; M^4 \; \frac{f(1/u)}{u}$$

with

$$C_T = C_V = n^* \; \frac{\Gamma\left(\frac{5}{2}\right)}{\pi^{\frac{5}{2}}}.$$

$n^* = 8D_0$ is the number of the initially massless boson/fermion pairs

In this particular example the coefficients C_T and C_V are equal due to the underlying gravitino mass/temperature duality.

For fixed u the first term stands for the thermal contribution to the pressure, $P_{\rm thermal}$ the second term stands for minus the effective potential, $-V_{\rm eff}$

$$P = P_{\rm thermal} \; - \; V_{\rm eff}$$

- The coefficient C_T is fixed and positive as it is determined by the number of all massless boson/fermion pairs in the initially SUSY theory.
- The coefficient C_V will depend on the way the SUSY-breaking operator Q_R couples to the left- and right- movers.
- In general $Q_F \neq Q_R$ and the temperature/ gravitino duality will be broken. C_V can be either positive or negative.
- For models with $N = 2$ initial SUSY a with $Q_R \neq Q_F$,the contribution of the twisted sector is negative $(-)^{(Q_R - Q_F)} = (-)^H = -1$.

The change of sign indicates that in the twisted sector the states that become massive are the bosons rather than the fermions.

Adding small SUSY mass scales from Wilson line deformations, a new supersymmetric scale appears M_Y.

Incorporating the effects of the Wilson lines up to quadratic order.

$$P = C_T \, T^4 \, f_{\frac{5}{2}}(u) \; - \; D_T \, T^2 \, M_Y^2 \, f_{\frac{3}{2}}(u)$$

$$+ \, C_V \, M^4 \, \frac{f_{\frac{5}{2}}(1/u)}{u} \; - \; D_V \, M^2 \, M_Y^2 \, \frac{f_{\frac{3}{2}}(1/u)}{u}$$

M_Y introduces a new supersymmetric scale in the theory, which is qualitatively different from the SUSY-breaking scales T and M.

Scaling Properties of the Thermal Effective Potential

The final expression for P contains three mass scales: M, T and M_Y. The first identity follows from the definition of P

$$\left(T\frac{\partial}{\partial T} + M\frac{\partial}{\partial M} + M_Y\frac{\partial}{\partial M_Y} \right) \; P = 4P$$

$$P \equiv T^4 \; I_4(u) \; + \; T^2 \; M_Y^2 \; I_2(u) = P_4 + P_2, \quad u = \frac{M}{T}$$

$$\rho \equiv T\frac{\partial}{\partial T}P - P = \rho_4 + \rho_2$$

$$\rho_4 = \left(3P_4 - u\frac{\partial}{\partial u}P_4 \right) \qquad \rho_2 = \left(P_2 - u\frac{\partial}{\partial u}P_2 \right)$$

5. Gravitational Equations and the Critical Solution

We are now in the position to investigate the back-reaction to the initially flat metric and moduli fields allowing the SUSY-breaking scales T and M to vary with time while fixing the SUSY mass scale M_Y and u.

The gravitino scale M is given in terms of the no-scale modulus Φ

$$M = e^{\alpha\Phi}, \qquad \alpha = \sqrt{\frac{3}{2}},$$

Since $-P$ play the role of the one loop effective potential we obtain the Φ-field equation

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{\partial}{\partial\Phi}P = \alpha u\frac{\partial}{\partial u}P = -\alpha\left(\rho_4 - 3P_4 + \rho_2 - P_2\right)$$

Assuming that the space time metric is homogeneous and isotropic

$$ds^2 = -dt^2 + a(t)^2\, d\Omega_k^2, \qquad H = \left(\frac{\dot{a}}{a}\right)$$

we can derive the gravitational equations in terms of the no-scale modulus Φ and in terms of ρ and P. Ω_k denotes the three dimensional space with curvature k

$$3H^2 = \frac{1}{2}\dot{\Phi}^2 + \rho - \frac{3k}{a^2}$$

$$2\dot{H} + 3H^2 = -\frac{k}{a^2} - P - \frac{1}{2}\dot{\Phi}^2$$

We find useful to use the linear sum of the above two equations instead of the second.

$$\dot{H} + 3H^2 = -\frac{2k}{a^2} + \frac{1}{2}(\rho - P)$$

The Critical Solution

The fundamental ingredients in our analysis are the scaling properties of the thermal effective potential $-P = -P_4 - P_2$ at finite T.

Their structure suggests to search for a solution where the mass scales of the system, $M(\Phi)$, T and $(1/a)$ remain proportional during their evolution in time

$$e^{\alpha\Phi} \equiv M(\Phi) = \frac{1}{\gamma a} \quad \longrightarrow \quad H = -\alpha\dot{\Phi}, \quad M(\Phi) = u\,T$$

Our aim is thus to determine the constants γ and $u = 1/\xi$.

On the critical trajectory,

$$r_4 = \rho_4/T^4, \;\; p_4 = P_4/T^4, \;\; r_2 = \rho_2/T^2, \;\; p_2 = P_2/T^2$$

remains constants.

The compatibility of the Φ-equation and the gravity equation along the critical trajectory implies an identification of the coefficients of the monomials in M.
The quartic terms determines $\xi = 1/u$.
The quadratic terms determine k

$$r_4 = \frac{6\alpha^2 - 1}{2\alpha^2 - 1}\, p_4$$

$$-2k\gamma^2 = \frac{2\alpha^2 - 1}{2}\, (r_2 - p_2)\, \xi^2 M_Y^2$$

The Friedman-Hubble equation in the background $\dot{\Phi}^2 = (H^2/\alpha^2)$

$$\left(\frac{6\alpha^2 - 1}{6\alpha^2}\right)\, 3H^2 = -\frac{3k}{a^2} + \rho = -\frac{3k}{a^2} + \rho_4 + \rho_2$$

The factor in front of $3H^2$ can be absorbed in the definition of $\hat{k}$ and r_4

$$3H^2 = -\frac{3\hat{k}}{a^2} + \frac{C_R}{a^4}$$

$$3\hat{k} = -\frac{\xi^2 M_Y^2}{\gamma^2} \, \frac{6\alpha^2}{6\alpha^2 - 1} \left(\frac{3(2\alpha^2 - 1)}{4}(r_2 - p_2) + r_2 \right) .$$

$$C_R = \frac{\xi^4}{\gamma^4} \, \frac{6\alpha^2}{6\alpha^2 - 1} \, r_4 \, = \frac{\xi^4}{\gamma^4} \, \frac{6\alpha^2}{2\alpha^2 - 1} \, p_4$$

Remarks

The plausible existence of cosmological

super-string solutions

Inflationary (with initial SUSY $N < 2$) or not

which are generated dynamically

at the quantum sting level

from a flat classical space-time and

spontaneously broken supersymmetry

(no-scale radiative-induced cosmology).

6. Resolution of the Hagedorn transition in Type II

In type II superstings , both left- and right- movers contribute to the space-time fermion number: $F = F_L + F_R$. (In the heterotic, F_R is always even)

In the heterotic string, the thermal partition function is obtained by the temperature phase insertion

$$(-)^{\,mF_L + n\tilde{F}_L + mn}.$$

This definition is indeed unique, as it is dictated by the spin-statistics connection and modular invariance.

In the type II closed string the thermal phase insertion

$$(-)^{m(F_L+F_R)+n(\tilde{F}_L+\tilde{F}_R)}.$$

breaks the initial $N \leq 8$ supersymetry giving rise to a non trivial free energy density similar to the heterotic.

In the type II case the free energy is well defined as soon as

$$T \ << \ T_H, \qquad R_0 >> \ R_H = \frac{\sqrt{2}}{2}$$

The breaking of SUSY via "graviphoton magnetic-flux" is achieved associated to an R-symmetry charge Q by inserting the SUSY breaking phase :

$$(-)^{m_1(Q_L+Q_R)+n_1(\tilde{Q}_L+\tilde{Q}_R)}$$

introducing non trivial coupling of Q to the wrapping numbers (m_1, n_1) of $S^1_{R_1}$ cycle

If we choose a symmetric coupling $Q_L = F_L, \ \ Q_R = -F_R$, Then the situation is similar to the heterotic giving rise to a finite thermal free energy density below the Hagedorn transition.

$$R_0, \ \ R_1 \ << \ \frac{\sqrt{2}}{2}$$

Is it possible to go beyond Hagedorn transition?
If yes, what is the initial state of our universe?
As I will show, by an explicit example in type IIB orientifolds, the answer is YES.

The construction is based to an asymmetric "thermal" breaking of SUSY

$$(-)^{m_0 F_L \,+\, n_0 \tilde{F}_L \,+\, m_0 n_0}$$

$$(-)^{m_1 F_R \,+\, n_1 \tilde{F}_R \,+\, m_1 n_1}$$

The proposed thermal partition function is

$$Z = \int_{\mathcal{F}} \frac{d^2\tau}{4\mathrm{Im}\tau} \frac{\Gamma_{(8,8)}}{(\eta\bar{\eta})^{12}} \left[\frac{1}{2} \sum_{a,b} (-)^{a+b+ab} \theta \begin{bmatrix} a \\ b \end{bmatrix}^4 \; \Gamma_0 \begin{bmatrix} a \\ b \end{bmatrix} \right] \left[\frac{1}{2} \sum_{\bar{a},\bar{b}} (-)^{\bar{a}+\bar{b}+\bar{a}\bar{b}} \bar{\theta} \begin{bmatrix} \bar{a} \\ \bar{b} \end{bmatrix}^4 \; \Gamma_1 \begin{bmatrix} \bar{a} \\ \bar{b} \end{bmatrix} \right]$$

where the shifted lattices $\Gamma_0 \begin{bmatrix} a \\ b \end{bmatrix}$ and $\Gamma_1 \begin{bmatrix} \bar{a} \\ \bar{b} \end{bmatrix}$ are:

$$\Gamma_0 \begin{bmatrix} a \\ b \end{bmatrix} = \sum_{m_0, n_0} \frac{R_0}{\sqrt{\mathrm{Im}\tau}} \; e^{-\pi R_0^2 \frac{|m_0 + n_0 \tau|^2}{\mathrm{Im}\tau}} \; (-)^{m_0 a + n_0 b + m_0 n_0}.$$

$$\Gamma_1 \begin{bmatrix} \bar{a} \\ \bar{b} \end{bmatrix} = \sum_{m_1, n_1} \frac{R_1}{\sqrt{\mathrm{Im}\tau}} \; e^{-\pi R_1^2 \frac{|m_1 + n_1 \tau|^2}{\mathrm{Im}\tau}} \; (-)^{m_1 \bar{a} + n_1 \bar{b} + m_1 n_1}.$$

The $\Gamma_{(8,8)}$ lattice depend on all moduli of the eight dimensional space.

The $\Gamma_{(1,1)}(R_0)$ lattice is "thermally" coupled to the left-movers
The $\Gamma_{(1,1)}(R_1)$ lattice is "thermally" coupled to the right-movers

Setting $n_0 \longrightarrow 2n_0 + h_0$ and $n_1 \longrightarrow 2n_1 + h_1$, where h_0, h_1 are 0,1 modulo 2, and performing Poisson re-summations over the wrapping numbers m_0, m_1, The left- and right-moving shifted momenta are: ($a + \bar{a}$=1 or 0 correspond to Fermions or Bosons)

$$\sqrt{2}p_L^0 = \frac{2m_0 + (a - h_0)}{\sqrt{2}R_0} \; + \; (2n_0 + h_0)\sqrt{2}R_0$$

$$\sqrt{2}p_R^0 = \frac{2m_0 + (a - h_0)}{\sqrt{2}R_0} \; - \; (2n_0 + h_0)\sqrt{2}R_0$$

$$\sqrt{2}p_L^1 = \frac{2m_1 + (\bar{a} - h_1)}{\sqrt{2}R_1} \; + \; (2n_1 + h_1)\sqrt{2}R_1,$$

$$\sqrt{2}p_R^1 = \frac{2m_1 + (\bar{a} - h_1)}{\sqrt{2}R_1} \; - \; (2n_1 + h_1)\sqrt{2}R_1.$$

In addition, the left GSO projection is reversed in the h_0-odd winding sector
the right GSO projection is reversed in the h_1-odd winding sector

The partition function is given by

$$Z = \int_{\mathcal{F}} \frac{d^2\tau}{4\mathrm{Im}\tau} \frac{\Gamma_{(8,8)}}{(\eta\bar{\eta})^8}$$

$$\left[\sum_{m_0,n_0} \Gamma_{2m_0,2n_0}\chi_V + \Gamma_{2m_0-1,2n_0+1}\chi_O - \Gamma_{2m_0+1,2n_0}\chi_S - \Gamma_{2m_0,2n_0+1}\chi_C \right]$$

$$\times \left[\sum_{m_1,n_1} \Gamma_{2m_1,2n_1}\bar{\chi}_V + \Gamma_{2m_1-1,2n_1+1}\bar{\chi}_O - \Gamma_{2m_1+1,2n_1}\bar{\chi}_S - \Gamma_{2m_1,2n_1+1}\bar{\chi}_C \right],$$

χ_V, χ_O, χ_S, χ_C are the usual $SO(8)$ characters

$$\chi_O = \frac{1}{2\eta^4}(\theta_3^4 + \theta_4^4), \quad \chi_V = \frac{1}{2\eta^4}(\theta_3^4 - \theta_4^4)$$
$$\chi_S = \frac{1}{2\eta^4}(\theta_2^4 + \theta_1^4), \quad \chi_C = \frac{1}{2\eta^4}(\theta_2^4 - \theta_1^4).$$

The asymmetric thermal IIB model have several novel features.

• There is no tachyonic state in the spectrum. A tachyon could only arise from the term $|\chi_O|^2$, with $h_0 = h_1 = 1$. For generic values ofi R_0 and R_1, the lightest states

$$\frac{1}{2}m^2_{\text{lightest}} = \frac{1}{4}\left(\frac{1}{2R_0^2} + 2R_0^2 + \frac{1}{2R_1^2} + 2R_1^2\right) - 1$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{2}R_0} - \sqrt{2}R_0\right)^2 + \frac{1}{2}\left(\frac{1}{\sqrt{2}R_1} - \sqrt{2}R_1\right)^2 .$$

These states never become tachyonic. They become massless when $R_0 = R_1 = 1/\sqrt{2}$; that is when both radii are at the fermionic point.

• All fermions are massive

$$m^2_S = \frac{1}{2R_0^2}, \quad m^2_{\bar{S}} = \frac{1}{2R_1^2}, \quad m^2_C = 2R_0^2, \quad m^2_{\bar{C}} = 2R_1^2$$

• The RR fields are also massive

$$m^2_{S\bar{S}} = \frac{1}{2R_0^2} + \frac{1}{2R_1^2}, \quad m^2_{C\bar{C}} = 2R_0^2 + 2R_1^2,$$

The reason is that these fields are charged under both the Z_2 symmetries $(-1)^{F_L}$ and $(-1)^{F_R}$. The asymmetric breaking of supersymmetry also leads to a spontaneous breaking of the $U(1)$ gauge symmetries associated to the RR antisymmetric tensors.

• The full spectrum of the theory is T-duality invariant under

$$\sqrt{2}R_0 \to \frac{1}{\sqrt{2}R_0}, \quad \sqrt{2}R_1 \to \frac{1}{\sqrt{2}R_1}$$

• The appearance of extra massless states at the fermionic point $\sqrt{2}R_0 = \sqrt{2}R_1 = 1$ is a signal of enhanced gauged symmetry namely

$$[SU(2)_2 \times SU(2)_2]_{left} \times [SU(2)_2 \times SU(2)_2]_{right}$$

• When $R_0 = R_1$ there is an extra Z_2 symmetry which will be used to give to define the open sector via an orientifold projection.

The thermal interpretation of the model becomes clear along the $R_0 = R_1 = R$ line of moduli space. In this case, the asymmetric breaking of supersymmetry amounts to decompose the $\Gamma_{(2,2)}$ torus lattice in terms of the diagonal and anti-diagonal $\Gamma^d_{(1,1)}$ and $\Gamma^a_{(1,1)}$ lattice quantum numbers

$$m_0 + m_1 = 2m_d + g, \;\; m_0 - m_1 = 2m_a - g$$

$$n_0 + n_1 = 2n_d + h, \;\; n_0 - n_1 = 2n_a - h$$

The result is given in terms of two coupled *thermal* $\Gamma_{(1,1)}$ lattices at radii $R_d = \sqrt{2}R$ and $R_a = \sqrt{2}R$ respectively.

In the large R radius limit, the contributions of the odd winding sectors $h = 1$ are exponentially suppressed. Neglecting these, and thanks to the initial left- and right-supersymmetry g must be zero as well. Thus in this limit the two thermal lattices decouple.

We identify $\Gamma^{d}_{(1,1)}$ with the lattice corresponding to the Euclidean time cycle. In the large radius limit, the lattice quantum numbers are precisely coupled to the space-time fermion number $F = F_L + F_R$ as required by the spin-statistics connection.

The $\Gamma^{a}_{(1,1)}$ lattice quantum numbers couple to the (anti-)fermion number $F = F_L - F_R$. It can be associated with a spatial cycle along which particular boundary conditions are used to break space-time supersymmetry via graviphoton magnetic flux.

Thus, in the large radius limit, the model describes a thermal system, where supersymmetry is first spontaneously broken via the Scherk-Schwarz mechanism along a spatial cycle, and then the resulting system is heated up. The effective temperature is given by $2\pi T = 1/\sqrt{2}R_d$ which is equal to the supersymmetry breaking scale $2\pi M = 1/\sqrt{2}R_a$

7. The Open String Sector

When $R_0 = R_1$ the thermal IIB partition function becomes a perfect square. This remark indicates how to define the orientifold projection $\mathbb{Z}_2$

$$\mathbb{Z}_2 \ : \ \Omega \cdot \mathcal{P}_{12}$$

Ω : interchanges the left- and right- movers

$\mathcal{P}_{12}$: interchanges the coordinates $x^1 \leftrightarrow x^2$

The torus partition function $\mathcal{T}$ counts with the invariant linear sum of a state under $\mathbb{Z}_2$ and as long as they are distinct. To complete the closed string states counting, one introduces the Klein bottle $\mathcal{K}$ in which only $\mathbb{Z}_2$ invariant states appear.

Each product of complex conjugate characters in $\mathcal{T}$ descend to a character in $\mathcal{K}$, with argument $2t = 2i\tau_2$.

They are dressed by the contributions of the momenta of the $\Gamma_{8,8}$ such that

$$p_L^I = p_R^I, \quad I = 3, ...9, \qquad p_L^0 = p_R^1, \quad p_L^1 = p_R^0 \longrightarrow$$

$$n_I = 0 \;\; I = 3, ..., 9 \;, \quad m_0 = m_1 = m, \quad n_0 = -n_1 = n$$

At the end the Klein bottle amplitude in the transverse channel is:

$$\mathcal{K} = \frac{2^4}{2} \int_0^{+\infty} \frac{d\tau}{4\mathrm{Im}\tau^2} \frac{\Gamma_8(2\tau)}{\eta(2\tau)^8} \left[\Gamma_e^d \, \Gamma_e^a \, \chi_V + \Gamma_o^d \, \Gamma_o^a \, \chi_O - \Gamma_o^d \, \Gamma_e^a \, \chi_S - \Gamma_e^d \, \Gamma_o^a \, \chi_C\right] (2\tau)$$

$$\Gamma_{e,o}^d = \Gamma_{e,o}^d \left(\sqrt{2}R\right), \qquad \Gamma_{e,o}^a = \Gamma_{e,o}^a \left(\frac{1}{\sqrt{2}R}\right),$$

It turns out that the annulus amplitude $\mathcal{A}$ in the transverse has the same form :

$$\mathcal{A} = \frac{2^{-4}N^2}{2} \int_0^{+\infty} \frac{d\tau}{4\mathrm{Im}\tau^2} \frac{\Gamma_8(2\tau)}{\eta(2\tau)^8} \left[\Gamma_e^d \, \Gamma_e^a \, \chi_V + \Gamma_o^d \, \Gamma_o^a \, \chi_O - \Gamma_o^d \, \Gamma_e^a \, \chi_S - \Gamma_e^d \, \Gamma_o^a \, \chi_C\right] (2\tau)$$

The factor N^2 is introduced to account for Chan-Paton degeneracies.

Finally the Mobius amplitude in the transverse channel becomes:

$$\mathcal{M} = -\frac{2N}{2}\int_0^{+\infty} \frac{d\tau}{4\text{Im}\tau^2}\frac{\Gamma_8(2\tau)}{\eta(2\tau)^8}\left[\Gamma_e^d\,\Gamma_e^a\,\chi_V + \Gamma_o^d\,\Gamma_o^a\,\chi_O - \Gamma_o^d\,\Gamma_e^a\,\chi_S - \Gamma_e^d\,\Gamma_o^a\,\chi_C\right](2\tau+1)$$

The choice of $N = 16$ eliminates all tadpoles for any $R:\ \sqrt{2}R \neq 1$

$$2^4 + 2^{-4}N^2 - 2N = 0 \qquad \Longrightarrow \qquad N = 16$$

The gauge group in the open sector is $SO(16)$

8. The Initial Phase of the Universe

At low Temperature T the Thermal Free Energy is identical to the one of the effective field theory spontaneously broken by M. At this regime only the V and S characters survives.

Thanks to T-duality, the very "High temperature" regime with V and S is dual to the "Low Temperature" regime with the dual representations V and C.

At the would be "Hagedorn singularity" at the Fermionic point, nothing singular is happening. Around this point the relevant representations are now V and O while both S and C are massive with identical mass spectrum.

At the self dual point (initial state of the universe) the gauge symmetry is extended $U(1) \times U(1) \to SU(2)_d \times SU(2)_a = SO(4)$.

This precise space is the initial non-singular phase of the Universe.

9. String structure of the Initial Phase of the Universe

In stringy gravity and cosmology new interesting phenomena occur.
Conventional notions from general relativity like :

Geometry and Topology

are well defined only as *low energy and/or small curvature approximations* of the stringy setup.

• At very small distances and at strong curvature scales, purely stringy phenomena imply that the physics can be quite different from what one might expect from the "naive" field theory approximation.

• New possibilities in the context of quantum cosmology and especially in the context of the "Stringy Big-Bang" picture

versus

"the initial singularity of the Big-Bang picture in General Relativity" .

Assuming for instance a compact space and sufficiently close to the singularity, the typical scale of the universe reaches at these early times the gravitational scale M_{string}.

At this early epoch classical gravity is no longer valid and has to be replaced by a more fundamental singularity-free theory such as (super-)string theory.

- The main obstruction in the stringy cosmological framework is
  the Hagedorn temperature limitation $T < T_H$.

It is well known that for high temperatures, $T > T_H$,
the string partition function diverges

A thermal winding state becomes tachyonic.
However, this is not a pathology in string theory.
It is a signal of a phase transition towards to a new vacuum.

Many proposals were made about the *"High Temperature Phase of the Universe".*

The Hagedorn-like singularities have to be resolved :

• by a stringy phase transition

OR

• by choosing Hagedorn-free string vacua in the early stage of the universe.

It is of fundamental importance to show that :

• the space of Hagedorn-free vacua is not empty

and that

• their existence is *at least equally natural* as the Hagedorn - singular ones.

• As we explained previously, a noticeable progress has been made in constructing Hagedorn-free string vacua which are characterized by the presence of non-trivial magnetic fluxes.

• Also, stringy vacua with a "Massive boson-fermion Spectrum-Degeneracy Symmetry, $MSDS$" are proposed recently to describe the early *"Stringy non-geometric era"*.

10. The Maximally Symmetric $MSDS$ -vacua

The proposed $MSDS$-vacua have at least 8 extremely small compact dimensions, close to the string scale. $\longrightarrow$ $d \leq 2$ target space

Their connection to the "higher-dimensional universes " in late cosmological times is achieved via large marginal deformations of current-current type : $M_{IJ}\ J_L^I \times J_R^J$

The large M_{IJ}-deformation limit *"induces an effective higher-dimensional space"*

In this limit one recovers a geometric field theory description in terms of an effective "higher-dimensional" conventional superstring theory

- The space-time supersymmetry appears to be *spontaneously broken via*

"Geometrical" and "Thermal" fluxes.

In the most symmetric $MSDS$-vacua all compact space coordinates are expressed in terms of free 2d-world-sheet fermions rather than the conventional compact bosonic coordinates.

The advantage of this fermionization lies in the consistent separation of left- and right-moving world-sheet degrees of freedom in terms of left- and right-moving 2d-fermions that permit easier manipulations of the left-right asymmetric (and even non-geometrical) constructions of vacua in string theory.

Type II degrees of freedom

In the "critical" Type II theories the left- and right- moving degrees of freedom are:

- The light-cone degrees: $(\partial X^0,\ \Psi^0)$, $(\partial X^L,\ \Psi^L)$
- The super-reparametrization ghosts: (b, c), (β, γ)
- The transverse super-coordinates: $(\partial X^I \equiv i y^I w^I,\ \ \Psi^I),\ I = 1, ...8$

The transverse super-coordinates $(\partial X^I,\ \Psi^I)$ are replaced by (y^I, w^I, Ψ^I) so that for every $I = 1, ..., 8$, $\{y^I, w^I, \Psi^I\}$ define the adjoint representation of a $SU(2)_{k=2}$.

In a more general fermionization the transverse super-coordinates are replaced by a set of free fermions in the adjoint representation of a semi-simple gauge group H :

$$\{\chi^a\},\ a = 1, ...n,\ \ n = \dim[\, H\,] = 24$$

The simplest choice of H is: $\quad H = SU(2)^8$

Other non-trivial choices of fermionization are also possible:

$$H = SU(5),\ \ H = SO(7) \times SU(2),\ \ H = G_2 \times Sp(4),$$
$$H = SU(4) \times SU(2)^3,\ \ H = SU(3)^3.$$

For simplicity I will restrict to the choice $H = SU(2)^8$ for both left- and right- moving transverse degrees of freedom.

Heterotic degrees of freedom

- The left-moving sector is identical to that of Type II theories.

The right-moving degrees of freedom are:

- The light-cone degrees: $(\partial X_0,\ \partial X_L)$
- The reparametrization ghosts: (b, c)
- The transverse coordinates: $(\partial X^I,\ I = 1, ...8)$
- The extra 32 right-moving fermions $(\psi^A,\ A = 1, 2, ...32)$

In total there are 48 free fermions in the right moving sector $\{\bar{\chi}^a,\ a = 1, 2, ...48\}$

i) 16 $(y^I, w^I,\ I = 1, 2, ..., 8)$ from coordinate fermionization $i\partial X^I = y^I w^I$

ii) extra 32 right-moving fermions $(\psi^A,\ A = 1, 2, ...32)$, for the anomaly cancelation.

The basic left- and right-moving chiral operators and partition functions

In both Type II and Heterotic theories the left-moving T_B and T_F have the same form:

$$T_B = -\frac{1}{2}(\partial X_0)^2 - \frac{1}{2}\Psi_0\partial\Psi_0 + \frac{1}{2}(\partial X_L)^2 + \frac{1}{2}\Psi_L\partial\Psi_L \; + \; \sum_{a=1}^{24} \frac{1}{2}\,\chi^a\partial\chi^a$$

$$T_F = i\partial X_0\Psi_0 \; + \; i\partial X_1\Psi_1 \; + \; \sum_{a,b,c} f_{abc}\,\chi^a\chi^b\chi^c \; ,$$

f_{abc} are the structure constants of the group $H_L = SU(2)^8$ and $\{\chi^a\}$ $(a = 1, 2, ..., 24)$

The heterotic right-moving $\bar{T}_B(\bar{z})$:

$$\bar{T}_B = -\frac{1}{2}(\bar{\partial}X_0)^2 + \frac{1}{2}(\bar{\partial}X_L)^2 + \sum_{a=1}^{48} \frac{1}{2}\,\bar{\chi}^a\bar{\partial}\bar{\chi}^a \, .$$

Following the rules of the fermionic construction

i) $H_L \times H_R = SU(2)^8 \times SU(2)^8$ in type II

ii) $H_L \times H_R = SU(2)^8 \times SO(48)$ in the heterotic

iff the choice of boundary conditions respects the global existence of the $H_L \times H_R$ symmetry, then the latter is promoted to a local gauge symmetry on the target space-time, both in Type II and the Heterotic

- We can construct very special tachyon free vacua, with left–right holomorphic factorization of the partition function.

In terms of the $SO(2n)$ characters ($n = 12$ or $n = 24$) :

$$V_{2n} = \frac{\theta_3^n - \theta_4^n}{2\eta^n}, \qquad O_{2n} = \frac{\theta_3^n + \theta_4^n}{2\eta^n}, \qquad S_{2n} = \frac{\theta_2^n - \theta_1^n}{2\eta^n}, \qquad C_{2n} = \frac{\theta_2^n + \theta_1^n}{2\eta^n},$$

Type II and Heterotic partition functions :

$$Z_{II} = \int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2} \left(V_{24} - S_{24}\right)\left(\overline{V}_{24} - \overline{S}_{24}\right)$$

$$Z_{Het} = \int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2} \left(V_{24} - S_{24}\right)\left(\overline{O}_{48} + \overline{C}_{48}\right).$$

• The expression for Z_{II} remains the same for any choice of left- and right-moving H-group H_L, H_R, since the dimension of each is always equal to 24. In this respect, Z_{II} is a unique tachyon-free partition function (modulo the chirality of the left- and right-spinors) *that respects the $H_L \times H_R$ gauge symmetry.*

• The expression of the left-moving part in Z_{Het} remains the same as well. The right-moving part, depends on the choice of H_R:

$$H_R \quad \rightarrow \quad SO(48), \qquad E_8 \times SO(32), \qquad E_8^3$$

• Both Z_{II} and Z_{Het} show a Massive Spectrum Degeneracy Symmetry.
This spectacular property reflects the relations between the characters of the $SO(24)$:

$$V_{24} \; - \; S_{24} = \text{constant} = 24 \; .$$

This follows from the well-known Jacobi identities between theta functions:

$$\theta_3^4 - \theta_4^4 - \theta_2^4 = 0, \qquad \theta_1^4 = 0, \qquad \theta_2\theta_3\theta_4 = 2\eta^3,$$

$$\longrightarrow \qquad \frac{\theta_3^{12} - \theta_4^{12}}{2\eta^{12}} - \frac{\theta_2^{12} - \theta_1^{12}}{2\eta^{12}} = 24$$

• The spectrum of massive bosons and fermions is identical to all string mass levels! This is similar to the analogous property of supersymmetric theories.

• In the massless level, however, although there are 24 left-moving bosonic degrees of freedom there are no massless fermionic states.

• In Type II there are 24 right-moving bosonic states as well, so in total there are 24×24 scalar bosons at the massless level transforming under the adjoint of $H_L \times H_R$.

• The integrated type II partition function :

$$Z_{II} = \frac{\pi^2}{3}\, d(H_L) \times d(H_R), \qquad \mathcal{I} = \int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2} = \frac{\pi^2}{3}$$

• In the Heterotic the left-moving sector gives constant contribution as in the Type II ($d(H_L) = 24$). The right-moving massive states are expressed in terms of the unique holomorphic modular invariant function $j(\tau)$:

$$Z_{Het} = \int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2}\, d(H_L) \times \{d(H_R) + [j(\bar\tau) - 744]\} = \frac{\pi^2}{3}\, d(H_L) \times d(H_R)$$

• The contribution of the anti-holomorphic function $[j(\bar\tau) - 744]$ vanishes when integrated over the fundamental domain.

• The final integrated expression for both Z_{Het} and Z_{II} are proportional to the number of massless states of the models.

$$Z = \frac{\pi^2}{3}\, d(H_L) \times d(H_R).$$

Depending on the choice of H_R in the Heterotic, the number of the massless states is :

$d(H_L) \times d[SO(48)] = 24 \times 1128$

$d(H_L) \times d[E_8 \times SO(32)] = 24 \times 744$

$d(H_L) \times d[E_8^3] = 24 \times 744$

• The massive boson-fermion degeneracy symmetry of the $MSDS$-vacua is not an accidental property of the above constructions.
It follows from the existence of a new superconformal symmetry.

11. Chiral superconformal algebra and spectral flow in $MSDS$

The symmetry operators of the $MSDS$ vacuum are the usual holomorphic (anti-holomorphic) operators T_B, T_F ($\bar{T}_B$, $\bar{T}_F$) giving rise to the standard $\mathcal{N} = (1,1)$ world-sheet superconformal symmetry in type II and the $\mathcal{N} = (1,0)$ in the heterotic

The extra symmetry operators are the currents of conformal weight $h_J = 1$, associated with the H_L- and H_R-affine algebras:

$$J^a \equiv f^a_{bc}\,\chi^b\chi^c \qquad \text{and} \qquad \bar{J}^a \equiv \bar{f}^a_{bc}\,\bar{\chi}^b\bar{\chi}^c$$

Furthermore, there are two $SO(24)$ spin-field operators with conformal weight $\frac{3}{2}$ and opposite chirality :

$$C = Sp\{\chi^a\}_+ \qquad \text{and} \qquad S = Sp\{\chi^a\}_-$$

The existence of the chiral operator C, of conformal weight $h_C = \frac{3}{2}$, together with T_B, T_F, J^a, χ^a, form a *new chiral superconformal algebra* implying the massive boson-fermion degeneracy of the spectrum.

One needs to utilize the OPE relations between C and S:

$$C(z)\ C(w) \sim \frac{1}{(z-w)} \left\{ \frac{\mathbf{1}}{(z-w)^2} + \frac{\hat{\chi}\hat{\chi}}{(z-w)} + \ \dots \right\},$$

$$S(z)\ S(w) \sim \frac{1}{(z-w)} \left\{ \frac{\mathbf{1}}{(z-w)^2} + \frac{\hat{\chi}\hat{\chi}}{(z-w)} + \ \dots \right\},$$

$$C(z)\ S(w) \sim \frac{1}{(z-w)^{\frac{1}{2}}} \left\{ \frac{\hat{\chi}}{(z-w)^2} + \frac{\partial\hat{\chi} + \hat{\chi}\hat{\chi}\hat{\chi}}{(z-w)} + \ \dots \right\},$$

$\hat{\chi} \equiv \gamma^a \chi_a \quad , \quad \gamma^a \to \gamma$-matrices of $SO(24)$

$C(z)S(w)$ OPE implies a boson-fermion Spectral Flow which guaranties the massive boson-fermion degeneracy of the Vacuum.

Spectral flow and the MSDS operator-relations

The vertex operators are dressed by the super-reparametrization ghost Φ:

$$e^{q\Phi} \quad \longrightarrow \quad \text{with} \quad h_q = -\frac{1}{2}\, q\,(q+2)$$

- Space-time boson vertices are expressed either in the 0 or the (-1) ghost picture.

$$\mathbf{V}_{(\mathbf{0})} \;=\; e^{-\Phi}\,\hat{\chi}, \qquad\qquad \mathbf{V}_{(\mathbf{1})} \equiv e^{-\Phi}\,(\,\partial\hat{\chi} \;+\; \hat{\chi}\,\hat{\chi}\,\hat{\chi}\,)$$

$\mathbf{V}_{(\mathbf{0})}, \mathbf{V}_{(\mathbf{1})}$ have conformal weight $h_0 = 1, h_1 = 2$.

The string spectrum of bosons starts from a massless sector which is described by $\mathbf{V}_{(\mathbf{0})}$.

$\mathbf{V}_{(\mathbf{1})}$ define a massive bosonic vertex at mass level 1.

• Space-time fermions are in the $(-\frac{1}{2})$ or $(-\frac{3}{2})$ pictures.

$$\mathbf{S} \;=\; e^{-\frac{1}{2}\Phi-\frac{1}{2}iH_0}\, S \quad \text{or} \quad \mathbf{S} \;=\; e^{-\frac{3}{2}\Phi+\frac{1}{2}iH_0}\, S$$

H_0 is the usual helicity field defined via bosonization $i\partial H_0 = \Psi_0\Psi_L$.

$\mathbf{S}$ has weight $h_S = 2$ in both the $(-\frac{1}{2})$ and $(-\frac{3}{2})$ pictures

$\longrightarrow$ all space-time fermions are massive, starting from mass level 1.

The flow of $\mathbf{V}_{(\mathbf{0}),(\mathbf{1})}$ states to $\mathbf{S}$-states is expressed by the action of a "Spectral-flow operator" $\mathbf{C}$:

$$\mathbf{C} \;\equiv\; e^{\frac{1}{2}(\Phi-iH_0)}\, C \; .$$

$\mathbf{C}$ is written in the $(+\frac{1}{2})$ ghost picture. It has conformal dimension $h_{\mathbf{C}} = 1$ and $(-1/2)$ helicity charge.

$\mathbf{C}$ acting on "physical" bosonic states produces "physical" fermionic states at the same string level and vice-versa.

Although the $\mathbf{C}$-action looks like a space-time supersymmetry transformation, the actual situation turns out to be drastically different.

The $\mathbf{C}$-action leaves the massless bosonic states of the theory invariant
$\longrightarrow$ the boson-to-fermion mapping does not exist for the massless states.

$$\mathbf{C}(z)\ \mathbf{V}_0(w) \ \sim \ \mathbf{S}\,, \quad \text{finite as } z \to w.$$

The absence of singular terms in $(z-w)$ shows clearly that the massless states are invariant under the $\mathbf{C}$-transformation.

$\mathbf{C}$ acts non-trivially on the massive states:

$$\mathbf{C}(z)\ \mathbf{V}_{\mathbf{1}}(w) \ \sim \ \frac{\mathbf{S}(w)}{(z-w)} \ + \ \text{finite terms.}$$

$$\mathbf{C}(z)\ \mathbf{S}(w) \ \sim \ \frac{\mathbf{V}_{(\mathbf{1})}(w)}{(z-w)} \ + \ \text{finite terms}$$

massive bosonic states are mapped into the *massive fermionic states* and vise-versa.

12. Marginal deformation of the $MSDS$ vacua

The massless states of $MSDS$-vacua are $d_L \times d_R$ scalars parametrizing a manifold similar to that gauged supergravities with $G = H_L \times H_R$:

$$\mathcal{K} = \frac{SO(d_L,\ d_R)}{SO(d_L) \times SO(d_R)}.$$

The non-abelian structure of $H_L \times H_R$ implies that the only marginal deformations are those that correspond to the Cartan sub-algebra.

The moduli space of these deformations: $M_{IJ}\ J_L^I \times J_R^J$ is reduced to:

$$\mathcal{M} = \frac{SO(r_L,\ r_R)}{SO(r_L) \times SO(r_R)}\ .$$

The maximal number of the moduli M_{IJ} is when:

$$H_L = H_R\ =\ SU(2)^8_{k=2}, \quad \text{with} \quad r_L = r_R = 8\,.$$

The deformed partition function factors out *a shifted lattice* $\Gamma_{8,8}(M)\left[{a,\bar{a} \atop b,\bar{b}}\right]$

The *shifted lattice* couples non-trivially to the "parafermion numbers" defined by the gauged WZW-cosets :

$$\prod_{I_L=1,\dots 8}\left(\frac{SU(2)_{k=2}}{U(1)}\right)_{I_L}\times\prod_{I_R=1,\dots 8}\left(\frac{SU(2)_{k=2}}{U(1)}\right)_{I_R}$$

For $k=2$, the above coset structure is equivalent to 8 left-moving world sheet fermions, Ψ_{I_L} and 8 right-moving ones, Ψ_{I_R} in type II.

At the end, the shifted lattice couples nontrivially to the R-symmetry charges of the conventional type II superstings: $\{a_I, b_I\ ;\ \bar{a}_I, \bar{b}_I\}$ of $\{\Psi_{I_{L,R}}\}$!

$$Z_{II}=\frac{1}{2}\sum_{a,b}\,(-)^{a+b}\,\frac{\theta[^a_b]^4}{\eta^{12}}\times\Gamma_{8,8}(M)\left[{a,\bar{a} \atop b,\bar{b}}\right]\times\frac{1}{2}\sum_{\bar{a},\bar{b}}\,(-)^{\bar{a}+\bar{b}}\,\frac{\bar{\theta}[^{\bar{a}}_{\bar{b}}]^4}{\bar{\eta}^{12}}$$

• In the large moduli limit (modulo S, T, U-dualities), the $\Gamma_{8,8}$ lattice decompactifies and the correlations with the R-symmetry charges become irrelevant.
One recovers the conventional ten dimensional type II supersymmetric vacua !

• For large but not infinitely large deformations, the obtained vacua are those of "spontaneously broken supersymmetric vacua in the presence of geometrical fluxes".

• Euclidian versions of the models, correspond to "thermal stringy vacua" with non-trivial left-right asymmetric "gravito-magnetic fluxes".
(see talk of N. Toumbas in this conference)

• The would be "initial" classical singularity of general relativity as well as the stringy Hagedorn-like singularities are both resolved by these fluxes !

The above generic properties of the deformed $MSDS$ vacua, strongly suggest the following *Cosmological Conjecture*.

Cosmological Conjecture

The $MSDS$ vacua, or even less symmetric orbifold versions
are potential candidates able to describe
the early non-singular phase of a stringy cosmological universe

- During the cosmological evolution $M_{IJ} \rightarrow M_{IJ}(t)$ evolves with the time.
  Once $M_{IJ}(t)$ are sufficiently large (modulo S, T, U-dualities)
  an effective field theory description emerges with an induced
  "space-time geometry" of an *"effective higher dimensional space-time"* .

- The relevant degrees of freedom and interactions are well described by some
  "no-scale" gauged supergravity theories of the conventional superstrings.

The effects of the initial $MSDS$ structure induces at the large moduli limit non-trivial
"geometrical" fluxes which in the language of the effective supergravity give rise to a
spontaneous breaking of supersymmetry and to finite temperature effects.

13. Orbifold reduction of the $MSDS$ structure

The originally proposed $MSDS$-vacua and in particular the ones with $H_L \equiv SU(2)^8$, are too symmetric to be phenomenologically viable.

• In the extreme large-M deformation limit (decompactification limit), the induced effective theory is that of *non-chiral* extended gauge supergravities, implemented with a well-defined set of geometrical fluxes.

• From our cosmological viewpoint, the strongly deformed $MSDS$-vacua should consistently represent our late time universe $\longrightarrow$ It should contain:

• A non-trivial net number of chiral families

• A reduced gauge group unifying in the most realistic manner the standard model interactions

In collaboration with I. Florakis, we able to classify all possible $\mathbb{Z}_2^N$-(asymmetric) orbifolds with reduced $MSDS$ symmetry.

In all proposed models the massive boson and fermion degrees of freedom exhibit sector by sector (untwisted and twisted) Massive Spectrum Degeneracy Symmetry.

Sector by sector, the number of massless bosons $n_I(b)$ and massless fermions $n_I(f)$ are different; $n_I(b) \neq n_I(f)$.

These remarkable properties follow from shifted versions of θ^{12}-identity.

For instance in Z_2-orbifold:

$$\text{Untwisted sectors}: \qquad V_{16}O_8 - S_{16}C_8 = 16\,, \qquad O_{16}V_8 - C_{16}S_8 = 8$$

$$\text{Twisted sectors}: \qquad V_{16}C_8 - S_{16}O_8 = 0\,, \qquad O_{16}S_8 - C_{16}V_8 = 8$$

The spectral-flow operator C_{24}, is truncated by Z_2 :

$$Z_2 \ : \ C_{24} = C_{16}C_8 + S_{16}S_8 \ \longrightarrow \ C_{Z_2} = C_{16}C_8$$

The global existence of C_{Z_2}, along with the truncated chiral algebra, are sufficient to guarantee massive supersymmetry of the spectrum.

In heterotic orbifold $MSDS$-vacua the anti-holomorphic contributions to the partition function are also constant numbers modulo a part proportional to $\bar{j}(\bar{\tau})$.

$$Z_{het,A} = n_A + m_A \, [\, \bar{j}(\bar{\tau}) - 744 \,] \,, \qquad n_A = n_A(b) - n_A(f)$$

- There is a plethora of reduced $MSDS$ orbifold vacua in the heterotic framework.

The classification rules are given in the work done in collaboration with I. Florakis.

• Among the heterotic $MSDS$ orbifolds are those which are connected via large moduli deformations to semi-realistic four-dimensional heterotic chiral models. For instance, those with: (see A. Faraggi, C. Kounnas and I. Rizos) $H_R = SO(10) \times U(1)^3 \times SO(16)$ gauge group and with non-zero chiral families.

A representative example of this class of $MSDS$-orbifolds is the one with:

i) Holomorphic partition function:

$$Z\begin{bmatrix} h_1 & h_2 \\ g_1 & g_2 \end{bmatrix} = \frac{1}{2} \sum_{a,b} (-)^{a+b} \frac{\theta\begin{bmatrix} a \\ b \end{bmatrix}^6 \theta\begin{bmatrix} a+h_1 \\ b+g_1 \end{bmatrix}^2 \theta\begin{bmatrix} a+h_2 \\ b+g_2 \end{bmatrix}^2 \theta\begin{bmatrix} a-h_1-h_2 \\ b-g_1-g_2 \end{bmatrix}^2}{\eta^{12}} \ ,$$

ii) Anti-holomorphic partition function:

$$\bar{Z}\begin{bmatrix} h_1 & h_2 \\ g_1 & g_2 \end{bmatrix} = \frac{1}{2^3\,\bar{\eta}^{24}} \sum_{\gamma,\delta} \bar{\theta}\begin{bmatrix} \gamma \\ \delta \end{bmatrix}^5 \bar{\theta}\begin{bmatrix} \gamma+h_1 \\ \delta+g_1 \end{bmatrix} \bar{\theta}\begin{bmatrix} \gamma+h_2 \\ \delta+g_2 \end{bmatrix} \bar{\theta}\begin{bmatrix} \gamma-h_1-h_2 \\ \delta-g_1-g_2 \end{bmatrix} \sum_{\epsilon,\zeta} \bar{\theta}\begin{bmatrix} \epsilon \\ \zeta \end{bmatrix}^5 \bar{\theta}\begin{bmatrix} \epsilon+h_1 \\ \zeta+g_1 \end{bmatrix} \bar{\theta}\begin{bmatrix} \epsilon+h_2 \\ \zeta+g_2 \end{bmatrix} \bar{\theta}\begin{bmatrix} \epsilon-h_1-h_2 \\ \zeta-g_1-g_2 \end{bmatrix} \sum_{\bar{a},\bar{b}} \bar{\theta}\begin{bmatrix} \bar{a} \\ \bar{b} \end{bmatrix}^8$$

The full partition function can be written in a conventional *shifted and twisted* "$\Gamma_{8,8}$-lattice form" :

$$Z = \frac{1}{2^6\,\eta^{12}\bar\eta^{24}} \sum_{a,b,\gamma,\delta,h_i,g_i} (-)^{a+b}\,\theta[^a_b]\,\theta[^{a+h_1}_{b+g_1}]\,\theta[^{a+h_2}_{b+g_2}]\,\theta[^{a-h_1-h_2}_{b-g_1-g_2}]\;\times$$

$$\times\,\Gamma_{8,8}\left[\begin{matrix} a & , & \gamma \\ b & , & \delta \end{matrix}\,\middle|\,\begin{matrix} h_i \\ g_i \end{matrix}\right]\;\times \sum_{\gamma,\delta}\bar\theta[^\gamma_\delta]^5\,\bar\theta[^{\gamma+h_1}_{\delta+g_1}]\,\bar\theta[^{\gamma+h_2}_{\delta+g_2}]\,\bar\theta[^{\gamma-h_1-h_2}_{\delta-g_1-g_2}]\,\sum_{\bar a,\bar b}\bar\theta[^{\bar a}_{\bar b}]^8\,,$$

$\Gamma_{8,8}\left[\begin{matrix} a & , & \gamma \\ b & , & \delta \end{matrix}\,\middle|\,\begin{matrix} h_i \\ g_i \end{matrix}\right]$ indicates the contribution of the eight fermionized coordinates

$$\{y^I,\omega^I\,|\,\bar y^I,\bar\omega^I\}$$

The $MSDS$-structure follows from the holomorphic side.
The full partition function for this representative example is :

$$Z = 12\bar j(\bar\tau) = 12\;\times 744 + 12\;[\,\bar j(\bar\tau) - 744\,]$$

Inserting in the representative model all possible discrete torsion coefficients permitted by the fermionic construction, a plethora of $MSDS$ Heterotic models can be obtained.

The resulting models will in general exhibit different bosonic and fermionic massless spectra in different representations of the chiral (right-moving) gauge group

$$H_R = SO(10) \times U(1)^3 \times SO(16)$$

The moduli space contains a subspace of would-be *geometrical* M_{IJ}*-deformations* associated with the conventional supersymmetric $Z_2 \times Z_2$ freely acting orbifolds.

The $Z_2 \times Z_2$ action reduces the deformation space:

$$Z_2 \times Z_2 : \ \frac{SO(8,8)}{SO(8) \times SO(8)} \longrightarrow \frac{SO(4,4)}{SO(4) \times SO(4)} \times \frac{SO(2,2)}{SO(2) \times SO(2)} \times \frac{SO(2,2)}{SO(2) \times SO(2)}$$

Assuming very large deformations in the $(2,2)$ sub-space of $SO(4,4)$, a 4d flat space-time is generated, together with an internal 6-dimensional compact space described by

$$\frac{T^6}{Z_2 \times Z_2}$$

.

This class of models is connected with the 4d semi-realistic $N = 1$ chiral vacua based on $SO(10)$. The $N = 1$ supersymmetry appears broken spontaneously by very specific geometrical fluxes!

In the Euclidian version the deformed $MSDS$ correspond to "thermal stringy vacua" in the presence of non-trivial left-right asymmetric "gravito-magnetic fluxes".

The deformed integrated partition function becomes a non trivial function of the

- Temperature scale T
- SUSY breaking scale M
- All other moduli μ_I

14. Cosmological evolution in late times

i) Exit from $MSDS$ era:

Once the free energy is positive (negative pressure) the $MSDS$ vacuum is unstable
$\longrightarrow$ the moduli evolves towards larger values such that:

$$M, T << M_{string} \quad \longrightarrow \quad \text{Deformed } MSDS\text{-vacua at } t = t_{exit}.$$

• This transition will occur when $n(f) > n(b)$ so that the $MSDS$ partition function is negative.

ii) Intermediate cosmological era $t_{exit} \leq t \leq t_w$:

After the "$MSDS$ transition exit" $t \geq t_{exit}$
and before the electroweak symmetry breaking phase transition $t \leq t_w$

This cosmological phase was extensively studied in collaboration with:
F. Bourliot, J. Estes and H. Partouche
T. Catelin-J, H. Partouche and N. Toumbas

We show that cosmological evolution is attracted to "radiation-like" evolution in an effective d-dimensional space-time :

$$RDS^d \ : \qquad M(t) \sim T(t) \sim 1/a(t) \sim t^{-2/d}, \quad \text{for} \quad t \geq t_{exit},$$

• This evolution is unique and stable at late times in certain physically relevant SUSY breaking schemes (structure of the fluxes).

Furthermore, for $T, M << M_{string}$ only the SUSY breaking moduli $M(t)$ and $T(t)$ can give a relevant contribution to the free energy $\mathcal{F}(T, M)$.

All other moduli, μ_I

- are attracted and stabilized to the extended gauge symmetry points, $\mu_I \sim M_{\rm string}$

OR

- are effectively frozen to an arbitrary value such that $\mu_I \gg T, M$

In both cases, their contribution to $\mathcal{F}$ is exponentially suppressed.

$$\mathcal{F}(T, M, \mu_I) = \mathcal{F}(T, M) + \mathcal{O}\left[\exp(-\frac{\mu_I}{T}), \exp(-\frac{\mu_I}{M})\right]$$

Finally the limitation $t \leq t_w$ follows from the appearance of a new scale in low energies:
"The infrared renormalisation group invariant transmutation scale Q"
of the effective field theory.

At this scale the $(\text{mass})^2$ of the SUSY standard model Higgs becomes negative:

$\longrightarrow$ no-scale radiative breaking of $SU(2) \times U(1) \to U(1)_{\rm em}$.

- Q is irrelevant when $M(t), T(t) \gg Q$

- Q becomes relevant and stops the evolution of $M(t)$ when $M \sim Q$ at $t \sim t_w$ and the electroweak breaking phase transition takes place.

The physics for $t \gg t_w$ is of main importance in particle physics and cosmology.

- Unfortunately the infrared phase at $t \gg t_w$, depends strongly on the specific initial $MSDS$-vacuum data as well as on the specific evolution of the deformation moduli.

$\longrightarrow$ A lot of work is necessary to select the initial $MSDS$-vacuum that leads to the late-time precise structure of our universe.

On the other hand, I would like to stress that the qualitative infrared behavior of the effective "no-scale" field theory, strongly suggests that we are definitely in an interesting "non-singular string evolutionary scenario connecting particle physics and cosmology"