Cosmological phases of the string thermal effective potential

John Estes[1] François Bourliot [1] Costas Kounnas [2] Hervé Partouche [1]

[1]Centre de Physique Théorique, Ecole Polytechnique,
F–91128 Palaiseau cedex, France

[2]Laboratoire de Physique Théorique, Ecole Normale Supérieure,
24 rue Lhomond, F–75231 Paris cedex 05, France

September 11, 2009

Introduction

Previous talks:

- Back-reaction of gas of strings at finite temperature on static flat background
- Leads to quasi-static evolution:
  - Radiation dominated Universes
  - Ratio of supersymmetry breaking scale to temperature stabilized
  - Other moduli were taken fixed near the string scale

This talk:

- Relax fixed moduli condition and study the dynamics of the spectator moduli.
- In particular, we study the internal radii-moduli of the Heterotic string.

Pure thermal case

Initial background:

- Flat four-dimensional Minkowski space-time
  - $ds_4^2 = -dt^2 + a(t)^2 ds_3^2$
  - Without thermal effects $a(t) = \text{const}$
- Six-dimensional internal space of the form:
  - Case (I): $S^1(R_4) \times \mathcal{M}_5$
  - Case (II): $(S^1(R_4) \times T^3)/\mathbb{Z}_2 \times \mathcal{M}_2$

Introduce temperature: $S(R_0) \times T^3(\text{space-time}) \times \mathcal{M}_6(R_4)$

- Temperature: $T \sim e^{\phi}/2\pi R_0$
- Back-reaction of temperature/gas of strings captured by the free energy/string partition function
  - $\mathcal{F} = -Z_{\text{connected}}/V_4$

Equations of motion

Thermodynamics

- Pressure:
  $$P = -\mathcal{F}$$
- Equation of state:
  $$\rho = T\frac{\partial P}{\partial T} - P$$
- For $P \sim T^4$ we find $\rho = 3P$, equation of state for radiation

Variational principle

- $S = S_{\text{classical}} - \int d^4x\sqrt{g}\mathcal{F}$
- Thermal energy-momentum tensor: $\mathcal{T}_{\text{therm}\,\mu\nu} = -g_{\mu\nu}\mathcal{F} + 2\frac{\partial\mathcal{F}}{\partial g^{\mu\nu}}$
  - $a^2P = \mathcal{T}_{\text{therm}\,ii} = -a^2\mathcal{F}$
  - $\rho = \mathcal{T}_{\text{therm}\,00} = \mathcal{F} - 2T^2\frac{\partial\mathcal{F}}{\partial T^2}$

Equations of motion

Variables

- $\zeta = \log R_4$
- $H = \dot{a}/a$

Einstein equations

- $3H^2 = \frac{1}{2}(\dot{\phi}^2 + \dot{\zeta}^2) + \rho$ Friedmann equation
- $3H^2 + \dot{H} = \frac{1}{2}\rho - \frac{1}{2}P$

$\Rightarrow \partial_t \ln(\frac{\rho+P}{T^4}) + 3H + 3\dot{T}/T = 0$ Conservation of energy/entropy

Scalars

- $\ddot{\phi} + 3H\dot{\phi} = P_\phi$
- $\ddot{\zeta} + 3H\dot{\zeta} = P_\zeta$

Partition function/free energy

Case I: circle $\mathcal{M}_6 = S^1(R_4) \times \mathcal{M}_5$

$$\mathcal{F}_{(I)} = -Z_{\text{conn}}/V_4 = -T^4 n_T[\tfrac{\pi^4}{48} + k_T(T, |\zeta|, \phi)] - T^4 \tilde{n}_T g_T(T, \zeta, \phi)$$

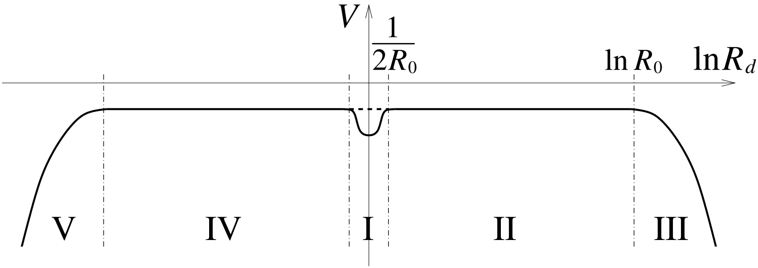
- Exact up to exponentially suppressed terms: $\mathcal{O}(e^{-R_0})$
- k_T is not suppressed for $R_4 > R_0$ or $R_4^{-1} > R_0$
- g_T is not suppressed for $|R_4 - 1| < \frac{1}{2R_0}$
- $R_0 = e^{\phi/\sqrt{2}}/2\pi T$

Definitions

$$k_T(T, \zeta, \phi) = \sum_{k, m \neq 0} \left(\frac{m}{2k+1} R_0 e^{-\zeta} \right)^2 K_2\left(|(2k+1)m| R_0 e^{-\zeta} \right)$$

$$g_T(T, \zeta, \phi) = \sum_k \left(\frac{e^{-\zeta} - e^{\zeta}}{2k+1} R_0 \right)^2 K_2\left(2\pi |(2k+1)(e^{-\zeta} - e^{\zeta})| R_0 \right)$$

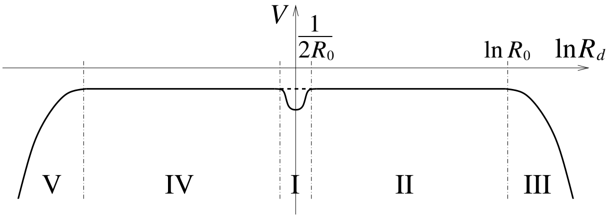
Effective potential for ζ: $V = -P$



Five regions

- (I): Enhanced symmetry region
- (II): Modulus region
- (III): Higher dimensional region
- (IV): T-dual modulus region
- (V): T-dual higher dimensional region

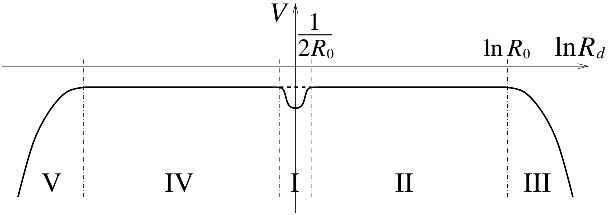
Region (I): enhanced symmetry point



Region (I)

- $P \simeq T^4(n_T + \tilde{n}_T)[\frac{\pi^4}{48}] - T^2\tilde{n}_T e^{\sqrt{2}\phi}\zeta^2[\frac{\pi^2}{4}] + \mathcal{O}(\zeta^4)$
- $\zeta = 0$ is solution with $\rho \sim 3P \Rightarrow$ radiation dominated solution
- First order correction away from $\zeta = 0$ pushes ζ back towards zero
- Potential becomes steeper as T decreases
- $\ddot{\zeta} + 3H\dot{\zeta} = P_\zeta$

Region (II): plateau



Region (II)

- $P \simeq T^4 n_T[\frac{\pi^4}{48}]$
- $\dot{\zeta} = 0$ is a solution with $\rho \sim 3P \Rightarrow$ radiation dominated solution
- ζ is a modulus taking any value on the plateau
- Marginally stabilized by ”gravitational friction"
- $\ddot{\zeta} + 3H\dot{\zeta} = P_\zeta$

Careful analysis

Difficulty

Scalar equations depend on $a(t)$ through H and T dependence

- Re-parameterize the time "t" in terms of "ln a" so that $\dot{\phi} = H\mathring{\phi}$
- $\rho = T^4 r(\zeta, \phi, a)$
- $P = T^4 p(\zeta, \phi, a)$

Scalar equations

- $h\overset{\circ\circ}{\phi} + \frac{1}{2}(1 - \frac{p}{r})\mathring{\phi} = p_\phi / r$
- $h\overset{\circ\circ}{\zeta} + \frac{1}{2}(1 - \frac{p}{r})\mathring{\zeta} = p_\zeta / r$
- $h = \frac{1}{3 - \frac{1}{2}(\mathring{\phi}^2 + \mathring{\zeta}^2)}$
- Can show analytically that perturbations around RDS are stable

Region (III): large ζ behavior

Region (III)

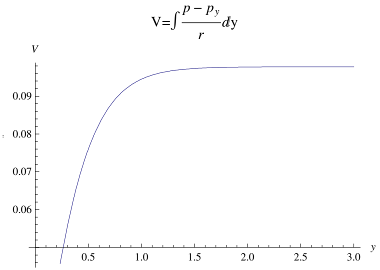
$P \simeq T^4 n_T \sum_{m,k} \frac{3}{4} \frac{e^y}{[(2k+1)^2 + m^2 e^{2y}]^{5/2}}$

- Introduce: $e^y = R_4/R_0 = e^\zeta 2\pi T e^{-\phi/\sqrt{2}}$
- $P_\zeta \neq 0$ so $\dot{\zeta} = 0$ is not a solution
- $P_y = P_\zeta = -\sqrt{2} P_\phi \Rightarrow \phi_\perp \equiv \zeta + \sqrt{2}\phi$ is a modulus

Change variables to $\phi_\perp$ and y

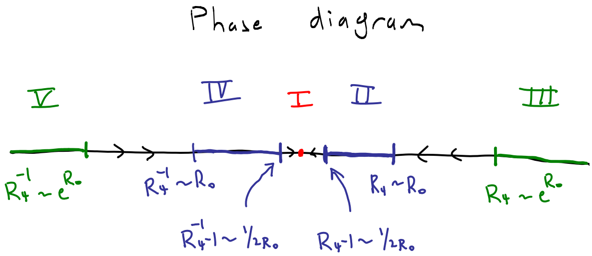
- $h\left(\overset{\circ\circ}{y} + \frac{1}{3}\ln(\overset{\circ\circ}{r} + p)\right) + \frac{1}{2}(1 - \frac{p}{r})\left(\overset{\circ}{y} + \frac{1}{3}\ln(\overset{\circ}{r} + p)\right) = \frac{p_y - p}{r}$
- $h\overset{\circ\circ}{\phi}_\perp + \frac{1}{2}(1 - \frac{p}{r})\overset{\circ}{\phi}_\perp = 0$
- Equation of state: $\rho = T\frac{\partial P}{\partial T} + \frac{\partial P}{\partial y} - P$

Region (III): large ζ behavior



- $V_y = \frac{p - p_y}{r} > 0 \Rightarrow$ force pushing y towards negative values
- For large y: $V_y \simeq e^{-4y}$
- For e^{4y} negligible, $V_y \simeq 0$ with $\rho = 4P \Rightarrow$ five-dimensional RDS
- For e^{4y} not negligible, y decreases and we enter region (II)
- Recall, we have already dropped terms of order e^{-R_0} in the partition function

Phase diagram



Five radiation dominated phases

- (I): Enhanced symmetry phase
- (II): Modulus phase
- (III): Higher dimensional phase (stable when $(R_0/R_4)^d$ is negligible)
- (IV): T-dual modulus phase
- (V): T-dual higher dimensional phase (stable when $(R_0 R_4)^d$ is negligible)

Orbifold

Case II: orbifold $\mathcal{M}_6 = (S^1(R_4) \times T^3)/\mathbb{Z}_2 \times \mathcal{M}_2$

$$\mathcal{F}_{(II)} = \frac{1}{2}\mathcal{F}_{\text{un-twisted}} + \frac{1}{2}\mathcal{F}_{\text{twisted}}$$

$$= -T^4 \frac{n_T}{2}[\frac{\pi^4}{48} + k_T(T, |\zeta|, \phi)] - T^4 \frac{\tilde{n}_T}{2} g_T(T, \zeta, \phi) - T^4 \frac{n_T^t}{2} \frac{\pi^4}{48}$$

Modifications of the effective potential for $\zeta = \ln(R_4)$

- Region (I): $n_T + \tilde{n}_T \to \frac{1}{2}(n_T + \tilde{n}_T + n_T^t)$
- Region (II): $n_T \to \frac{1}{2}(n_T + n_T^t)$
- Region (III): Repulsion from higher dimensional phase is stronger. For large y, $V_y \sim e^{-y}$ so flat potential phase when e^{-y} can be neglected

Susy breaking

Introduce temperature and SUSY breaking: $S(R_0) \times T^3(\text{space-time}) \times \mathcal{M}_6(R_4, R_5)$

- Temperature: $T \sim e^{\phi}/2\pi R_0$
- SUSY breaking scale $M \sim e^{\phi}/2\pi R_5$
- $\mathcal{M}_6(R_4, R_5) = S^1(R_4) \times S^1_S(R_5) \times \mathcal{M}_4$
- $z = M/T$

Scalar equations of motion

- $\ddot{\phi}_\perp + 3H\dot{\phi}_\perp = P_{\phi_\perp}$
- $\ddot{\zeta} + 3H\dot{\zeta} = P_\zeta$
- $\left(\ddot{z} + \frac{1}{3}\ln(\ddot{\frac{\rho+P}{T^4}})\right) + 3H\left(\dot{z} + \frac{1}{3}\ln(\dot{\frac{\rho+P}{T^4}})\right) = P_z + P$

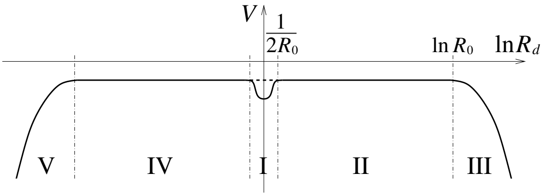
Partition function/free energy

$\mathcal{M}_6(R_4, R_5) = S^1(R_4) \times S^1_S(R_5) \times \mathcal{M}_4$

$$P = T^4 n_T[f_T(z) + k_T(z, \eta - |\zeta|)] + T^4 n_V[f_V(z) + k_v(z, \eta - |\zeta|)] \\ + T^4 \tilde{n}_T g_T(z, \eta, |\zeta|) + T^4 \tilde{n}_V g_V(z, \eta, |\zeta|)$$

- $z = R_0/R_5 \qquad \zeta = \ln R_4 \qquad \eta = \ln R_5$
- $n_T, \tilde{n}_T, \tilde{n}_V > 0$ while n_V may take negative values
- Exact up to exponentially suppressed terms: $\mathcal{O}(e^{-R_0})$
- $g_{T(V)}$ is suppressed for $|R_4 - 1| > \frac{1}{2R_{0(5)}}$
- $k_{T(V)}$ is suppressed for $R_4 < R_{0(5)}$

Effective potential for ζ: $V = -P$



Regions (I) and (II)

- Story for ζ same as in the pure thermal case
- $\tilde{n}_T, \tilde{n}_V > 0 \Rightarrow$ always a minimum at enhanced symmetry point
- Region (I): RDS solution at $\zeta = 0$ with $z = z_c$ for $-\frac{1}{15} < \frac{n_V + \tilde{n}_V}{n_T + \tilde{n}_T} < 0$
- Region (II): RDS solution with $z = z_c$ for $-\frac{1}{15} < \frac{n_V}{n_T} < 0$
- z_c defined by $\rho(z_c) = 4P(z_c)$

Large ζ behavior

Change variables to $y = R_4/R_0$

- $h\left(\overset{\circ\circ}{y} + \frac{1}{3}\ln(\overset{\circ\circ}{r+p})\right) + \frac{1}{2}(1 - \frac{p}{r})\left(\overset{\circ}{y} + \frac{1}{3}\ln(\overset{\circ}{r+p})\right) = \frac{p_y - p}{r}$

Region (III)

- For large y, $V_y = \frac{p - p_y}{r} \simeq e^{-4y}(n_T + n_V) > 0 \Rightarrow$ force always pushes y towards negative values
- For e^{4y} negligible, $V_y \simeq 0$, five-dimensional RDS with z_c defined by $\rho(z_c) = 5P(z_c)$ for $-\frac{1}{31} < \frac{n_V + \tilde{n}_V}{n_T + \tilde{n}_T} < 0$
- For e^{4y} not negligible, y decreases and we enter region (II)

Type II theories

Perturbative

- No enhancement of massless states at self-dual point
- $\tilde{n}_T = \tilde{n}_V = 0 \Rightarrow$ region (I) becomes flat with regions (II) and (IV)
- By Heterotic-Type II duality we expect the $SU(2)$ phase to exist

Proposal for non-perturbative

- Enhancement of massless states obtained by considering D-branes with separation of branes playing the role of R_4
- For the branes close to each other, there is an attraction as in region (I)
- For the branes far enough apart their separation becomes stable as in region (II)
- Further increasing their distance the branes start to seperate as in region (III)

Summary of results

- Five phases
  - (I): Enhanced symmetry phase
  - (II): Modulus phase
  - (III): Higher dimensional phase (stable when $(R_0/R_d)^d$ is negligible)
  - (IV): T-dual modulus phase
  - (V): T-dual higher dimensional phase (stable when $(R_0 R_d)^d$ is negligible)
- Stabilization at enhanced symmetry points
- Compact directions never de-compactify