Gravitational anomaly and fundamental forces

J. J. van der Bij
Institut für Physik
Albert-Ludwigs Universität Freiburg

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Is there a reason for the choice of gauge group and representations?

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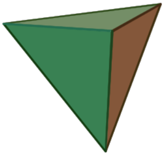
I. Rabi: Who ordered that?

on the discovery of the muon

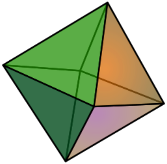
A. Einstein: Did God have a choice when he created the world?

Leukippos, Demokritos
Emptiness $\longrightarrow$ space
Fullness $\longrightarrow$ atoms

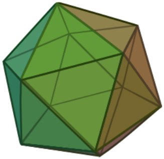
Plato, Empedokles, Aristoteles
Elements



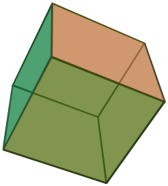
Tetrahedron
(fire)



Octahedron
(air)



Icosahedron
(water)



Cube
(earth)

Dodekahedron $\longrightarrow$ fifth element (quintessence)
prediction of the theory
mathematical basis

Space $\longrightarrow$ Spacetime

with structure: $R_{\mu\nu\alpha\beta}$

and dynamic: $R_{\mu\nu} = \kappa T_{\mu\nu}$ (Einstein)

Matter: gauge theories (symmetry)

Groups and representations
Many possibilities
Constraint: anomaly cancellation

Symmetry breaking (Higgs): not considered here

Ellis, Gaillard, Zumino (1980)

$N = 8$ Supergravity
$SO(8)$ symmetry too small
Hidden $SU(8)$

assumption 1: $SU(8)$ becomes dynamical
assumption 2: also superpartners dynamical
assumption 3: anomaly free subset of $SU(5)$
assumption 4: non-chiral part mass to Planck mass
Leaves $3(5 + \bar{5}) + 9(1)$
and supersymmetry, which has to be broken

Nowadays $N = 8$ supergravity is not anymore considered fundamental (non-renormalizable).

Gross, Harvey, Martinec, Rohm (1985)

Heterotic string
Anomaly free superstring 10 dimensions
Gauge group $E(8) \times E(8)$

assumption 1: compactification to 4-D
assumption 2: one $E(8)$ disappears
assumption 3: Calabi-Yau manifold to break $E(8)$
assumption 4: topology to get 3 generations
assumption 5: some form of supersymmetry breaking

Nowadays string theory has many vacua and no unique gauge theory is expected.

What do we learn?

- Assuming one knows the fundamental laws of physics and only has to construct the standard model out of these is not very promising. Therefore use experiment.
- anomalies are important
- topology is important

Vector bosons

gauge group seen: $SU(3) \times SU(2) \times U(1)$
natural embedding in $SU(5)$

$$SU(3) \times SU(2) = \begin{pmatrix} * & * & * & & \\ * & * & * & & \\ * & * & * & & \\ & & & * & * \\ & & & * & * \end{pmatrix}$$

$$U(1) = \begin{pmatrix} +2 & & & & \\ & +2 & & & \\ & & +2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

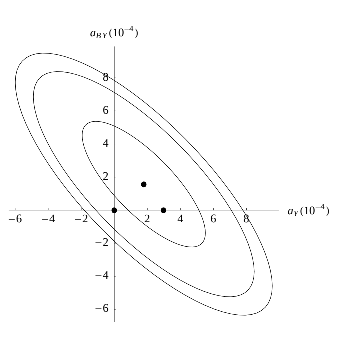
No sign for a higher rank gauge group

Fitting analysis: χ^2 plot (I)

- Significance plot for the parameter space a_Y, a_{BY} for $P_{10}(\chi^2)$. The contours acumulate 68%, 90% and 95% CL.

The three dots correspond to the models' *best point*:

- SM at origin
- a_Y at x-axis,
- a_Y, a_{BY} at contour center



Fermions (lefthanded) under $SU(5)$

$$\bar{5} = \begin{pmatrix} \bar{d} \\ \bar{d} \\ \bar{d} \\ e^- \\ \nu_e \end{pmatrix}$$

$$10 = \begin{pmatrix} 0 & \bar{u} & -\bar{u} & u & d \\ & 0 & \bar{u} & u & d \\ & & 0 & u & d \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}$$

$$1 = \bar{\nu}_e$$

$10 + \bar{5} + 1 = 16$ of $SO(10)$
Therefore automatically (chiral) anomaly free

we have to explain

- Vektorbosons $\rightarrow SU(5)$
- Fermions $\rightarrow SO(10)$
- 3 generations of fermions

  $Z \rightarrow \nu\bar{\nu}$

  $\delta\rho$

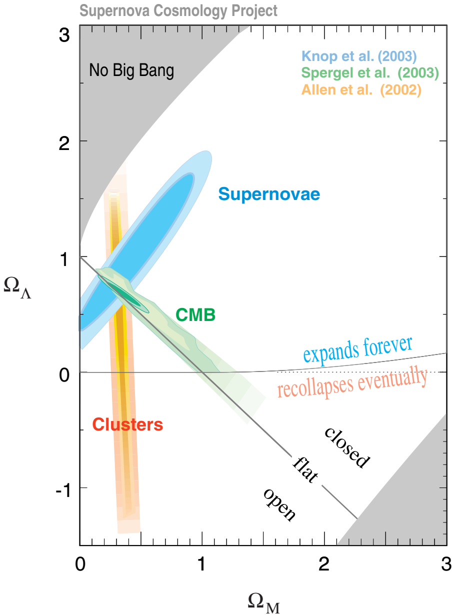
  δM_Z

The only known indication is an anomaly, possibly related to topology

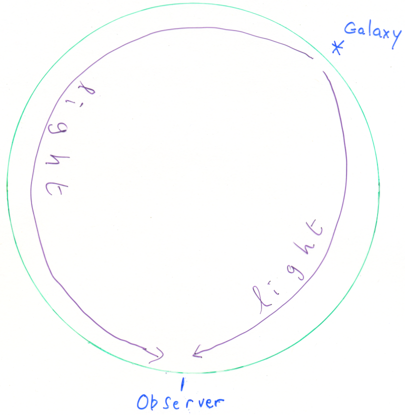
Example of topology in the universe

Higher dimensional Kaluza-Klein universe
Example: $M_4 \times U(1)^n$ with radius of U(1) going to zero
Difficult in practice

Therefore we try the opposite and assume that the universe was three dimensional


Supernova Cosmology Project
No Big Bang
Knop et al. (2003)
Spergel et al. (2003)
Allen et al. (2002)
Supernovae
CMB
expands forever
recollapses eventually
Clusters
closed
flat
open
Ω_Λ
Ω_M
3
2
1
0
-1
0
1
2
3

Cosmic topology



- multiple images: difficult, evolution
- cosmic microwave background: circles in the sky (lack of power in quadrupole ??)

Theory: In flat space, topology can always be at too large a scale to be seen directly

Bianchi-I universes

Flat, homogeneous, non-isotropic
Pancake picture

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2) + b^2(t)dz^2$$

dx,dy topology R^2; dz topology S_1

Late in time $\frac{\dot{a}}{a} \sim \frac{\dot{b}}{b}$, therefore isotropy

This is generically true when there is a positive cosmological constant (Wald's theorem 1983)
So present day isotropy says little about the (very) early universe.

At small t approximate Kasner solution (1925)

$$ds^2 \cong -dt^2 + dx^2 + dy^2 + t^2 dz^2$$

therefore the third dimension gets compactified to zero at early times

For instance exact dust universe ($\lambda = 0$)

$$g_{zz} = M^{1/3} t/(t + \Sigma)^{1/3}$$

$$g_{xx} = g_{yy} = M^{2/3}(t + \Sigma)^{2/3}$$

M and Σ are integration constants

Suggested topology of the universe $M_3 \times S_1$

S_1 The radius may be too large to see the topology at the present time
However a preferred direction may be visible

There appears to be an allignment of low multipoles along a preferred axis in the data

This could be explained in an inflationary Bianchi-I model

3 dimensional Yang-Mills theory

$$\mathcal{L} = -\frac{1}{2} Tr\, F_{\mu\nu} F^{\mu\nu} - i\, m\, \epsilon^{\mu\nu\rho}\, Tr(A_\mu \partial_\nu A_\rho + A_\mu A \nu A \rho)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$$

There is a gauge invariance under $A_\mu \to U^{-1}(\partial_\mu / g + A_\mu) U$
Under large gauge transformations the action shifts by
$8\pi^2 im/g^2$

So full invariance leads to a quantization condition

$g_{YM} = \frac{4\pi m}{g^2}$ must be integer

Renormalization

$$q_{YM}^{ren} = q_{YM}^{0} + C(G) + sign(m_f) N_f C(R)$$

$SU(N)$: $C(G) = N$
fundamental fermion: $C(R) = 1/2$
So one needs an even number of fermions. This stays true even when $m_f = 0$.

In four dimensions there is a similar effect when one starts with Weyl-fermions in a $M_3 \times S_1$ spacetime. There a Chern-Simons like term is generated

$$\mathcal{L}_{CSlike} = m_{ph} n^{\alpha} \epsilon_{\alpha\mu\nu\rho} A^{\mu} F^{\nu\rho}$$

Three dimensional gravity

$$\mathcal{L} = -(1/\kappa^2)\sqrt{g}R - \frac{i}{4\kappa^2\mu}\epsilon^{\mu\nu\lambda}(R_{\mu\nu ab}\omega_\lambda^{ab} + \frac{2}{3}\omega_{\mu a}^{b}\omega_{\nu b}^{c}\omega_{\lambda c}^{a}).$$

$q_{gr} = \frac{6\pi}{\mu\kappa^2}$ must be integer

Renormalization

$$q_{gr}^{ren} = q_{gr}^{0} + \frac{1}{8}N_g\ sign(m_g) - \frac{1}{16}\,N_f\,sign(m_f)$$

N_g is the number of vector bosons
N_f is the number of fermions

assume $q_{gr} = 0$ (Einstein equations)
consistency: $N_f \mp 2N_g = 0 \mod(16)$

Stronger conditions

isotropization: $q_{gr}^{ren} = 0$

vectors and fermions separately consistent:

$$N_g = 0 \mod(8)$$

$$N_f = 0 \mod(16)$$

In combination vectors $SU(5)$: 24
fermions $SO(10)$: 16

$$2 \times 24 - 3 \times 16 = 0$$

Basically unique if also:
1) fermions automatically anomaly free, i.e. no $SU(n)$:
2) fermions in fundamental representation

Speculations

- Symmetry breaking:
  $SU(5)$ decomposition: $16 = 10 + \bar{5} + 1$.

$$SU(3) \to +, \quad SU(2) \to -, \quad U(1) \to +$$

$$10 \to +, \qquad \bar{5} \to -, \qquad 1 \to -$$

  $2 \times (8 - 3 + 1) - 3 \times (10 - 5 - 1) = 0$
  possible: $SU(5) \to SU(3) \times SU(2) \times U(1)$
  impossible: $SU(5) \to SU(4) \times U(1)$
- more conditions
- other compactifications
- underlying structure
- quantum gravity

Rabi's question: who ordered that?

Answer: the early universe.

Einstein's question: did God have a choice?

Answer: no, because He has to use perfect symmetry.
However the devil may have had something to do with the Higgs sector.