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Orientifolds of N=2 gauged linear sigma-models

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Outline

- Linear sigma models
- Parity symmetries of linear sigma models
- Moduli spaces of orientifolds (bulk moduli)
- D-branes in linear sigma-models and orientifolds
- Summary, conclusions

Based on arXiv:0812.2880 (with Manfred Herbst)

Orientifolds and Calabi-Yau compactifications

- Geometric description: formulate orientifold using an involution of the target space geometry
  - B-type: holomorphic involution
  - A-type: anti-holomorphic involution
- Stringy regime: At Gepner point description in terms of rational conformal field theory possible Sagnotti etal
  - Construct boundary and crosscap states
  - Solve tadpole conditions
  - Read off low energy spectra
- Look for a framework that allows to interpolate between stringy and geometric regime: Linear sigma model.
- Study dependence on Kähler moduli.

Linear sigma models

- Two-dimensional gauge theory
  - Gauge group $U(1)^k$
  - Matter chiral superfields Φ_i , charged under the various $U(1)$
  - Action

$$S = S_{gauge} + S_{matter-kin} + S_{FI} + S_W$$

- S_{FI} contains the complexified Fayet-Iliopoulos parameter

$$t^a = r^a - i\theta^a,$$

  t parametrizes the Kähler moduli space.
- The potential for the scalar fields has been analyzed by Witten (1993). It depends on r^a, and for different limits of r^a one obtains different "phases" of the theory.

Phases of the linear sigma model

- Gauge group $U(1)^k$, N chiral superfields Φ_i with scalar components ϕ_i, charges Q_i^a.
- D-term $\sum_{i=1}^N Q_i^a |\phi_i|^2 = r^a$
- F-term $\partial_i W = 0$
- Example quintic: 6 fields $X_1, \ldots X_5, P$, charges $q(X_i) = 1$, $q(P) = -5$
$$\sum |x_i|^2 - 5|p|^2 - r = 0$$
- $r >> 0 \Rightarrow x_i$ cannot all be 0, they become coordinates of CP_4, together with the F-term we obtain the quintic hypersurface in $CP_4 \rightarrow$ geometrical phase.
- $r << 0 \Rightarrow p$ cannot be 0, gauge symmetry is broken to Z_5.$\rightarrow$ Landau-Ginzburg (orbifold) phase.

Parity actions in $N = (2,2)$ supersymmetric theories

- Two-dimensional superspace: $\sigma^{\pm} = \tau \pm \sigma$, $\theta^{\pm}, \bar{\theta}^{\pm}$.
- Superderivatives:

$$D_{\pm} = \frac{\partial}{\partial\theta^{\pm}} - i\bar{\theta}^{\pm}\frac{\partial}{\partial\sigma^{\pm}}, \quad \bar{D}_{\pm} = -\frac{\partial}{\partial\bar{\theta}^{\pm}} + i\theta^{\pm}\frac{\partial}{\partial\sigma^{\pm}}$$

- B-parity:$\sigma^{\pm} \to \sigma^{\mp}$, $\theta^{\pm} \to \theta^{\mp}$, $\bar{\theta}^{\pm} \to \bar{\theta}^{\mp}$
- Chiral fields $\bar{D}_{\pm}\Phi = 0$ get mapped to chiral fields
- Twisted chiral fields $\bar{D}_{+}T = D_{-}T = 0$ get mapped to twisted anti-chiral fields fulfilling $\bar{D}_{-}T = D_{+}T = 0$

Parity symmetries in linear sigma models

- We have an explicit Lagrangian, and this way can determine the available parity symmetries.
- The superpotential term:

$$\int d\theta^+ d\theta^- W(P, X_i)$$

  W is a polynomial in the chiral matter fields. Invariance under B-parity requires $W \to -W$.
- The twisted chiral superpotential (FI-part of the action)

$$\tilde{W} = t^a \Sigma_a,$$

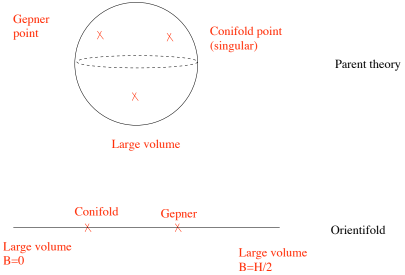
  where Σ_a is the field strength of the $U(1)_a$ gauge group.
- Important consequence of invariance under a B-parity:

$$t^a = \bar{t^a} \mod 2\pi i \to \theta = 0, \pi$$

- The Kähler moduli space is divided into real slices.

Example: B-type orientifold of the quintic

- The Kähler moduli space of the quintic $x_1^5 + \cdots + x_5^5 = 0$.



- The stringy Gepner point is connected to large volume with $B \neq 0$, but separated by a singular point from $B = 0$.
- No smooth extrapolation of the physics through the singular point.

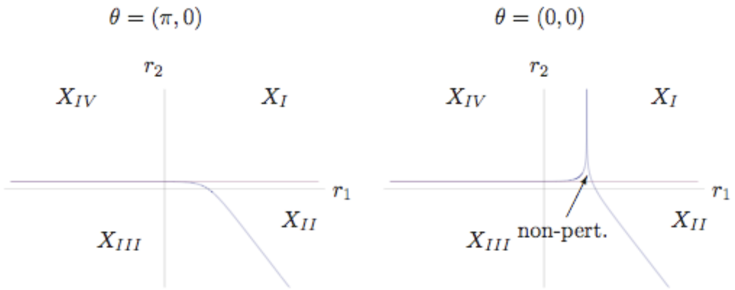
Example: A two-parameter model

- In the geometric phase, the model corresponds to the degree 8 hypersurface in the weighted projective space $P_{(11222)}$:

$$x_1^8 + x_2^8 + x_3^4 + x_4^4 + x_5^4 = 0$$

- In the linear sigma model, it corresponds to a theory with gauge group $U(1)^2$ and chiral superfields $X_1, \ldots, X_6, P$.
- The model has four different phases, two of which are geometric.
- Depending on the discrete values for θ the orientifold moduli space does (or doesn't) intersect with the singular locus.
- For $\theta = (0, \pi), (\pi, \pi)$ one can move smoothly between the phases
- For $\theta = (0, 0), (\pi, 0)$ this is not possible.

Phases of the two-parameter orientifold



- The figure shows the intersection of the orientifold moduli space with the singular set.
- The moduli space is divided into distinct sectors and one cannot cross from one part to the other.
- On the right hand side, there is a region in the moduli space that is disconnected from all perturbative limits.

Linear sigma models with boundary

- To study D-branes we must look at linear sigma models on surfaces with boundary. Herbst, Hori, Page
- Add Wilson line terms

$$P\ exp\left\{i\int_{\partial\Sigma} ds\mathcal{A}\right\}$$

  [carries a representation of the gauge group]
- General D-brane: Pile up a stack of Wilson line branes and turn on a tachyon profile Q among the individual components.
- To preserve B-type supersymmetry, Q must fulfill $Q^2 = W$.
- The linear sigma model allows to move between large and small radius regime. One can recover Landau-Ginzburg branes (given by matrix factorizations + representation labels of the discrete orbifold group) in the stringy regime and geometric B-branes in the large radius regime.

Parity action on D-branes

- If parity is combined with an involution τ then

$$Q(\Phi_i) \rightarrow -\tau^* Q(\Phi_i)^T$$

  T denotes a graded version of the ordinary transpose.
- Likewise, open string fields get mapped to their transpose

$$\psi(\Phi_i) \rightarrow \tau^* \psi(\Phi)^T$$

- We have learned how to obtain D-branes compatible with the orientifold at any point in moduli space. We can transport D-branes from one point to another in orientifold moduli space, generalizing results of Herbst-Hori-Page. [Recover results on orientifold brane category in the extreme limits. Diaconescu etal, Hori-Walcher]

Type of orientifold

- Orientifolds can be of SO- or Sp-type.
- Using our framework we can determine the type using probe D-branes on top of the (different components of) the orientifold fixed point set at any point in Kähler moduli space.
- This includes e.g. different B-fields in the large radius regime.
- The type of orientifold depends on the Kähler moduli, at large volume "compactification without vector structure".
- It can in particular change when connecting two different large volume points on a path in the stringy regime (involving special points).

Conclusions

- The linear sigma model provides a way to study the moduli space of Calabi-Yau orientifold theories
- For B-type parities, the Kähler moduli space consists of several real slices. These can intersect with singular loci such as conifold points, leading to a "complicated" structure of the moduli space.
- Using probe D-branes, we can derive the type of the orientifold anywhere in moduli space.
- Components of the orientifold can change type when navigating through the stringy regime.
- Aside: We can provide a IIB worldsheet explanation for effects observed in the work of Collinucci-Denef-Esole and Braun-Hebecker-Triendl in the F-theory context.