Orientifolds of N=2 gauged linear sigma-models

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Outline

- Linear sigma models
- Parity symmetries of linear sigma models
- Moduli spaces of orientifolds (bulk moduli)
- D-branes in linear sigma-models and orientifolds
- Summary, conclusions

Based on arXiv:0812.2880 (with Manfred Herbst)
Orientifolds and Calabi-Yau compactifications

- Geometric description: formulate orientifold using an involution of the target space geometry
  - B-type: holomorphic involution
  - A-type: anti-holomorphic involution

- Stringy regime: At Gepner point description in terms of rational conformal field theory possible
  - Construct boundary and crosscap states
  - Solve tadpole conditions
  - Read off low energy spectra

- Look for a framework that allows to interpolate between stringy and geometric regime: Linear sigma model.

- Study dependence on Kähler moduli.
Linear sigma models

- Two-dimensional gauge theory
  - Gauge group $U(1)^k$
  - Matter chiral superfields $\Phi_i$, charged under the various $U(1)$
  - Action
    \[ S = S_{\text{gauge}} + S_{\text{matter-kin}} + S_{\text{FI}} + S_W \]
- $S_{\text{FI}}$ contains the complexified Fayet-Iliopoulos parameter
  \[ t^a = r^a - i\theta^a, \]
  $t$ parametrizes the Kähler moduli space.
- The potential for the scalar fields has been analyzed by Witten (1993). It depends on $r^a$, and for different limits of $r^a$ one obtains different “phases” of the theory.
Gauge group $U(1)^k$, $N$ chiral superfields $\Phi_i$ with scalar components $\phi_i$, charges $Q^a_i$.

D-term $\sum_{i=1}^{N} Q^a_i |\phi_i|^2 = r^a$

F-term $\partial_i W = 0$

Example quintic: 6 fields $X_1, \ldots X_5, P$, charges $q(X_i) = 1$, $q(P) = -5$

$$\sum |x_i|^2 - 5|p|^2 - r = 0$$

$r \gg 0 \Rightarrow x_i$ cannot all be 0, they become coordinates of $CP_4$, together with the F-term we obtain the quintic hypersurface in $CP_4 \rightarrow$ geometrical phase.

$r \ll 0 \Rightarrow p$ cannot be 0, gauge symmetry is broken to $Z_5.\rightarrow$ Landau-Ginzburg (orbifold) phase.
Parity actions in $N = (2, 2)$ supersymmetric theories

- Two-dimensional superspace: $\sigma^\pm = \tau \pm \sigma, \theta^\pm, \bar{\theta}^\pm$.
- Superderivatives:

\[ D_\pm = \frac{\partial}{\partial \theta^\pm} - i\bar{\theta}^\pm \frac{\partial}{\partial \sigma^\pm}, \quad \bar{D}_\pm = -\frac{\partial}{\partial \bar{\theta}^\pm} + i\theta^\pm \frac{\partial}{\partial \sigma^\pm} \]

- B-parity: $\sigma^\pm \to \sigma^\mp, \theta^\pm \to \theta^\mp, \bar{\theta}^\pm \to \bar{\theta}^\mp$
- Chiral fields $\bar{D}_\pm \Phi = 0$ get mapped to chiral fields
- Twisted chiral fields $\bar{D}_+ T = D_- T = 0$ get mapped to twisted anti-chiral fields fulfilling $\bar{D}_- T = D_+ T = 0$
We have an explicit Lagrangian, and this way can determine the available parity symmetries.

The superpotential term:

\[ \int d\theta^+ d\theta^- W(P, X_i) \]

\( W \) is a polynomial in the chiral matter fields. Invariance under B-parity requires \( W \rightarrow -W \).

The twisted chiral superpotential (FI-part of the action)

\[ \tilde{W} = t^a \Sigma_a, \]

where \( \Sigma_a \) is the field strength of the U(1)\(_a\) gauge group.

Important consequence of invariance under a B-parity:

\[ t^a = \bar{t}^a \mod 2\pi i \rightarrow \theta = 0, \pi \]

The Kähler moduli space is divided into real slices.
Example: B-type orientifold of the quintic

- The Kähler moduli space of the quintic $x_1^5 + \cdots + x_5^5 = 0$.

- The stringy Gepner point is connected to large volume with $B \neq 0$, but separated by a singular point from $B = 0$.

- No smooth extrapolation of the physics through the singular point.
Example: A two-parameter model

- In the geometric phase, the model corresponds to the degree 8 hypersurface in the weighted projective space $P_{(11222)}$:

\[ x_1^8 + x_2^8 + x_3^4 + x_4^4 + x_5^4 = 0 \]

- In the linear sigma model, it corresponds to a theory with gauge group $U(1)^2$ and chiral superfields $X_1, \ldots, X_6, P$.

- The model has four different phases, two of which are geometric.

- Depending on the discrete values for $\theta$ the orientifold moduli space does (or doesn't) intersect with the singular locus.

- For $\theta = (0, \pi), (\pi, \pi)$ one can move smoothly between the phases.

- For $\theta = (0, 0), (\pi, 0)$ this is not possible.
Phases of the two-parameter orientifold

The figure shows the intersection of the orientifold moduli space with the singular set.

- The moduli space is divided into distinct sectors and one cannot cross from one part to the other.
- On the right hand side, there is a region in the moduli space that is disconnected from all perturbative limits.
To study D-branes we must look at linear sigma models on surfaces with boundary. Herbst, Hori, Page

Add Wilson line terms

\[ P \exp \left\{ i \int_{\partial \Sigma} ds A \right\} \]

[carries a representation of the gauge group]

General D-brane: Pile up a stack of Wilson line branes and turn on a tachyon profile \( Q \) among the individual components.

To preserve B-type supersymmetry, \( Q \) must fulfill \( Q^2 = W \).

The linear sigma model allows to move between large and small radius regime. One can recover Landau-Ginzburg branes (given by matrix factorizations + representation labels of the discrete orbifold group) in the stringy regime and geometric B-branes in the large radius regime.
If parity is combined with an involution $\tau$ then

$$Q(\Phi_i) \rightarrow -\tau^* Q(\Phi_i)^T$$

$T$ denotes a graded version of the ordinary transpose.

Likewise, open string fields get mapped to their transpose

$$\psi(\Phi_i) \rightarrow \tau^* \psi(\Phi)^T$$

We have learned how to obtain D-branes compatible with the orientifold at any point in moduli space. We can transport D-branes from one point to another in orientifold moduli space, generalizing results of Herbst-Hori-Page. [Recover results on orientifold brane category in the extreme limits. Diaconescu et al, Hori-Walcher]
Orientifolds can be of SO- or Sp-type.

Using our framework we can determine the type using probe D-branes on top of the (different components of) the orientifold fixed point set at any point in Kähler moduli space.

This includes e.g. different B-fields in the large radius regime.

The type of orientifold depends on the Kähler moduli, at large volume “compactification without vector structure”.

It can in particular change when connecting two different large volume points on a path in the stringy regime (involving special points).
Conclusions

- The linear sigma model provides a way to study the moduli space of Calabi-Yau orientifold theories.
- For B-type parities, the Kähler moduli space consists of several real slices. These can intersect with singular loci such as conifold points, leading to a “complicated” structure of the moduli space.
- Using probe D-branes, we can derive the type of the orientifold anywhere in moduli space.
- Components of the orientifold can change type when navigating through the stringy regime.
- Aside: We can provide a IIB worldsheet explanation for effects observed in the work of Collinucci-Denef-Esole and Braun-Hebecker-Triendl in the F-theory context.