Attractions to radiation eras in superstring cosmologies

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Introduction

- A small percent of the energy density of the Universe is radiation : The C.M.B. at $T = 3$K.
- In the past, the Universe was smaller, hotter. There has been a Radiation Dominated Era.
- This looks like a black body :
  - A black body is a 3D box T^3, of volume $V = (2\pi R_{\text{box}})^3$, filled with a gas of massless states at temperature T.
  - In quantum canonical ensemble :

$$Z_{\text{th}} = \text{Tr}\, e^{-\beta H} \quad \Longrightarrow \quad F = -\frac{\ln Z_{\text{th}}}{\beta} \quad \Longrightarrow \quad \begin{array}{c}\text{Stefan's law}\\ \rho = 3P \propto T^4\end{array}$$

  - With strings, we have a thermal gas of all (massive) states.

- For the Universe, the pressure pushes the walls of the box : $R_{\rm box} \to R_{\rm box}+dR_{\rm box}$.
- We recompute everything in the torus of radius $R_{\rm box}+dR_{\rm box}$ $\Rightarrow$ Quasi-static evolution $R_{\rm box}(t)$, $P(t)$, $\rho(t)$, $T(t)$.
- Consistent if P, ρ are perturbations : We'll work at weak coupling and 1-loop.

- The final aim is to apply this to models compatible with astroparticle physics and phenomenology :
  - Models with spontaneous susy breaking, $\mathcal{N}=1 \to 0$ in 4D at late times.
  - They have 3 scales : The temperature $T(t)$, the susy breaking scale $M(t)$, the scale factor of the Universe $a(t)$.

- The general picture is :

Intermediate Era :

The evolution is attracted to a Radiation Dominated Solution :

$$T(t) \propto M(t) \propto \frac{1}{a(t)}$$

Late Time Era :

• $T(t)$ approaches the Infrared Renormalization Group invariant scale $Q_{\rm ew}$, at which the MSSM Higgs (mass)^2 becomes negative :

• Radiative breaking $SU(2)\times U(1)\to U(1)_{\rm em}$.

• $M(t)$ is stabilized around $Q_{\rm ew}$. [In sugra : Kounnas, Pavel, Zwirner]

• The MSSM particles get masses $\to$ Matter Dominated Era.

The general picture is :

Early Era :

• $T(t)$ approaches some Hagedorn temperature T_H, where Tr $e^{-\beta H}$ diverges.

• Implement phase transition or use models free of Hagedorn singularity.

For arbitrary I.B.C. at the end of the Hagedorn era, the cosmological evolution is attracted to the RDS.

Intermediate Era :

The evolution is attracted to a Radiation Dominated Solution :

$$T(t) \propto M(t) \propto \frac{1}{a(t)}$$

Plan

- To fix the ideas : Supersymmetric models, at finite T (no M).
- Models with $\mathcal{N} = 1$ or $2 \to 0$, at finite T :
  - Attraction to a Radiation Dominated Solution in 4D.
  - Or dynamical change of space-time dimension $\to$ 5D.

Pure thermal case

- $Z_{\text{th}} = \text{Tr}\, e^{-\beta H} = \int \mathcal{D}\phi\, e^{-S_E[\phi]}$

In Euclidean $S^1(R_0) \times T^3(R_{\text{box}})$
$\beta = 2\pi R_0$

$$F = -\frac{\ln Z_{\text{th}}}{\beta} = -\frac{1}{\beta}\left(Z^{\text{c}}_{\text{1-loop}} + \cdots\right) \simeq -\frac{1}{\beta}\bigcirc \implies -\frac{1}{\beta}$$

In $S^1(R_0) \times T^3(R_{\text{box}}) \times CY$

$$\int_{\mathcal{F}} \frac{d\tau_1 d\tau_2}{2\tau_2} \frac{\beta V}{(2\pi)^4 \tau_2^2} \sum e^{-\pi\tau_2 (Mass)^2} \sum_{\tilde{m}_0, n_0} e^{-\frac{\pi R_0^2}{\tau_2}|\tilde{m}_0 + n_0 \tau|^2} (-)^{a\tilde{m}_0 + b n_0 + \tilde{m}_0 n_0}$$

Oscillators + Internal lattice

Euclidean time lattice

- Finite in the UV (not in field theory) :

$$Z_{\text{genus-1}} = \beta V \frac{1}{(2\pi R_0)^4} \left(n_T \sum_{\tilde{m}_0} \frac{1}{\pi^2 (2\tilde{m}_0 + 1)^4} + \mathcal{O}(e^{-2\pi R_0 (\cdots)}) \right)$$

If $R_0 \gg 1$ ($i.e.$ after Hagedorn era)
and $R_I \simeq 1$ (see next talk)

$1/R_I$ or R_I	for internal KK or windings
1	for oscillators
R_0	for Euclidean time windings

- We could compute P, ρ from F using statistical physics formulas and impose them by hand as sources in Einstein gravity.

- Or, write the effective action of the Lorentzian model :

$$S = \int d^4x \sqrt{-G} \left[e^{-2\phi_{\text{dil}}} \left(\frac{R}{2} + 2(\partial\phi_{\text{dil}})^2 \right) + \frac{Z_{\text{genus-1}}}{\beta V} \right]$$

with the (quasi-static) background of $S^1(R_0) \times T^3(R_{\text{box}})$, back to Lorentzian signature.

In Einstein frame :

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 \left((dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right) , \quad \phi_{\text{dil}}(t)$$

where $N = 2\pi R_0\, e^{-\phi_{\text{dil}}} \equiv \dfrac{1}{T}$ is the laps function

and $a = 2\pi R_{\text{box}}\, e^{-\phi_{\text{dil}}}$ is the scale factor.

- Variation of N, a $\Longrightarrow$ $T_\mu{}^\nu = \mathrm{diag}(-\rho^{\mathrm{tot}}, P^{\mathrm{tot}}, P^{\mathrm{tot}}, P^{\mathrm{tot}})_\mu{}^\nu$

$$= \phi_{\mathrm{dil}} \text{ kinetic term} + \text{1-loop part}$$

$$P = -\frac{Z_{\text{genus-1}}}{\beta V} \quad = -\left(\frac{\partial F}{\partial V}\right)_\beta e^{4\phi_{\mathrm{dil}}}$$

$$\rho = -P + T\,\frac{\partial P}{\partial T} = \frac{1}{V}\left(\frac{\partial(\beta F)}{\partial \beta}\right)_V e^{4\phi_{\mathrm{dil}}}$$

In the present case : $\rho = 3P = T^4\, n_T\, \dfrac{\pi^2}{48}$

- Equations of motions :
  - For the dilaton $\Longrightarrow$ $\dot\phi_{\mathrm{dil}} = \dfrac{c_{\mathrm{dil}}}{a^3}$
  - Conservation of $T_\mu{}^\nu$ $\Longrightarrow$ $T = \dfrac{c}{a}$
  - Friedmann $\Longrightarrow$ $3H^2 = \dfrac{c_{\mathrm{rad}}}{a^4} + \dfrac{c_{\mathrm{dil}}^2}{a^6} \underset{t\to+\infty}{\longrightarrow} \dfrac{c_{\mathrm{rad}}}{a^4}$

  i.e. attraction to the **Radiation Dominated Solution**

Susy breaking & Temperature

- $\mathcal{N} = 2$ or $1 \to 0$ at finite T :

$$S^1(R_0) \times T^3(R_{\mathrm{box}}) \times S^1(R_4) \times S^1 \times \frac{T^4}{\mathbb{Z}_2}$$

$$S^1(R_0) \times T^3(R_{\mathrm{box}}) \times \frac{S^1(R_4) \times T^3}{\mathbb{Z}_2} \times T^2$$

$\Big($or $\dfrac{S^1(R_4) \times T^5}{\mathbb{Z}_2 \times \mathbb{Z}_2}$ for $\mathcal{N} = 1\Big)$

- Boundary conditions :

$$(-)^{a\tilde m_0} \quad \Longrightarrow \quad \frac{m_0}{R_0} + \frac{a}{2R_0} \quad : \quad \text{Mass shift } T$$

$$(-)^{(a+Q)\tilde m_0} \quad \Longrightarrow \quad \frac{m_4}{R_4} + \frac{a+Q}{2R_4} \quad : \quad \text{Mass shift } M$$

- Hypothesis : $\begin{cases} R_0, R_4 \gg 1 \quad i.e. \quad T, M \ll T_H \\ R_{I\neq 4} \simeq 1 \end{cases}$

The n_T KK towers of $S^1(R_0)$ and $S^1(R_4)$ contribute.

Corrections $\cancel{e^{-2\pi\frac{R_0}{R_I}}}$, $\cancel{e^{-2\pi\frac{R_4}{R_I}}}$ are negligible, except $e^{-2\pi\frac{R_0}{R_4}}$.

- $$Z_{\text{genus-1}} = \beta V \, \frac{1}{(2\pi R_0)^4} \, p(z) \qquad \text{where} \qquad e^z = \frac{R_0}{R_4}$$

$$p(z) = n_T \, f_T(z) + n_V \, f_V(z) + n_T^t \, \frac{\pi^2}{48}$$

$$n_V = \sum_{s=1}^{n_T} (-)^{Q_s}$$

Twisted sector KK towers of $S^1(R_0)$

$$f_T(z) = \sum_{\tilde{m}_0, \tilde{m}_4} \frac{e^{4z} \, \frac{\Gamma(5/2)}{\pi^{5/2}}}{\left(e^{2z}(2\tilde{m}_0+1)^2 + (2\tilde{m}_4)^2\right)^{5/2}} \, , \quad f_V(z) = e^{3z} \, f_T(-z)$$

- In Einstein frame :

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} - \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}(\partial\Phi)^2 + \frac{e^{4\phi_{\text{dil}}} Z_{\text{genus-1}}}{\beta V} \right]$$

linear combinations of ϕ_{dil} and $\ln R_4$

- The fields are : $T(t)$, $a(t)$, $\varphi(t)$, $M(t) \equiv \dfrac{e^{\sqrt{\frac{3}{2}}\Phi(t)}}{2\pi}$

- The energy-momentum tensor gives :

$$P = T^4 \, p(z) \, , \quad \rho = T^4 \, \big(3p(z) - p_z(z)\big)$$

$$\boxed{e^z = \frac{R_0}{R_4} = \frac{M}{T}}$$

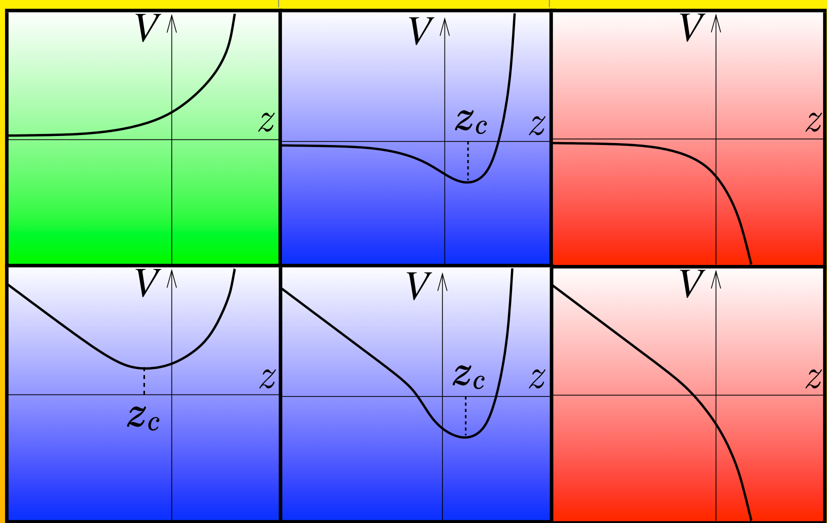
- Equations of motion :
  - Conservation of $T_\mu{}^\nu$: $\quad T = \dfrac{\mathcal{A}(z)}{a}$
  - Friedmann : $\quad H^2 = \dfrac{\mathcal{C}_1}{a^4}$ $\quad \mathcal{C}_i(z; \mathring{z}, \mathring{\varphi})$
  - Scalars : $\begin{cases} \mathcal{C}_2 \,\mathring{\mathring{z}} + \mathcal{C}_3\, \mathring{z}^2 + \mathcal{C}_4\, \mathring{z} + \dfrac{dV}{dz}(z) = 0 \\ \mathcal{C}_5\, \mathring{\mathring{\varphi}} + \mathcal{C}_6\, \mathring{\varphi} = 0 \end{cases}$

$$\frac{dz}{d\ln a} \qquad -p(z) - p_z(z)$$

- If $\quad e^z = \dfrac{M}{T} \equiv e^{z_c}\ , \qquad \varphi = cst$

$$\Longrightarrow \quad T \propto M \propto \frac{1}{a} \quad \text{with} \quad H^2 \propto \frac{1}{a^4}$$ **: Radiation Era**

	$\dfrac{n_V}{n_T} < -\dfrac{1}{15}$	$-\dfrac{1}{15} < \dfrac{n_V}{n_T} < 0$	$0 < \dfrac{n_V}{n_T}$
non-orbifold $p = n_T\, f_T + n_V\, f_V$			
orbifold $+n_T^t\, \dfrac{\pi^2}{48}$			



- Perturbations around the Radiation Era :
  $z = z_c + \varepsilon_{(z)}\ , \quad \varphi = cst + \varepsilon_{(\varphi)} \quad \Longrightarrow \quad \varepsilon_{(z)}, \varepsilon_{(\varphi)} \xrightarrow[t\to+\infty]{} 0$
  Also true for large oscillations (numerically).
- The evolution is always attracted to a contracting phase.

- z slides so that $e^z = \frac{R_0}{R_4} \ll 1$ *i.e.* $R_{\rm box} \gg R_0,\ R_4 \gg 1$
  - Redo analysis in 5D,

$$S = \int d^5x\sqrt{-G'}\left[e^{-2\phi'_{\rm dil}}\left(\frac{R'}{2}+2(\partial\phi'_{\rm dil})^2\right)+\frac{Z_{\text{genus-1}}}{\beta V(2\pi R_4)}\right]$$

with Einstein frame background :

$$ds'^2 = -N'(t)^2dt^2 + a'(t)^2\Big((dx^1)^2+(dx^2)^2+(dx^3)^2\Big)+b(t)^2(dx^4)^2\ , \quad \phi'_{\rm dil}(t)$$

$$N' = 2\pi R_0\, e^{-\frac{2}{3}\phi'_{\rm dil}} \equiv \frac{1}{T'}\ ,\ a' = 2\pi R_{\rm box}\, e^{-\frac{2}{3}\phi'_{\rm dil}}\ ,\ b = 2\pi R_4\, e^{-\frac{2}{3}\phi'_{\rm dil}}$$

  - Use the fields $e^\xi = \frac{R_4}{R_{\rm box}} = \frac{b}{a'}\ , \quad e^z = \frac{R_0}{R_4} = \frac{1}{b\,T'}$
  - Using $\frac{Z_{\text{genus-1}}}{\beta V(2\pi R_4)} = \frac{1}{(2\pi R_0)^5}\, n_T\,\big(c + e^{4z}\,c' + \mathcal{O}(e^{-2\pi\frac{R_4}{R_0}})\big)$

    one shows $\xi\ ,\ z\ ,\ \phi'_{\rm dil} \underset{t\to+\infty}{\longrightarrow} cst$ *i.e.* Attraction to

    $T' \propto \frac{1}{a'} \propto \frac{1}{b}$ with $H^2 \propto \frac{1}{a'^5}$ *i.e.* **Radiation Era in 5D**

Summary

- We focussed on eras of the Universe which are thermalized.
- Start from a flat classical susy background.
  - Switch on finite temperature *i.e.* a thermal gas of all string states.
  - This breaks spontaneously susy $\Longrightarrow$ vacuum energy at genus 1 (the genus 0 cannot wrap the Euclidean time) $\Longrightarrow$ cosmology.
  - Also true for flat classical backgrounds where susy is spontaneously broken.
- Attractions to Radiation Eras.
- The space-time dimension for a given model is dynamically determined.