Matrix Models and Emergent Gravity

Harold Steinacker

Department of Physics, University of Vienna

Corfu, September 2009

Motivation

- expect quantum structure of space-time at Planck scale

  due to $\boxed{\text{Gravity} \leftrightarrow \text{Quantum Mechanics}}$

- cosmology: "dark matter, dark energy" ... ??

  cosmological constant problem

$\Rightarrow$ perhaps gravity is modified:

Matrix Models $\leftrightarrow$ noncommutative (=quantized) space-time

Outline:

- Geometry from Matrix Models
- Relation with NC gauge theory
  Nonabelian gauge fields
- Quantization
- Cosmological solution without fine-tuning
- Newtonian limit and long-distance modifications

Matrix Models

candidate for quantum theory of fundamental interactions

$$S = -Tr\left([X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} \; + \bar{\Psi}\Gamma^a[X_a, \Psi]\right)$$

$$X^a \in Mat(N,\mathbb{C}) \quad N \to \infty, \qquad a = 1, ..., 10$$

(IKKT Model 1996)

- no geometrical pre-requisites, extremely simple
- $\left\{ \begin{array}{l} \text{NC space-time} \\ \text{metric (=gravity)} \end{array} \right\}$ emerge
- $\left\{ \begin{array}{l} \text{nonabelian gauge fields} \\ \text{gravitons} \end{array} \right\}$ ... fluctuations of NC space
- well-behaved under quantization

  new perspectives for dark energy / dark matter !

Rivelles 2002, Yang 2006, H.S. 2007 ff, ...

Space-time & gravity from matrix models:

e.o.m.: $\delta S = 0 \;\Rightarrow\; [X^a,[X^{a'},X^{b'}]]\eta_{aa'} = 0$

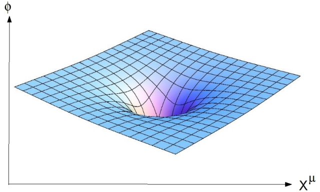
solutions:

- $[X^a, X^b] = i\theta^{ab}\,\mathbb{1}$, "quantum plane"
- $[X^a, X^b] \sim i\theta^{ab}(x)$, generic quantum space

$D = 10$ required for quantization (maximal SUSY)

$\rightarrow$ space-time as 3+1-dimensional brane solution $\mathcal{M}^4 \subset \mathbb{R}^{10}$

$$X^a = (X^\mu, \Phi^i), \qquad \mu = 1, ..., 4; \qquad X^\mu \sim x^\mu$$
$$\Phi^i = \Phi^i(x^\mu)$$



Noncommutative spaces and Poisson structure

$(\mathcal{M}, \theta^{\mu\nu}(x))$ ... $2n$-dimensional manifold with Poisson structure

Its quantization $\mathcal{M}_\theta$ is NC algebra such that

$$\begin{aligned} \mathcal{C}(\mathcal{M}) &\to \mathcal{A} \subset Mat(\infty, \mathbb{C}) \\ f(x) &\mapsto \hat{f}(X) \\ x^i &\mapsto X^i \end{aligned}$$

such that $$[\hat{f}(X), \hat{g}(X)] = i\{f(x), g(x)\} + O(\theta^2)$$

<u>furthermore:</u>

$$(2\pi)^2\, Tr(\phi(X)) \sim \int d^4x\, \rho(x)\, \phi(x)$$

$\rho(x)$ ... symplectic volume

(cf. Bohr-Sommerfeld quantization)

Effective geometry:

consider scalar field coupled to Matrix Model ("test particle")
use $[X^\mu,\varphi] \sim i\theta^{\mu\nu}(x)\partial_\nu\varphi \qquad \Rightarrow$

$$\begin{aligned} S[\varphi] &= Tr\,[X^a,\varphi][X^b,\varphi]\,\eta_{ab} \qquad (U(\mathcal{H}) \quad \text{gauge inv.!}) \\ &\sim \int d^4x\,\sqrt{|G_{\mu\nu}|}\,G^{\mu\nu}(x)\,\partial_\mu\varphi\partial_\nu\varphi \end{aligned}$$

$$\begin{aligned} G^{\mu\nu}(x) &= e^{-\sigma}\theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\,g_{\mu'\nu'}(x) \qquad \text{effective metric} \\ g_{\mu\nu}(x) &= \eta_{\mu\nu} + \partial_\mu\Phi^i\partial_\nu\Phi^j\delta_{ij} \qquad \text{induced metric on } \mathcal{M}^4_\theta \end{aligned}$$

$$e^{-2\sigma} = \frac{|\theta^{-1}_{\mu\nu}|}{|g_{\mu\nu}|}, \qquad |G_{\mu\nu}| = |g_{\mu\nu}|$$

φ couples to metric $G^{\mu\nu}(x)$, determined by $\theta^{\mu\nu}(x)$ & embedding ϕ^i

same for gauge fields, fermions

... quantized Poisson manifold with metric $(\mathcal{M},\theta^{\mu\nu}(x),G_{\mu\nu}(x))$

Equations of motion: can show

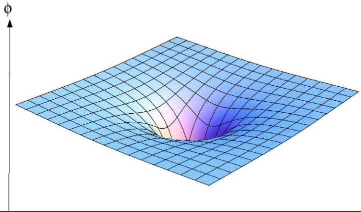
matrix e.o.m: $[X^a,[X^b,X^{a'}]]\eta_{aa'} = 0 \quad \Longleftrightarrow$ (H.S., NPB 810 (2009))

$$\begin{aligned} \Delta_G \Phi^i &= 0, \quad \Delta_G x^\mu = 0 \\ \nabla^\mu(e^\sigma \theta^{-1}_{\mu\nu}) &= e^{-\sigma}\, G_{\rho\nu}\theta^{\rho\mu}\partial_\mu \eta \\ \eta &= e^\sigma\, G^{\mu\nu} g_{\mu\nu} \end{aligned}$$

covariant formulation in semi-classical limit

furthermore:

$\mathcal{M}^4 \hookrightarrow \mathbb{R}^D$ is harmonic embedding (w.r.t. $G_{\mu\nu}$)
minimal surface



$su(n)$ gauge fields: same model, new vacuum

$$Y^a = \begin{pmatrix} Y^\mu \\ Y^i \end{pmatrix} = \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n \end{pmatrix}$$

include fluctuations:

$$Y^a = (1 + \mathcal{A}^\rho \partial_\rho) \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n \end{pmatrix}$$

where

$$\begin{aligned} \mathcal{A}^\mu &= -\theta^{\mu\nu} A_{\nu,\alpha} \otimes \lambda^\alpha, \qquad \lambda^\alpha \in \mathfrak{su}(n) \\ \Phi^i &= \Phi^i_\alpha \otimes \lambda^\alpha \end{aligned}$$

$\Rightarrow$ effective action:

$$\boxed{S_{YM} = \int d^4x \sqrt{G}\, e^\sigma\, G^{\mu\mu'} G^{\nu\nu'}\, tr\, F_{\mu\nu} F_{\mu'\nu'} + 2 \int \eta(x)\, tr\, F \wedge F}$$

(H.S., JHEP 0712:049 (2007), JHEP 0902:044,(2009))

... $\mathfrak{su}(n)$ Yang-Mills coupled to metric $G^{\mu\nu}(x)$

Symmetries & Noether theorem

Matrix model: translational symmetry $X^a \to X^a + c^a \mathbb{1}$

$\Rightarrow$ conserved "matrix current"

$$[X^a, T^{bc}]\,\eta_{ab} = 0$$

$$T^{ab} = [X^a, X^c][X^b, X^{c'}]\eta_{cc'} - \frac{1}{4}\eta^{ab}[X^d, X^c][X^{d'}, X^{c'}]\eta_{dd'}\eta_{cc'} + (a \leftrightarrow b)$$

semi-classical limit:

- $U(1)$ component:

$$\nabla^\mu(e^\sigma \theta^{-1}_{\mu\nu}) = e^{-\sigma}\, G_{\rho\nu}\theta^{\rho\mu}\partial_\mu\eta$$ ... NC $\leftrightarrow$ gravity

- $SU(n)$ component: H.S., JHEP 0902:044,2009.

$$0 = -\sqrt{G}(\nabla_\rho + i[A_\rho, .])(e^\sigma F^{\rho\nu}) - 2F_{\alpha\beta}\varepsilon^{\nu\alpha\beta\rho}\partial_\rho\eta$$

... e.o.m. for Yang-Mills + "would-be top." coupled to $G_{\mu\nu}$

$\Rightarrow$ expect to be valid at quantum level !

Symmetries & Noether theorem

Matrix model: translational symmetry $X^a \to X^a + c^a \mathbb{1}$
$\Rightarrow$ conserved "matrix current"

$$[X^a, T^{bc}]\,\eta_{ab} = 0$$

$$T^{ab} = [X^a, X^c][X^b, X^{c'}]\eta_{cc'} - \frac{1}{4}\eta^{ab}[X^d, X^c][X^{d'}, X^{c'}]\eta_{dd'}\eta_{cc'} + (a \leftrightarrow b)$$

semi-classical limit:

- $U(1)$ component:

$$\nabla^\mu(e^\sigma \theta^{-1}_{\mu\nu}) = e^{-\sigma}\, G_{\rho\nu}\theta^{\rho\mu}\partial_\mu\eta$$ ... NC $\leftrightarrow$ gravity

- $SU(n)$ component: H.S., JHEP 0902:044,2009.

$$0 = -\sqrt{G}\,(\nabla_\rho + i[A_\rho, .])(e^\sigma F^{\rho\nu}) - 2F_{\alpha\beta}\varepsilon^{\nu\alpha\beta\rho}\partial_\rho\eta$$

... e.o.m. for Yang-Mills + "would-be top." coupled to $G_{\mu\nu}$

$\Rightarrow$ expect to be valid at quantum level !

Quantization and the cosmological constant

Quantization of matrix model:

$$Z = \int dX^a d\Psi\, e^{-S[X]-S[\Psi]} \;=\; e^{-S_{eff}}$$

$\Rightarrow$ effective action

$$S_{eff} \sim \int d^4x \sqrt{|G|}\, (\Lambda^4 + c\Lambda^2 R[G] + ...)$$

($R[G]$ due to UV/IR mixing in NC gauge theory)

cosm.const. problem in G.R.:

$\int d^4x \sqrt{|G|}\, \Lambda^4$ = huge vacuum energy, needs fine-tuning

here: metric $G_{\mu\nu}$ "composite" (emergent)

vacuum energy $\Rightarrow$ space-time = minimal surface $\mathcal{M}^4 \subset \mathbb{R}^{10}$

special class of solutions:

$$\begin{aligned} g_{\mu\nu} &= G_{\mu\nu}, \\ \Delta_G \phi^i &= 0 \\ \nabla^\mu \theta^{-1}_{\mu\nu} &= 0 \end{aligned}$$

holds for (anti)self-dual symplectic structure $\theta^{-1}_{\mu\nu}$,

$\star(\theta^{-1}) = \pm\theta^{-1}$ Euclidean

$\star(\theta^{-1}) = \pm i\theta^{-1}$ Minkowski (Wick rotation $X^0 \to it$)

then

$$S_{MM} \sim Tr[X^a, X^b][X^{a'}, X^{\beta'}] = \int d^4x \sqrt{|g_{\mu\nu}|}$$

... same structure as vacuum energy, “brane tension”.

Dynamics of emergent NC gravity

effective action

$$S = \int d^4x \sqrt{|g|}\,(\Lambda_4^2 R - 2\Lambda_1^4) + S_{\rm matter}$$

leads to

$$\begin{aligned} \delta S &= \int d^4x \sqrt{|g|}\, \delta g_{\mu\nu}(-\Lambda_1^4 g^{\mu\nu} + 8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu}) \\ &= -2\int \delta\phi^i \partial_\mu(\sqrt{|g|}\,(-\Lambda_1^4 g^{\mu\nu} + 8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu}))\partial_\nu \phi^i \end{aligned}$$

since $g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu\phi^i\partial_\nu\phi^i$

1. "Einstein branch"

$$\Lambda_1^4 g^{\mu\nu} + \Lambda_4^2 \mathcal{G}^{\mu\nu} = 8\pi T^{\mu\nu}$$

2. "harmonic branch"

$$\Lambda_1^4 \Box_g \phi = (8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu})\nabla_\mu\partial_\nu\phi$$

prototype: flat space $\mathbb{R}^4_\theta \subset \mathbb{R}^{10}$, even for $\Lambda_1 > 0$!

Cosmological solution

D. Klammer, H. S., PRL 102 (2009)

assume: vacuum energy $\Lambda^4 \gg$ energy density ρ

$\Rightarrow$ look for harmonic embedding $\Delta x^a = 0$ of FRW metric

$$ds^2 = -dt^2 + a(t)^2(d\chi^2 + \sinh^2(\chi)d\Omega^2),$$

Ansatz

$$x^a(t,\chi,\theta,\varphi) = \left(\begin{array}{c} a(t) \begin{pmatrix} \cos\psi(t) \\ \sin\psi(t) \end{pmatrix} \otimes \begin{pmatrix} \sinh(\chi)\sin\theta\cos\varphi \\ \sinh(\chi)\sin\theta\sin\varphi \\ \sinh(\chi)\cos\theta \\ \cosh(\chi) \end{pmatrix} \\ 0 \\ x_c(t) \end{array} \right) \in \mathbb{R}^{10}$$

(cf. B. Nielsen, JGP **4**, (1987))

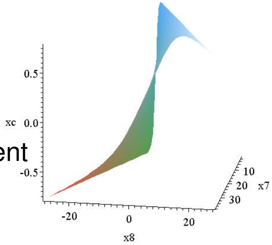
Evolution $a(t), \Psi(t), x_c(t)$ determined by $\Delta x^a = 0$

solution of M.M + leading term $\int d^4x\sqrt{G}\Lambda^4$ in Γ_{1-loop}

harmonic embedding $\Delta_g x^a = 0$ leads to

analog of Friedmann equations

$$\begin{aligned} H^2 = \frac{\dot{a}^2}{a^2} &= -b^2 a^{-10} + d^2 a^{-8} - \frac{k}{a^2}. \\ \frac{\ddot{a}}{a} &= -3d^2 a^{-8} + 4b^2 a^{-10}. \end{aligned}$$



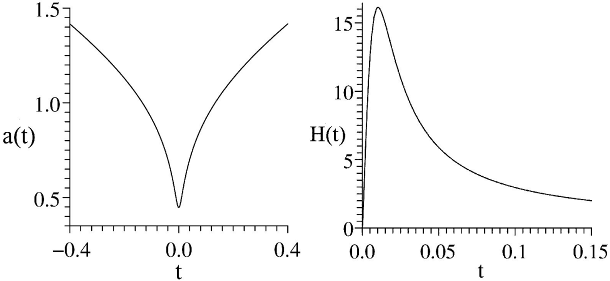
largely independent of detailed matter/energy content
as long as $\Lambda^4 \gg \rho$

$k = -1$ (negative spatial curvature) most interesting

Implications:

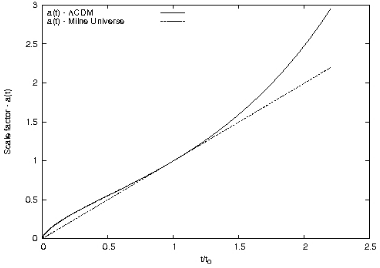
1) early universe:

- big bounce: $\dot{a} = 0$ for $a = a_{min} \sim b^{1/4}$
  ($\exists$ bound for energy density ρ vs. vacuum energy Λ^4)
- inflation-like phase $a(t) \sim t^2$, ends at $a(t_{\rm exit}) = \sqrt{\frac{4}{3}\frac{b}{d}}$
  geometric mechanism (no scalar field required),
  no fine-tuning



2) late evolution (now): $\dot{a} \to 1$

approaches Milne-like universe ($k = -1$, spatial curvature),



in remarkably good agreement with observation
(age $13.8 \cdot 10^9\ yr$, type Ia supernovae)
different physics for early universe (recombination etc.)

A. Benoit-Levy and G. Chardin, [arXiv:0903.2446]

CMB acoustic peak argued to be at correct scale (?)

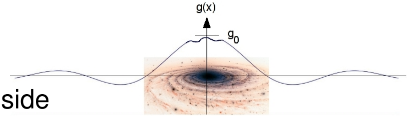
no fine-tuning of cosm. const., no need for dark energy !

Newtonian limit & long-distance modifications

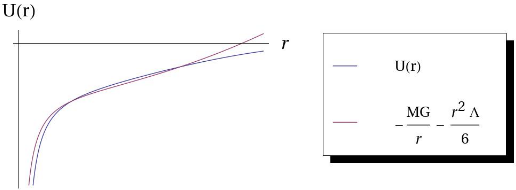
localized mass $\Rightarrow$ local harmonic deformations of flat embedding

H. S., arXiv 0909.xxxx

large-scale mass clusters embedded in harmonic deformations of space-time ("gravity bags")



- recover Newtonian gravity inside



- screening of gravity $U(r) \sim \frac{1}{r^2}$ at long distances $r > L_\omega$
  enhancement of (galactic) rotation curves

e.o.m. $\Lambda_1^4 \Box_g \phi = 8\pi\rho\nabla_0^2\phi$ $\qquad (T^{00} = \rho,\ T^{ij} \sim 0)$

<u>Ansatz:</u> $\phi^i(x,t) = g(x)e^{i\omega t} = g(x)\begin{pmatrix}\cos(\omega t)\\ \sin(\omega t)\end{pmatrix}$ "gravity bag"

$\Rightarrow$ static effective metric

$$ds^2 = -(1-\omega^2 g^2)\, dt^2 + (\delta_{ij} + \partial_i g \partial_j g) dx^i dx^j.$$

spher. symm. mass M at origin; $g(r) = g_0 \frac{\sin(\omega r+\delta)}{\omega r}, \qquad \delta \sim M$

$\Rightarrow$ Newtonian gravity, long-distance screening $U(x) \sim \omega^2 g^2 \sim \frac{1}{r^2}$.

gravitational field due to localized mass:

$$\begin{aligned} g_{00} &\approx -\left(1 + 2U_0 - \frac{2GM}{r} - \frac{1}{3}\Lambda_{\rm eff} r^2\right) \\ \Delta U &= 4\pi G\left(\rho(x) + \frac{\Lambda_1^4}{8\pi}\right) \\ G &= \frac{2g_0^2\omega^4}{\Lambda_1^4}, \qquad \Lambda_{\rm eff} = -\frac{1}{2}G\Lambda_1^4 \end{aligned}$$

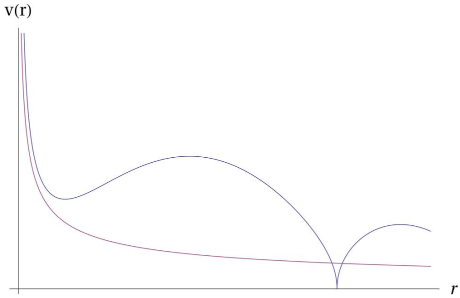
Newton constant G, cutoff L_ω dynamical,
determined by largest structures; might differ between galaxies

(galactic) rotation curves:

orbital velocities $v(r)$ larger for larger distances,

similar to observations ($\leftrightarrow$ "dark matter" ?!)

for point mass:



note:

- Newtonian gravity without using E-H term!
  $\Rightarrow$ cutoffs can be much lower than Planck scale, even $O(TeV)$
- remarkably close to “what we see” without fine-tuning
- solar system precision tests not clear
  (non-standard g_{rr}, needs refinement)
- gauge couplings will be different in early universe
- non-standard spin connection (D. Klammer, H.S 2008, 2009)
- add cubic terms to matrix model $\Rightarrow$ extra-dim. fuzzy S^2, interesting low-energy gauge groups
  (P. Aschieri, T. Grammatikopoulos, H.S., G. Zoupanos JHEP 0609:026,2006; Madore, Manousselis; Aoki, Azuma, Iso, ...; H. Grosse, F. Lizzi, H.S. in preparation)

Summary and Conclusion

- matrix-model $Tr[X^a, X^b][X^{a'}, X^{b'}]\,\eta_{aa'}\eta_{bb'}$

  dynamical NC spaces $\leftrightarrow$ emergent gravity & gauge thy
- *not* same as G.R., long-distance corrections
- intriguing cosmological solutions,
  physics of vacuum energy different from GR
  less fine-tuning
- suitable for quantizing gravity
  (IKKT model, $N = 4$ SUSY in $D = 4$)
- ... more work is needed; solar system constraints ?