Higher Derivative Effects on η/s with Finite Chemical Potential

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Based on arXiv:0903.3244[hep-th] with Sera Cremonini, James Liu and Phillip Szepietowski

September 8th, 2009

AdS/CFT correspondence: Maldacena, GKP, Witten

Type IIB superstring on $AdS_5 \times SE_5$

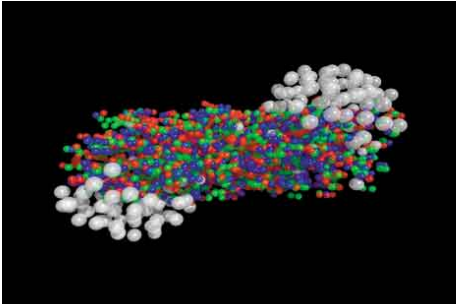
$$\Updownarrow$$

$\mathcal{N} = 1$ superconformal field theory

- Strong-weak duality — Difficult to prove
- Alternative tool to analyze strongly coupled QFT
- "Phenomenological test" of AdS/CFT correspondence

Quark-gluon plasma at RHIC

- Au-Au collision at $\sqrt{s}/N \sim 200\,GeV$
- Intermediate region — Not confined, not free
- Quark-gluon plasma formed and thermalized
- QGP behaves like a nearly perfect fluid



Stolen from CERN press release

Quantity of interest

Shear viscosity "η" : Measure of viscousness of a fluid

Traditional approaches don't work very well...

- Perturbation theory unreliable
- "Real-time" quantities hard to extract from lattice calculation

Goal

Calculate $\eta(/s)$ using AdS/CFT

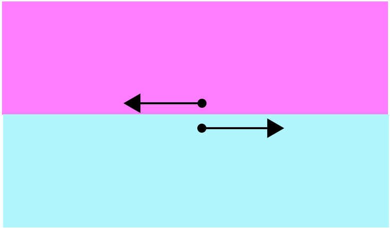
Plan of the Talk

- Holographic hydrodynamics
- KSS bound
- Finite N corrections and violation of KSS bound

What is shear viscosity?

$$T_{ij} = \delta_{ij} p - \eta \left(\partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \partial_k u_k \right) - \zeta \delta_{ij} \partial_k u_k$$

- η: Resistance of a fluid under shear stress
- η/s: Dimensionless measure of viscousness



η/s ratio for various objects:

- Water under normal condition ~ 100
- Liquid Helium ~ 1
- Quark-gluon plasma at RHIC $\sim$ 0-0.2
  (Least viscous object known in nature)

Perturbative calculation of QGP

$$\frac{\eta}{s} \sim 1$$

What about AdS/CFT calculation?

Big picture

Kubo formula

$$\eta = \lim_{\omega\to 0}\frac{1}{\omega}\text{Im}\left(-i\int dtd\vec{x}e^{i\omega t}\theta(t)\langle[T_{xy}(t,\vec{x}),T_{xy}(0,0)]\rangle\right)$$

GKP-Witten relation

$$Z_{SUGRA} := e^{-I_{SUGRA}[\phi(x,z)|_{z=0}=\phi_0(x)]} = \langle\exp(i\int d^4x\phi_0(x)\mathcal{O}(x))\rangle$$

- $\mathcal{O}(x) = T_{\mu\nu}(x)$ for $\phi(x) = h^{\mu\nu}(x)$
- Roughly, $\frac{\delta^2 Z_{SUGRA}}{\delta h^{xy}(x)\delta h^{xy}(0)}$ gives the integrand of Kubo formula

Summary of the calculation

1. Find a background solution
   - Plasma with finite temperature $\rightarrow$ AdS black hole (brane)
2. Derive the partition function
   - Partition fn. depends only on the boundary values of the fields
   - Integrate out the bulk degrees of freedom using the e.o.m.
3. Read off the (retarded) Green's function
4. Use Kubo formula to get the shear viscosity

$\mathcal{N} = 4$ SYM, infinite λ and N: Kovtun-Son-Starinets

$$\eta/s = \frac{1}{4\pi}\ (\sim 0.08)$$

- The value is within the experimental bound! (0-0.2)
- $\mathcal{N} = 4$ and conformal, but...

Universality

KSS formula valid for

- $\mathcal{N} = 1$ SCFT: Kovtun-Son-Starinets
- Non-conformal theories: Kovtun-Son-Starinets, Buchel-Liu
- Fundamental matters: Mateos-Myers-Thomson
- Finite chemical potential (for R-charges):

  Mas, Son-Starinets, Maeda-Natsuume-Okamura
- Time-dependent background: Janik

$$+$$

All the known materials has larger ratio

$$\Downarrow$$

KSS bound

$\eta/s \geq 1/4\pi$ for all materials

KSS conjecture valid for finite λ and N ?

Finite λ corrections

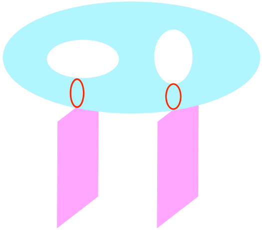
R^4 corrections from string theory

$$S_{R^4} = \frac{1}{16\pi G_{10}} \int d^{10}x \frac{\zeta(3)\alpha'^3}{8} e^{-\frac{3}{2}\phi} C^4$$

KSS formula is modified, but the bound not violated: Buchel-Liu-Starinets

$$\frac{\eta}{s} = \frac{1}{4\pi} \left(1 + 15\zeta(3)\lambda^{-3/2} + \cdots\right)$$

How about finite N corrections?



7-branes wrapping 3-cycles in SE_5: Aharony-Pawelczyk-Theisen-Yankielowicz

$$\int C_{(4)} \wedge \mathrm{tr}(R \wedge R) \Rightarrow \int A \wedge \mathrm{tr}(R \wedge R) \Leftrightarrow \int R^2$$

The coefficient c_2 determined from trace/R anomaly

$$c_2 = \frac{c - a}{a} \sim \mathcal{O}\left(\frac{1}{N}\right)$$

Finite N corrections

Viscosity-entropy ratio: Kats-Petrov, Brigante-Liu-Myers-Shenker-Yaida

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{c-a}{a}\right)$$

- $c - a \geq 0$ for any interacting CFT at large N?: Buchel-Myers-Sinha
- KSS bound always violated!

Finite chemical potential?

Finite R-charge chemical potential

Need to know...

- Chemical potential in CFT $\sim$ BH charge in AdS
- R^2, RF^2 and F^4 terms in 5d SUGRA: KH-Ohashi-Tachikawa
- Additional counter terms $\longrightarrow$ Order by order: Buchel-Liu-Starinets

Corrections to η/s: Cremonini-KH-Liu-Szepietowski, Paulos-Myers-Sinha

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{c-a}{a}\left(1 + \frac{3}{2\bar{\Phi}^2}\left(1 + \frac{4\bar{\Phi}^2}{3} - \sqrt{1 + \frac{8\bar{\Phi}^2}{3}}\right)\right)\right)$$

The range of η/s

$$\frac{1}{4\pi}\left(1 - 3\frac{c-a}{a}\right) \leq \frac{\eta}{s} \leq \frac{1}{4\pi}\left(1 - \frac{c-a}{a}\right)$$

Conclusion

- KSS bound can be violated by finite N corrections
- Violation is enhanced in the presence of chemical potential

- Chemical potential for baryon number?
- Subleading corrections?
- Microcausality constraints on η/s? Brigante-Liu-Myers-Shenker-Yaida
- Proof of $c - a \geq 0$? Counterexample?