Asymptotic Helicity Conservation in SUSY

G.J. Gounaris

with
J. Layssac and
F.M. Renard

Summary

•**HCns** applies to any 2-to-2 process $a_{\lambda_1} b_{\lambda_2} \to c_{\lambda_3} d_{\lambda_4}$

•**HCns** is exact in **MSSM** and strongly simplifies the asymptotic 2-to-2 amplitudes.

• In **SM** it is often approximately true, provided $F_{Born} \neq 0$.

•**Application 1**: to 1loop EW amplitudes for $ug \to dW^+$ and $ug \to \tilde{d}_L \tilde{\chi}_i^+$.
In MSSM, HCns provides relations among subprocess cross sections, which may even be relevant at LHC.

•**Application 2**: to 1loop EW amplitudes for gg→VH, HH' in MSSM and SM

• In **SM**, examples where HCns is strongly violated, have been identified.

Meaning of HCns

Renard+G: PRL 94:131601(2005),
PR D73:097301(2006)

- **In MSSM** (with R-party), for any 2-to-2 processes,

at **$s \gg M^2_{SUSY}$** and **fixed angles**,

all non-vanishing helicity amplitudes must satisfy the **rule**

$$F(a_{\lambda_1} b_{\lambda_2} \to c_{\lambda_3} d_{\lambda_4}) \Leftrightarrow \lambda_1 + \lambda_2 = \lambda_3 + \lambda_4 \quad .$$

In SUSY, HCns is exact. All amplitudes violating it, must vanish exactly, in this asymptotic limit. **It strongly simplifies the asymptotic 2-to-2 amplitudes.**

- **In SM, HCns** is **approximately true,** provided **$F_{Born} \neq 0$.**

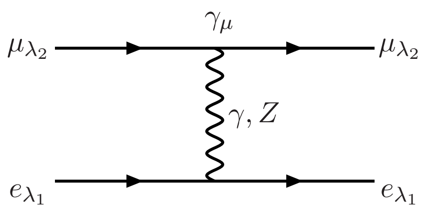
Thus, to the **1-loop Leading Log level**, provided **$(s, |t|, |u|) \gg m^2_W$** ,
the Helicity Violating (HV) amplitudes go asymptotically to (usually non-vanishing) small constants, which are much smaller than the Helicity Conserving (HC) amplitudes.

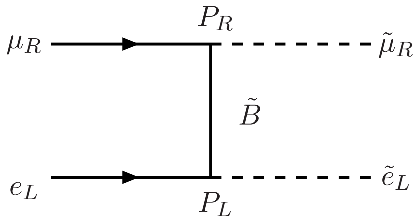
If **$F_{Born}=0$**, no general statement exists in SM.

HCns at the Born level

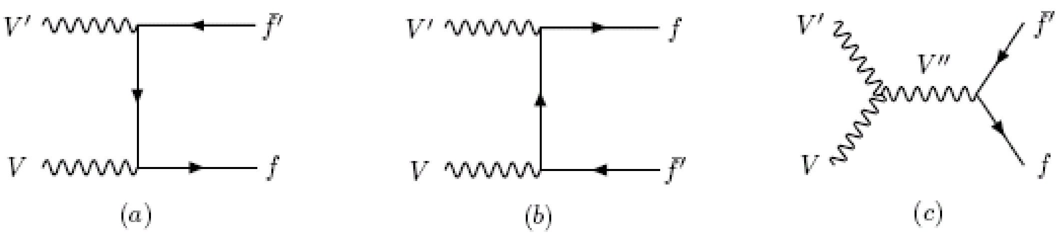
$(s, |t|, |u|) \gg$ all masses, s/t=fixed

If no external gauge bosons appear, the validity of HCns is almost trivial.





In the case of external gauge bosons, large gauge cancellations among the diagrams appear.



The couplings must be the standard renormalizable ones, for HCns to be true. Anomalous couplings always violate HCns.

At Born level, HCns is valid, in both SM and MSSM

HCns at the 1-loop level

with Layssac, Porfyriadis, Renard, Diakonidis

$(s, |t|, |u|) \gg$ all masses
s/t=fixed

$\gamma\gamma \to \gamma\gamma$, $\gamma\gamma \to ZZ$, $\gamma\gamma \to \gamma Z$

- The helicity conserving (HC) amplitudes are much larger than the helicity violating (HV) ones and predominantly imaginary in both, SM and MSSM.
- In SM, the helicity violating amplitudes go to **(small) constants**.
- In MSSM, the helicity violating amplitudes **vanish exactly**.

Take e.g F_{+++-}.

In **SM** $\quad F_{+++-} \to c(\theta) \leftrightarrow$ small

In **MSSM** $\quad F_{+++-} \to 0$

The standard particle loop is exactly cancelled by the supersymmetric one.

Proof of HCns, to all orders in MSSM (R-parity was assumed).

• Neglect all dimensional parameters: soft terms and the μ term.

• A new U(1) is identified, respected by all vertices, except the f-f-scalar Yukawa terms.

•This leads to **HCns** for all **2-to-2** processes,
with external **fermions or scalars only**.

•It is then observed that the SUSY transformation, when projected on the one-particle states, relates

gauge ↔ gaugino with helicities of the **same sign**

•This then leads to HCns for all **2-to-2** processes.

•It should be straightforward to extend the proof to any reasonable NMSSM

•For multiparticle final states, the situation is more complicated and HCns is not generally valid.…

1a. Application to 1loop EW for

$$u(\lambda_u)+g(\lambda_g)\rightarrow d(\lambda_d)+W^+(\lambda_W)$$

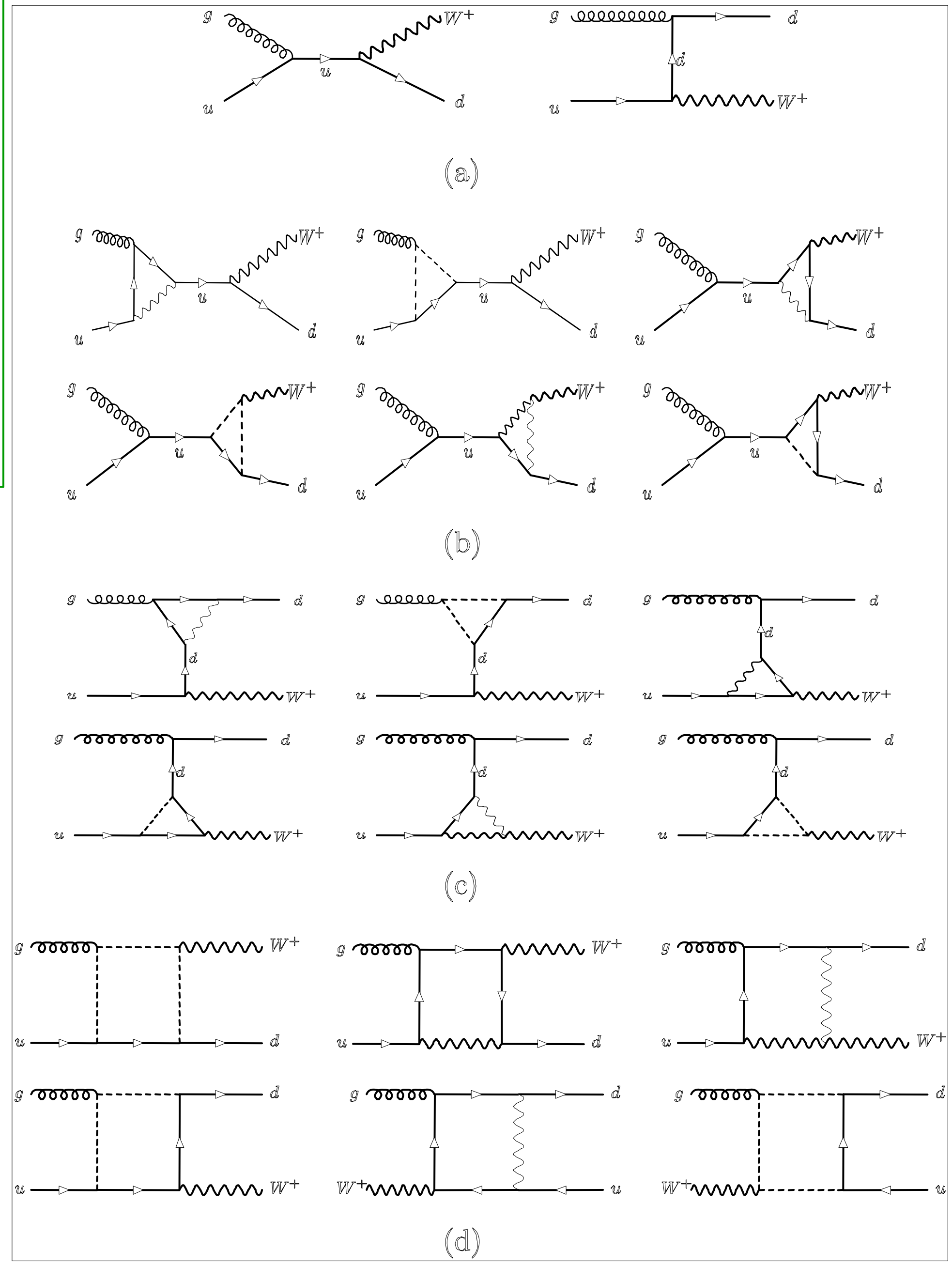
$$F_{\lambda_u\lambda_g\lambda_d\lambda_W} \Rightarrow F_{-\lambda_g-\lambda_W} ,$$

$$(\lambda_g=\pm1) , (\lambda_W=\pm1,0)$$

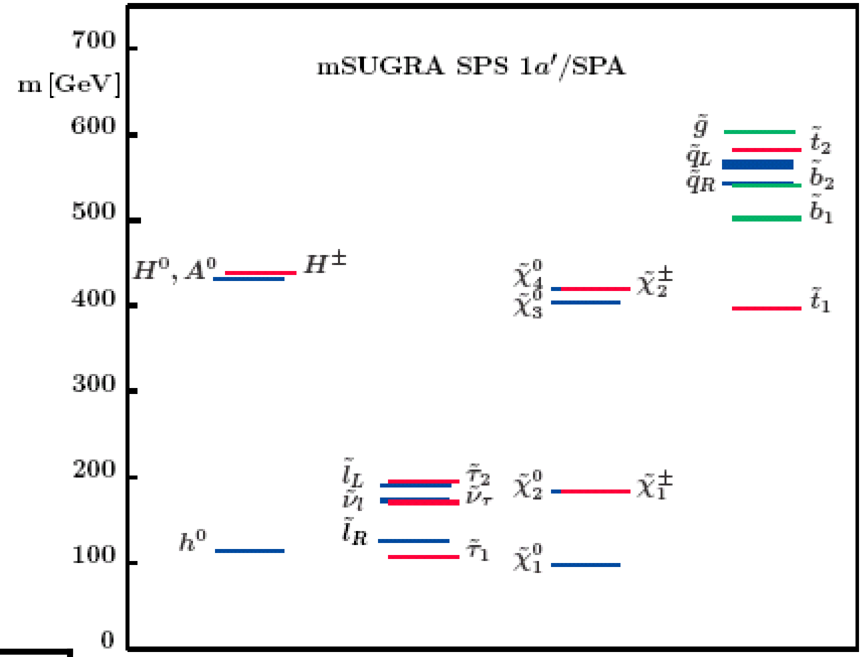
HCns $\Rightarrow \lambda_g=\lambda_W$

The independent amplitudes are

HC	F_{----} F_{-+-+}	Born exists large
HV1	F_{---+} , F_{---0}	No Born very small
HV2	F_{-+--} F_{-+-0}	Born exists small

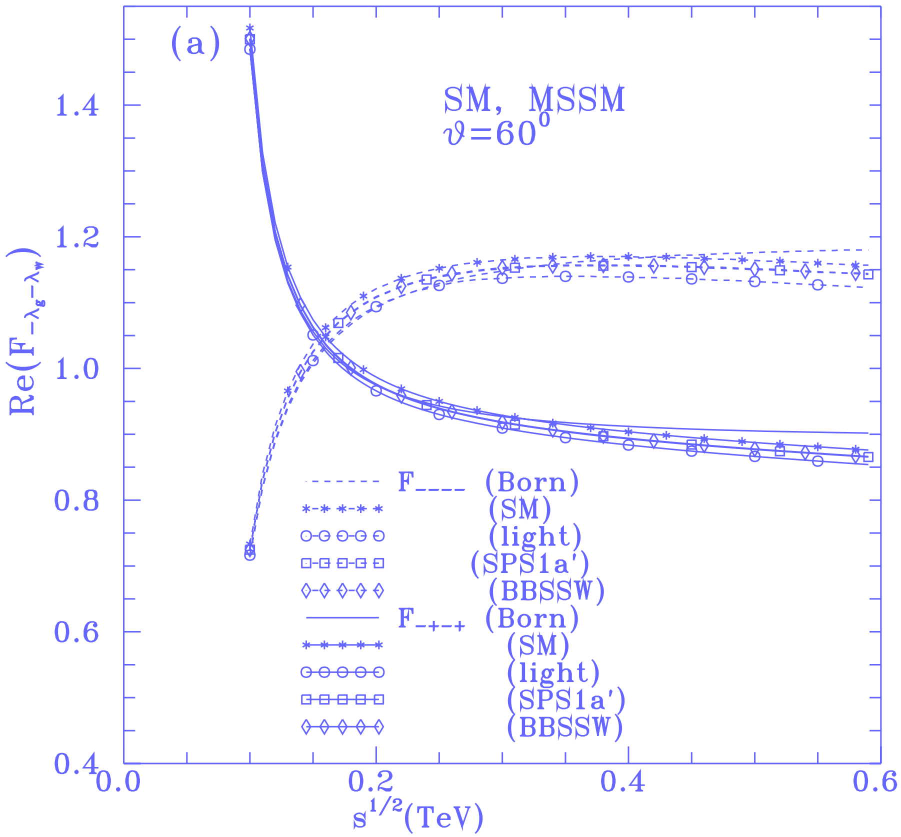


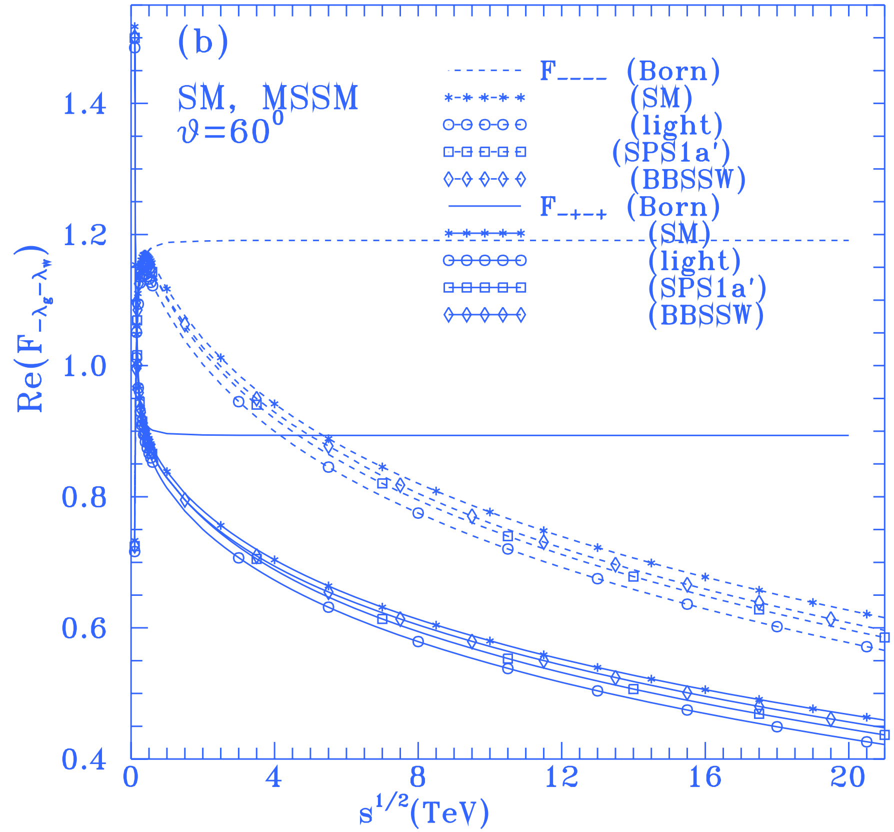
We need benchmarks covering a wide range of SUSY masses. BSSW, FLN mSP4 and SPS1a' are consistent with everything known; (Baer, Nath, SPA).



GeV	BBSSW	FLN mSP4	SPS1a'	“light”
$m_{1/2}$	900	137	250	50
m_0	4716	1674	70	60
A_0	0	1985	-300	0
$\tan\beta$	30	18.6	10	10
M_{SUSY}	700	1500	350	40

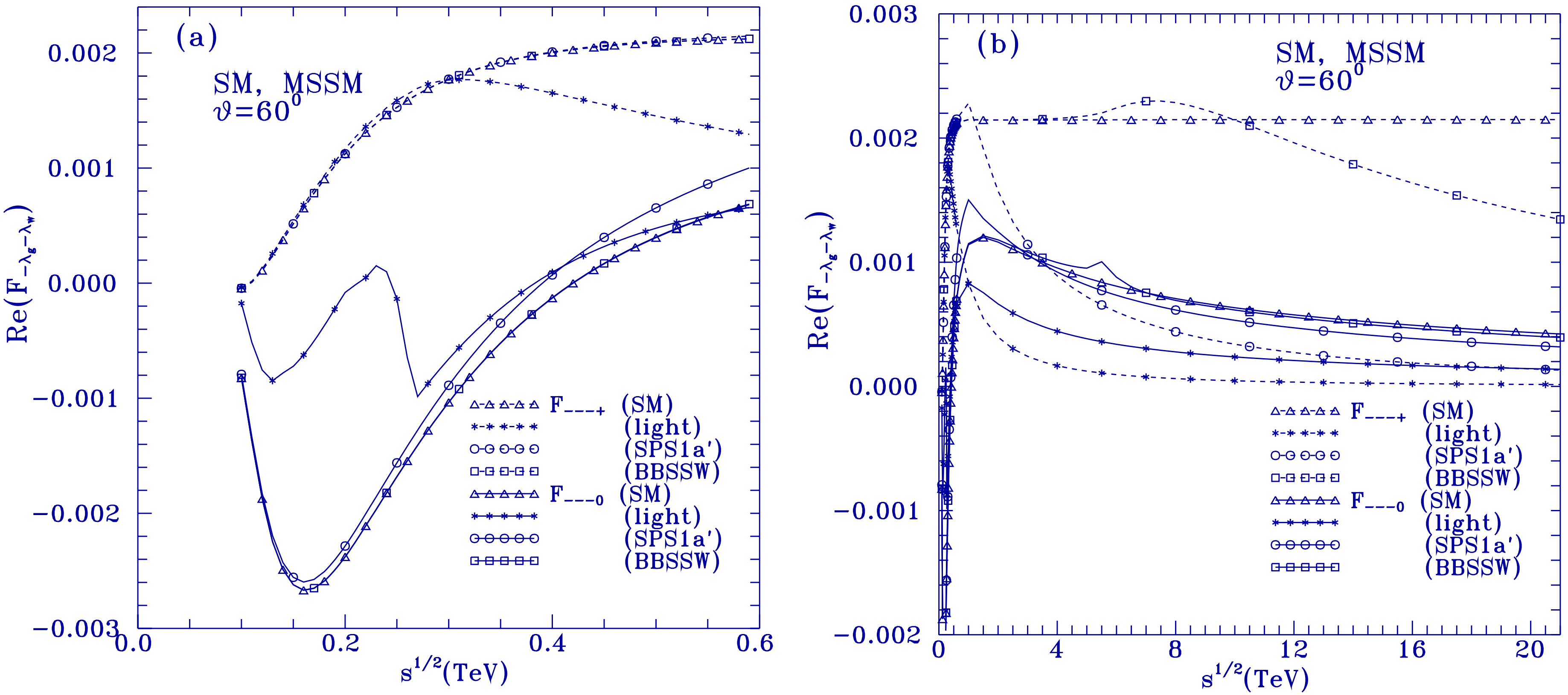
The most important $u\, g \rightarrow d\, W^{+}$ amplitudes in SM and MSSM are the **HC** ones



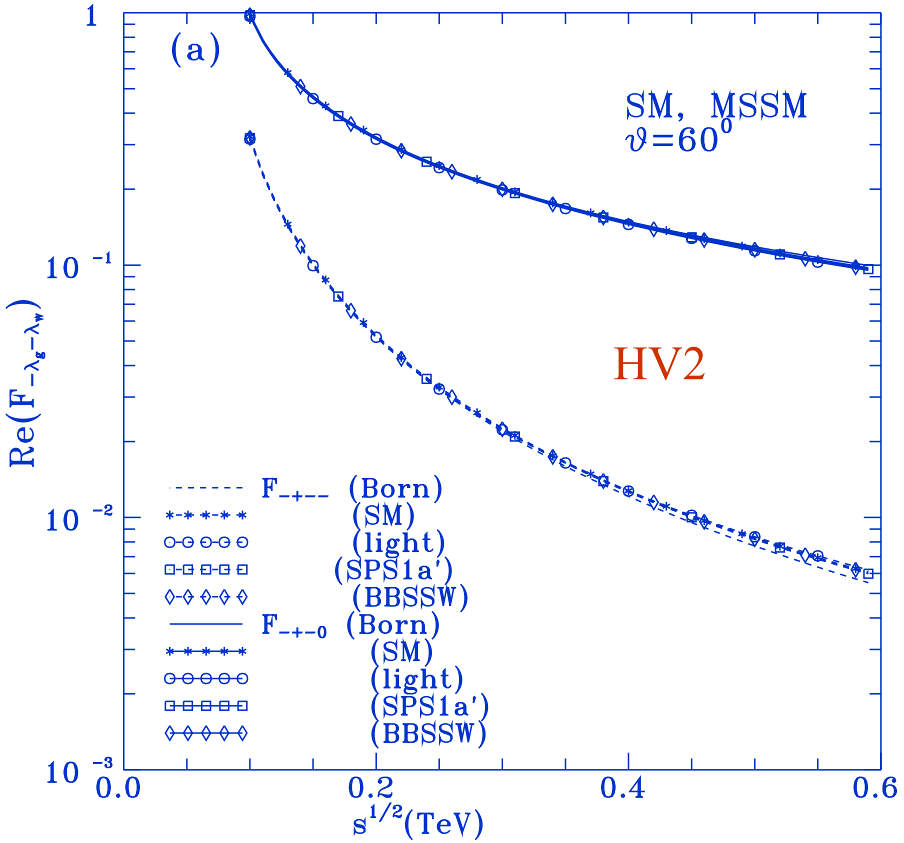


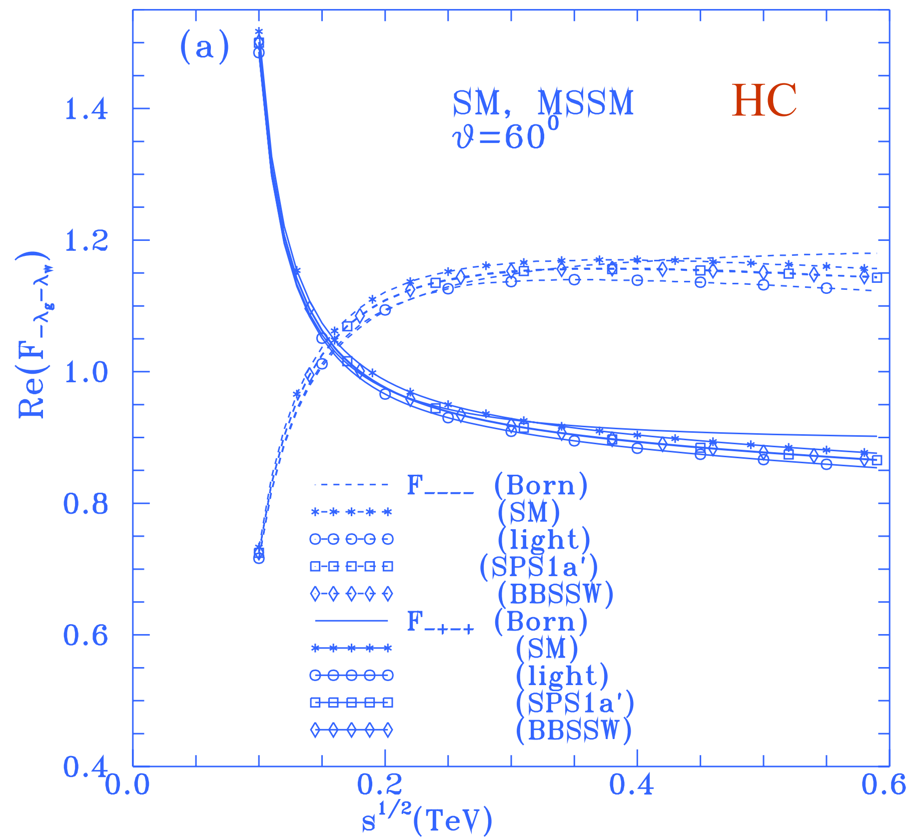
Im parts are much smaller than the Real parts..
Loop effects become very important above 1TeV.

The HV1 $u\,g \to d\,W^+$ amplitudes are very small. (No Born contributions for them.)



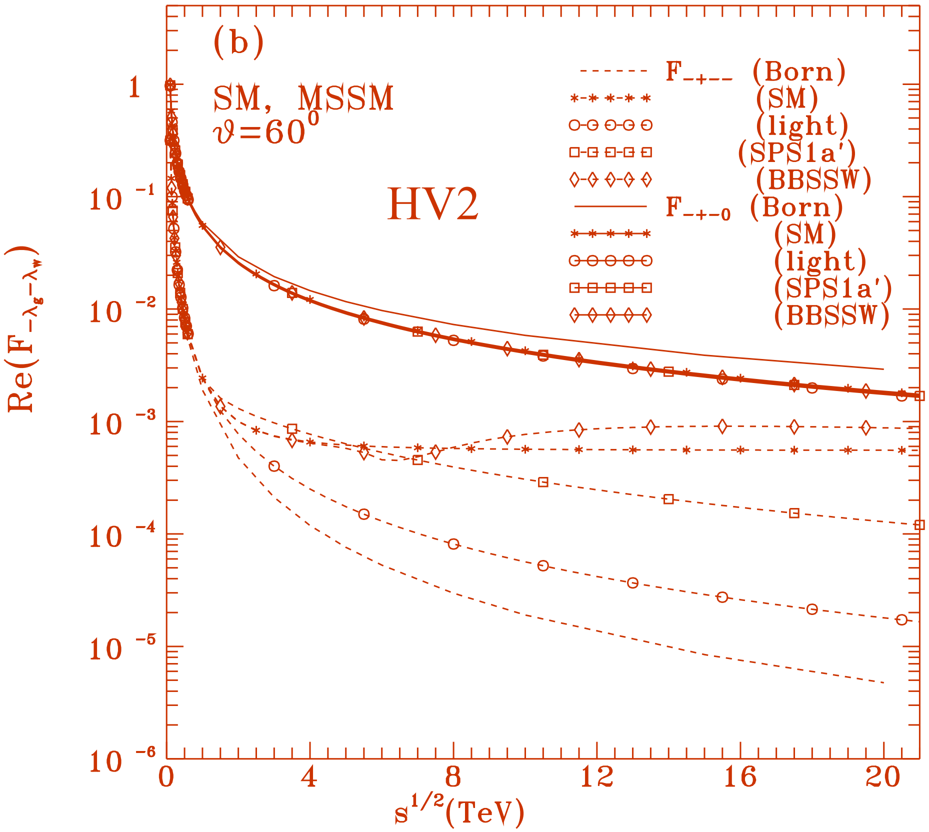
For $\sqrt{s} \lesssim 0.3$TeV, the HV2 $u\, g \to d\, W^+$ amplitudes may be comparable to the HC ones.

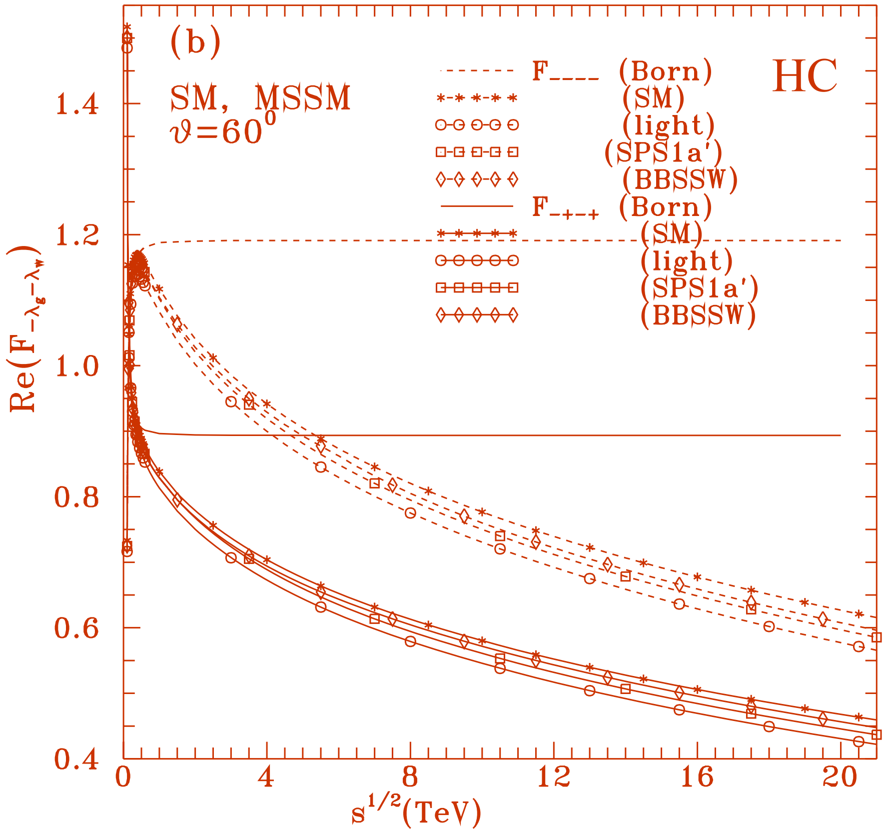




Im parts are much smaller than the Real parts. They become negligible above ~2TeV. Born approximation should be adequate for them.

At higher energies, HV2≪HC for $u\, g \to d\, W^+$





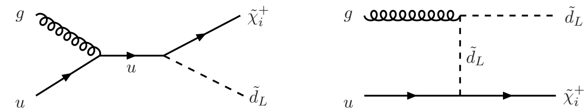
1b. Application to 1loop EW corrections for

$$u(\lambda_u)+g(\lambda_g)\rightarrow \tilde{d}_L+\tilde{\chi}_i(\lambda_i)$$

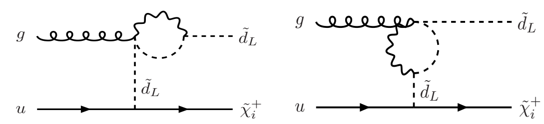
$$\lambda_u=-\frac{1}{2}\,,\quad \lambda_g=\pm 1\,,\quad \lambda_i=\pm\frac{1}{2}$$

$$F_{\lambda_u\lambda_g\lambda_i}\Rightarrow F_{---}\,,\ F_{--+}\,,\ F_{-+-}\,,\ F_{-++}$$

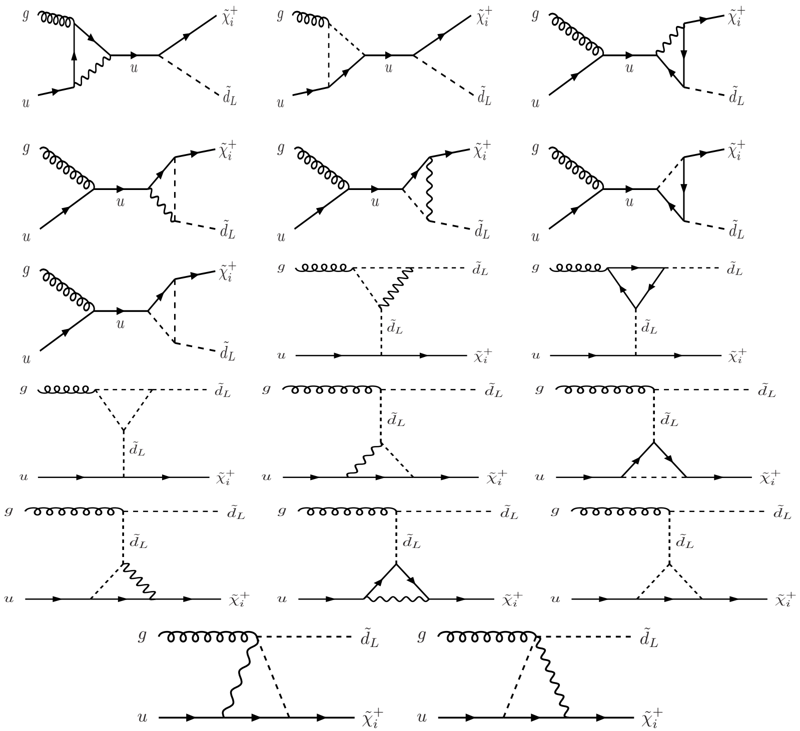
Four independent helicity amplitudes.
Only F_{-++} respects **HCns**



(a)



(b)



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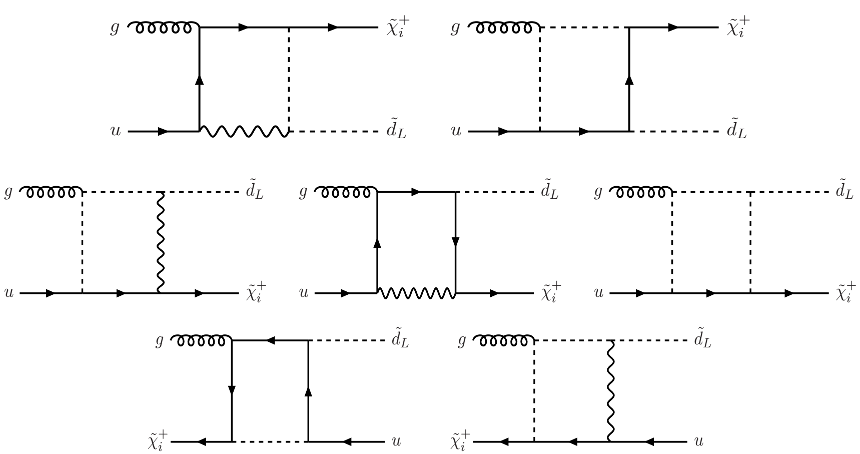
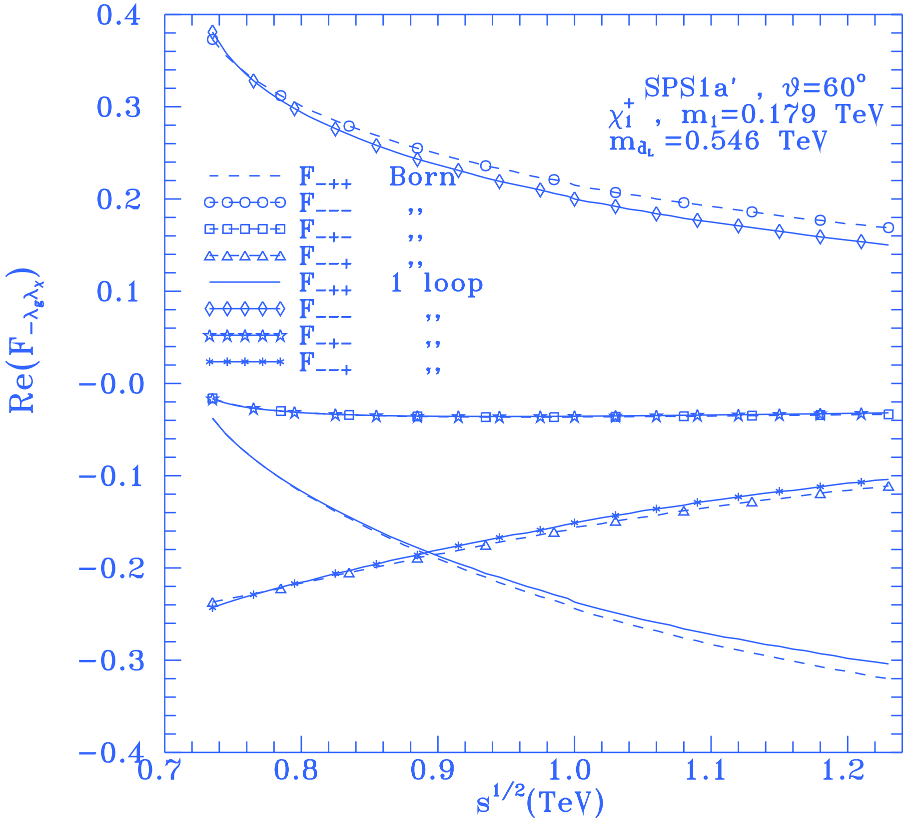
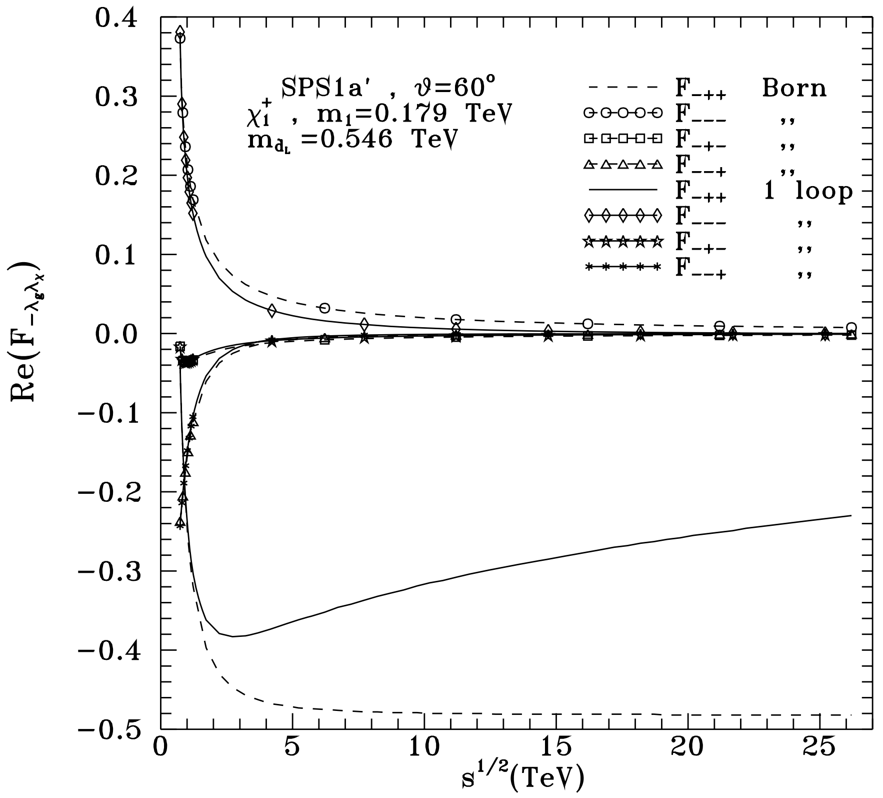


Energy dependence of the helicity amplitudes for SPS1a' at θ=60°

$$u(\lambda_u) + g(\lambda_g) \rightarrow \tilde{d}_L + \tilde{\chi}_i(\lambda_i)$$

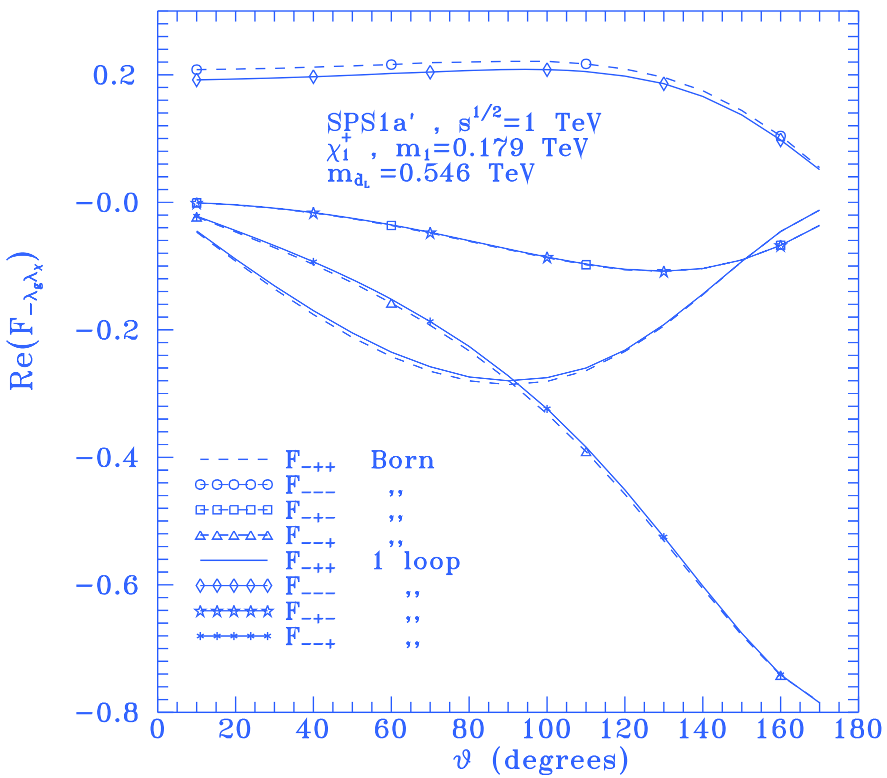


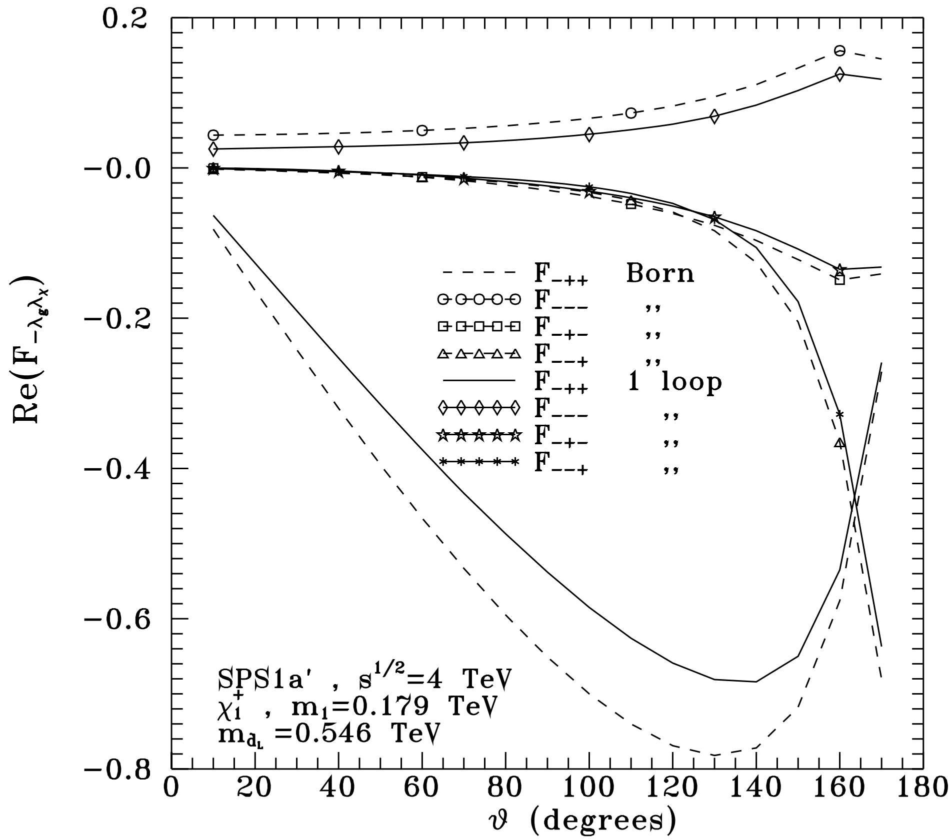


For $\sqrt{s} \gtrsim 5$TeV, HC is approximately established in SPS1a'

Angular dependence of the helicity amplitudes for SPS1a' at 1 and 4 TeV

$$u(\lambda_u) + g(\lambda_g) \rightarrow \tilde{d}_L + \tilde{\chi}_i(\lambda_i)$$





HC is faster established away from the forward and backward regions for SPS1a'

SUSY relations among $ug \to dW^+$ and $ug \to \tilde{d}_L \tilde{\chi}_i$ at asymptotic energies

$$a_{\tilde{\chi}_i} = \frac{\alpha}{4\pi} \frac{(1+26c_w^2)}{72c_w^2 s_w^2} \ln\left(\frac{M_{SUSY}^2}{m_z^2}\right)$$

$$\cos(\theta/2) F_{----}^{dW^+} = \frac{F_{-+-+}^{dW^+}}{\cos(\theta/2)} = \frac{F_{-++}^{\tilde{d}_L \tilde{\chi}_i^+}}{\sin(\theta/2) Z_{1i}^- (1+a_{\tilde{\chi}_i})}$$

SUSY F-relation

$$\frac{d\sigma(ug \to dW^+)}{d\cos\theta} \approx \frac{1}{R_{iW}} \frac{d\sigma(ug \to \tilde{d}_L \tilde{\chi}_i^+)}{d\cos\theta}$$

SUSY σ-relation

$$R_{iW} = \frac{\left\{\left[s-(m_{\tilde{\chi}_i^+}+m_{\tilde{d}_L})^2\right]^{1/2}\left[s-(m_{\tilde{\chi}_i^+}-m_{\tilde{d}_L})^2\right]^{1/2}\right\}}{s-m_w^2} \left|Z_{1i}^-\right|^2 \frac{(1+a_{\tilde{\chi}_i})^2 \sin^2\theta}{5+2\cos\theta+\cos^2\theta}$$

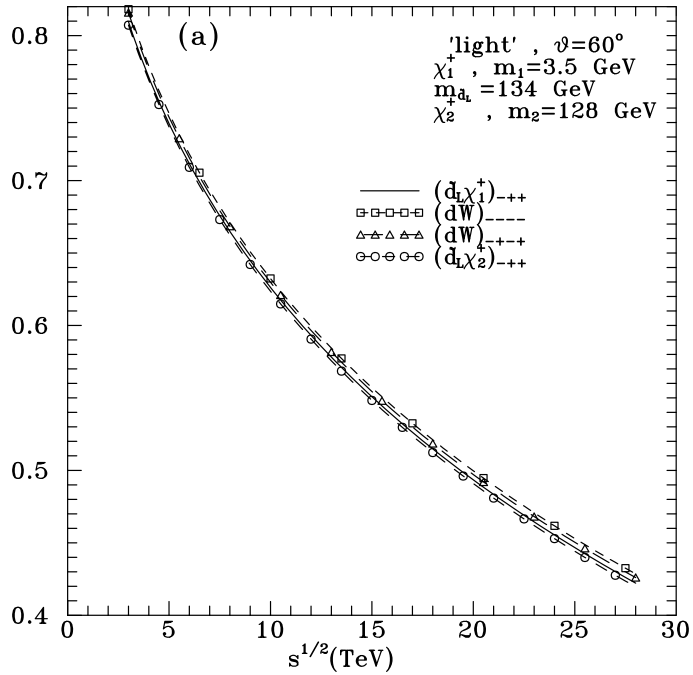
These relations have been derived to 1loop EW order.
At finite energies, they are violated by "constant" and mass-suppressed terms.
In the **SUSY σ-relation**, violation comes also from HV amplitudes.

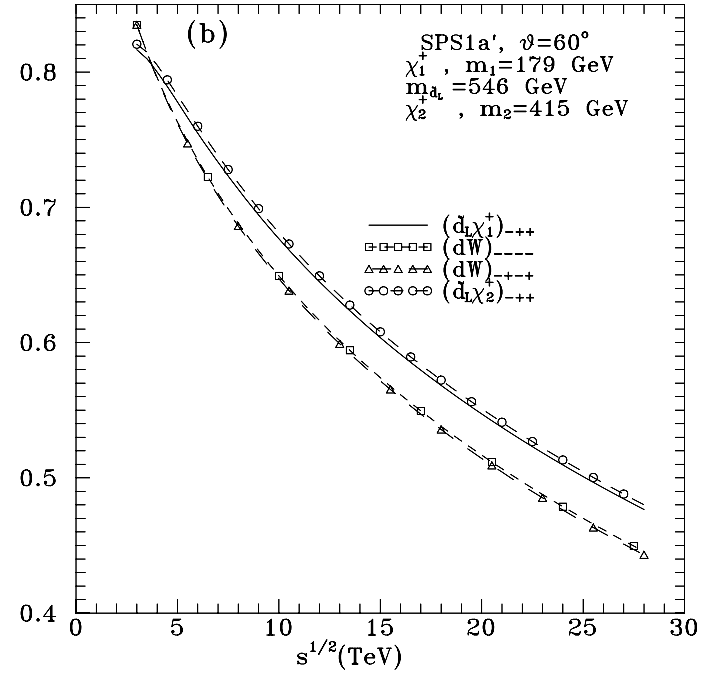
SUSY F-relations

$$\frac{F_{-++}^{\tilde{d}_L \tilde{\chi}_i^+}}{\sin(\theta/2) Z_{1i}^{-}(1+a_{\tilde{\chi}_i})}$$

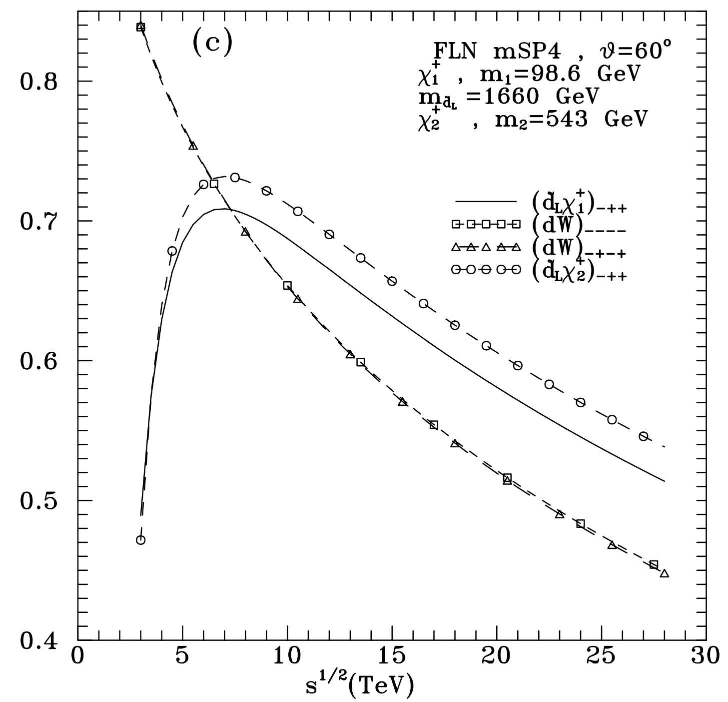
$$\cos(\theta/2) F_{----}^{dW^+}$$

$$\frac{F_{-+-+}^{dW^+}}{\cos(\theta/2)}$$





F-relations affect only the asymptotically dominant HC amplitudes.
They are almost perfect for the “light” MSSM model, and worsen as we move to higher SUSY masses in SPS1a' and FLN mSP4.



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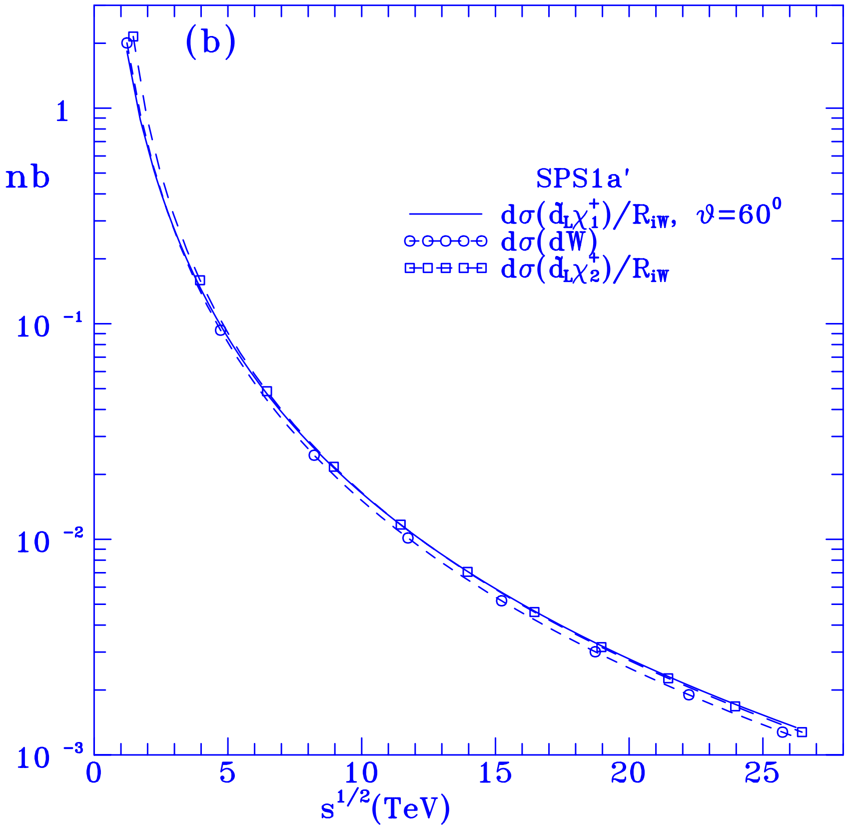
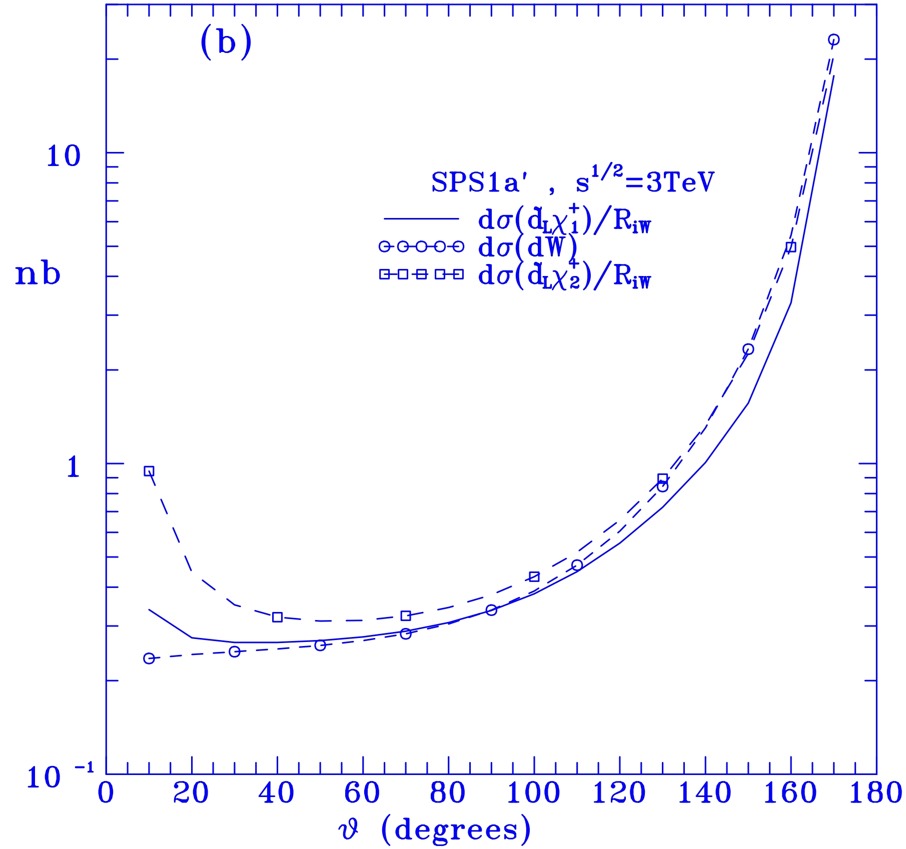
The SUSY σ-relation for SPS1a'.
M_{SUSY} =0.35TeV

$$\frac{d\sigma(ug \to dW^+)}{d\cos\theta}\ ,\ \frac{1}{R_{iW}}\frac{d\sigma(ug \to \tilde{d}_L \tilde{\chi}_i^+)}{d\cos\theta}$$

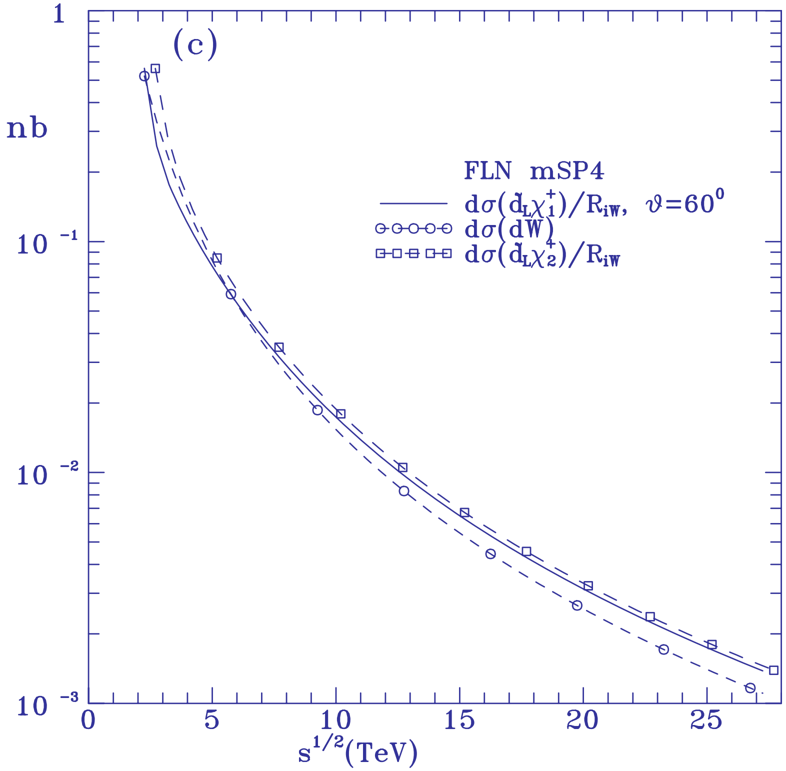


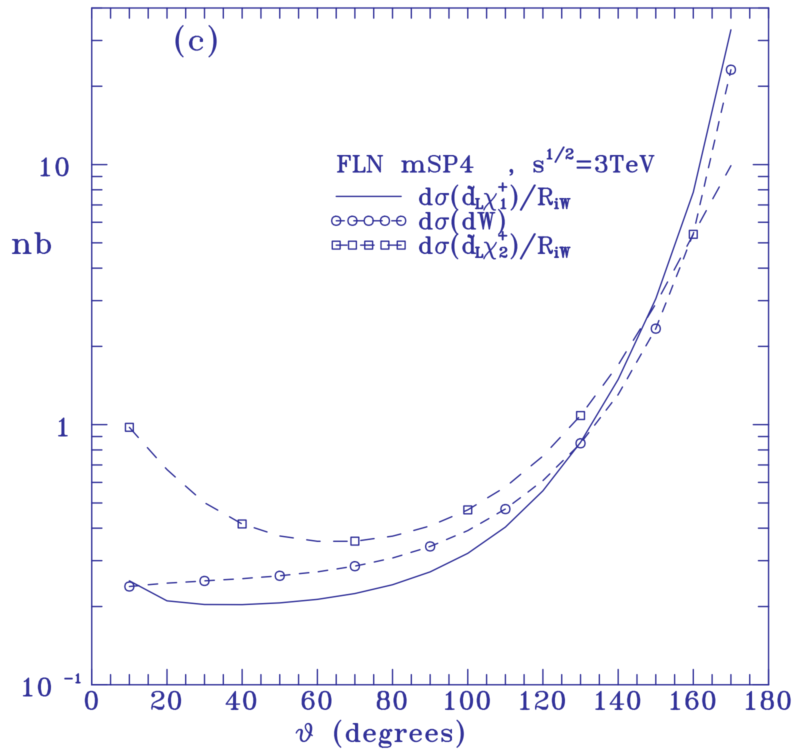


In principle, the SUSY σ-relations could be much worse than the F-relations, at non-asymptotic energies, since they also involve the squares of the HV amplitudes. **Actually, they are considerably better!**

The SUSY σ-relation for FLN mSP4.
M_{SUSY} =1.5TeV

$$\frac{d\sigma(ug \to dW^+)}{d\cos\theta}\ , \quad \frac{1}{R_{iW}}\frac{d\sigma(ug \to \tilde{d}_L \tilde{\chi}_i^+)}{d\cos\theta}$$





In these models, the SUSY σ-relations at non-asymptotic energies, are much better than the SUSY F-relations.
For some reason, the sub-dominant HV amplitudes reduce the discrepancies.

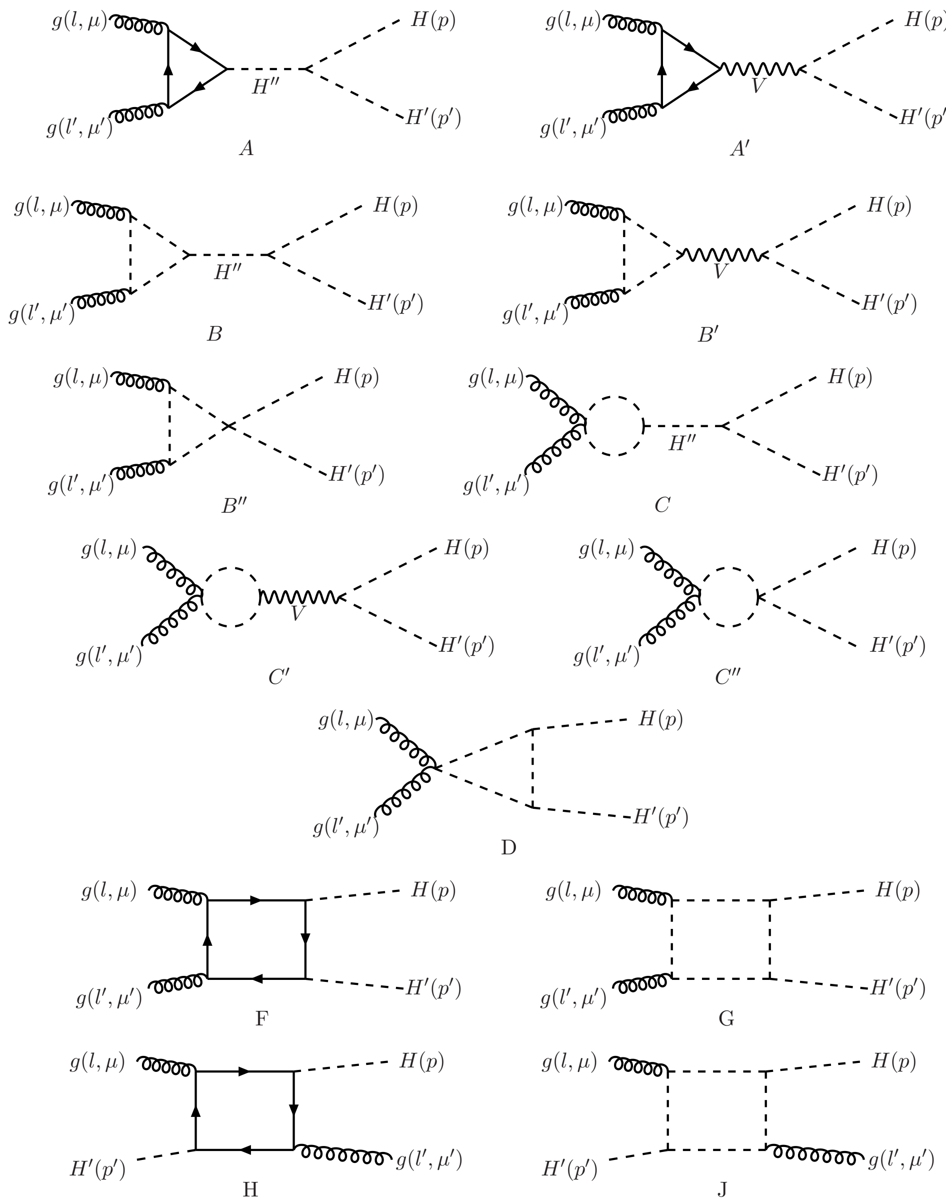
2a. Processes $g(\lambda_g) + g(\lambda_g') \to H + H' \Rightarrow F_{\lambda_g \lambda_g'}$, HC $\Rightarrow \lambda_g = -\lambda_g'$

1loop EW diagrams for
gg→HH'
in SM and MSSM.

No Born contribution this time.

In MSSM, the dominant amplitude F_{+-} tends to an energy-independent limit.

The other independent amplitude $F_{++} \to 0$.



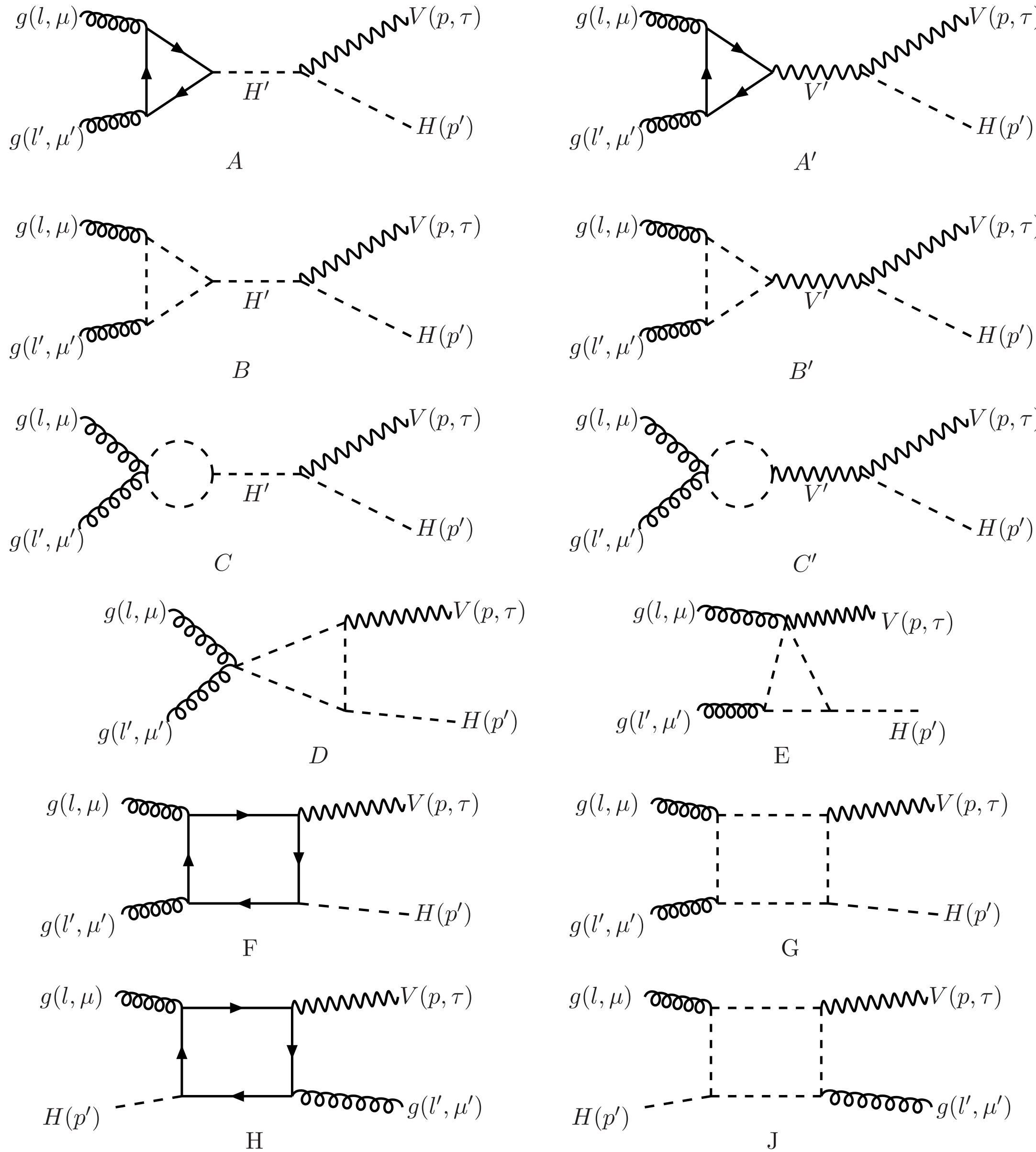
$2b.\ g(\lambda_g)+g(\lambda_g^{'})\rightarrow V(\lambda_V)+H \ \Rightarrow\ F_{\lambda_g\lambda_g^{'}\lambda_V}$, HC $\Rightarrow\ \lambda_g=-\lambda_g^{'}\ ,\ \lambda_V=0$

1loop EW diagrams for
gg→VH
in SM and MSSM.

No Born contribution this time.

The dominant amplitude F_{+-0} tends to energy-independent limit.

All others tend to 0…



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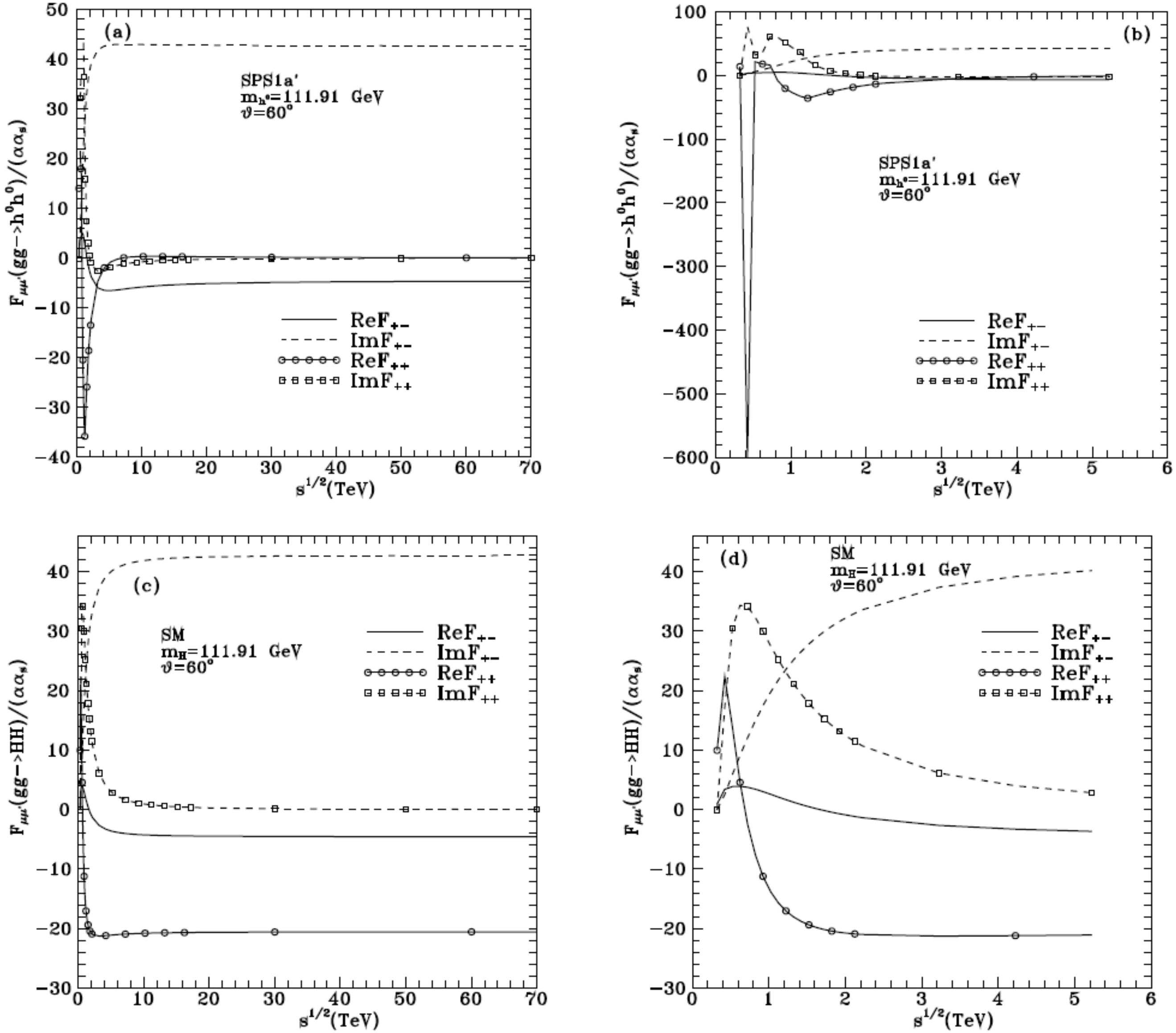
For $gg \to h^0 h^0$

HCns is strongly violated in SM.

Asymptotic $F_{++} \neq 0$ in SM



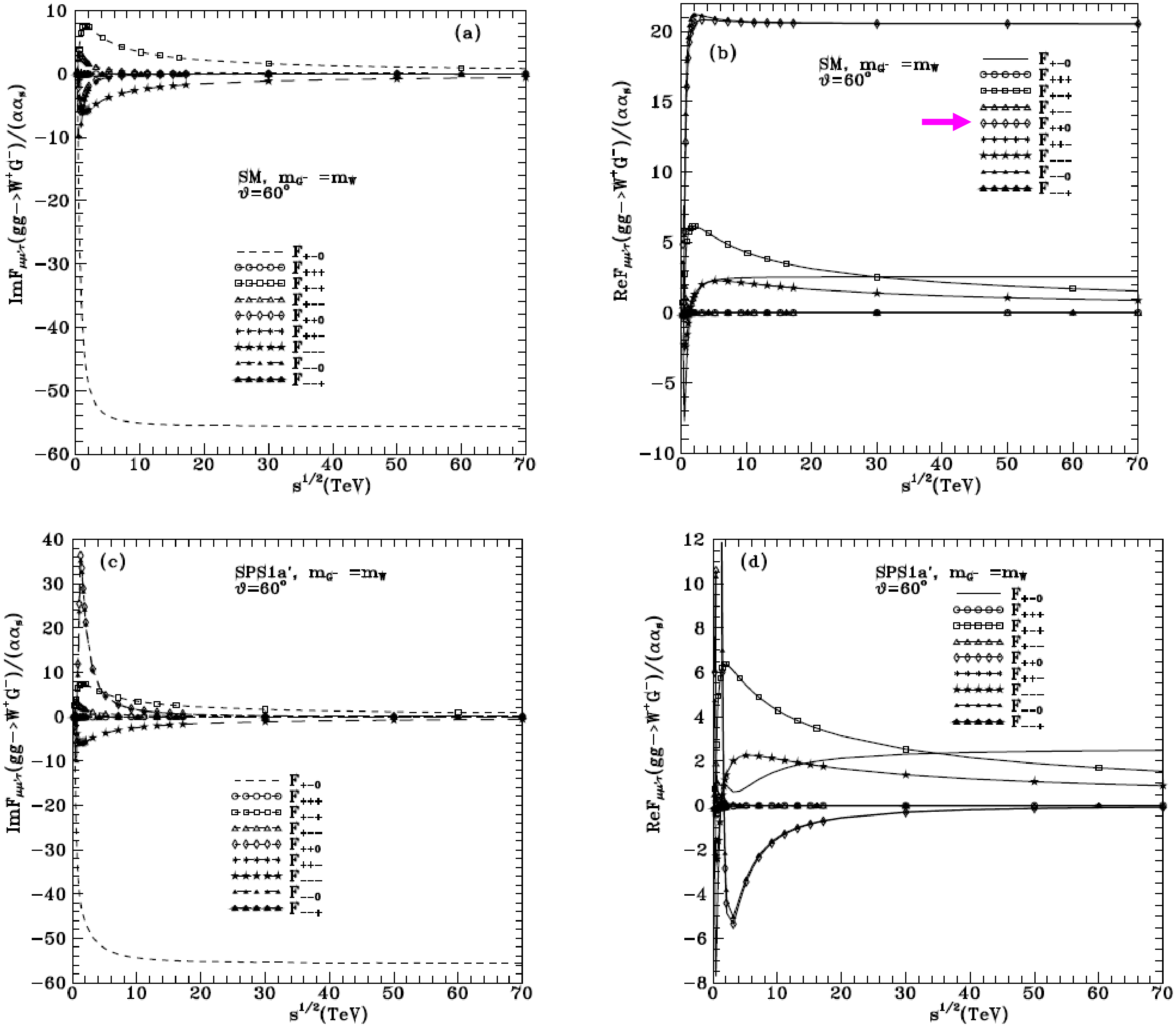
But HCns is always respected in MSSM, leading to $F_{++} \to 0$

For $gg \to W^+G^-$

HCns is strongly violated in SM.

Asymptotic $F_{++0} \neq 0$ in SM.

But HCns always respected in MSSM leading to $F_{++0} \to 0$



The relations among the HC asymptotic amplitudes, may be transformed to asymptotic relations among the subprocess cross sections, like

$$R_1 \Rightarrow \tilde{\sigma}(gg \to G^0 G^0) \approx \tilde{\sigma}(gg \to G^0 A^0)\left(\frac{R_{a1}}{R_{a2}}\right)^2 \approx \tilde{\sigma}(gg \to A^0 A^0)\left(\frac{R_{a1}}{R_{a3}}\right)^2$$

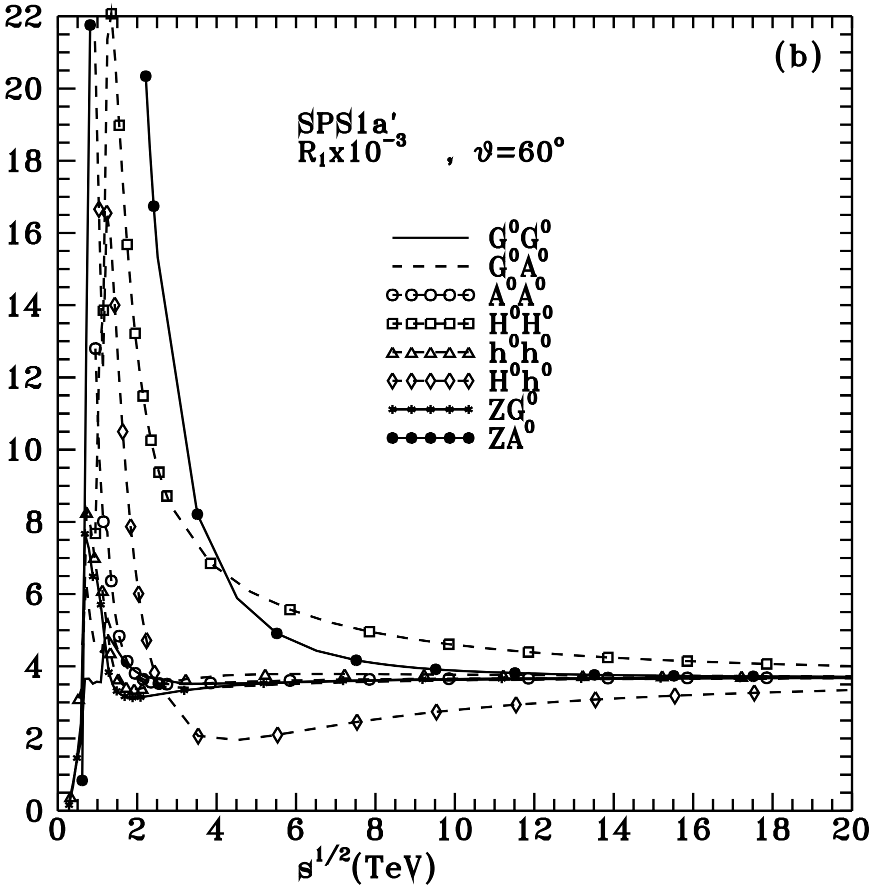
$$\approx \tilde{\sigma}(gg \to H^0 H^0)\left(\frac{R_{a1}}{R_{a4}}\right)^2 \approx \tilde{\sigma}(gg \to h^0 h^0)\left(\frac{R_{a1}}{R_{a5}}\right)^2 \approx \tilde{\sigma}(gg \to H^0 h^0)\left(\frac{R_{a1}}{R_{a6}}\right)^2$$

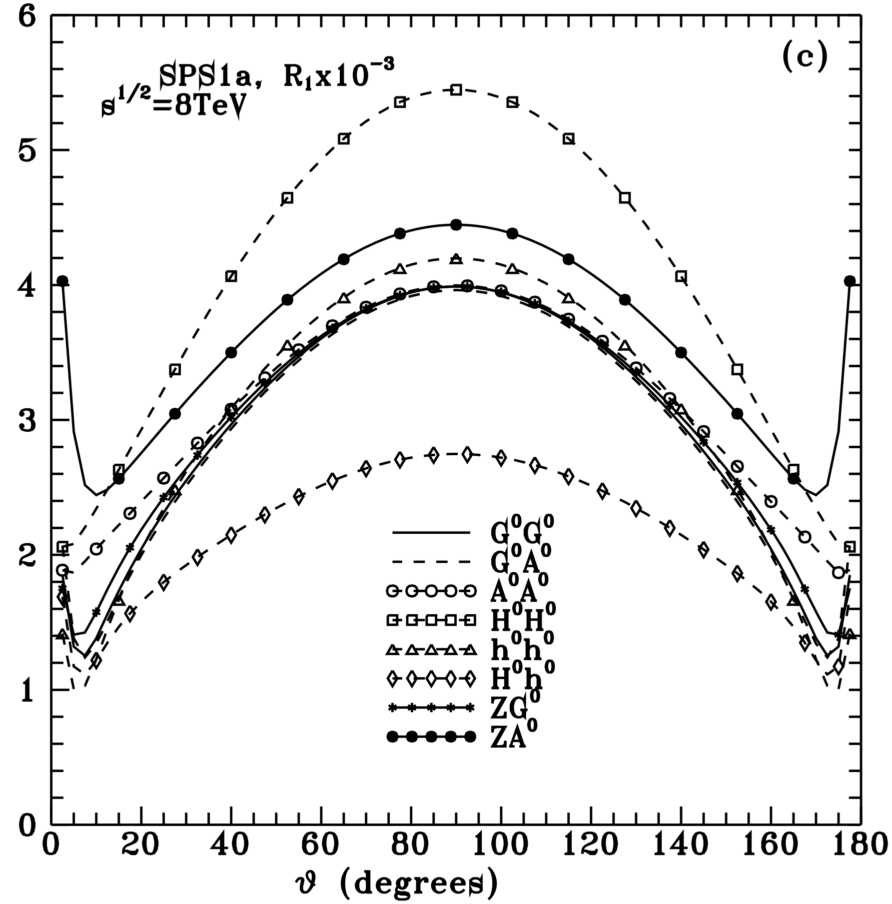
$$\approx \tilde{\sigma}(gg \to Z^0 G^0) \approx \tilde{\sigma}(gg \to Z^0 A^0)\left(\frac{R_{a1}}{R_{a2}}\right)^2$$

R_{aj} depend only on m_t, m_b, and the scalar sector parameters β and α

$$\tilde{\sigma}(gg \to ab) \equiv \frac{512\pi}{\alpha^2 \alpha_s^2} \frac{s^{3/2}}{p} \frac{d\sigma(gg \to ab)}{d\cos\theta}$$

They are valid to 1loop EW order. In deriving them "constant" asymptotic contributions to the PV functions have been retained. Only mass-suppressed terms have been neglected.





In SPS1a‘, at a subprocess c.m. energy $s^{1/2} \simeq 8$TeV, the R_1 relations are only partially satisfied.

We have derived many more such 1loop EW relations.

Conclusions

• **HCns is a genuine asymptotic SUSY property,**
which strongly simplifies the asymptotic 2-to-2 amplitudes.
It solely depends on the symmetry. Not on its breaking!

•**HCns should be considered on the same footing as the other basic SUSY properties, like e.g. the unification of couplings for TeV SUSY scale, the cancellation of the quadratic divergencies, and the inclusion of DM candidates.**

•**SM : If $F_{Born} \neq 0$, HCns is established, to the usually dominant 1loop leading-Log order.**
But for $F_{Born} = 0$, we have found examples where HCns is strongly violated in SM.

• **HCns provides many asymptotic relations among various subprocess cross sections. If the SUSY scale is not too high, they may be useful for LHC, or a future higher energy machine.**

•Codes for the amplitudes of the 1loop EW process used in this work, are available in http://users.auth.gr/gounaris/ FORTRANcodes.