Loop Quantum Gravity
Reduced Phase Space Quantisation

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NORDITA

Plan of the Talk

- Dynamics in Loop Quantum Gravity
- Reduced Phase space for General Relativity as a classical starting point for LQG
- Quantisation: Algebra of observables, its dynamics and semiclassical properties
- Summary & Conclusions

Dynamics in Loop Quantum Gravity

Classical Starting Point

- General Relativity, Classical Einstein Equations in its canonical form
- Perform a 3+1 split of spacetime, q_{ab}, p^{ab}
- h_{can} and constraints $c(q,p), c_a(q,p)$, moreover $h_{can} \approx 0$

Possibilities to quantise systems with constraints

- Consider classically symmetry reduced sector and quantise it (LQC lectures by Ashtekar)

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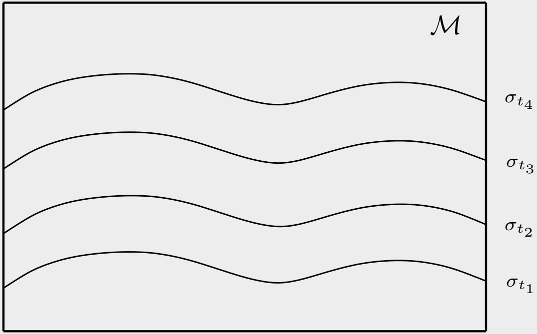
Foliation of Space Time

(3+1) Split into Space & Time



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Dynamics in Loop Quantum Gravity

Option 1: (standard Dirac procedure)

- Quantum Einstein's equations, constraints become operators $\hat{\mathrm{c}}, \hat{\mathrm{c}}_{\mathrm{a}}$
- Gauge dof are quantised, kinematical algebra, kinem. Hilbert space $\mathcal{H}_{\mathrm{kin}}$
- Solutions of $\hat{\mathrm{c}}\psi = 0$ and $\hat{\mathrm{c}}_{\mathrm{a}}\psi = 0$, physical Hilbert space

Option 2:

- Gauge dof are reduced at the classical level, Construction of observables
- Quantum Einstein's equations involving physical Hamiltonian $\widehat{\mathrm{H}}_{\mathrm{phys}} \neq 0$
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Option 2: Reduced Phase Space Approach

- Task 1: Construct observables for General Relativity
- Task 2: Discuss their evolution which cannot be generated by $\mathbf{h}_{\mathrm{can}}$
- Task 3: Quantisation of algebra of observables

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Clocks for General Relativity

Explicit Construction of Observables for GR

- Problem of time in GR: Gauge and physical evolution, $\mathrm{h}_{\mathrm{can}} \approx 0$
- Physical evolution can be defined in relational way [Bergmann'50][Rovelli '90]
- Introduction of reference fields
- Choose clock and ruler to give time & space physical meaning
- Choose clocks which lead to (partially) deparametrised form of GR
- 4 scalar fields, 4 dust fields ... dynamically coupled observer

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Brown-Kuchař-Mechanism ['95]

Add Dust Lagrangian to Gravity + Standard Model

Dust action

$$S_{\mathrm{dust}} = -\frac{1}{2}\int_M d^4X\sqrt{|\det(g)|}\rho(g^{\mu\nu}U_\mu U_\nu + 1)$$

where $U_\mu = -T_{,\mu} + W_J S^J_{,\mu}$ is the four velocity, $J = 1, 2, 3$

- After solving second class constraints for ρ and W_J we are left with T, S^J
- $U^\mu = g^{\mu\nu}U_\nu$ is a geodesic, fields S^J are constant along geodesics, T defines proper time along each geodesic
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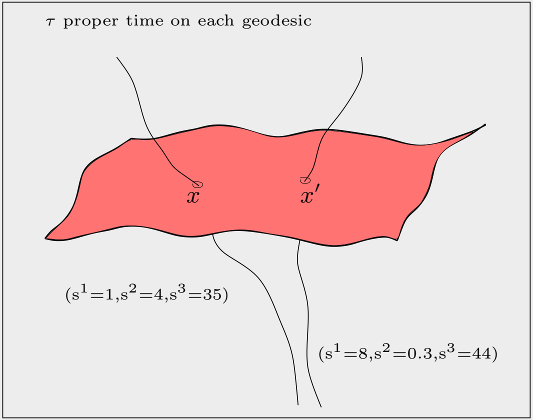
Observables with respect to Dust Clock & Rulers

Space time points are labelled by τ and s^j



Deparametrisation of the Constraints in GR [Brown – Kuchar '90s]

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- Canonical (3+1) – split:

  $(\mathrm{P}, \mathrm{T}), (\mathrm{S}^{\mathrm{J}}, \mathrm{P}_{\mathrm{J}})$ dust variables

  (q_{ab}, p^{ab}) etc. (gravity and any standard matter)
- Constraints:

$$\begin{aligned} c^{tot} &= c^{nd} + c^{dust}, \quad c^{dust} = -\sqrt{P^2 + q^{ab} c^{dust}_a c^{dust}_b} \\ c^{tot}_a &= c^{nd}_a + c^{dust}_a, \quad c^{dust}_a = PT_{,a} + P_J S^J_{,a} \end{aligned}$$

- Idea: Solve constraints for dust momenta P and P_j

$$\begin{aligned} \tilde{c}^{tot} &= P + h(q_{ab}, p^{ab}), \qquad h = \sqrt{(c^{nd})^2 - q^{ab} c^{nd}_a c^{nd}_b} \\ \tilde{c}^{tot}_J &= P_J + h_J(T, S^J, q_{ab}, p^{ab}), \quad h_J = S^a_J(c^{nd}_a - hT_{,a}) \end{aligned}$$

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Observables & Physical Hamiltonian

Task 1: Observables: Gravity + any other standard matter

- Relational formalism [Rovelli '90],
  Power series expression for observables [Dittrich '05]
- For any f not depending on dust dof we construct observables

$$O_{f,\{T,S^J\}}(\tau,s^J) := \exp(\{h_\tau,.\})\exp(\int_\sigma d^3x\beta^J\{c_J^{tot},.\})\cdot f(t,x)\big|_{\beta^J=s^J-S^J}$$

$$h_\tau := \int_{\mathcal{S}} d^3s(\tau-T)h(s)$$

For simplicity denote observables by capital letters: $f(t,x) \longrightarrow F(\tau,s)$

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$$\mathbf{H}_{phys} = \int_{\mathcal{S}} d^3s\, H(s) \quad \text{with} \quad H(s) = \sqrt{(C^{nd})^2 - Q^{ab}C^{nd}_a C^{nd}_b} \quad \mathbf{H}_{phys} \neq 0$$

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Quantisation

Reduced Phase Space Quantisation

- Algebra of observables is in general more complicated than the kinematical one
- Here we have

$$\{P^{IJ}(\tau, s)\,,\, Q_{KL}(\tau, s')\} = \delta^I_K \delta^J_L \delta^3(s, s')$$

- Easy to find representations: Quantisation trivial? Fock space possible?
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Reduced Phase Space Quantisation

- Algebra of observables is in general more complicated than the kinematical one
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Elementary kinematical variables

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- Additional Gauß Constraint:

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- Constraints closer to lattice gauge theory

$$\begin{aligned} G^{grav}_j &= \mathcal{D}_I E^I_j, \quad C^{grav}_I = \mathrm{Tr}(F_{IJ}E^J) \\ C^{grav} &= \frac{\mathrm{Tr}(F_{IJ}[E^I,E^J])}{\sqrt{|\det(E)|}} + \ldots \end{aligned}$$

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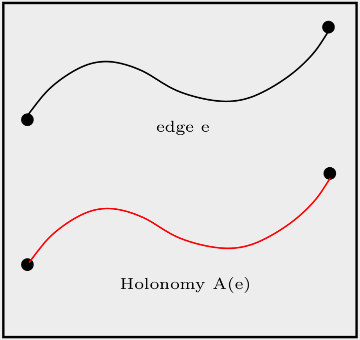

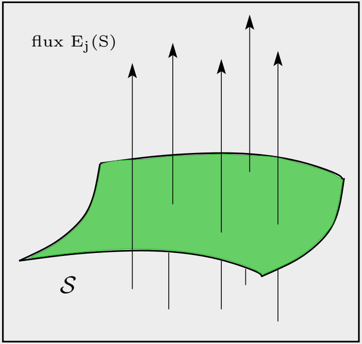

Classical physical phase space for reduced LQG

Recall: Symmetries of $\mathbf{H}_{\text{phys}}$

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  [K.G., Thiemann '06]
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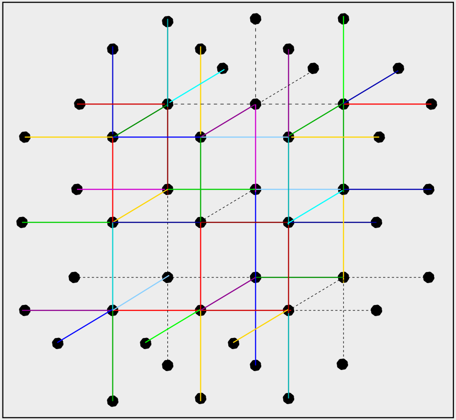
Quantisation of $\widehat{\mathbf{H}}_{\mathrm{phys}}$

Example: Cubic Graph



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Operator for $\mathbf{H}_{\mathrm{phys}}$

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$$\widehat{\mathbf{H}}_{\mathrm{phys}} = \frac{\hbar}{\ell_{\mathrm{p}}^4} \sum_{\mathrm{v}\in V(\alpha)} \sqrt{\left| \sum_{\mu=0}^{4} \eta^{\mu\mu} \left[\sum_{\mathrm{a}=1}^{3} \mathrm{Tr}\Big(\tau_\mu A(\alpha_{\mathrm{v}}^{\mathrm{a}}) A(e_{\mathrm{v}}^{\mathrm{a}}) \big[A(e_{\mathrm{v}}^{\mathrm{a}})^{-1}, V_{\mathrm{v}} \big] \Big) \right]^2 \right|}$$

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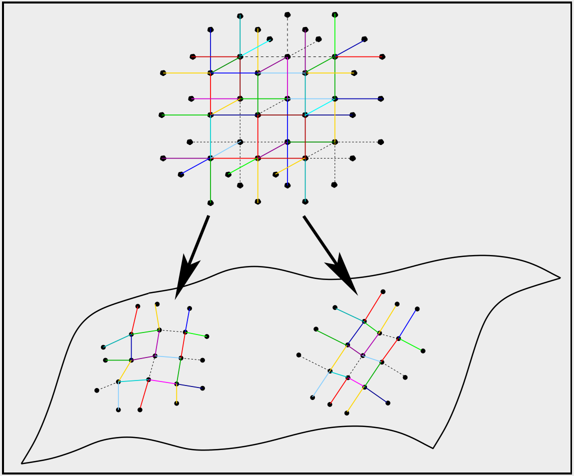
Fundamental Algebraic Graph

Information about embedding are encoded in coherent states



Semiclassical Analysis

Coherent States

- Choose manifold σ and an embedding X of the algebraic graph α, $\mathrm{X}(\alpha) = \gamma$
- Choose cell complex γ^*, dual to γ s.t. $\mathrm{e} \leftrightarrow \mathrm{S_e}$
- Choose classical field configuration $(\mathrm{A_0}, \mathrm{E_0})$
- Coherent States [Hall 90's], [Sahlmann, Thiemann, Winkler 00's]

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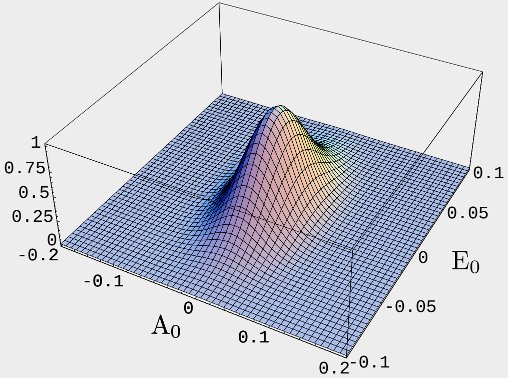
Coherent States [Hall '90s] [Sahlmann, Thiemann, Winkler '00s]

Coherent States peaked around classical phase space point (A_0, E_0)



Semiclassical Limit [K.G., Thiemann 06 – '07]

Theorem: For any sufficiently fine $X(\alpha)$ and any (A_0, E_0)

1. Exp. value:

$$\langle \psi_{(A_0,E_0)}\,,\, \widehat{\mathbf{H}}_{\mathrm{phys}}\, \psi_{(A_0,E_0)} \rangle = \mathbf{H}_{\mathrm{phys}}(A_0, E_0) + O(\hbar)$$

2. Fluctuations:

$$\langle \widehat{\mathbf{H}}^2_{\mathrm{phys}} \rangle_{\psi_{(A_0,E_0)}} - \left(\langle \widehat{\mathbf{H}}_{\mathrm{phys}} \rangle_{\psi_{(A_0,E_0)}} \right)^2 = O(\hbar)$$

Corollary

i. Quantum Hamiltonian is correctly implemented

ii. For sufficiently small τ

$$e^{i\tau \widehat{\mathbf{H}}_{\mathrm{phys}}} \psi_{(A_0,E_0)} \approx \psi_{(A_0(\tau),E_0(\tau))}$$

Semiclassical Limit [K.G., Thiemann 06 – '07]

Theorem: For any sufficiently fine $\mathrm{X}(\alpha)$ and any $(\mathrm{A}_0, \mathrm{E}_0)$

1. Exp. value:

$$\langle \psi_{(\mathrm{A}_0,\mathrm{E}_0)}\,,\, \widehat{\mathbf{H}}_{\mathrm{phys}}\, \psi_{(\mathrm{A}_0,\mathrm{E}_0)} \rangle = \mathbf{H}_{\mathrm{phys}}(\mathrm{A}_0, \mathrm{E}_0) + \mathrm{O}(\hbar)$$

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$$\langle \widehat{\mathbf{H}}^2_{\mathrm{phys}} \rangle_{\psi_{(\mathrm{A}_0,\mathrm{E}_0)}} - \big(\langle \widehat{\mathbf{H}}_{\mathrm{phys}} \rangle_{\psi_{(\mathrm{A}_0,\mathrm{E}_0)}}\big)^2 = \mathrm{O}(\hbar)$$

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Summary

Reduced Phase Space Quantisation for LQG [K.G.,Thiemann '07]

- By means of additional matter component (dust) constraints of GR can be reduced
- Gauge invariant analogue of Einstein's equation with true Hamiltonian
- Reduced phase space approach provides direct access to $\mathcal{H}_{\rm phys}$
- Algebra of observables and $\mathbf{H}_{\rm phys}$ can be quantised
- Semiclassical limit of $\widehat{\mathbf{H}}_{\rm phys}$ correct
- Reduced LQG is formulated as (background independent) Hamiltonian Lattice Gauge Theory

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- Anomalies of $\widehat{\mathrm{H}}_{\mathrm{phys}}$: Naive quantisation [work in progress K.G, Thiemann]
- Observer – dependent QFT, Unitary equivalence between different observers at the quantum level
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