Wolfenstein parametrization

Inspired by the fact that bottom quark couples predominantly to the charm quark, i.e.

$$|V_{cb}|^2 \gg |V_{ub}|^2$$

Wolfeinstein suggested an approximate parametrization of V:

$$V = \begin{bmatrix} (1-\lambda^2/2) & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & (1-\lambda^2/2) & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + O(\lambda^4)$$

The parameter λ serves as an expansion parameter.

Usefulness of the Wolfenstein parametrization:

One can estimate the order of magnitude of any function of V^{CKM} by considering the leading term of its expansion in powers of λ.

Exact Wolfenstein-type parametrization

It is possible to have a parametrization of the Wolfenstein-type (where each matrix element can be expanded in powers of a small parameter) which is exact.

Lavoura, G.C.B

- Adopt the following phase convention:

$$V_{ud} \; ; \; V_{us} \; ; \; V_{cs} \; , \; V_{cb} \; , \; V_{tb}$$

are real and positive.

Exercise – Show explicitly that this is always possible.

- Define λ, A, μ, ϕ by the following exact relations:

$$\lambda \equiv V_{us} \quad ; \quad A \equiv \frac{V_{cb}}{V_{us}^2} \quad ; \quad \mu \equiv \left| \frac{V_{ub}}{V_{us} V_{cb}} \right|$$

$$\phi \equiv \arg Q_{uscb}$$

One can then reconstruct the full CKM matrix in terms of the parameters λ, A, μ, ϕ :

$$V_{ud} = \sqrt{1 - \lambda^2 - A^2\mu^2\lambda^6}$$

$$V_{tb} = \sqrt{1 - A^2\lambda^4 - A^2\mu^2\lambda^6}$$

$$V_{cs} = \left\{ -A^2\mu\lambda^6 \cos\phi + \left[1 - \lambda^2 - A^2\lambda^4 + A^2(1-2\mu^2)\lambda^6 + \right.\right.$$
$$\left.\left. + A^2\mu^2\lambda^8 + A^4\mu^2\lambda^{10} + A^4\mu^2(\mu^2 - \sin^2\phi)\lambda^{12} \right]^{1/2} \right\} \Big/ 1 - A^2\mu^2\lambda^6$$

V_{cd} , V_{ts} , V_{td} can be readily obtained using the unitarity relations:

$$V_{cd} = \frac{-V_{cs}V_{us}^* - V_{cb}V_{ub}^*}{V_{ud}^*}$$

$$V_{ts} = \frac{-V_{us}V_{ub}^* - V_{cs}V_{cb}^*}{V_{tb}^*}$$

$$V_{td} = \frac{-V_{ts}V_{us}^* - V_{tb}V_{ub}^*}{V_{ud}^*}$$

One can then perform an expansion as a series in λ to any desired order.

It is convenient to introduce $\rho \equiv \mu \cos\phi$, $\eta \equiv \mu \sin\phi$ in order to easily compare with the Wolfenstein parametrization.

$$V_{us} = \lambda \quad ; \quad V_{cb} = A\lambda^2 \quad ; \quad V_{ub} = A\mu\lambda^3 e^{-i\phi}$$

$$V_{ud} = 1 - 1/2\,\lambda^2 - \frac{1}{8}\lambda^4 + O(\lambda^6)$$

$$V_{cd} = -\lambda + A^2\left(\frac{1}{2} - \rho - i\eta\right)\lambda^5 + O(\lambda^7)$$

$$V_{cs} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}(1 + 4A^2)\lambda^4 + O(\lambda^6)$$

$$V_{td} = A(1 - \rho - i\eta)\lambda^3 + \frac{1}{2}A(\rho + i\eta)\lambda^5 + O(\lambda^7)$$

$$V_{ts} = -A\lambda^2 + A\left(\frac{1}{2} - \rho - i\eta\right)\lambda^4 + O(\lambda^6)$$

$$V_{tb} = 1 - \frac{1}{2}A^2\lambda^4 + O(\lambda^6)$$

This coincides with the original Wolfenstein parametrization up to order λ^3.

Rephasing invariant parametrizations

Can one parametrize V^{CKM}, using only rephasing invariant quantities? Yes!

Examples:

1) Parametrization using moduli of V^{CKM}

One can parametrize V^{CKM} using 4 independent moduli. A convenient choice is:

$$|V_{us}| \ ; \ |V_{ub}| \ ; \ |V_{cb}| \ ; \ |V_{td}|$$

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

One can use normalization of columns and rows of V^{CKM} to compute the value of the remaining five moduli

One can evaluate the strength of CP violation (i.e. J) from the general formula:

$$J^2 = U_{\alpha i} U_{\beta j} U_{\alpha j} U_{\beta i} - \frac{1}{4}\left[2\,\mathrm{Re}\,Q_{\alpha i \beta j}\right]^2$$

where

$$2\,\mathrm{Re}\,Q_{\alpha i \beta j} = 1 - U_{\alpha i} - U_{\beta j} - U_{\alpha j} - U_{\beta i}$$

$$+ U_{\alpha i} U_{\beta j} + U_{\alpha j} U_{\beta i}$$

$U_{\alpha i} \equiv |V_{\alpha i}|^2$

With our choice of moduli, it is convenient to use the quartet

$$\boxed{Q_{uscb}}$$

i.e. $\alpha = u$, $i = s$; $\beta = c$, $j = b$.

Note that only $|J|$ can be obtained from input moduli, i.e there is a two-fold ambiguity.

Caution: Choose the moduli carefully!!

Bjorken - Dunietz parametrization

B., D. suggest a parametrization using

one rephasing invariant phase and three moduli:

$$|V_{us}|, |V_{ub}|, |V_{cb}| \quad \arg Q_{uscb}$$

Exercise: Show that one can reconstruct the full CKM matrix from the above 4 input values, using unitarity of V^{CKM}

Question:

Can one parametrize V^{CKM}, using 4 rephasing invariant phases as input?

Aleksan – Kayser – London parametrization

Consider the standard model with n_g generations. For the moment let us not impose unitarity of V^{CKM}:

$$\begin{bmatrix} \bar{u} & \bar{c} & \bar{t} & \cdots \end{bmatrix} \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & \cdots \\ V_{cd} & V_{cs} & V_{cb} & \cdots \\ V_{td} & V_{ts} & V_{tb} & \cdots \\ \vdots & & & \ddots \end{bmatrix} \begin{bmatrix} d \\ s \\ b \\ \vdots \end{bmatrix}$$

Obviously, there are n_g^2 phases in V^{CKM}.

Of these, one can eliminate $(2n_g - 1)$ phases through rephasing of quark fields. The number of independent rephasing invariant phases (without imposing unitarity!) is thus

$$\boxed{N_{phases} = n_g^2 - (2n_g - 1) = (n_g - 1)^2}$$

Note that

$$N_{phases} = N_{parameters} \quad !!$$

Special feature of the SM!!

Aleksan - Kayser and London have shown that one can fully reconstruct V^{CKM} from 4 independent rephasing invariant phases, using unitarity of V^{CKM}. An interesting choice of rephasing invariant phases is:

$$\beta \equiv \arg\left(-Q_{tbcd}\right)$$

$$\gamma \equiv \arg\left(-Q_{cbud}\right)$$

$$\beta_s \leftarrow \chi \equiv \arg\left(-V_{cb} V_{ts} V_{cs}^* V_{tb}^*\right)$$

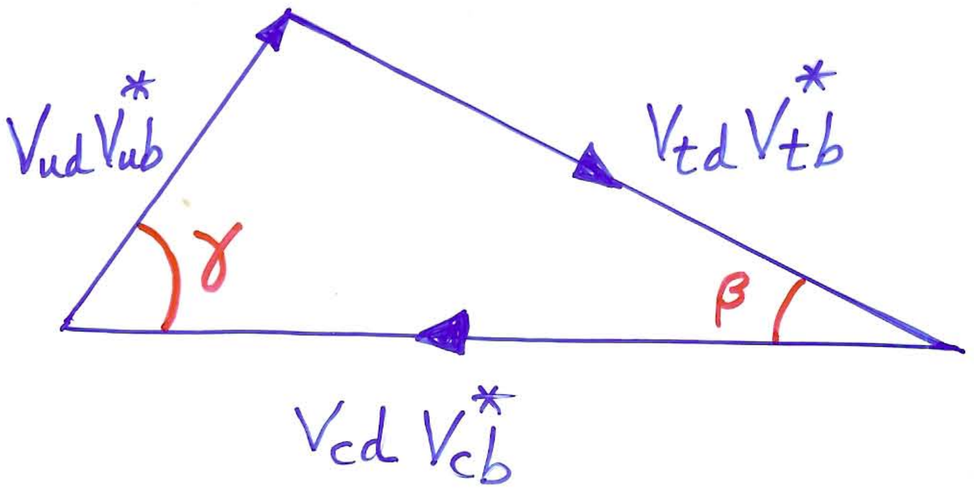
$$\chi' \equiv \arg\left(-V_{us} V_{cd} V_{ud}^* V_{cs}^*\right)$$

In the SM, β and γ may be large while χ and χ' have to be small.

Exercise: Prove that one can reconstruct the full CKM matrix, using $\beta, \gamma, \chi, \chi'$, and unitarity of V^{CKM}.

The usual unitarity triangle:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



The usual Unitarity triangle fit consists of checking wether all data on

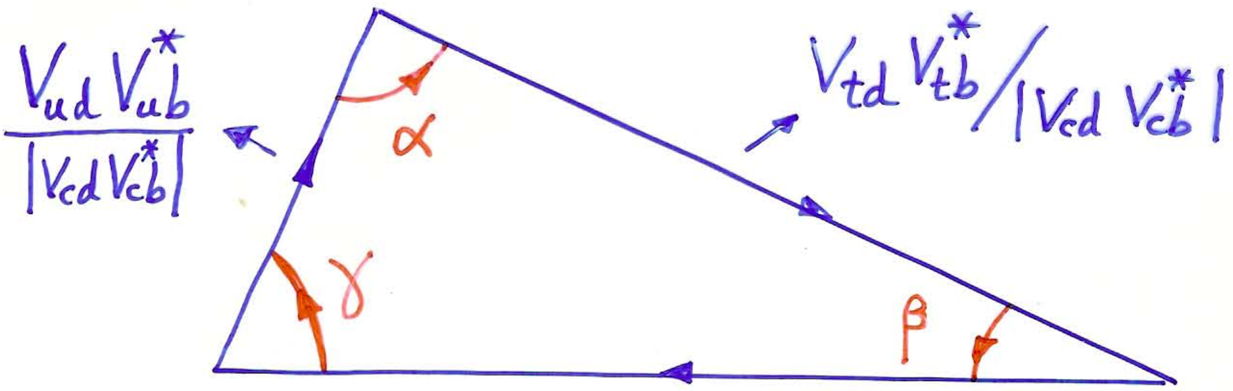
$|V_{ub}|$; $|V_{td}|$, β, γ, ε_K etc.

can be simultaneously fit

The unitarity Triangle

Orthogonality of 1st and 3rd column

$$|V_{ud}||V_{ub}|e^{i\gamma} - |V_{cd}||V_{cb}| + |V_{td}||V_{tb}|e^{-i\beta} = 0$$



$$\frac{V_{cd}V_{cb}^*}{|V_{cd}V_{cb}^*|} = -1$$

$$\alpha \equiv \arg\left(-V_{td}V_{ub}V_{tb}^*V_{ud}^*\right)$$
$$\beta \equiv \arg\left(-V_{cd}V_{tb}V_{cb}^*V_{td}^*\right)$$
$$\gamma \equiv \arg\left(-V_{ud}V_{cb}V_{ub}^*V_{cd}^*\right)$$

; $\alpha+\beta+\gamma = \pi$ by definition !!

In the Wolfenstein parametrization, to leading order :

$$\frac{V_{ud}V_{ub}^*}{|V_{cd}V_{cb}^*|} = \rho + i\eta \qquad \frac{V_{td}V_{tb}^*}{|V_{cd}V_{cb}^*|} = 1-\rho-i\eta$$

can be used as the definition of ρ, η

At present, the **CKM** mechanism for flavour mixing and CP Violation is in "agreement" with all available experimental data. This is a remarkable success of the SM.

$$V^{CKM} = \begin{pmatrix} \overset{V_{ud}}{c_{12}c_{13}} & \overset{V_{us}}{c_{13}s_{12}} & \overset{V_{ub}}{s_{13}e^{-i\delta_{13}}} \\ \overset{V_{cd}}{-c_{23}s_{12} - c_{12}s_{23}s_{13}e^{i\delta_{13}}} & \overset{V_{cs}}{c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}}} & \overset{V_{cb}}{c_{13}s_{23}} \\ \overset{V_{td}}{s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}}} & \overset{V_{ts}}{-c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta_{13}}} & \overset{V_{tb}}{c_{23}c_{13}} \end{pmatrix}$$

$$|V_{us}| \longrightarrow \boxed{s_{12}} \;;\; |V_{cb}| \longrightarrow \boxed{s_{23}} \;;\; |V_{ub}| \longrightarrow \boxed{s_{13}}$$

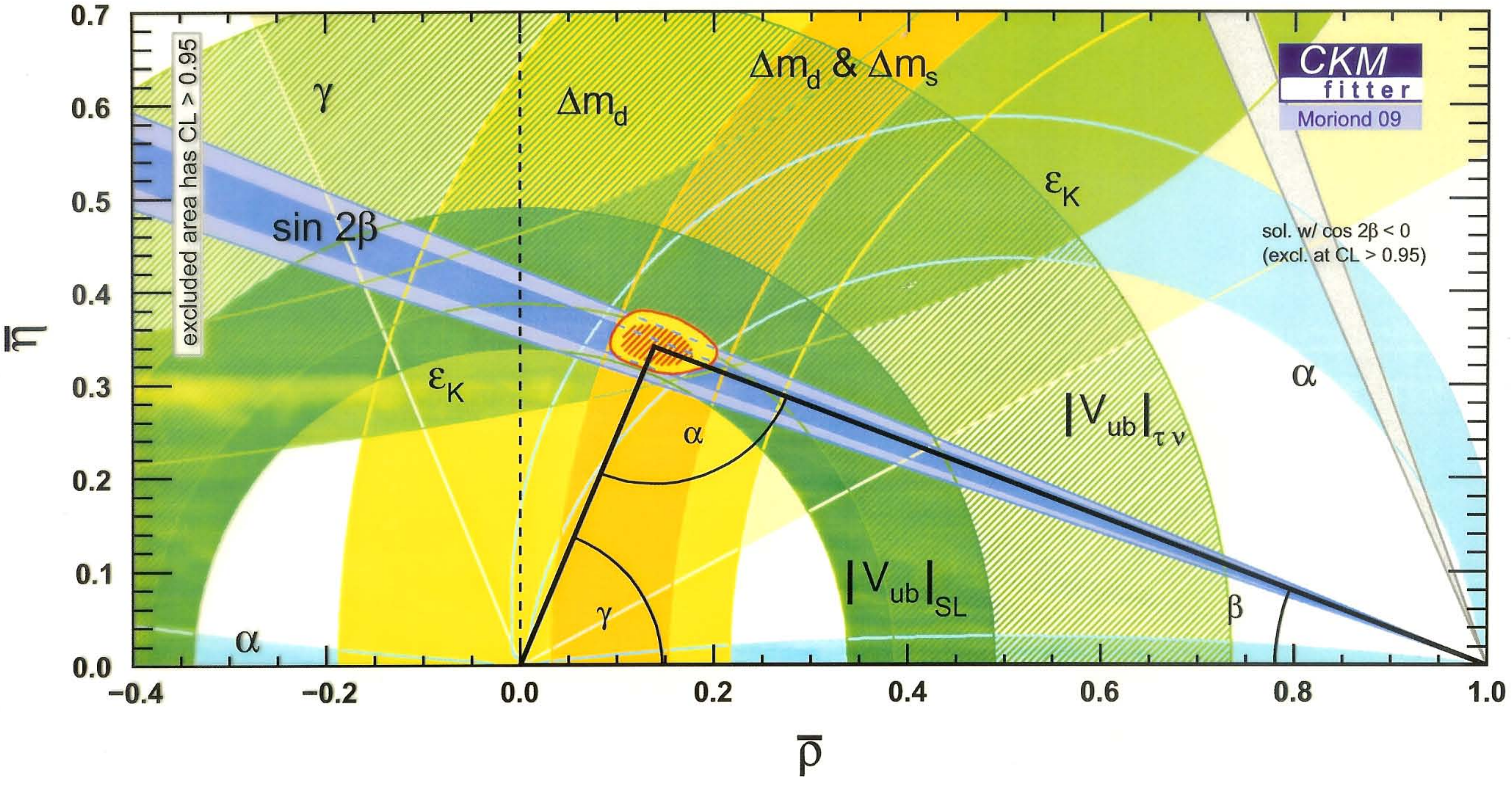
A large number of experimental quantities have to be fit with only one parameter (δ_{13}):

ε_K ; $\sin 2\beta$; ΔM_{B_d} , ΔM_{B_s} etc

ε'/ε , γ , $K^+ \to \pi^+ \nu\bar{\nu}$

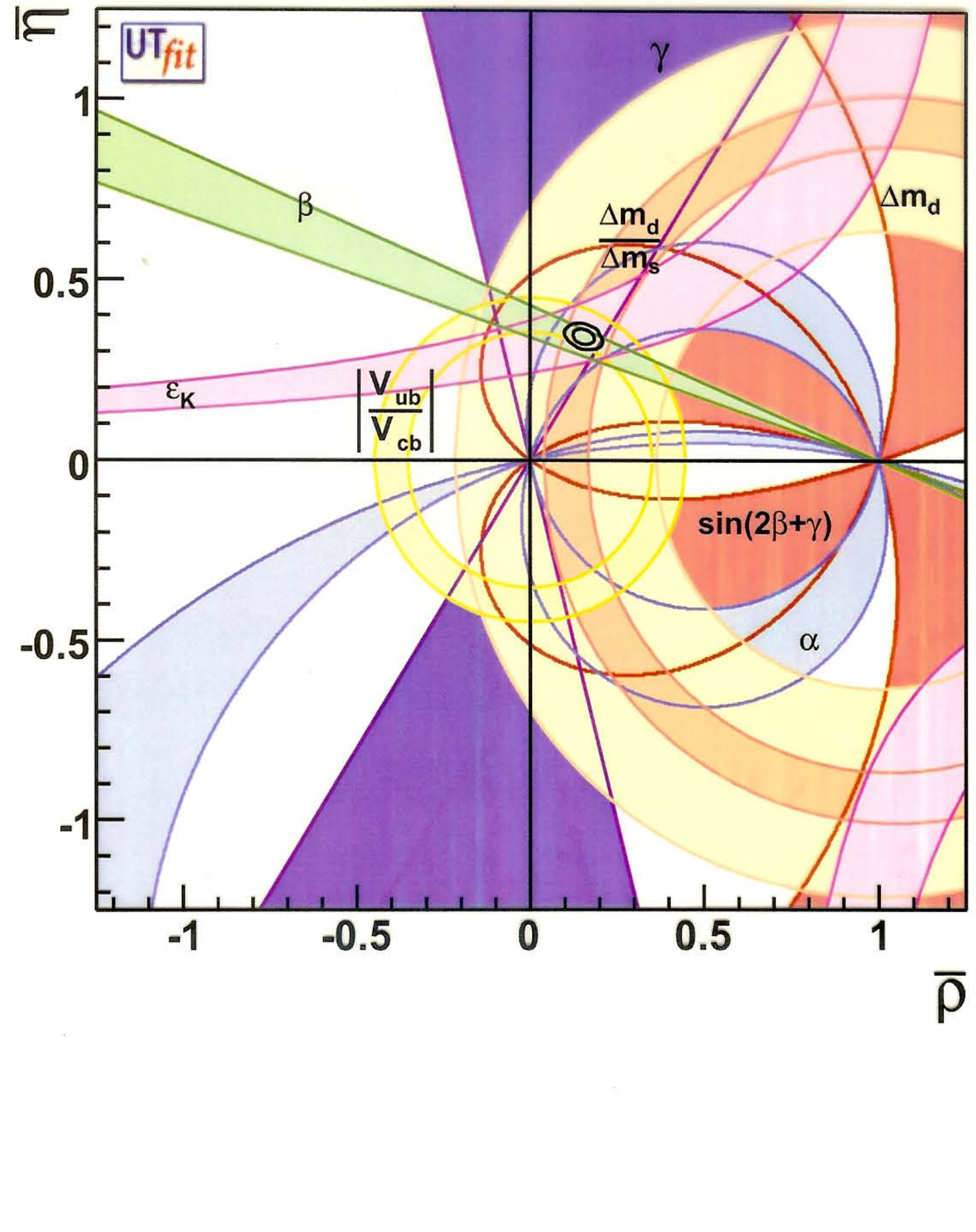
$K_L \to \pi^0 \nu\bar{\nu}$

etc.



CKM
fitter
Moriond 09
excluded area has CL > 0.95
γ
Δm_d
Δm_d & Δm_s
ε_K
sin 2β
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
ε_K
$|V_{ub}|_{\tau\nu}$
$|V_{ub}|_{SL}$
β
$\bar{\eta}$
$\bar{\rho}$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0

50



UTfit
$\bar{\eta}$
$\bar{\rho}$
γ
β
ε_K
$\left|\frac{V_{ub}}{V_{cb}}\right|$
$\frac{\Delta m_d}{\Delta m_s}$
Δm_d
$\sin(2\beta+\gamma)$
α
1
0.5
0
-0.5
-1

Testing the Standard Model and Searching for New Physics

Consider the quark mixing matrix:

$$V^{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The only phases which are physically meaningful are the arguments of rephasing invariant quartets. There are four independent rephasing invariant phases which can be chosen to be:

$$\gamma \equiv \arg\left(-V_{ud} V_{cb} V_{ub}^* V_{cd}^*\right)$$

$$\beta \equiv \arg\left(-V_{cd} V_{tb} V_{cb}^* V_{td}^*\right)$$

$$\beta_s \leftarrow \chi \equiv \arg\left(-V_{cb} V_{ts} V_{cs}^* V_{tb}^*\right)$$

$$\chi' \equiv \arg\left(-V_{us} V_{cd} V_{ud}^* V_{cs}^*\right)$$

Together with the 9 moduli, one has 13 physical quantities.

In the Standard model these 13 quantities are related by 3x3 unitarity.

- One can derive, using 3x3 unitarity, a series of exact equations relating these 13 physical quantities.
- These relations provide a stringent test of the SM, with the potential of uncovering New Physics

In deriving exact unitarity relations it is very useful to adopt a convenient phase convention :

$$\arg V^{CKM} = \begin{bmatrix} 0 & \chi' & -\gamma \\ \pi & 0 & 0 \\ -\beta & \pi+\chi & 0 \end{bmatrix}$$

Examples of exact relations implied by 3×3 unitarity : G.C.B., F.Botella, M.Nebot, M.N.Rebelo

$$\boxed{\frac{|V_{ub}|}{|V_{td}|} = \frac{\sin\beta}{\sin\gamma}\,\frac{|V_{tb}|}{|V_{ud}|}}$$ (db triangle)

$$\boxed{\sin\chi = \frac{|V_{td}|}{|V_{ts}|}\,\frac{|V_{cd}|}{|V_{cs}|}\,\sin\beta}$$ (ct triangle)

Important future test of the SM.

χ is of order λ^2 in the SM

$$\boxed{\sin \chi' = \frac{|V_{ub}|}{|V_{us}|}\frac{|V_{cb}|}{|V_{cs}|} \sin \gamma}$$

it shows that $\chi' = O(\lambda^4)$ in the SM

Another interesting exact relation:

$$\boxed{\sin \chi = \frac{|V_{us}||V_{cd}||V_{cb}|}{|V_{ts}||V_{tb}||V_{ud}|} \frac{\sin\beta \sin(\gamma + \chi')}{\sin(\gamma+\beta)}}$$

This can be very well approximated by:

$$\boxed{\sin \chi \simeq \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{\sin\beta \sin\gamma}{\sin(\gamma+\beta)}}$$

Silva, Wolfenstein

These exact relations provide a simple way of testing the SM, which goes beyond the usual Unitarity triangle fit.

In testing the SM and searching for New Physics, it is reasonable to assume that:

(i) Tree level weak decays are dominated by the charged weak interactions of the SM. This means that the extraction of $V_{ud}, V_{us}, V_{ub}, V_{cd}, V_{cs}, V_{cb}$ are not affected by the eventual presence of New Physics

(ii) There may be significant New Physics contributions to $B_d-\bar{B}_d$, $B_s-\bar{B}_s$ mixings. This may affect the extraction of $|V_{td}|$ and $|V_{ts}|$ from the experimental value of $B_d-\bar{B}_d$ and $B_s-\bar{B}_s$ mixings

- At present the Unitarity Triangle Fit is consistent with the SM.

- New Physics contributions to $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings at 10-20% level are still allowed.

- The is strong evidence that V^{CKM} is complex (i.e. unitarity triangles do not collapse) even if one allows for the presence of New Physics

- Recently some experimental evidence hint at potential deviations from the SM predictions in mixing induced CP violation in $B_s \to J/\psi\, \phi$

Recall that in the SM χ is small $\to O(\lambda^2)$

The Origin of CP Violation

Question: Does the evidence for a complex V^{CKM} imply that the origin of **CP violation** is "complex Yukawa couplings"?

Answer: Absolutely not!!

It is possible to have "reasonable models" where the Lagrangian is CP invariant but the vaccuum breaks CP spontaneously.

This breaking of CP can generate a complex V^{CKM} even if Yukawa couplings are all real.

Spontaneous CP Violation

Spontaneous CP violation occurs when CP is a symmetry of the Lagrangian but the vacuum is not CP invariant.

This means that :

- There is a transformation that may be physically interpreted as CP, under which the Lagrangian is invariant
- There is no transformation that may be physically interpreted as CP under which both the Lagrangian and the vacuum are invariant.

Let us consider a multi-Higgs extension of the SM with n Higgs doublets. Let us assume that the minimum of the potential is at:

$$\langle 0 | \phi_j | 0 \rangle = v_j e^{i\theta_j}$$

What are the conditions for this vacuum to be CP invariant, i.e.

$$CP|0\rangle = |0\rangle$$

$$\langle 0 | (CP)^\dagger (CP) \phi_j (CP)^\dagger CP | 0 \rangle = v_j e^{i\theta_j}$$

If under CP:

$$CP\, \phi_j\, (CP)^\dagger = U_{jk} \phi_k^*$$

$U \rightarrow$ unitary matrix acting in Higgs space

one obtains

$$\boxed{U_{jk} v_k e^{-i\theta_k} = v_j e^{i\theta_j}}$$

If there is no U which satisfies this equation, there is Spontaneous CP violation

T. D. Lee Model of Spontaneous CP violation in 2009

Scalar potential for 2 Higgs doublet model, with no extra symmetries.

$$V = Y_{ab}\,\phi_a^\dagger \phi_b + Z_{abcd}\,(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

Hermiticity of V implies:

$$Y_{ab} = Y_{ba}^*$$

$$Z_{abcd} = Z_{badc}^*$$

Impose CP symmetry at Lagrangian level by requiring Y_{ab}, Z_{abcd} to be real. T. D. Lee has carefully shown that there is a region of the parameters where:

$$\langle 0|\phi_1|0\rangle = \begin{bmatrix} 0 \\ v_1 \end{bmatrix} \; ; \; \langle 0|\phi_2|0\rangle = \begin{bmatrix} 0 \\ v_2 e^{i\theta} \end{bmatrix}$$

Conserves charge !!!

This vacuum violates CP and T invariance. The crucial point is that in this case, U_{jk} appearing in

$$CP\,\phi_j\,(CP)^\dagger = U_{jk}\,\phi_k^*$$

is trivial, namely $U_{jk} = \delta_{jk}$.

Question : Does the phase θ generate a complex CKM matrix?

Answer :: For 3 or more fermion generations, Yes !!

$$M_d = Y_d^1\, v_1 + Y_d^2\, v_2\, e^{i\theta}$$

$$M_u = Y_u^1\, v_1 + Y_u^2\, v_2\, e^{-i\theta}$$

$$M_d M_d^\dagger = Y_d^1 Y_d^{1\dagger} + Y_d^2 Y_d^{2\dagger} + v_1 v_2 \left(Y_d^1 Y_d^{2\dagger} + Y_d^2 Y^{1\dagger}\right)\cos\theta$$

$$+\, i\, v_1 v_2 \left(Y_d^2 Y_d^{1\dagger} - Y_d^1 Y_d^{2\dagger}\right)\operatorname{sen}\theta$$

Similarly for $M_u M_u^\dagger$.

One can explicitly check that in general for the Lee's model for three generations

$$\mathrm{tr}\left[H_u, H_d\right]^3 \neq 0 \Rightarrow V_{CKM} \text{ complex}$$

Two sources of CP violation:

KM mechanism + Higgs exchange

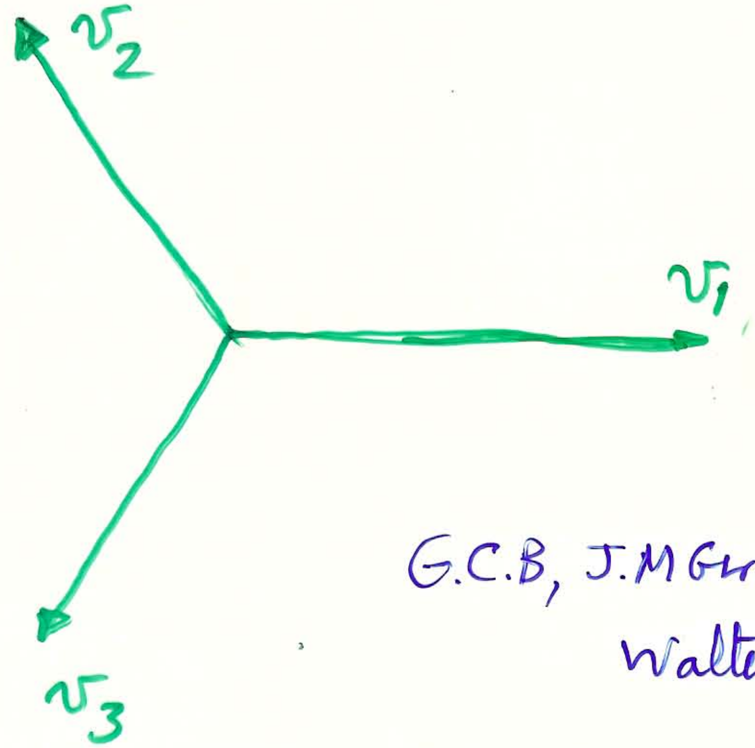
Difficulty of the Model

FCNC in the Higgs sector at tree level!

Exercise – Show that if one introduces NFC in the Higgs sector à la Glashow–Weinberg, one kills the possibility of having spontaneous CP violation

Models with "family" symmetries often lead to vacua with "geometrical phases".

For example S_3 has minima of the type



G.C.B, J.M Gerard, Walter Grimus

Usually these minima do not violate CP. The same symmetry which leads to these vacua, allows for special CP transformations which may the vacua CP invariant

There are various models of Spontaneous CP violation which avoid FCNC in Higgs sector while generating a complex CKM matrix

- Non-minimal supersymmetric extensions of SM
- Extensions with vector-like quarks

etc.

Invariant Approach to leptonic CP violation

For definiteness, let us consider again the case of 3 left-handed neutrinos and assume that a Majorana left-handed mass is generated by some lepton-number violating mechanism.

We have seen that in order for CP invariance to hold, the following relations have to be satisfied:

$$U_L^T \; m_L \; U_L = - m_L^*$$

$$U_L^\dagger \; h_\ell \; U_L = h_\ell^*$$

where $h_\ell \equiv m_\ell m_\ell^\dagger$ and U_L is a unitary matrix acting in lepton flavour space

To simplify the notation, let us drop the subscripts L, l :

$$\boxed{\begin{array}{l} U^T \, m \, U = - m^* \\ U^\dagger \, h \, U = h^* \end{array}}$$

We want to build from these two equations necessary conditions for CP invariance to hold, written in terms of weak-basis invariants. Recall that for Majorana neutrinos, the number of independent CP violating phases is

$$n_\phi = \frac{1}{2} n(n-1)$$

So for $n = 2$, one has

$$n_\phi = \frac{1}{2} 2(2-1) = 1$$

It should then be possible for two generations of Majorana neutrinos to find a Weak-basis invariant which would provide a necessary and sufficient condition for CP invariance in the leptonic sector. This would be the analogous to

$$\mathrm{tr}\left[H_u, H_d\right]^3$$

in the quark sector.

It is straightforward to build WB invariants which have to vanish in order to CP invariance to hold. But most of these invariants are trivially satisfied for 2 generations.

$$U^T \, m \, U = -m^*$$

$$U^\dagger \, h \, U = h^*$$

$$U^\dagger \, m^* \, U^* = -m$$

$$U^T \, h^* \, U^* = h$$

From the above equations one derives:

$$\left(U^\dagger h U\right)\left(U^\dagger m^* U^*\right)\left(U^T m U\right)\left(U^\dagger m^* U^*\right) \times$$

$$\times \left(U^T h^* U^*\right)\left(U^T m U\right) =$$

$$\left(h^* m m^* m h m^*\right)$$

This equation implies:

$$\boxed{\operatorname{Im} \operatorname{tr}\left(h m^* m m^* h^* m\right) = 0}$$

Let us call

$$Q \equiv h m^* m m^* h^* m$$

The condition:

$$\boxed{\operatorname{Im} \operatorname{tr} Q = 0}$$

is a necessary and sufficient condition to have CP invariance in the leptonic sector for two Majorana neutrinos.

Since $\operatorname{tr} Q$ is a WB invariant it should be possible to express it in terms of physical, measurable quantities, namely

masses, mixing angles, phases

Let us parametrize the leptonic mixing matrix as :

$$K = \begin{bmatrix} \cos\theta & -\sin\theta \exp(i\gamma) \\ \sin\theta \exp(-i\gamma) & \cos\theta \end{bmatrix}$$

$\theta \rightarrow$ mixing angle

$\gamma \rightarrow$ Majorana-type CP-violating phase

One can then derive :

$$\boxed{\mathrm{Im}\, \mathrm{tr}\, Q = \frac{1}{4} m_1 m_2 \left(m_2^2 - m_1^2\right)\left(m_\mu^2 - m_e^2\right)^2 \times \sin^2 2\theta \sin 2\gamma}$$

$\Rightarrow$ CP violation requires

$\gamma \neq \frac{\pi}{2}, 0$

$m_i \neq 0$

$m_\mu^2 \neq m_e^2$

$\theta \neq \pi/2, 0$

Compare to the results obtained in the quark sector

The case of 3 generations of Majorana neutrinos

In the case of three generations, the number of CP violating phases is

$$\frac{1}{2}\, n(n-1) = \frac{1}{2}\, 3(3-1) = 3$$

Examples of invariant conditions for CP invariance, valid for any number of generations:

$$\operatorname{Im} \operatorname{tr}\left[h\, m^* m\, m^* H^* m\right] = 0$$

$$\operatorname{Im} \operatorname{tr}\left[h\, (m^* m)^2 (m^* H^* m)\right] = 0$$

$$\operatorname{Im} \operatorname{tr}\left[h\, (m^* m)^2 (m^* h^* m)(m^* m)\right] = 0$$

$$\operatorname{Im} \det\left[(m\, h\, m^*) + (h^*\, m\, m^*)\right] = 0$$

Dirac and Majorana triangles in Leptonic mixing:

Consider the V^{PMNS} mixing matrix

$$\begin{pmatrix} \bar{e} & \bar{\mu} & \bar{\tau} \end{pmatrix} \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Since V^{PMNS} is an unitary matrix, one can contruct unitarity triangles. It is useful to consider separately Dirac and Majorana triangles, corresponding to orthogonality of rows and columns of $V^{P,MNS}$, respectively.

Majorana-type unitarity triangles

$$U_{e1} U_{e2}^* + U_{\mu 1} U_{\mu 2}^* + U_{\tau 1} U_{\tau 2}^* = 0$$

$$U_{e1} U_{e3}^* + U_{\mu 1} U_{\mu 3}^* + U_{\tau 1} U_{\tau 3}^* = 0$$

$$U_{e2} U_{e3}^* + U_{\mu 2} U_{\mu 3}^* + U_{\tau 2} U_{\tau 3}^* = 0$$

Under rephasing these triangles do not rotate, since only charged-lepton fields can be rephased. Therefore, contrary to what happens with Dirac triangles, the orientation of Majorana triangles does have physical meaning, which is related to the presence of Majorana phases

The Majorana triangles provide the necessary and sufficient conditions for CP invariance in the leptonic sector with Majorana neutrinos:

- vanishing of their common area
- orientation of all Majorana triangles along the direction of the real or imaginary axes

In the quark sector, we have seen that the minimal non-trivial rephasing invariant quantities are the quartets. In the case of Majorana neutrinos, quantities like:

$\mathrm{Im}\,(U_{ij}\, U^*_{ik})$ are physically meaningful

Neutrino Sector

Let us consider now the situation in the lepton sector. For simplicity, let us work in the weak-basis where charged-lepton mass matrix is diagonal, real

Number of physical parameters contained in m_ν:

3 neutrino masses
3 mixing angles
2 Majorana-type CP violating phases
1 Dirac-type phase

9 parameters

This number has to be compared with the number of measurable quantities through feasible experiments

Measurable quantities through feasible experiments

2 squared mass differences → neutrino oscillations experiments

3 mixing angles → neutrino oscillation experiments

1 Dirac-type phase → CP violation in neutrino oscillation experiments

1 $\sum U_{ei}^2 m_i$ → neutrinoless double-beta decay

---

7 parameters

$7 < 9$

Glashow et al

⇓

We arrive at the dreadful conclusion that no presently conceivable set of feasible experiments can fully determine the neutrino mass matrix.

Possible ways out:

Two suggestions

1. Glashow, Frampton and Marfatia:

Considered the possibility that a restricted class of neutrino mass matrices may suffice to describe current data, namely complex symmetric matrices with texture zeros

2. G.C. Branco, R. Gonzalez-Felipe, F.R. Joaquim, T. Yanagida

suggested the weak-basis independent condition:

$$\boxed{\det m_\nu = 0}$$

Checking that the condition:

$$\boxed{\det m_\nu = 0}$$

removes the ambiguities in the determination of neutrino mass matrix from feasible experiments:

Physical parameters

2 neutrino masses
3 mixing angles
1 Dirac-type phase
1 Majorana-type phase

7

Parameters measurable through feasible experiments

2 squared mass differences
3 mixing angles
1 Dirac-type phase
1 $\Sigma U_{ei}^2 m_i$

7

$7 = 7$!!

q.e.d.

Conclusions

- The SM is very far from being the final theory. Particle Physics is in its infancy!!

- The Flavour Problem or, in Feynman's words the Mass Problem continues being an Open Fundamental Question

- As Franck Wilczek has said, with advent of LHC we are entering a

  New Golden Age in Particle Physics!