String models with Massive Boson-Fermion Degeneracy

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& on work in progress with: A. Faraggi, J. Rizos, N. Toumbas

Physics at the Planck scale may be quite surprising!

Classical Cosmology (General Relativity) “predicts” an initial-time singularity - “Big Bang”.

- Physically unacceptable → need quantum gravity.
- Replace General Relativity with more fundamental singularity-free theory → String Theory.

It turns out that classical notions such as geometry, topology, etc. only appear as low energy approximations of the underlying string theory.

- Physics at strong curvature deviates from the field theory approximation.
- Several examples of purely stringy phenomena, where notions of geometry and field-theory break down → String gas cosmology.
  [Brandenberger, Vafa 1989], [Kiritsis, Kounnas 1994, etc.]

Hagedorn singularities

• However, even in the stringy approach, one still encounters instabilities since the (naive) introduction of temperature breaks supersymmetry spontaneously, and the density of bosonic minus fermionic states grows exponentially for large mass levels.

• In the case of Cosmology the instability is of thermal nature. Equivalently, it is marked by the presence of tachyonic modes, since the introduction of temperature spontaneously breaks space-time supersymmetry by coupling spacetime fermion numbers to the momentum lattice:

$$Z_{II} = \frac{1}{2^2 \eta^{12} \bar{\eta}^{12}} \sum_{a,b,\bar{a},\bar{b}=0,1} (-)^{a+b+\bar{a}+\bar{b}} \; \theta[^a_b]^4 \; \bar{\theta}[^{\bar{a}}_{\bar{b}}]^4 \; \Gamma_{(1,1)}[^{a,\bar{a}}_{b,\bar{b}}],$$

$$\Gamma_{(1,1)}[^{a,\bar{a}}_{b,\bar{b}}] = \sum_{m,n \in \mathbb{Z}} e^{-\frac{\beta^2}{4\pi \mathrm{Im}\tau} |m+\tau n|^2} (-)^{m(a+\bar{a})+n(b+\bar{b})}$$

Towards a Resolution...

There are mainly two courses of action:

- Study the corresponding stringy phase transition.
- Alternatively, try to construct stable vacua, free of Hagedorn/tachyon singularities.

• Recently, considerable progress has been made in the construction of such Hagedorn-free vacua by utilising non-trivial gravito-magnetic fluxes.
[Angelantonj, Kounnas, Partouche, Toumbas 2008, arXiv:0808.1357]
[See talk by N. Toumbas]

• Natural question:
→ Is there any *systematic* way to construct stable thermal vacua ?

Asymptotic boson-fermion degeneracy

• Absence of Hagedorn singularities $\longrightarrow$ asymptotic boson-fermion degeneracy of the spectrum. This is equivalent to the absence of (physical) tachyons from the spectrum:
[Kutasov, Seiberg 1991]

$$n(b) - n(f) \rightarrow 0\ ,\ \text{as } m^2 \rightarrow \infty \Leftrightarrow \text{NO Tachyons!}$$

• This is indeed satisfied by the Hagedorn-free vacua constructed in [AKPT 2008].
• Can we directly construct vacua with broken supersymmetry at low mass levels, while asymptotically satisfying the above boson-fermion degeneracy condition ?

This provided the motivation for the construction of the maximally symmetric *MSDS* vacuum.

The 'Prototype Model'

Such a model has been constructed as an exact CFT of free fermions.

- In its maximally symmetric form, the 'prototype' vacuum is constructed in 2 flat spacetime dimensions, the rest are compactified at the string scale $R_i = \sqrt{\alpha'/2}$ ("fermionic" radius).
- Higher-dimensional vacua can be obtained by marginal deformations of the original *MSDS* vacuum.

Extended Symmetry & Fermionization

Worldsheet degrees of freedom (e.g. in Heterotic models):

- Lightcone $(\partial X^0, \Psi^0), (\partial X^L, \Psi^L) \mid (\bar\partial X^{0,L})$
- (Super)ghosts $(b, c), (\beta, \gamma) \mid (\bar b, \bar c)$
- Transverse supercoordinates $(\partial X^I, \chi^I), \mid (\bar\partial X^I), I = 1, \ldots, 8$
- Extra fermions $\bar\psi^A$, $A = 1, \ldots, 16$: (anomaly cancelation)

Fermionization amounts to the realization of the $[U(1)]^8$ current algebra of transverse bosons in terms of 16 additional free worldsheet fermions:

$$i\partial X^I =: y^I \omega^I :$$

$$y^I = \sqrt{2} : \cos X^I : \ , \ \omega^I = \sqrt{2} : \sin X^I :$$

Global symmetry group in $\{\chi^I, y^I, \omega^I\}$ realizes $\widehat{SO}(24)_{k=1}$.

- Non-linear realization of worldsheet supersymmetry.

[di Vechia, Knizhnik, Petersen, Rossi 1985]

- Consistency: fermions transform in the adjoint representation of some semi-simple Lie group H (dim$H = 24$) [Antoniadis, Bachas, Kounnas, Windey 1986]

$$J^a_{\text{local}} = f^{abc} : \chi^b \chi^c : \ \rightarrow \ \{T_B, T_F, J^a\}$$

$\longrightarrow$ We use the simplest gauging of maximal rank $H = [\widehat{SU}(2)_{k=2}]^8$.

Other possible choices are: $SU(5)$, $SO(7) \times SU(2)$, $G_2 \times Sp(4)$, $SU(4) \times SU(2)^3$, $SU(3)^3$.

Partition functions of maximal MSDS vacua

For Type II theory:

$$Z_{II} = \frac{1}{2^2} \sum_{a,b=0,1} (-)^{a+b} \frac{\theta[^a_b]^{12}}{\eta^{12}} \sum_{\bar{a},\bar{b}=0,1} (-)^{\bar{a}+\bar{b}} \frac{\bar{\theta}[^{\bar{a}}_{\bar{b}}]^{12}}{\bar{\eta}^{12}} = (V_{24} - S_{24}) \left(\bar{V}_{24} - \bar{S}_{24}\right)$$
$$= 24 \times \overline{24} = 576$$

For Heterotic theory:

$$Z_{het} = \frac{1}{2} \sum_{a,b=0,1} (-)^{a+b} \frac{\theta[^a_b]^{12}}{\eta^{12}} \, \Gamma[H_R] = 24 \times \left(d(H_R) + [\, \bar{j}(\bar{\tau}) - 744 \,] \right)$$

where the number of massless states is $d(SO(48)) = 1128$ and $d(SO(32) \times E_8) = d(E_8^3) = 744$.

• At the fermionic point the partition functions of the maximally symmetric *MSDS* vacua are simply *constants*.

(modulo Klein j-invariant in heterotic - eliminated by integration)

Massive Spectral boson-fermion Degeneracy Symmetry

The 'prototype models' satisfy in an *exact* way the required spectrum degeneracy between bosonic and fermionic *d.o.f.*

- Only $d(H_L) \times d(H_R)$ bosons at the massless level.
- All massive levels are degenerate: equal number of bosons and fermions $\rightarrow$ "*massive supersymmetry*".
- The situation seems *very different* from that of ordinary supersymmetry. The connection will be seen in terms of the underlying *CFT*.
- The case of 2 dimensions is marginal: the Supersymmetry algebra itself allows singlet states.

$MSDS$ structure I

Consider the general structure of modular invariant partition functions in Type II or Heterotic theories at the fermionic point:

$$Z(q,\bar{q}) = (\text{const.}) + f(q\bar{q}) + \text{Spur}(q,\bar{q}) + Q(\bar{j}(\bar{\tau}))$$

- (const.) : massless physical states.
- $f(q\bar{q})$: massive physical states, satisfying level-matching.
- $\text{Spur}(q,\bar{q})$: massive spurious states, descendants of physical states that violate level-matching.
- $Q(\bar{j}(\bar{\tau}))$: rational function of the Klein j-invariant, corresponding to spurious excitations of the $|0\rangle_R$ vacuum.

$MSDS$ structure II

• For models with $MSDS$ structure the massive bosonic-fermionic physical states give vanishing contribution $\longrightarrow$ $f(q\bar{q}) = 0$.

• However, the algebraic construction of spurious states as descendants of physical ones forces them to obtain the *same* degeneracy structure $\longrightarrow$ $\mathrm{Spur}(q, \bar{q}) = 0$.

• In Type II theories the pole structure would require $Q \sim \sqrt{\bar{j}(\bar{\tau})}$. But this breaks modular invariance $(\Delta_L - \Delta_R \in \mathbb{Z})$! Therefore, $Q = 0$.

• In Heterotic theories the pole structure requires $Q \sim \bar{j}(\bar{\tau})$ $\longrightarrow$ allowed by modular invariance.

The partition function of $MSDS$ models at the fermionic point is of the general form:

$$Z_{MSDS} = m + n\left[\,\bar{j}(\bar{\tau}) - 744\,\right] \;\;,\;\; m, n \in \mathbb{Z}$$

$SO(24)$-characters & identities

- Ordinary SUSY manifests itself through the *triality* of $SO(8)$. In terms of characters: $V_8 - S_8 = 0$.
- $MSDS$ appears as: $V_{24} - S_{24} = 24$.

Such holomorphic identities between $SO(24)$ characters involve only constant numbers → can always be proven by use of Jacobi 'abstrusa' and 'triple product' identies alone:

$$\theta_3^4 - \theta_4^4 - \theta_2^4 = 0 \quad , \quad \theta_2\,\theta_3\,\theta_4 = 2\eta^3$$

- $MSDS$ structure → 'generalized Jacobi' identities for $SO(24)$ characters.
- The *holomorphic* structure of those identities hints at the presence of a *chiral superconformal algebra*.

Other perverse identities

When we consider $\mathbb{Z}_2$-orbifolds of the original vacuum we will find several other identities:

$$\begin{aligned} V_{16}O_8 - S_{16}C_8 &= 16 \\ O_{16}V_8 - C_{16}S_8 &= 8 \\ V_{16}C_8 - S_{16}O_8 &= 0 \\ O_{16}S_8 - C_{16}V_8 &= 8 \end{aligned}$$

In the case of $\mathbb{Z}_2 \times \mathbb{Z}_2$-orbifolds :

$$\begin{aligned} V_8O_8O_8 - S_8C_8C_8 &= 8 \\ O_8S_8O_8 - C_8V_8C_8 &= 8 \\ V_{12}O_4O_4O_4 + O_{12}V_4V_4V_4 - S_{12}C_4C_4C_4 - C_{12}S_4S_4S_4 &= 12 \\ V_{12}O_4V_4V_4 + O_{12}V_4O_4O_4 - S_{12}C_4S_4S_4 - C_{12}S_4C_4C_4 &= 4 \\ V_{12}O_4S_4S_4 + O_{12}V_4C_4C_4 - S_{12}C_4V_4V_4 - C_{12}S_4O_4O_4 &= 0 \\ V_{12}V_4S_4C_4 + O_{12}O_4C_4S_4 - S_{12}S_4V_4O_4 - C_{12}C_4O_4V_4 &= 4 \end{aligned}$$

$$\vdots \qquad \vdots \qquad \vdots$$

The chiral MSDS algebra I

• By analogy to ordinary supersymmetry, we expect the existence of a chiral operator $j_{\text{MSDS}}(z)$ which maps massive bosonic states to fermionic ones and vice versa, while *leaving massless states invariant*.

• Such an operator exists in the (holomorphic) maximally symmetric *MSDS* model:

$$j_\alpha(z) = \underbrace{e^{\frac{1}{2}\Phi - \frac{i}{2}H_0}}_{-5/8+1/8} C_{24,\alpha}(z)$$

[Ghost charge: $e^{q\Phi} \to \Delta = -\frac{q(q+2)}{2}$]

- Ghost and longitudinal dressing $\to \Delta_j = 1$, *MSDS current*.
- Its zero mode yields *BRST* invariant (conserved) *MSDS charge* :

$$Q_{MSDS} = \oint \frac{dz}{2\pi i}\, j(z)$$

- Ensures the mapping is between states at the *same mass level*.

The chiral *MSDS* algebra II

- The chiral $\mathcal{N}=1$ superconformal algebra of $\{T_B, T_F, J^a\}$, with fermionization current $J^a = f^a{}_{bc}\chi^b\chi^c$, admits a spectral flow generated by the spinorial *MSDS* currents $j_\alpha(z) = e^{\frac{1}{2}\Phi - \frac{i}{2}H_0}\, C_\alpha(z)$.

- The $(-\frac{5}{2})$-picture of the *MSDS* currents

$$j_\alpha(z) = e^{-\frac{5}{2}\Phi + \frac{i}{2}H_0}\, C_\alpha(z)$$

can be used to calculate the non-trivial OPEs between the currents:

$$J^{ab}(z) j_\alpha(w) = \frac{(\sigma^{ab})_{\alpha\beta}}{z-w}\, j_\beta(w) + \text{reg.}$$

$$j_\alpha(z) j_\beta(w) = e^{-2\Phi}\left(\frac{\delta_{\alpha\beta}}{(z-w)^2} + \frac{(\sigma^{ab})_{\alpha\beta}}{z-w}\, J^{ab}(w) + \frac{\epsilon_{\mu\nu}\Psi^\mu\Psi^\nu(w)}{z-w} + \text{reg.}\right)$$

$$j_\alpha(z)\bar{j}_{\dot\alpha}(w) = \frac{1}{z-w} : \left(e^{-\Phi}\gamma^a_{\alpha\dot\alpha}\chi^a\right)\left(e^{-\Phi}\Gamma^\mu\Psi_\mu\right)_{(-1)} : (w) + \text{reg.} \rightarrow \widehat{\Gamma}^\mu_{\alpha\dot\alpha}\partial X_\mu$$

where $J^{ab} \equiv \chi^a\chi^b$ are the global $SO(24)$ currents and $\bar{j}_{\dot\alpha}(z) = e^{\frac{1}{2}\Phi + \frac{i}{2}H_0} S_{\dot\alpha}(z)$.

MSDS Spectral Flow

• Consider the primary operators generating the $SO(24)$ vectorial $\mathbf{V} \equiv e^{-\Phi}\hat{\chi}$ and spinorial $\mathbf{S}_\alpha \equiv e^{-\frac{1}{2}\Phi - \frac{i}{2}H_0} S_\alpha$ representations. The *MSDS* current acts trivially on the vectorial:

$$j(z)\,\mathbf{V}(w) \sim \mathbf{S}(w) + \ldots = \text{reg.}$$

→ massless states are invariant under the *MSDS* charge.

• However, the first massive descendants $[\mathbf{V}]_{(1)} \equiv e^{-\Phi}(\partial\hat{\chi} + \hat{\chi}\hat{\chi}\hat{\chi})$ of the vectorial family transform non-trivially:

$$j(z)\,[\mathbf{V}]_{(1)}(w) \sim \frac{\mathbf{S}(w)}{z-w} + \text{reg.}$$

$$j(z)\,\mathbf{S}(w) \sim \frac{[\mathbf{V}]_{(1)}(w)}{z-w} + \text{reg.}$$

→ the massive bosonic states are mapped into the massive fermionic ones and vice-versa.

- Only the massless states contribute to the (holomorphic) partition function: $V_{24} - S_{24} = 24$.

Orbifold reductions

• To obtain models of phenomenological interest one needs to *reduce* the initial $H_L \times H_R$ symmetry while maintaining *MSDS* symmetry.
• Question:
→ Under what conditions do $\mathbb{Z}_2$-type orbifolds preserve the *MSDS* structure ?
• Answer:

- If at least *some* of the *MSDS* charges $Q_{MSDS,\alpha}$ *survive* the orbifold projections and
- If their chiral action on the representations of the spectrum is *well-defined* (single-valued OPEs for relevant currents),

then the *MSDS* structure is inherited by the twisted sectors.

Classification of fermionic MSDS vacua I

Consider the general $(\mathbb{Z}_2)^N$-twist.

- Suppose that the action of the *MSDS* charge on a state $|\mathcal{A}_{(q)}\rangle$ produces another state $|\mathcal{B}_{(q')}\rangle$.
- Assume, furthermore, that $|\mathcal{A}_{(q)}\rangle$ is created by the q-th descendant operator $[\mathcal{A}]_{(q)}$ in the conformal family of the primary field $\mathcal{A}(w)$ and, similarly, that $|\mathcal{B}_{(q)}\rangle$ is created by $[\mathcal{B}]_{(q')}$ in the conformal family of $\mathcal{B}(w)$.

The fusion rule in terms of the *MSDS* current is:

$$j(z)\cdot[\mathcal{A}]_{(q)}(w) \sim \sum_{q'\geq 0} \frac{[\mathcal{B}]_{(q')}(w)}{(z-w)^{\Delta_{\mathcal{A}}-\Delta_{\mathcal{B}}+q-q'+1}}$$

where $\Delta_{\mathcal{A}}, \Delta_{\mathcal{B}}$ are the conformal weights of the *primary* fields $\mathcal{A}$ and $\mathcal{B}$, respectively.

Classification of fermionic $MSDS$ vacua II

• Only *simple pole* terms in the spectral-flow OPE contribute to commutators of the *MSDS* charge. The general *MSDS* mapping condition is then:

$$\Delta_{\mathcal{A}} - \Delta_{\mathcal{B}} = q' - q \quad , \text{ where } q, q' \in \mathbb{N} \equiv \{0, 1, \ldots\} \tag{1}$$

This implies:

- $\Delta_{\mathcal{A}} - \Delta_{\mathcal{B}} \in \mathbb{Z}$ for any primaries $\mathcal{A}, \mathcal{B}$, otherwise the OPE is *non-local* and Q_{MSDS} is not well-defined.
- The massless states ($\Delta_{\mathcal{A}} = 1/2$ and $q = 0$) are *the only ones to violate* condition (1) for any $q' \geq 0$ and so they do not transform.

Classification of fermionic MSDS vacua III

• In the case of a general $(\mathbb{Z}_2)^N$-twist, the primary operators $\mathcal{A}$, $\mathcal{B}$ in the untwisted sector will be of the form:

$$\mathcal{A} = \prod_{n_i}^{n} \mathbf{V}_{n_i} \prod_{m_j}^{m} \mathbf{O}_{m_j} \overset{MSDS}{\longrightarrow} \mathcal{B} = \prod_{n_i}^{n} \mathbf{S}_{n_i} \prod_{m_j}^{m} \mathbf{C}_{m_j}$$

where consistency requires $\sum_{n_i}^{n} n_i + \sum_{m_j}^{m} m_j = 24$. The *MSDS* mapping condition (1) in the untwisted sector, $\frac{n-3}{2} = q' - q$, is trivially satisfied because of the odd parity forced by the GSO projections.

• In the twisted sector we have:

$$\mathcal{A}' = \prod_{n_i}^{n-a} \mathbf{V}_{n_i} \prod_{n_i}^{a} \mathbf{S}_{n_i} \prod_{m_j}^{m-b} \mathbf{O}_{m_j} \prod_{m_j}^{b} \mathbf{C}_{m_j} \overset{MSDS}{\longrightarrow} \quad \mathcal{B}' = \prod_{n_i}^{n-a} \mathbf{S}_{n_i} \prod_{n_i}^{a} \mathbf{V}_{n_i} \prod_{m_j}^{m-b} \mathbf{C}_{m_j} \prod_{m_j}^{b} \mathbf{O}_{m_j}$$

and the mapping condition (1) becomes: $\frac{1}{8}\left(\sum_{n_i}^{a} n_i + \sum_{m_j}^{b} m_j\right) + \frac{n-3}{2} - a = q' - q$.

- $q' - q \in \mathbb{Z}$ now becomes: $\sum_{n_i}^{a} n_i + \sum_{m_j}^{b} m_j = 0(\text{mod}\, 8)$.
- Ensures the *MSDS* mapping for all *massive* twisted states.

Classification of fermionic MSDS vacua VI

The *necessary* and *sufficient* conditions for the construction of *MSDS* vacua are:

- The choice of boundary conditions and GSO-projections in the theory must respect the *global chiral definition* of the *MSDS* spectral-flow operator $j_\alpha(z)$.
- The boundary condition vectors b_i should satisfy, in addition to the usual modular invariance constraints, the extra *holomorphic* constraint: $n_L(b_i) = 0 (\mathrm{mod}\, 8)$.

• In Type II theories, overall modular invariance and the above holomorphic constraint imply that the analogous *anti-holomorphic* constraint $n_R(b_i) = 0 (\mathrm{mod}\, 8)$ is also satisfied $\rightarrow$ the *MSDS* partition function in Type II is *always constant*, as mentioned before.

Example I: $\mathbb{Z}_2$-twisted Type II

• Simple $\mathbb{Z}_2$-twist in a Type II theory constructed by the basis elements and breaking set, respectively:

$$H_L = \{\chi^{1\ldots 8}, y^{1\ldots 8}, w^{1\ldots 8}\}\ ,\ H_R = \{\overline{\chi}^{1\ldots 8}, \overline{y}^{1\ldots 8}, \overline{w}^{1\ldots 8}\}$$
$$b = \{\chi^{5\ldots 8}, y^{5\ldots 8} | \overline{\chi}^{5\ldots 8}, \overline{y}^{5\ldots 8}\}$$

This corresponds to a $\mathbb{Z}_2$-twist of the four internal coordinates ∂X^I and $\bar{\partial} X^I$, $I = 5, 6, 7, 8$.

$$Z_{II,b} = \frac{1}{2^2} \sum_{a,b,\bar{a},\bar{b}} \frac{1}{2} \sum_{h,g} (-)^{a+b} \frac{\theta[^a_b]^8\, \theta[^{a+h}_{b+g}]^4}{\eta^{12}} (-)^{\bar{a}+\bar{b}} \frac{\bar{\theta}[^{\bar{a}}_{\bar{b}}]^8\, \bar{\theta}[^{\bar{a}+h}_{\bar{b}+g}]^4}{\bar{\eta}^{12}}$$
$$= 16 \times \overline{16} + 8 \times \overline{8} + 0 \times \overline{0} + 8 \times \overline{8} = 384$$

- Massless states from $V_{16} O_8 \overline{V}_{16} \overline{O}_8$, $O_{16} V_8 \overline{O}_{16} \overline{V}_8$ and $O_{16} S_8 \overline{O}_{16} \overline{S}_8$.
- *MSDS* structure originates separately sector by sector.
- The $\mathbb{Z}_2$-invariant spectral flow operator is $j(z) = e^{\frac{1}{2}\Phi - \frac{i}{2} H_0}\, C_{16} C_8(z)$.

Example II: $\mathbb{Z}_2 \times \mathbb{Z}_2$ *Heterotic*

• An example of $T^8/\mathbb{Z}_2 \times \mathbb{Z}_2$-twist of the internal ∂X^I, $\bar\partial X^I$, $I = 1, \ldots, 6$ coordinates, combined with independent left and right- shifts $X^I \to X^I + \pi$, $\bar X^I \to \bar X^I + \pi$.

$$Z = \frac{1}{\eta^{12}\bar\eta^{24}} \frac{1}{2^8} \sum_{\Gamma_\alpha, \Delta_\beta} \sum_{h_a, g_b} (-)^{a+b+HG+\Phi} \, \theta[^a_b]\theta[^{a+h_1}_{b+g_1}]\theta[^{a+h_2}_{b+g_2}]\theta[^{a-h_1-h_2}_{b-g_1-g_2}]$$

$$\times \Gamma_{(8,8)} \left[\begin{array}{c|c} a,k & P,h_1,h_2,\psi \\ b,l & Q,g_1,g_2,\omega \end{array} \right] \underbrace{\bar\theta[^k_l]^5}_{SO(10)} \underbrace{\bar\theta[^{k+h_1}_{l+g_1}]\bar\theta[^{k+h_2}_{l+g_2}]\bar\theta[^{k-h_1-h_2}_{l-g_1-g_2}]}_{U(1)\times U(1)\times U(1)} \times \underbrace{\bar\theta[^\rho_\sigma]^4 \bar\theta[^{\rho+H}_{\sigma+G}]^4}_{SO(8)\times SO(8)}$$

Here, $\Gamma_{(8,8)}$ is an asymmetrically shifted $(8,8)$ momentum lattice:

$$\Gamma_{(8,8)} \equiv \theta[^{a+P}_{b+Q}]^5 \theta[^{a+P+h_1}_{b+Q+g_1}]\theta[^{a+P+h_2}_{b+Q+g_2}]\theta[^{a+P-h_1-h_2}_{b+Q-g_1-g_2}]$$
$$\times \bar\theta[^{k+\psi}_{l+\omega}]^5 \bar\theta[^{k+\psi+h_1}_{l+\omega+g_1}]\bar\theta[^{k+\psi+h_2}_{l+\omega+g_2}]\bar\theta[^{k+\psi-h_1-h_2}_{l+\omega-g_1-g_2}]$$

$$\Gamma_\alpha = (a, k, \rho), \;\; \Delta_\beta = (b, l, \sigma), \;\; h_a = (P, h_1, h_2, \psi, H), \;\; g_b = (Q, g_1, g_2, \omega, G)$$

There are $2^{\frac{8\times7}{2}+1} = 2^{29}$ independent models, determined by the choice of GSO coefficients. This reflects the freedom of choosing the generic modular invariant phase Φ.

Example II: $\mathbb{Z}_2 \times \mathbb{Z}_2$ *Heterotic*

• The general phase Φ may be decomposed into 29 irreducible (modular invariant) phases:

$$\begin{aligned} \Gamma_\alpha g_a + \Delta_\alpha h_a + h_a g_a &: 15\ (10) \\ h_a g_b + h_b g_a \quad a < b &: 10 \\ \Gamma_1\Delta_2 + \Gamma_2\Delta_3 + \Gamma_3\Delta_1 + (\Gamma \leftrightarrow \Delta) &: 1\ \ (0) \\ \Gamma_\alpha\Delta_\alpha &: 3 \end{aligned}$$

Only 23 of the above phases are compatible with the conditions for a chiral algebra.

• These models are naively supersymmetric, unless the choice of GSO projections gives string-scale masses to the gravitini. In that case, the vacuum may still have *MSDS* symmetry, provided that the projections respect the *MSDS* conditions.

• A non-trivial class of models was very recently constructed in collaboration with C. Kounnas and J. Rizos. For special choices of the GSO projections one may generate *MSDS* vacua where the massless spectrum is dominated by the fermionic rather than the bosonic d.o.f.:

$$n(b) - n(f) < 0 \ \ , \ \text{at } m^2 = 0$$

Example II: $\mathbb{Z}_2 \times \mathbb{Z}_2$ Heterotic

As an example, consider the previous heterotic partition function with:

$$\begin{aligned}\Phi &= a(g_1+g_2)+b(h_1+h_2)+k(Q+\omega)+l(P+\psi)+PG+QH\\ &+\rho(Q+g_1+g_2+\omega)+\sigma(P+h_1+h_2+\psi)\\ &+(h_1+h_2)\omega+(g_1+g_2)\psi+\psi G+\omega H\end{aligned}$$

• This phase breaks supersymmetry, but preserves *MSDS*. In addition, it generates fermions from the twisted sectors which eventually dominate the massless spectrum:

$$\begin{aligned}Z &= 592-256\times 4+64\times(4-2)+192+128\times(1-2)+\frac{1}{\bar{q}}+\mathcal{O}(\bar{q})\\ &= -240+2(\bar{j}(\bar{\tau})-744)\end{aligned}$$

• Upon integration, the partition function (1-loop) gives a positive contribution to the effective potential: backreaction can provide a natural cosmological flow of the various moduli towards supersymmetric vacua.

[See corresponding talks by C. Kounnas and H. Partouche.]

Example II: $\mathbb{Z}_2 \times \mathbb{Z}_2$ Heterotic

Now consider marginal deformations : $M_{IJ} J^I \times \bar{J}^J$.

• Ignore Wilson lines and work only on deformations of the $(8,8)$-sublattice:

$$\frac{SO(8,8)}{SO(8)\times SO(8)} \to \frac{SO(4,4)}{SO(4)\times SO(4)} \times \frac{SO(2,2)}{SO(2)\times SO(2)} \times \frac{SO(2,2)}{SO(2)\times SO(2)}$$

The only sector dangerous of producing tachyons is:

$$a = k = \rho = 0,\ h_1 = h_2 = 0,\ H = 1,\ P \neq \psi$$

The phase $\Phi \to l + \sigma + Q + \omega + G$ reverses the GSO projections in these sectors.

$\longrightarrow$ the theory is free of tachyons for this subclass of ‘almost geometric’ deformations !

MSDS as a stringy Higgs phenomenon

Consider now the case where the partition function can be separated into contributions from the R-symmetry charges and the $(8,8)$ momentum lattice:

$$Z = \frac{1}{2\eta^{12}} \sum_{a,b} (-)^{a+b} \theta[^a_b]^4 \; \Gamma_{(8,8)}[^{a,\bar{a}}_{b,\bar{b}}] \times \overline{(\ldots)\,[^{\bar{a}}_{\bar{b}}]}$$

Then, we can truncate the *MSDS* operator down to:

$$j(z) = e^{\frac{1}{2}\Phi - \frac{i}{2}H_0} C_8 C_{16}(z)$$

However, the generator of "would-be" supersymmetry in the $(+\frac{1}{2})$-picture is:

$$Q(z) = e^{\frac{1}{2}\Phi - \frac{i}{2}H_0} C_8 \partial X^I(z) \quad \text{(does not transform states!)}$$

From this point of view the *MSDS* operator is an enhanced 2-dimensional "(super)symmetry":

$$\{\partial X^I\} \rightarrow \{\partial X^I \oplus e^{iv\cdot X}\} \ , \ v^2 = 2$$

This enhancement of $[U(1)]^8$ into a non-abelian Kac-Moody algebra is only possible for points in the moduli space where v_I belong to an even self-dual lattice, or equivalently, are roots of a simply-laced Lie algebra $\rightarrow$ E_8.

E_8-Lattice & SUSY breaking

A simple free-field realization of this E_8-enhancement is given in terms of:

- the 8 Cartan generators ∂X^I
- the 112 adjoint currents $J^{IJ} = e^{\pm i(X^I \pm X^J)}$, for $I \neq J$
- the 128 spinorial currents $C_{16} = e^{\frac{i}{2}(\pm X^1 + \pm X^2 + \ldots \pm X^8)}{}_{(+)}$ with positive chirality

Finally, the $(8,8)$-momentum lattice at the E_8-point can be written explicitly as:

$$\Gamma_{(8,8)}\left[{}^{a,\bar{a}}_{b,\bar{b}}\right] = \sum_{m_i, n_i \in \mathbb{Z}} e^{-\frac{\pi}{\tau_2}(G+B)_{ij}(m_i + \tau n_i)(m_j + \bar{\tau} n_j) + i\pi \mathcal{T}}$$

where G_{ij}, B_{ij} are the metric and parallelized torsion of $E_8 \times \overline{E}_8$, and $\mathcal{T}$ is the coupling to the R-symmetry charges:

$$\mathcal{T} = \underbrace{(MN + aM + bN)}_{Heterotic-like} + \underbrace{(a + \bar{a})m + (b + \bar{b})n}_{TypeII-like}$$

- We need to "infinitely" deform at most 2 moduli in order to "restore" (conventional) supersymmetry.

$\longrightarrow$ Connection of 2-dim. *MSDS* with 4-dim. Supersymmetric theories.

Cosmological scenario

• Suppose that in its early phase, the universe has $\hat{c} = 8$ dimensions compactified close to the string scale.
$\longrightarrow$ The *MSDS* models present natural candidates for the "initial Vacuum".
$\longrightarrow$ Introduce temperature -à la Matsubara- by compactifying time to a circle of radius $R = \beta/2\pi$.
• We recognize in the previous coupling of the $(8, 8)$-lattice to the left- and right-moving R-symmetry charges the well-known temperature couplings of Heterotic and Type II theories.
• These two moduli are identified as the temperature T and the SUSY breaking scale M (à la Sherk-Schwarz).
• The *MSDS* partition function then becomes the free energy $F(\beta, M, \ldots)$ of the corresponding thermal string theory, at a well-defined temperature of the order of the string scale $T \sim 1/\sqrt{\alpha'}$.
$\longrightarrow$ It is very interesting to study the evolution of the moduli $M_{IJ}(t)$ as a function of cosmological time, and determine which general features of the resulting theory (chiral matter, # of generations,...) are compatible with an evolution from an initial *MSDS* vacuum.

Ultimate Hope: provide a link between Particle Physics and Cosmology

Further development...

A number of very important issues are currently under investigation:

- Study of deformation space and stability - cosmological flow
- Tachyon condensation as a stabilization mechanism
- Generation of chirality & fermion mass hierarchy
- Minimal *MSDS* algebra & spectral flow
- Non-critical constructions

Due to 'finite time' constraints it was not possible to discuss here examples of more general ($\mathbb{Z}_N$-type) orbifolds preserving the *MSDS* structure.