Gauge Symmetry Breaking in Matrix Models

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This is the very last talk of a three weeks summer institute which comprised of three parts:

* School and Workshops on the Standard Model and Beyond - Standard Cosmology

* School and Workshops on Cosmology - Strings: Theory - Cosmology - Phenomenology

* 2nd School on Quantum Gravity and Quantum Geometry

Although I have not been here all the time I assume that people started talking in the first workshop of particle physics and the standard model, then went to theories beyond it, then a week of strings, and then we did not even show respect for the very structure of spacetime, rendering it a network, foamy and noncommutative

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You will see that I have a substantial holonomy...

Harold Steinacker has already introduce matrix models in this conference, but to keep my talk self-contained I will quickly introduce them again

Consider a $\boxed{U(1)}$ gauge theory in a space described by the Moyal $\boxed{\star}$ product

The theory is noncommutative (also in the $\boxed{U(1)}$ case), due to the noncommutativity of the product

$$\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}[A_\mu, A_\nu]_\star}$$

Consider as usual the action to be the square of the curvature

$$\boxed{S = -\frac{1}{4}\int \mathrm{d}x F_{\mu\nu} \star F^{\mu\nu}}$$

The theory is invariant for $\boxed{F \to U \star F \star U^\dagger}$ for $\boxed{U \star U^\dagger = 1}$

Considering $\boxed{A_\mu = A_\mu^\alpha \lambda^\alpha}$ for $\boxed{\lambda^\alpha}$ generators of $\boxed{U(n)}$ allows for noncommutative Yang-Mills theories

For the Moyal product $\boxed{\partial_\mu(f) = \mathrm{i}\theta^{-1}_{\mu\nu}[x^\nu, f]}$

If one defines (Madore, Schraml, Schupp, Wess) the covariant coordinates $\boxed{X^\mu = x^\mu + \theta^{\mu\nu}A_\nu}$ and

$$\boxed{D_\mu f = \mathrm{i}\theta^{-1}_{\mu\nu}[X^\mu, f]_\star = \partial_\mu f - \mathrm{i}[f, A_\mu]_\star}$$

we have

$$\boxed{F^{\mu\nu} = [D^\mu, D^\nu]_\star = [X^\mu, X^\nu]_\star + \theta^{\mu\nu}}$$

And the constant can be reabsorbed by a field redefinition. The action is the square of this quantity, integrated over spacetime

The objects we have defined are elements of a noncommutative algebra and we can always represent them as operators on a Hilbert space, in this case the integral becomes a trace and this suggests the use of the matrix action

$$\boxed{S = -\frac{1}{4g}\,\mathrm{Tr}\,[X^\mu, X^\nu][X^{\mu'}, X^{\nu'}]g_{\mu\mu'}g_{\nu\nu'}}$$

Where the $\boxed{X}$'s are operators (matrices) and the metric $\boxed{g_{\mu\mu'}}$ is the flat Minkowski (or Euclidean) metric

Harold has shown how gravity emerges from this action, and I will take it to be my starting point (but I reserve the right to add other, "soft', terms)

The equations of motion are

$$\boxed{[X^\mu,[X^\nu,X^{\mu'}]]g_{\mu\mu'}=0}$$

A possible vacuum (the $\boxed{U(1)}$ Moyal vacuum) given by a set of matrices $\boxed{X_0}$ such that $\boxed{[X_0^\mu,X_0^\nu]=\mathrm{i}\theta^{\mu\nu}}$ con $\boxed{\theta}$ constant

This is some sort of semiclassical vacuum and we can consider $\boxed{f(X_0)}$ as deformation of functions on a Moyal deformed space

This is the vacuum described earlier.

An alternative vacuum, still solution of the equations of motion, is

$$\boxed{\bar{X}_0^\mu = X_0^\mu \otimes \mathbb{1}_n}$$

Consider as the fluctuations

$$\boxed{X^\mu = \bar{X}_0^\mu = \bar{X}_0^\mu + A_0^\mu + A_\alpha^\mu \lambda_\alpha}$$

where in the fluctuations we have separated the traceless generators of $\boxed{SU(n)}$ from the trace part ($\boxed{A_0}$)

The $\boxed{U(1)}$ trace part of the fluctuation gives rise to the gravitational coupling, while the remaining $\boxed{A_\alpha}$ describe a $\boxed{SU(n)}$ gauge theory

We are slightly better than usual noncommutative geometry models which have $\boxed{U(n)}$ symmetry. How to get closer to the standard model?

The plan is to find a matrix model with some coordinates (a vacuum) and an action which reproduces, as close as possible, the standard model

We should not be shy of making as many assumptions as are needed. The game is not to find the standard model, but rather to find a noncommutative geometry which “fits” it

We have already managed to find a $\boxed{SU(n)}$ theory. We need two more stages, first a modification of the model to allow $\boxed{SU(3) \times SU(2) \times U(1)}$, and then a symmetry breaking mechanism

We also have to put fermions (in the right representation)

What we are proposing is in some sense a fully noncommutative version of the Connes-Lott and Chamseddine-Connes-Marcolli models

In their case the geometry is “almost commutative”, the product of ordinary spacetime by a finite dimensional matrix, and the action is either the square of the curvature (as basically is our case) or the spectral action

The model I will present is incomplete and can be considered as a first approximation. Remarkably however some key characteristics of the standard model emerge naturally, which makes us confident that a fully viable (and predictive) model is within reach

In this talk I will mostly concentrate on the first stage, and comments on the electroweak breaking at the end

The first stage can be accomplished by considering the following vacuum for which we add another coordinate, for which I will use the index $\boxed{\mathbf{\Phi}}$ and a different typeset to differentiate it from the usual coordinates

$$\boxed{\mathfrak{X}^{\Phi} = \begin{pmatrix} \alpha_1 \mathbb{1}_2 & & \\ & \alpha_2 \mathbb{1}_2 & \\ & & \alpha_3 \mathbb{1}_3 \end{pmatrix}}$$

with $\boxed{\alpha_i \in \mathbf{R}}$

Since $\boxed{[X^{\mu}, \mathfrak{X}^{\Phi}] = 0}$ the equations of motion are still satisfied, but the gauge symmetry is reduced to

$$\boxed{SU(2) \times SU(2) \times SU(3) \times U(1) \times U(1)}$$

The contribution to the action is a term of the kind

$$\boxed{F^{\mu\Phi} = [\bar{X}^\mu + A^\mu, A^\Phi] + [A^\mu, X^\Phi]}$$

with

$$\boxed{[\bar{X}^\mu + A^\mu, \mathfrak{X}^\Phi] = i\theta^{\mu\nu} D_\nu \mathfrak{X}^\phi = i\theta^{\mu\nu}(\partial_\nu + iA_\nu)\mathfrak{X}^\phi, =}$$

$$\boxed{-(2\pi)^2 \operatorname{Tr}[X^\mu, \mathfrak{X}^\phi][X^\nu, \mathfrak{X}^\phi]\eta_{\mu\nu} = \int d^4x G^{\mu\nu} \left(\partial_\mu \mathfrak{X}^\Phi \partial_\nu \mathfrak{X}^\Phi - [A_\mu, \mathfrak{X}^\Phi][A_\nu, \mathfrak{X}^\Phi]\right)}$$

The mixed terms vanish assuming the Lorentz gauge $\partial^\mu A_\mu = 0$. Since $\boxed{\mathfrak{X}^\Phi = \text{const}}$ the first term in the above integral vanish

We can separate the fluctuations of this extra dimension which are a field, the (high energy) Higgs field.

Consider the block form of A^μ

$$\boxed{A^\mu = \begin{pmatrix} A^\mu_{11} & A^\mu_{12} & A^\mu_{13} \\ A^\mu_{21} & A^\mu_{22} & A^\mu_{23} \\ A^\mu_{31} & A^\mu_{32} & A^\mu_{33} \end{pmatrix}}$$

The first term of the curvature is the covariant derivative

The second term instead is

$$\boxed{[A^\mu, \mathfrak{X}^\phi] = \begin{pmatrix} 0 & (\alpha_2 - \alpha_1)A^\mu_{12} & (\alpha_3 - \alpha_1)A^\mu_{13} \\ (\alpha_1 - \alpha_2)A^\mu_{21} & 0 & (\alpha_3 - \alpha_2)A^\mu_{23} \\ (\alpha_1 - \alpha_3)A^\mu_{31} & (\alpha_2 - \alpha_3)A^\mu_{32} & 0 \end{pmatrix}}$$

If the differences $\boxed{\alpha_1 - \alpha_2}$ is large, all non diagonal blocks of $\boxed{A^\mu}$ acquire large masses decoupling

This extra dimension can have an interpretation in terms of fuzzy spheres

The interpretation of matrix models as some sort of fuzzy sphere Kaluza-Klein is a fascinating idea that goes back some time, and the most relevant work in this context is a paper by Aschieri, Grammatikopoulos, Steinacker and Zoupanos `JHEP 0609:026,2006`.

Add to the action some "soft" (lower order) terms

$$\boxed{V_{soft}(\mathfrak{X}^i) = \mathrm{Tr}\left(c_2\mathfrak{X}^i\,\mathfrak{X}^j\delta_{ij} + ic_3\varepsilon_{ijk}\mathfrak{X}^i\,\mathfrak{X}^j\,\mathfrak{X}^k\right)}$$

Where the $\boxed{c}$'s are real constants

The equations of motions become

$$\boxed{\frac{1}{g^2}[\mathfrak{X}^j,[\mathfrak{X}^i,\mathfrak{X}^{j'}]]\delta_{jj'} + c_2\,\mathfrak{X}^i + \frac{3}{2}\mathrm{i}c_3\varepsilon_{ijk}[\mathfrak{X}^j,\mathfrak{X}^k] = 0}$$

with solution

$$\boxed{\mathfrak{X}^i = \alpha J_N^i}$$

with

$$\boxed{[J_N^i, J_N^j] = \mathrm{i}\varepsilon_{ijk}J_N^k \qquad J_N^i J_N^i = \frac{N^2-1}{4}}$$

with

$$\boxed{\frac{1}{g^2}\alpha^2 + 3c_3\alpha + c_2 = 0}$$

Now consider a "stack" of fuzzy spheres:

$$\boxed{\mathfrak{X}_i = \begin{pmatrix} a_1 J^i_{N_1} \otimes \mathbb{1}_2 & 0 & 0 \\ 0 & a_2 J^i_{N_2} \otimes \sigma_3 & 0 \\ 0 & 0 & a_3 J^i_{N_3} \otimes \mathbb{1}_3 \end{pmatrix}}$$

Then the symmetry breaks to $\boxed{SU(3) \times SU(2) \times U(1) \times U(1) \times U(1)}$. unless $\boxed{N_1 = N_2, \alpha_1 = \alpha_2}$, then the symmetry is $\boxed{SU(4) \times SU(3) \times U(1)}$ which may be phenomenologically relevant

As noted by AGSZ the off diagonal elements of $\boxed{A^\mu}$ acquire a large mass, in analogy with the previous case.

For the diagonal blocks only the $\boxed{l=0}$ mode of the decomposition of the matrices into fuzzy spherical harmonics remain massless, while all higher Kaluza-Klein modes acquire a mass $\boxed{m^2 \sim \alpha^2 l(l+1)}$

The low-energy sector of such a fuzzy sphere vacuum is essentially captured by the effective single-variable described earlier

This is a geometrical version of the usual Higgs effect. However, the fuzzy sphere scenario provides a natural origin of a Higgs potential with nontrivial minimum.

We now need to introduce fermions. They are described by the matrix

$$\boxed{\Psi = \begin{pmatrix} \mathcal{L}_{4\times 4} & \mathcal{Q} \\ \mathcal{Q}' & 0_{3\times 3} \end{pmatrix}}$$

L contains leptons (color-blin, $\boxed{Q}$ and $\boxed{Q'}$ contain quarks (which we assume to be in $\boxed{(\bar{3})}$ for convenience)

$$\boxed{\mathcal{L} = \begin{pmatrix} 0_{2\times 2} & L_L \\ L'_L & \begin{matrix} 0 & e_R \\ e'_R & 0 \end{matrix} \end{pmatrix}}$$

$$\boxed{L_L = \begin{pmatrix} \tilde{l}_L & l_L \end{pmatrix}, \qquad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \qquad \tilde{l}_L = \begin{pmatrix} \tilde{e}_L \\ \tilde{\nu}_L \end{pmatrix}}$$

Here $\boxed{l_L}$ is the standard (left-handed) leptons, $\boxed{e_R}$ the right-handed electron, $\boxed{\tilde{l}}$ corresponds to additional leptons with the same quantum numbers as Higgsinos in principle allowed by the model.

The fields with a prime may or may not be new independent fields. They provide some sort of "mirror sector", and can be set to zero)

The quark matrix is

$$\boxed{\mathcal{Q} = \begin{pmatrix} Q_L \\ Q_R \end{pmatrix}, \qquad Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad Q_R = \begin{pmatrix} d_R \\ u_R \end{pmatrix}}$$

The correct hypercharge, electric charge and baryon number are then reproduced by the following traceless generators

$$\boxed{Y = \begin{pmatrix} 0_{2\times 2} & & \\ & -\sigma_3 & \\ & & -\frac{1}{3}\mathbb{1}_{3\times 3} \end{pmatrix} - \frac{1}{7}}$$

$$\boxed{Q = T_3 + \frac{Y}{2} = \frac{1}{2}\begin{pmatrix} \sigma_3 & & \\ & -\sigma_3 & \\ & & -\frac{1}{3}\mathbb{1}_{3\times 3} \end{pmatrix} - \frac{2}{7}\mathbb{1}}$$

$$\boxed{B = \begin{pmatrix} 0 & & \\ & 0 & \\ & & -\frac{1}{3}\mathbb{1}_{3\times 3} \end{pmatrix} - \frac{1}{7}}$$

which act in the adjoint

Weak and colour interactions sit in the first and last diagonal blocks

The charges of all fermions turn out to be the correct ones, which is non trivial, not every charge of the fermions can be obtained

Moreover in this case the “gauge charge” problem does not appear

In usual noncommutative gauge theories all fermions transform under the same representation of the group (or the trivial one), including the case for which the group is $\boxed{U(1)}$

This is a problem because there are fermions with different hypercharge and electric charge

The proposed solution uses the Seiberg-Witten map and the enlargment of the theory to the universal enveloping algebra

In our case, at the level of matrix models the problem is not posed at all in the full noncommutative theory, and the limit gives the correct representations

The electroweak breaking can be accomplished by another extra coordinate

$$\boxed{\mathfrak{X}^{\varphi} = \begin{pmatrix} 0_{2\times 2} & \varphi & 0_{2\times 1} & 0_{2\times 1} & 0_{2\times 1} & 0_{2\times 1} \\ \varphi^{\dagger} & 0 & 0 & 0 & 0 & 0 \\ 0_{1\times 2} & 0 & 0 & 0 & 0 & 0 \\ 0_{1\times 2} & 0 & 0 & 0 & 0 & 0 \\ 0_{1\times 2} & 0 & 0 & 0 & 0 & 0 \\ 0_{1\times 2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}}$$

Where $\boxed{\varphi}$ is the usual 2-component Higgs with vacuum expectation value

$$\boxed{\langle \varphi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}}$$

I will not discuss in detail this other breaking for lack of time (see my Wroclaw talk), but I mention that this model gives the correct form of the Yukawa couplings

Fuzzy sphere breaking is also possible

Let me discuss now the “holonomy” of the full circle of this conference

We have a matrix model, based on a noncommutative spacetime, which reproduces gravity with an emergent mechanism and contains gauge theories

The same model, modulo some modifications (like soft terms in the action) contains a vacuum with a symmetry which resembles the correct gauge interactions

Therefore I claim we are “close” to phenomenology, and hence the full circle

But we are not there...

The gauge group is too big (also after the second breaking), we eliminate one $\boxed{U(1)}$ with gravity, but we still have unwanted generators

The Yukawa couplings pairs the correct left-right particles, but the couplings are all the same (before renormalization)

No generations

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Hopefully a better understanding of the model will indicate the necessary modifications to render trivial or vanishing the holonomy

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Thanks to all!!!