

# Minimal Standard Heterotic String Models

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- Recent results from free fermion heterotic–string model building

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# DATA $\rightarrow$ STANDARD MODEL

$$SU(3) \times SU(2) \times U(1)_Y \longrightarrow SU(5) \longrightarrow SO(10)$$

$$\left[ \begin{pmatrix} \nu \\ e \end{pmatrix} + D_L^c \right] + \left[ U_L^c + \begin{pmatrix} u \\ d \end{pmatrix} + E_L^c \right] + N_L^c$$
$$\bar{5} \quad + \quad 10 \quad + \quad 1 \quad \quad \quad \frac{\quad}{16}$$

## STANDARD MODEL $\rightarrow$ UNIFICATION

### ADDITIONAL EVIDENCE:

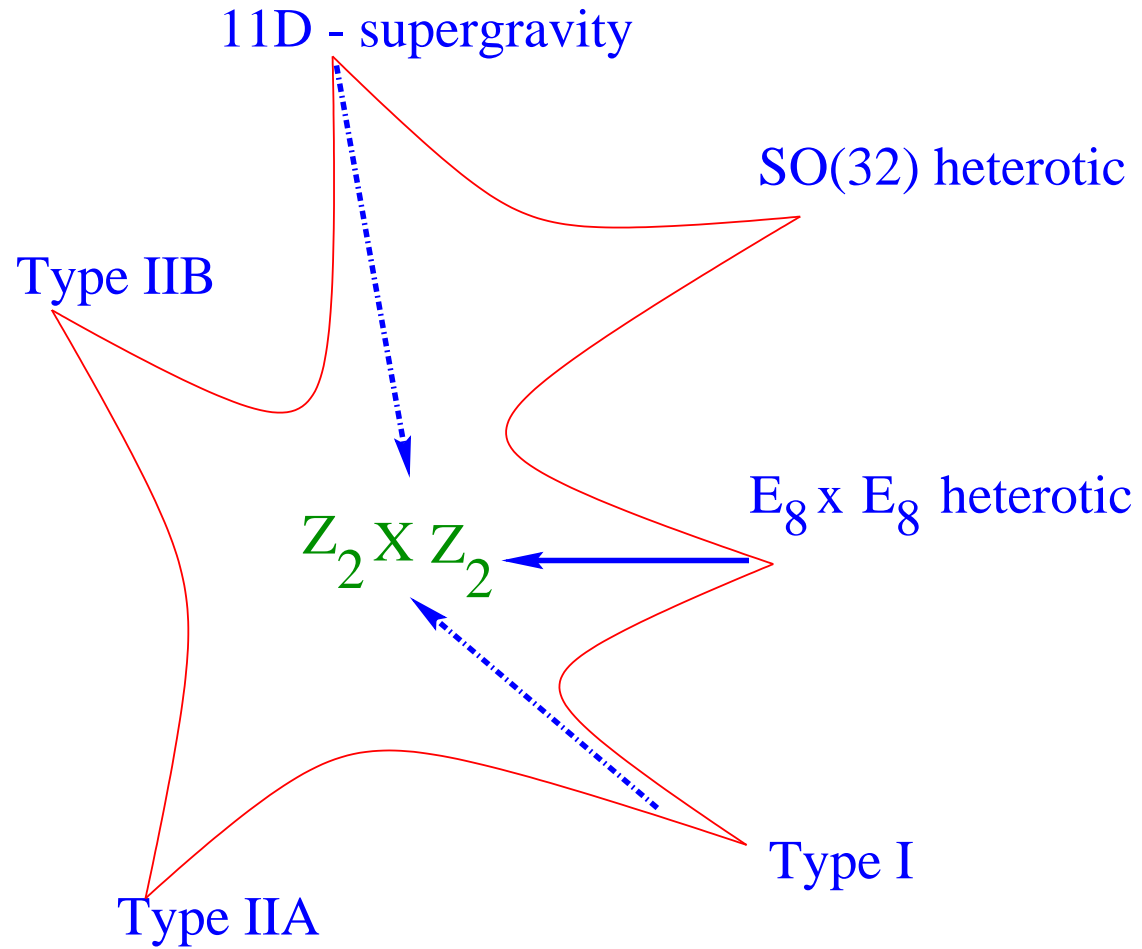
Logarithmic running, proton longevity, neutrino masses

### PRIMARY GUIDES:

3 generations

SO(10) embedding

Point, String, Membrane ....



## Realistic free fermionic models

### 'Phenomenology of the Standard Model and string unification'

- Top quark mass  $\sim 175\text{--}180\text{GeV}$  PLB 274 (1992) 47
- Generation mass hierarchy NPB 407 (1993) 57
- CKM mixing NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification NPB 457 (1993) 409 (with Dienes)
- Proton stability NPB 428 (1994) 111
- Squark degeneracy NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model PLB 455 (1999) 135  
(with Cleaver & Nanopoulos)
- Moduli fixing NPB 728 (2005) 83

# Free Fermionic Construction

Left-Movers:  $\psi_{1,2}^\mu$ ,  $\chi_i$ ,  $y_i$ ,  $\omega_i$  ( $i = 1, \dots, 6$ )

Right-Movers

$$\bar{\phi}_{A=1, \dots, 44} = \left\{ \begin{array}{ll} \bar{y}_i, \bar{\omega}_i & i = 1, \dots, 6 \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1, \dots, 5} \\ \bar{\phi}_{1, \dots, 8} \end{array} \right.$$

$$f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$V \longrightarrow V$$

$$Z = \sum_{\text{all spin structures}} c \left( \begin{array}{c} \vec{\alpha} \\ \vec{\beta} \end{array} \right) Z \left( \begin{array}{c} \vec{\alpha} \\ \vec{\beta} \end{array} \right)$$

Models  $\longleftrightarrow$  Basis vectors + one-loop phases

The NAHE set :  $\{ 1, S, b_1, b_2, b_3 \}$

$$N = 4 \rightarrow 2 \quad 1 \quad 1 \quad \text{vacua}$$

$Z_2 \times Z_2$  orbifold compactification

$$\implies \text{Gauge group } SO(10) \times SO(6)^{1,2,3} \times E_8$$

beyond the NAHE set      Add  $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \longrightarrow SU(3) \times SU(2) \times U(1)_{T_{3R}} \times U(1)_{B-L}$$

$$U(1)_Y = \frac{1}{2}(B - L) + T_{3R} \in SO(10) !$$

$$SO(6)^{1,2,3} \longrightarrow U(1)^{1,2,3} \times U(1)^{1,2,3}$$

## The massless spectrum

Three twisted generations

$b_1, b_2, b_3$

$h_{1,0,0}$

$\bar{h}_{1-1,0,0}$

Untwisted Higgs doublets

$h_{2,0,1,0}$

$\bar{h}_{2,0,-1,0}$

$h_{3,0,0,1}$

$\bar{h}_{3,0,0,-1}$

“standard”  $SO(10)$  representations

NAHE +  $\{ \alpha, \beta, \gamma \} \rightarrow$  exotic vector-like matter  $\rightarrow$  superheavy

$\oplus$  Quasi-realistic phenomenology

## Fermion mass hierarchy

### Fermion mass terms

$$c g f_i f_j h \left( \frac{\langle \phi \rangle}{M} \right)^{N-3}$$

c - calculable coefficients g - gauge coupling

$$f_i, f_j \in b_j \quad j = 1, 2, 3$$

$h \rightarrow$  light Higgs multiplets

$$M \sim 10^{18} \text{ GeV}$$

$\langle \phi \rangle$  generalized VEVs, several sources



## Top quark mass prediction

$$\text{only } \lambda_t = \langle t^c Q_t \bar{h}_1 \rangle = \sqrt{2}g \neq 0 \text{ at } N = 3$$

$$W_4 \longrightarrow b^c Q_b h_{\alpha\beta} \Phi_1 + \tau^c L_\tau h_{\alpha\beta} \Phi_1$$
$$\implies \lambda_b = \left( c_b \frac{\langle \phi \rangle}{M} \right) \quad \lambda_\tau = \left( c_\tau \frac{\langle \phi \rangle}{M} \right)$$

$$\longrightarrow \lambda_b = \lambda_\tau = 0.35g^3 \sim \frac{1}{8}\lambda_t$$

Evolve  $\lambda_t$  ,  $\lambda_b$  to low energies

$$m_t = \lambda_t v_1 = \lambda_t \frac{v_0}{\sqrt{2}} \sin \beta$$

$$m_b = \lambda_b v_2 = \lambda_b \frac{v_0}{\sqrt{2}} \cos \beta$$

where  $v_0 = \frac{2m_W}{g_2(M_Z)} = 246\text{GeV}$  and  $(v_1^2 + v_2^2) = \frac{v_0^2}{2}$

$$m_t = \lambda_t(m_t) \frac{v_0}{\sqrt{2}} \frac{\tan \beta}{(1 + \tan^2 \beta)^{\frac{1}{2}}} \implies m_t \sim 175\text{GeV} \quad \text{PLB274(1992)47}$$

Exotics : (in FNY model, NPB 335 (1990) 347)

$$W_2 = \frac{1}{\sqrt{2}} \{ H_1 H_2 \phi_4 + H_3 H_4 \bar{\phi}_4 + H_5 H_6 \bar{\phi}_4 + (H_7 H_8 + H_9 H_{10}) \phi'_4 + \\ (H_{11} + H_{12})(H_{13} + H_{14}) \bar{\phi}'_4 + V_{41} V_{42} \bar{\phi}_4 + V_{43} V_{44} \bar{\phi}_4 + \\ V_{45} V_{46} \phi_4 + (V_{47} V_{48} + V_{49} V_{50}) \bar{\phi}'_4 + V_{51} V_{52} \phi'_4 \}$$

$\langle \bar{\phi}_4, \bar{\phi}'_4, \phi_4, \phi'_4 \rangle \rightarrow$  massive exotic states at N=3 (PRD46 (1993) 3204)

CFN  $\rightarrow$  Classification of flat directions (PLB 455 (1999) 135)

All Standard Model charged states beyond MSSM  $\rightarrow \approx M_{\text{string}}$

MINIMAL STANDARD HETEROTIC STRING MODEL

John Rizos  $\rightarrow$  Exophobic Pati–Salam Models

# MINIMAL DOUBLET HIGGS CONTENT

(w Manno & Timirgaziu (SLM); Christodoulides (FSU5) in progress)

	$\psi^\mu$	$\chi^{12}$	$\chi^{34}$	$\chi^{56}$	$y^3 y^6$	$y^4 \bar{y}^4$	$y^5 \bar{y}^5$	$\bar{y}^3 \bar{y}^6$	$y^1 \omega^5$	$y^2 \bar{y}^2$	$\omega^6 \bar{\omega}^6$	$\bar{y}^1 \bar{\omega}^5$	$\omega^2 \omega^4$	$\omega^1 \bar{\omega}^1$	$\omega^3 \bar{\omega}^3$	$\bar{\omega}^2 \bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
$\alpha$	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0
$\beta$	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1
$\gamma$	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0

SYMMETRIC  $\leftrightarrow$  ASYMMETRIC

with respect to  $b_1$  &  $b_2$

$h_1, \bar{h}_1, D_1, \bar{D}_1, h_2, \bar{h}_2, D_2, \bar{D}_2$  are projected out

$h_3, \bar{h}_3$  remain in the spectrum

$\lambda_t Q_3 t_3^c \bar{h}_3$  with  $\lambda_t O(1)$

No Phenomenologically viable flat directions

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009

Classification of F and D flat directions in EMT reduced Higgs model

No D flat direction which is F-flat up to order eight in the superpotential  
no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models)

implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

quasi-realistic model: SLM; 3 gen;  $SO(10)$  embed; Higgs &  $\lambda_t \sim 1$ ; ...

vanishing one-loop partition function,

Fixed geometrical, twisted and SUSY moduli

NAHE  $\oplus$  ( $\xi_2 = \{\bar{\psi}_{1,\dots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1$ )  $\rightarrow \{1, S, \xi_1, \xi_2, b_1, b_2\}$

Gauge group:  $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$  and 24 generations.

toroidal compactification  $(6_L + 6_R)$   $g_{ij}, b_{ij}$

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \quad b_{ij} = \begin{cases} g_{ij} & i < j \\ 0 & i = j \\ -g_{ij} & i > j \end{cases}$$

$R_i \rightarrow$  the free fermionic point  $\rightarrow$  G.G.  $SO(12) \times E_8 \times E_8$

mod out by a  $Z_2 \times Z_2$  with standard embedding

$\Rightarrow SO(4)^3 \times E_6 \times U(1)^2 \times E_8$  with 24 generations

Exact correspondence

In the realistic free fermionic models

replace  $X = \{\bar{\psi}^{1,\dots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1$

with  $2\gamma = \{\bar{\psi}^{1,\dots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\dots,4}\} = 1$

Then  $\{\vec{1}, \vec{S}, \vec{\xi}_1 = \vec{1} + \vec{b}_1 + \vec{b}_2 + \vec{b}_3, 2\gamma\} \rightarrow$  N=4 SUSY and

$$SO(12) \times SO(16) \times SO(16)$$

apply  $b_1 \times b_2 \rightarrow Z_2 \times Z_2 \rightarrow$  N=1 SUSY and

$$SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$$

$$b_1, \quad b_2, \quad b_3 \quad \Rightarrow \quad (3 \times 8) \cdot 16 \quad \text{of} \quad SO(10)_O$$

$$b_1 + 2\gamma, \quad b_2 + 2\gamma, \quad b_3 + 2\gamma \quad \Rightarrow \quad (3 \times 8) \cdot 16 \quad \text{of} \quad SO(16)_H$$

$$\text{Alternatively, } c \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = +1 \quad \rightarrow \quad -1$$

## $Z_2 \times Z_2$ orbifolds

torus: One complex parameter  $Z = Z + n e_1 + m e_2$

$T^2 \times T^2 \times T^2 \longrightarrow$  Three complex coordinates  $z_1$ ,  $z_2$  and  $z_3$

$Z_2$  orbifold:  $Z = -Z + \sum_i m_i e_i \longrightarrow$  4 fixed points

$$Z = \{ 0, 1/2 e_1, 1/2 e_2, 1/2 (e_1 + e_2) \}$$

$$\frac{T^2 \times T^2 \times T^2}{Z_2 \times Z_2}$$

$$\begin{aligned} \alpha : (z_1, z_2, z_3) &\rightarrow (-z_1, -z_2, +z_3) \rightarrow 16 \\ \beta : (z_1, z_2, z_3) &\rightarrow (+z_1, -z_2, -z_3) \rightarrow 16 \\ \alpha\beta : (z_1, z_2, z_3) &\rightarrow (-z_1, +z_2, -z_3) \rightarrow \underline{16} \end{aligned}$$

48



$$\gamma : (z_1, z_2, z_3) \rightarrow (z_1+1/2, z_2+1/2, z_3+1/2) \longrightarrow 24$$

Basis vectors: consistent modular blocks 4,8 periodic fermions

$$1 = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i \mid \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \dots, 6, \quad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \quad N = 4 \rightarrow N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} \mid \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \quad N = 2 \rightarrow N = 1$$

Vector bosons: NS,  $z_{1,2}, z_1 + z_2, x = 1 + s + \sum e_i + z_1 + z_2$

impose: Gauge group  $SO(10) \times U(1)^3 \times \text{hidden}$



Independent phases  $c_{[v_i|v_j]}^{[v_i]} = \exp[i\pi(v_i|v_j)]:$  **upper block**

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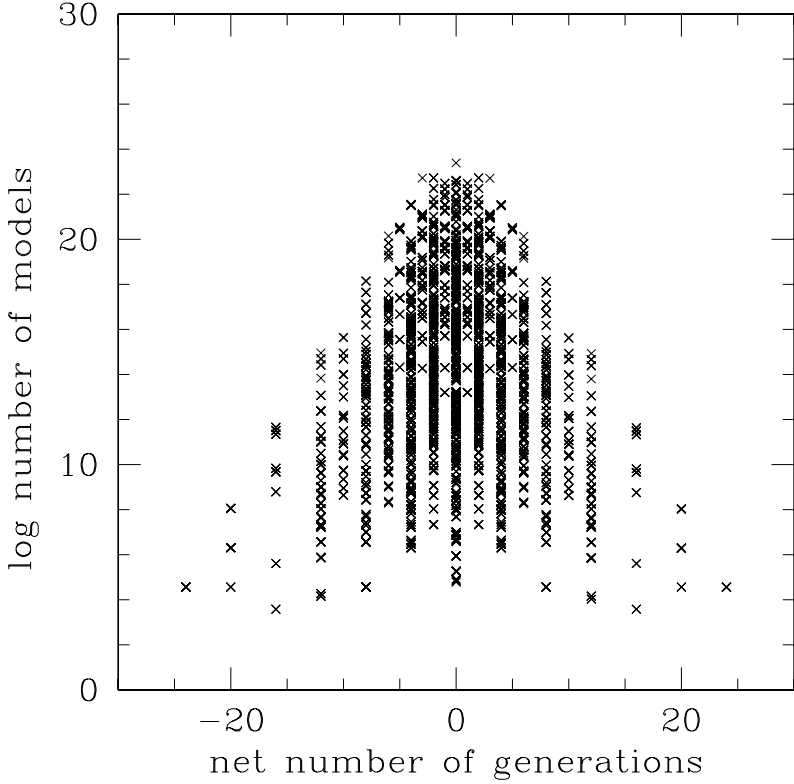
$$\begin{array}{c}
 1 \\
 S \\
 e_1 \\
 e_2 \\
 e_3 \\
 e_4 \\
 e_5 \\
 e_6 \\
 z_1 \\
 z_2 \\
 b_1 \\
 b_2
 \end{array}
 \left(
 \begin{array}{cccccccccccc}
 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 \\
 -1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\
 & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
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 \end{array}
 \right)$$

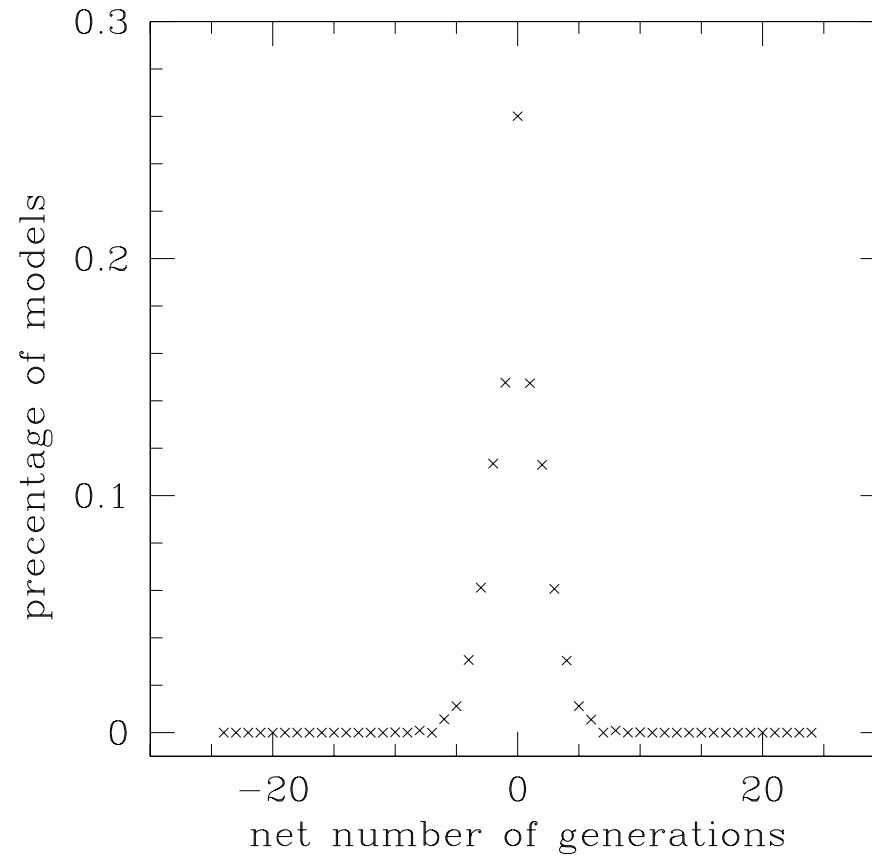
**Apriori 55 independent coefficients  $\rightarrow 2^{55}$  distinct vacua**

**Impose: Gauge group  $SO(10) \times U(1)^3 \times SO(8)^2$**

**$\rightarrow$  40 independent coefficients**

RESULTS:

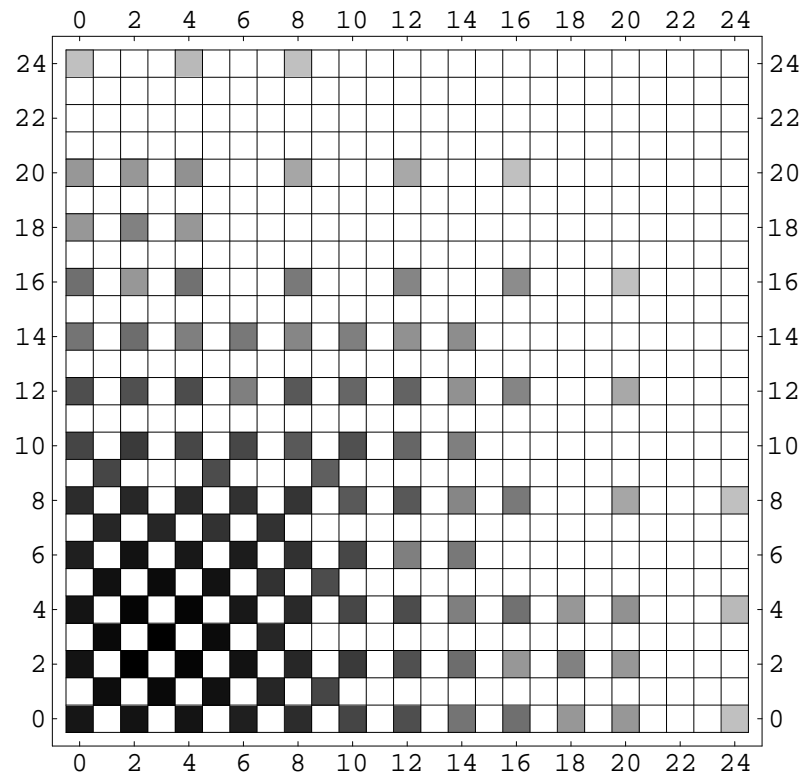




$7 \times 10^9$  models  $\sim$  15% with 3 gen FKRI

## Spinor–vector duality:

Invariance under exchange of  $\#(16 + \overline{16}) < - > \#(10)$



Symmetric under exchange of rows and columns

$$E_6 : \quad 27 = 16 + 10 + 1 \quad \overline{27} = \overline{16} + 10 + 1$$

Self-dual:  $\#(16 + \overline{16}) = \#(10)$  without  $E_6$  symmetry

## NAHE-based partition functions:

w Carlo Angelantonj, Mirian Tsulaia

Question:

$$\frac{T^6}{Z_2 \times Z_2} \rightarrow 48 \text{ fixed points}$$

$$\frac{SO(12)}{Z_2 \times Z_2} \rightarrow 24 \text{ fixed points}$$

$$Z_2 \text{ shift} : 48 \longleftrightarrow 24$$

Is this the same model? In general, no.

shift that reproduces the  $SO(12)$  lattice at the free fermionic point?

## Possible shifts:

$$A_1 : X_{L,R} \rightarrow X_{L,R} + \frac{1}{2}\pi R ,$$

$$A_2 : X_{L,R} \rightarrow X_{L,R} + \frac{1}{2} \left( \pi R \pm \frac{\pi\alpha'}{R} \right) ,$$

$$A_3 : X_{L,R} \rightarrow X_{L,R} \pm \frac{1}{2} \frac{\pi\alpha'}{R} .$$

Using the level-one  $SO(2n)$  characters

$$O_{2n} = \frac{1}{2} \left( \frac{\vartheta_3^n}{\eta^n} + \frac{\vartheta_4^n}{\eta^n} \right) ,$$

$$V_{2n} = \frac{1}{2} \left( \frac{\vartheta_3^n}{\eta^n} - \frac{\vartheta_4^n}{\eta^n} \right) ,$$

$$S_{2n} = \frac{1}{2} \left( \frac{\vartheta_2^n}{\eta^n} + i^{-n} \frac{\vartheta_1^n}{\eta^n} \right) ,$$

$$C_{2n} = \frac{1}{2} \left( \frac{\vartheta_2^n}{\eta^n} - i^{-n} \frac{\vartheta_1^n}{\eta^n} \right) .$$

## The partition function of the heterotic string on $SO(12)$ lattice:

$$Z_+ = (V_8 - S_8) \left[ |O_{12}|^2 + |V_{12}|^2 + |S_{12}|^2 + |C_{12}|^2 \right] (\bar{O}_{16} + \bar{S}_{16}) (\bar{O}_{16} + \bar{S}_{16}) ,$$

and

$$\begin{aligned} Z_- = (V_8 - S_8) & \left[ \left( |O_{12}|^2 + |V_{12}|^2 \right) (\bar{O}_{16}\bar{O}_{16} + \bar{C}_{16}\bar{C}_{16}) \right. \\ & + \left( |S_{12}|^2 + |C_{12}|^2 \right) (\bar{S}_{16}\bar{S}_{16} + \bar{V}_{16}\bar{V}_{16}) \\ & + (O_{12}\bar{V}_{12} + V_{12}\bar{O}_{12}) (\bar{S}_{16}\bar{V}_{16} + \bar{V}_{16}\bar{S}_{16}) \\ & \left. + (S_{12}\bar{C}_{12} + C_{12}\bar{S}_{12}) (\bar{O}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{O}_{16}) \right] . \end{aligned}$$

where  $\pm$  refers to

$$c \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \pm 1$$

connected by :  $Z_- = Z_+ / a \otimes b ,$

$$a = (-1)^{F_L^{\text{int}} + F_\xi^1} , \quad b = (-1)^{F_L^{\text{int}} + F_\xi^2} .$$

Starting from:

$$Z_+ = (V_8 - S_8) \left( \sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16}) (\bar{O}_{16} + \bar{S}_{16}) ,$$

where as usual, for each circle,

$$p_{L,R}^i = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} ,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p_L^2} \bar{q}^{\frac{\alpha'}{4} p_R^2}}{|\eta|^2} .$$

Add shifts :  $(A_1, A_1, A_1)$  ,  $(A_3, A_3, A_3)$

(48  $\rightarrow$  24 yes)

( $SO(12)$ ? no)



Uniquely:

$$g : (A_2, A_2, 0),$$

$$h : (0, A_2, A_2),$$

where each  $A_2$  acts on a complex coordinate

(48  $\rightarrow$  24 yes)

( $SO(12)$ ? yes)

$$R = \sqrt{\alpha'}$$

Uniquely in 10D:

$$c \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = -1 \implies N = 1 \rightarrow N = 0 \text{ SUSY}$$

Question : What is the relation between the  $10D$  and  $D < 10$  cases?

Answer :  $\rightarrow$

Interpolation among SUSY and non-SUSY vacua

Start with:  $Z_+ = (V_8 - S_8) \left( \sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16}) (\bar{O}_{16} + \bar{S}_{16})$

apply  $Z_2 \times Z'_2 : g \times g' = (-1)^{F_{\xi^1}} \delta_1 \times (-1)^{F_{\xi^2}} \delta_1$

with  $\delta_1 X^9 = X_9 + \pi R_9$ ,

$$Z_-^{9d} = (V_8 - S_8) \left[ \begin{aligned} & \Lambda_{2m,n} (\bar{O}_{16} \bar{O}_{16} + \bar{C}_{16} \bar{C}_{16}) \\ & + \Lambda_{2m+1,n} (\bar{S}_{16} \bar{S}_{16} + \bar{V}_{16} \bar{V}_{16}) \\ & + \Lambda_{2m,n+\frac{1}{2}} (\bar{S}_{16} \bar{V}_{16} + \bar{V}_{16} \bar{S}_{16}) \\ & + \Lambda_{2m+1,n+\frac{1}{2}} (\bar{O}_{16} \bar{C}_{16} + \bar{C}_{16} \bar{O}_{16}) \end{aligned} \right] .$$

Next, compactify on  $S^1_2$  moded by

$$Z''_2 : g'' = (-1)^{F+F_{\xi^1}+F_{\xi^2}} \delta_2$$

with  $\delta_2 : X_8 \rightarrow X_8 + \pi R_8$

$\lim R_8 \rightarrow 0 \Rightarrow$  non-SUSY  $(SO(16) \times SO(16))/(Z_2 \times Z'_2)$

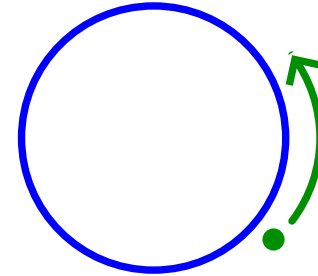
$\lim R_8 \rightarrow \infty \Rightarrow$  SUSY  $(SO(16) \times SO(16))/(Z_2 \times Z'_2)$

# T1 – COMPACTIFICATION

$X$



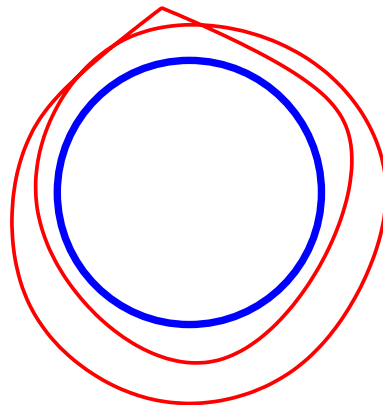
$X \sim X + 2\pi R m$



Point particle

$$\Psi \sim \text{Exp}(i P X) \Rightarrow P = \frac{m}{R}$$

String



$$P_{L,R} = \frac{m}{R} \pm \frac{n R}{\alpha'}$$

## T – DUALITY

$$\text{mass}^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{m R}{\alpha'}\right)^2$$

Invariant under

$$\frac{1}{R} \longleftrightarrow \frac{R}{\alpha'} \quad \text{with} \quad m \longleftrightarrow n$$

An exact symmetry in string perturbation theory!

Self-dual point  $R = \frac{\alpha'}{R}$  = free fermionic point

## Conclusions

Phenomenological string models produce interesting lessons

Spinor–vector duality

relevance of non–standard geometries

Free Fermionic Models  $\longrightarrow$   $Z_2 \times Z_2$  orbifold near the self–dual point

Duality & Self–Duality  $\Leftrightarrow$  String Vacuum Selection