Anomaly-Free Constraints in Neutrino Seesaw Models

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E. Franco, R. González Felipe, Phys. Rev. D 79 (2009) 115001

9th Corfu Summer Institute
of Elementary Particle Physics and Gravity

Workshops on Standard Model and Beyond and Standard Cosmology

Κέρκυρα, 5th September 2009

Outline

- Introduction

  Extra $\mathbf{U(1)_X}$ Gauge Group

  Seesaw mechanisms: **type-I**, **type-II**, **type-III**

- Anomaly-free Constraints

- Phenomenological Constraints

- A Supersymmetric Case: type-III Seesaw

- Conclusions

Motivations for an extra $\mathbf{U(1)_X}$ gauge symmetry

Extension of the Standard Model gauge group

[review see: Langacker 2008]

- The most simplest Abelian extension:
  $$\mathbf{SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X}$$
- GUT $\mathbf{SO(10)}$ or $\mathbf{E_6}$ have rank larger than 4
- Superstrings often involve large gauge symmetries which break to SM with many U(1) in 4D
- Little Higgs models which typically involve extended gauge structures

[Buchmüller, Covi, E.-C., Sören]

- Extra dimen. models, e.g.: orbifold GUT's $\mathbf{SO(10)}$ in 6D
- The $\mathbf{U(1)_X}$ symmetry associated with a possible heavy X would have profound implications for particle physics and cosmology
- Could be accessible and detectable at the LHC

Particle content and charge assignments

Multiplets	$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$	
$q \equiv (u,d)_L^T$	$(3,2,1/6)$	x_q
u_R	$(3,1,2/3)$	x_u
d_R	$(3,1,-1/3)$	x_d
$\ell \equiv (\nu,e)_L^T$	$(1,2,-1/2)$	x_ℓ
e_R	$(1,1,-1)$	x_e
ν_R	$(1,1,0)$	x_ν
T	$(1,3,0)$	x_T
$H_{u,\nu}$	$(1,2,-1/2)$	$z_{u,\nu}$
$H_{d,e}$	$(1,2,1/2)$	$z_{d,e}$
H_T	$(1,2,-1/2)$	z_T
H_δ	$(1,2,-1/2)$	z_δ
Δ	$(1,3,1)$	z_Δ
ϕ	$(1,1,0)$	z_ϕ

Yukawa Constraints

[Barr, Bednarz, Benesh;Babu, Mohapatra
Ma; Adhikari, Erler, Ma; ...]

Lagrangian for Yukawa Couplings

$$\begin{aligned}\mathscr{L}_Y =& Y_u\,\bar{q}_L u_R H_u + Y_d\,\bar{q}_L d_R H_d + Y_e\,\bar{\ell}_L e_R H_e + Y_\nu\,\bar{\ell}_L \nu_R H_\nu \\ &+ Y_T\,\bar{\ell}_L i\tau_2 T H_T + Y_{\mathrm{I}}\,\nu_R^T C \nu_R \phi + Y_{\mathrm{II}}\,\ell_L^T C i\tau_2 \ell_L \Delta \\ &+ Y_{\mathrm{III}}\,\mathrm{Tr}\left(T^T C T\right)\phi + \lambda_\Delta H_\delta H_\delta \Delta + \mathrm{H.c.}\end{aligned}$$

Bosonic Charges Constraints

$$\begin{aligned} -x_q + x_u + z_u &= 0 & -x_q + x_d + z_d &= 0 \\ -x_\ell + x_e + z_e &= 0 & -x_\ell + x_\nu + z_\nu &= 0 \\ -x_\ell + x_T + z_T &= 0 & 2x_\nu + z_\phi &= 0 \\ 2x_\ell + z_\Delta &= 0 & 2x_T + z_\phi &= 0 \\ 2z_\delta + z_\Delta &= 0 \end{aligned}$$

Yukawa Constraints

[Barr, Bednarz, Benesh;Babu, Mohapatra
Ma; Adhikari, Erler, Ma; ...]

Lagrangian for Yukawa Couplings

$$\begin{aligned}\mathscr{L}_Y =& Y_u\,\bar{q}_L u_R H_u + Y_d\,\bar{q}_L d_R H_d + Y_e\,\bar{\ell}_L e_R H_e + Y_\nu\,\bar{\ell}_L \nu_R H_\nu \\ &+ Y_T\,\bar{\ell}_L i\tau_2 T H_T + Y_{\mathrm{I}}\,\nu_R^T C \nu_R \phi + Y_{\mathrm{II}}\,\ell_L^T C i\tau_2 \ell_L \Delta \\ &+ Y_{\mathrm{III}}\,\mathrm{Tr}\left(T^T C T\right)\phi + \lambda_\Delta H_\delta H_\delta \Delta + \mathrm{H.c.}\end{aligned}$$

Bosonic Charges Constraints

$$\begin{aligned} z_u &= x_q - x_u & z_d &= x_q - x_d \\ z_e &= x_\ell - x_e & z_\nu &= x_\ell - x_\nu \\ z_T &= x_\ell - x_T & z_\phi &= -2x_\nu \\ z_\Delta &= -2x_\ell & z_\phi &= -2x_T \\ z_\delta &= x_\ell \end{aligned}$$

Local Anomaly-free Conditions

$\mathbf{U(1)_X\,[SU(3)_C]^2}$

$$A_1 = N_g\,(2\,x_q - x_u - x_d) = 0$$

$\mathbf{U(1)_X\,[SU(2)_L\,]^2}$

$$A_2 = \frac{N_g}{2}\,(x_\ell + 3x_q) - 2N_T\,x_T = 0$$

$\mathbf{U(1)_X\,[U(1)_Y]^2}$

$$A_3 = N_g\,\left(6y_q^2\,x_q\,+\,2y_\ell^2\,x_\ell\,-\,3y_u^2\,x_u\,-\,3y_d^2\,x_d\,-\,y_e^2\,x_e\right) = 0$$

$\mathbf{[U(1)_X]^2\,U(1)_Y}$

$$A_4 = N_g\,\left(6y_q\,x_q^2\,+\,2y_\ell\,x_\ell^2\,-\,3y_u\,x_u^2\,-\,3y_d\,x_d^2\,-\,y_e\,x_e^2\right) = 0$$

$\mathbf{[U(1)_X]^3}$

$$A_5 = N_g\,\left(6\,x_q^3 + 2\,x_\ell^3 - 3\,x_u^3 - 3\,x_d^3 - x_e^3\right) - N_R\,x_\nu^3 - 3\,N_T\,x_T^3 = 0$$

$\mathbf{U(1)_X} \times [\text{gravity}]^2$

$$A_6 = N_g\,(6x_q + 2\,x_\ell - 3x_u - 3x_d - x_e) - N_R\,x_\nu - 3N_T\,x_T = 0$$

Anomaly-free Constraints and Solutions

x_q, x_ℓ, x_u, x_d as functions of x_e, x_R, x_T

$$x_q = -\frac{N_g\, x_e + N_R\, x_\nu - 5\, N_T\, x_T}{6\, N_g}$$

$$x_\ell = \frac{N_g\, x_e + N_R\, x_\nu + 3\, N_T\, x_T}{2\, N_g}$$

$$x_u = -\frac{2\, N_g\, x_e - N_R\, x_\nu - N_T\, x_T}{3\, N_g}$$

$$x_d = \frac{N_g\, x_e - 2\, N_R\, x_\nu + 4\, N_T\, x_T}{3\, N_g}$$

Anomaly-free Conditions (cont.)

- A_4 anomaly-free condition

$$A_4 = -\frac{4\, N_R\, N_T\, x_\nu\, x_T}{N_g} = 0$$

- only the trivial solutions $N_R = 0$, $N_T = 0$, $x_\nu = 0$ or $x_T = 0$
- It is not possible to have an anomaly-free local $U(1)_X$ and, simultaneously, type-I ($N_R \neq 0$) and type-III ($N_T \neq 0$) seesaw models, unless $U(1)_X$ is proportional to the hypercharge $U(1)_Y$
- A_5 anomaly-free condition

$$\begin{aligned} A_5 =& (N_R^2 - N_g^2) N_R\, x_\nu + 3(N_T^2 - N_g^2) N_T\, x_T \\ & - \frac{3}{4} A_4\, (4x_e - N_R\, x_\nu + 5 N_T x_T) N_g^2 \end{aligned}$$

Anomaly-free Conditions (cont.)

- A_4 anomaly-free condition

$$A_4 = -\frac{4\, N_R\, N_T\, x_\nu\, x_T}{N_g} = 0$$

- only the trivial solutions $N_R = 0$, $N_T = 0$, $x_\nu = 0$ or $x_T = 0$
- It is not possible to have an anomaly-free local $U(1)_X$ and, simultaneously, type-I ($N_R \neq 0$) and type-III ($N_T \neq 0$) seesaw models, unless $U(1)_X$ is proportional to the hypercharge $U(1)_Y$
- A_5 anomaly-free condition

$$A_5 = (N_R^2 - N_g^2) N_R\, x_\nu + 3(N_T^2 - N_g^2) N_T\, x_T$$

Anomaly-free Conditions (cont.)

- A_4 anomaly-free condition

$$A_4 = -\frac{4\, N_R\, N_T\, x_\nu\, x_T}{N_g} = 0$$

- only the trivial solutions $N_R = 0$, $N_T = 0$, $x_\nu = 0$ or $x_T = 0$
- It is not possible to have an anomaly-free local $U(1)_X$ and, simultaneously, type-I ($N_R \neq 0$) and type-III ($N_T \neq 0$) seesaw models, unless $U(1)_X$ is proportional to the hypercharge $U(1)_Y$
- A_5 anomaly-free condition

$$A_5 = (N_R^2 - N_g^2) N_R\, x_\nu + 3(N_T^2 - N_g^2) N_T\, x_T$$

$$N_R = N_g \;\text{ or }\; N_T = N_g$$

- Combined **type-II** seesaw is always allowed

Anomaly-free Conditions

Ways Out

1. Both types of seesaw could coexist if new extra matter content is added to the theory. In such a case, at least two extra singlets charged under $U(1)_X$ are needed to cancel the A_4 and A_5 anomalies, respectively.

2. In the case of $N_T = 0$, can we have $N_R \neq N_g$? Yes! Provide we add an extra singlet that cancels the A_5 anomaly. $N_R = 2$ can accommodate oscillation neutrino data, predicts a massless neutrino and a "sterile" neutrino

Solutions

Type-I/II

$$x_u = (\alpha - 2)\, x_q$$
$$x_d = (4 - \alpha)\, x_q$$
$$x_\ell = -3\, x_q$$
$$x_\nu = (\alpha - 6)\, x_q$$

$$\alpha \equiv -x_e / x_q$$

Solutions

Type-I/II

$x_u = 4\, x_q$
$x_d = -2\, x_q$
$x_\ell = -3\, x_q$
$x_\nu = 0$

Replic of the Hypercharge

Solutions

Type-I/II

$x_u = x_q$

$x_d = x_q$

$x_\ell = -3\, x_q$

$x_\nu = -3\, x_q$

B-L Symmetry

$\alpha \equiv -x_e / x_q$

Solutions

Type-I/II

$$x_u = (\alpha - 2)\, x_q$$
$$x_d = (4 - \alpha)\, x_q$$
$$x_\ell = -3\, x_q$$
$$x_\nu = (\alpha - 6)\, x_q$$

$$z_u = (3 - \alpha)\, x_q$$
$$z_d = (\alpha - 3)\, x_q$$
$$z_e = (\alpha - 3)\, x_q$$
$$z_\nu = (3 - \alpha)\, x_q$$
$$z_\phi = 2\,(6 - \alpha)\, x_q$$
$$z_\Delta = 6\, x_q$$
$$z_\delta = -3\, x_q$$

$$\alpha \equiv -x_e / x_q$$

Solutions

Type-I/II

$$x_u = (\alpha - 2)\, x_q$$
$$x_d = (4 - \alpha)\, x_q$$
$$x_\ell = -3\, x_q$$
$$x_\nu = (\alpha - 6)\, x_q$$

$$z_u = (3 - \alpha)\, x_q$$
$$z_d = (\alpha - 3)\, x_q$$
$$z_e = (\alpha - 3)\, x_q$$
$$z_\nu = (3 - \alpha)\, x_q$$
$$z_\phi = 2\,(6 - \alpha)\, x_q$$
$$z_\Delta = 6\, x_q$$
$$z_\delta = -3\, x_q$$

Type-III/II

$$x_u = \frac{1}{5}\,(2 + 3\,\alpha)\, x_q$$
$$x_d = \frac{1}{5}\,(8 - 3\,\alpha)\, x_q$$
$$x_\ell = \frac{1}{5}\,(9 - 4\,\alpha)\, x_q$$
$$x_T = \frac{1}{5}\,(6 - \alpha)\, x_q$$

$$\alpha \equiv -x_e / x_q$$

Solutions

Type-I/II

$$x_u = (\alpha - 2)\, x_q$$
$$x_d = (4 - \alpha)\, x_q$$
$$x_\ell = -3\, x_q$$
$$x_\nu = (\alpha - 6)\, x_q$$

$$z_u = (3 - \alpha)\, x_q$$
$$z_d = (\alpha - 3)\, x_q$$
$$z_e = (\alpha - 3)\, x_q$$
$$z_\nu = (3 - \alpha)\, x_q$$
$$z_\phi = 2\,(6 - \alpha)\, x_q$$
$$z_\Delta = 6\, x_q$$
$$z_\delta = -3\, x_q$$

$$\alpha \equiv -x_e / x_q$$

Type-III/II

$$x_u = \frac{11}{5}\, x_q$$
$$x_d = -\frac{1}{5}\, x_q$$
$$x_\ell = -\frac{3}{5}\, x_q$$
$$x_T = \frac{3}{5}\, x_q$$

No B-L!

Solutions

Type-I/II

$$x_u = (\alpha - 2)\, x_q$$
$$x_d = (4 - \alpha)\, x_q$$
$$x_\ell = -3\, x_q$$
$$x_\nu = (\alpha - 6)\, x_q$$

$$z_u = (3 - \alpha)\, x_q$$
$$z_d = (\alpha - 3)\, x_q$$
$$z_e = (\alpha - 3)\, x_q$$
$$z_\nu = (3 - \alpha)\, x_q$$
$$z_\phi = 2\,(6 - \alpha)\, x_q$$
$$z_\Delta = 6\, x_q$$
$$z_\delta = -3\, x_q$$

Type-III/II

$$x_u = \frac{1}{5}\,(2 + 3\,\alpha)\, x_q$$
$$x_d = \frac{1}{5}\,(8 - 3\,\alpha)\, x_q$$
$$x_\ell = \frac{1}{5}\,(9 - 4\,\alpha)\, x_q$$
$$x_T = \frac{1}{5}\,(6 - \alpha)\, x_q$$

$$z_u = \frac{3}{5}\,(1 - \alpha)\, x_q$$
$$z_d = \frac{3}{5}\,(\alpha - 1)\, x_q$$
$$z_e = \frac{1}{5}\,(9 + \alpha)\, x_q$$
$$z_T = \frac{3}{5}\,(1 - \alpha)\, x_q$$
$$z_\phi = \frac{2}{5}\,(\alpha - 6)\, x_q$$
$$z_\Delta = \frac{2}{5}\,(4\,\alpha - 9)\, x_q$$
$$z_\delta = \frac{1}{5}\,(9 - 4\,\alpha)\, x_q$$

$$\alpha \equiv -x_e / x_q$$

Solutions

Type-I/II

$$x_u = (\alpha - 2)\, x_q$$
$$x_d = (4 - \alpha)\, x_q$$
$$x_\ell = -3\, x_q$$
$$x_\nu = (\alpha - 6)\, x_q$$

$$z_u = (3 - \alpha)\, x_q$$
$$z_d = (\alpha - 3)\, x_q$$
$$z_e = (\alpha - 3)\, x_q$$
$$z_\nu = (3 - \alpha)\, x_q$$
$$z_\phi = 2\,(6 - \alpha)\, x_q$$
$$z_\Delta = 6\, x_q$$
$$z_\delta = -3\, x_q$$

$$\alpha \equiv -x_e / x_q$$

Type-III/II

$$x_u = \frac{1}{5}\,(2 + 3\,\alpha)\, x_q$$
$$x_d = \frac{1}{5}\,(8 - 3\,\alpha)\, x_q$$
$$x_\ell = \frac{1}{5}\,(9 - 4\,\alpha)\, x_q$$
$$x_T = \frac{1}{5}\,(6 - \alpha)\, x_q$$

$$z_u = \frac{3}{5}\,(1 - \alpha)\, x_q$$
$$z_d = \frac{3}{5}\,(\alpha - 1)\, x_q$$
$$z_e = \frac{1}{5}\,(9 + \alpha)\, x_q$$
$$z_T = \frac{3}{5}\,(1 - \alpha)\, x_q$$
$$z_\phi = \frac{2}{5}\,(\alpha - 6)\, x_q$$
$$z_\Delta = \frac{2}{5}\,(4\,\alpha - 9)\, x_q$$
$$z_\delta = \frac{1}{5}\,(9 - 4\,\alpha)\, x_q$$

Type-I/II with $N_R = 2\,,\, N_S = 1$

$$x_u = (\alpha - 2)\, x_q$$
$$x_d = (4 - \alpha)\, x_q$$
$$x_\ell = -3\, x_q$$
$$x_\nu = 4\,(\alpha - 6)\, x_q$$
$$x_S = 5\,(6 - \alpha)\, x_q$$

$$z_u = (3 - \alpha)\, x_q$$
$$z_d = (\alpha - 3)\, x_q$$
$$z_e = (\alpha - 3)\, x_q$$
$$z_\nu = (21 - 4\,\alpha)\, x_q$$
$$z_\phi = 8\,(6 - \alpha)\, x_q$$
$$z_\Delta = 6\, x_q$$
$$z_\delta = -3\, x_q$$
$$z_S = 10\,(\alpha - 6)\, x_q$$

Phenomenological Constraints

- Theories with extra $U(1)$ gauge symmetries are phenomenological richer
- Spontaneous breaking of the symmetry leads to new massive neutral gauge bosons X or Z'
- Kinematically accessible and could be detectable at LHC
- $pp \to X \to b\,\bar{b}$ and $pp \to X \to t\,\bar{t}$ could be used to discriminate between different models
- branching ratios of quark to $\mu^+\mu^-$ production (reducing the theoretical uncertainties)

$$R_{b\mu} = \frac{\sigma(pp \to X \to b\,\bar{b})}{\sigma(pp \to X \to \mu^+\mu^-)} \simeq 3K_q \frac{x_q^2 + x_d^2}{x_\ell^2 + x_e^2}$$

$$R_{t\mu} = \frac{\sigma(pp \to X \to t\,\bar{t})}{\sigma(pp \to X \to \mu^+\mu^-)} \simeq 3K_q \frac{x_q^2 + x_u^2}{x_\ell^2 + x_e^2}$$

- $K_q \sim \mathcal{O}(1)$ is a constant which depends on QCD and electroweak correction factors.

Phenomenological Constraints (cont.)

Analysis of the $R_{t\mu} - R_{b\mu}$ parameter space would allow to distinguish the models (with different charges assignments)

Type-I seesaw cases

$$R_{b\mu} \simeq \frac{3(17 - 8\alpha + \alpha^2)}{9 + \alpha^2}$$

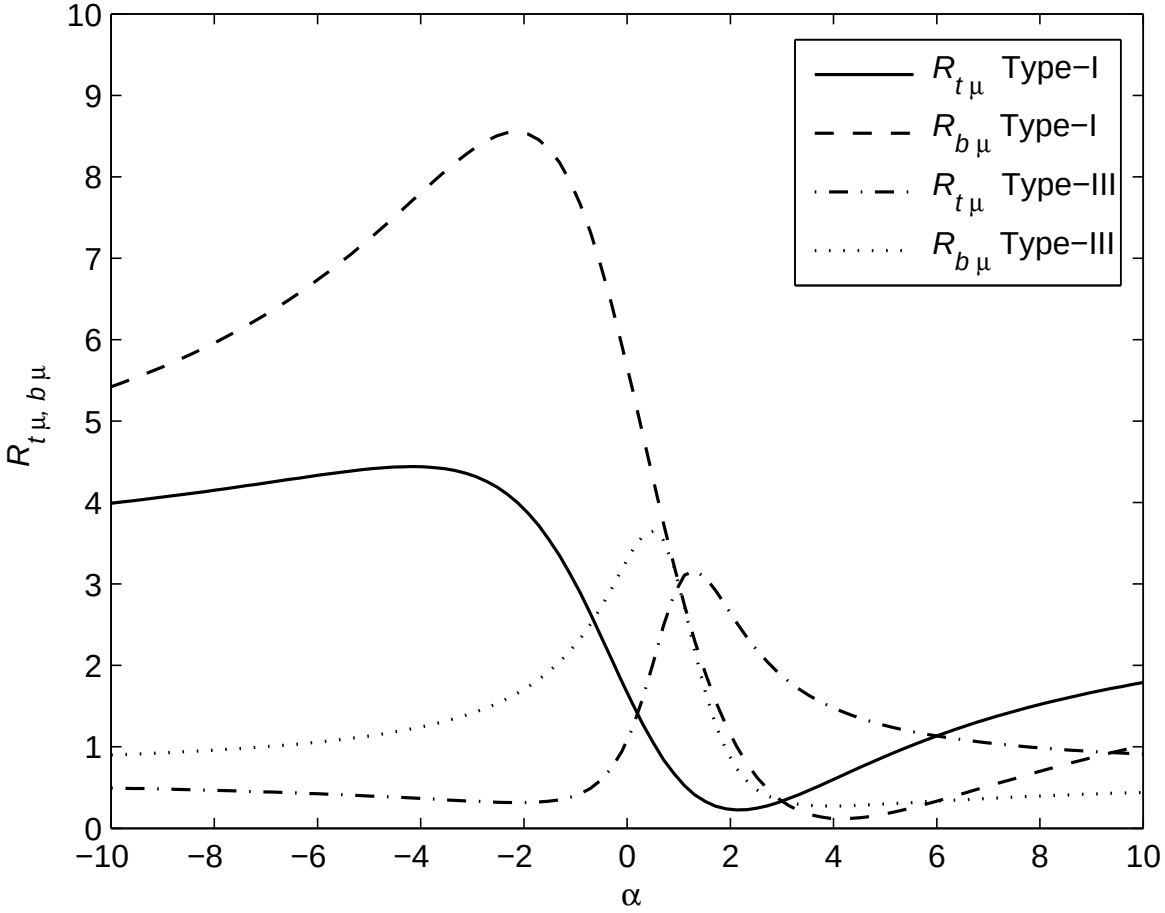
$$R_{t\mu} \simeq \frac{5 - 4\alpha + \alpha^2}{9 + \alpha^2}$$

Type-III seesaw case

$$R_{b\mu} \simeq \frac{3(89 - 48\alpha + 9\alpha^2)}{81 - 72\alpha + 41\alpha^2}$$

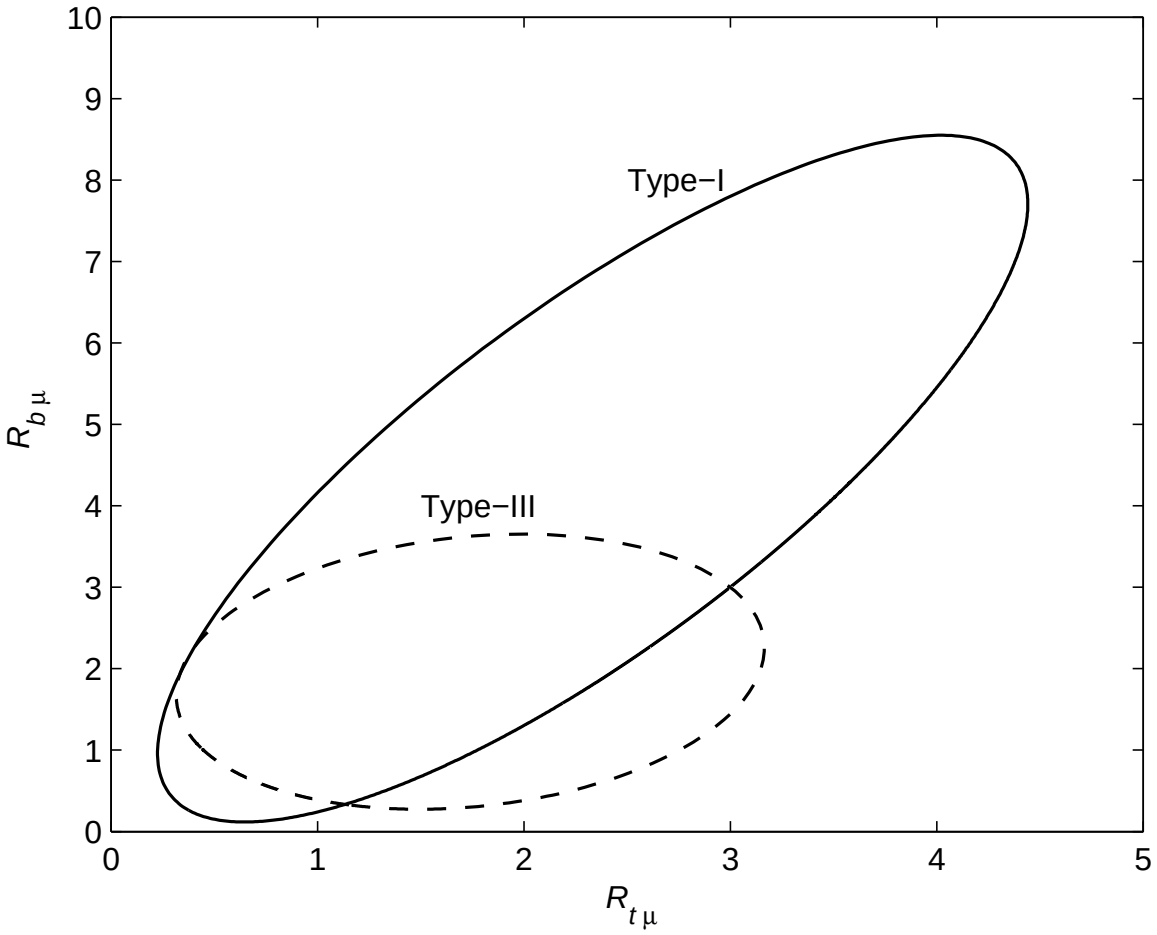
$$R_{t\mu} \simeq \frac{29 + 12\alpha + 9\alpha^2}{81 - 72\alpha + 41\alpha^2}$$

Phenomenological Constraints (cont.)



- Branching ratios of the X decays into quarks and muons as a function of the charge ratio α

Phenomenological Constraints (cont.)



- $R_{t\mu} - R_{b\mu}$ plane for type-I and type-III seesaw realizations

Phenomenological Constraints (cont.)

- EW precision data also severely constrains any mixing with Z boson to the sub-percent level
- $Z - X$ mixing $\propto g_Z g_X \sum_i y_i x_i v_i^2$ with $\sum_i v_i^2 = v^2$

  [review see: Langacker 2008]
- Vanishing $Z - X$ mixing implies ($r_u = v_u/v$)

Type-I/II seesaw

$$\alpha = 6 - \frac{3}{r_u^2}$$

Type-I seesaw, $N_R = 2$

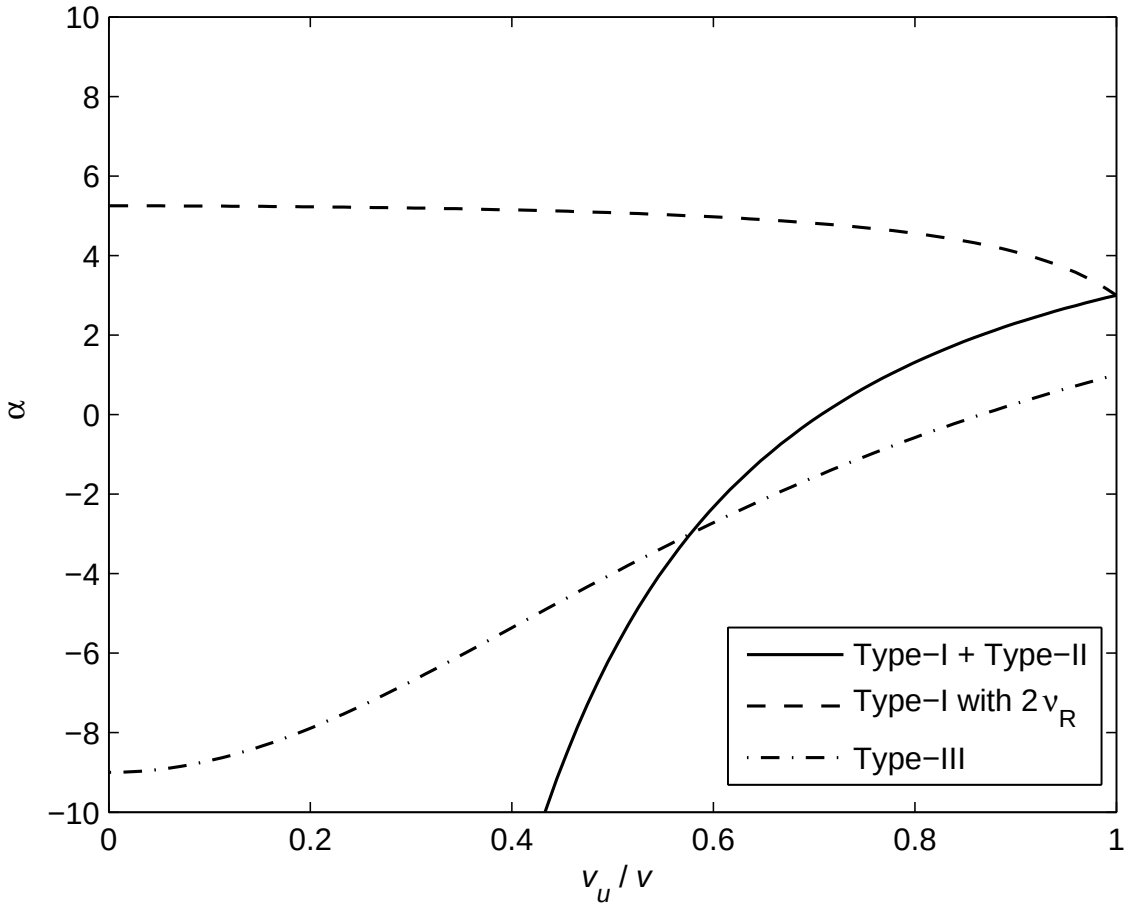
$$\alpha = \frac{21 - 18 r_u^2}{4 - 3 r_u^2}$$

Type-III seesaw

$$\alpha = \frac{12 r_u^2 - 9}{1 + 2 r_u^2}$$

- Heavy leptonic fields effects at LHC [Kersten,Smirnov; Abada et al. Del Águila, Aguilar-Saavedra]
  - TeV-scale seesaw mechanism could appear if some flavour symmetries: light neutrino masses and sizeable Yukawa's
  - More phenomelogy $pp \to X \to N\,N$, Multi-lepton Signals

Phenomenological Constraints (cont.)



- $U(1)_X$ charge ratio α as a function of the Higgs VEV ratio $r_u = v_u/v$ for vanishing $Z - X$ mixing
- These different models are clearly distinguishable

SUSY Type-III Seesaw with an Extra $U(1)_X$

[Mohapatra]

MSSM: sources of rapid proton decay

$$W = W_{MSSM} + \frac{\lambda_L}{M_{pl}} QQQL + \frac{\lambda_R}{M_{pl}} U^c U^c D^c E^c + (\not{R} \text{ terms})$$

$$\lambda_L \sim \lambda_R \sim 10^{-7} \qquad \lambda' \lambda'' < 10^{-24}$$

Type-III

$x_{U^c} = -\frac{1}{5}(2 + 3\alpha)\, x_q$
$x_{D^c} = -\frac{1}{5}(8 - 3\alpha)\, x_q$
$x_{E^c} = \alpha\, x_q$
$x_L = \frac{1}{5}(9 - 4\alpha)\, x_q$
$x_{T^c} = -\frac{1}{5}(6 - \alpha)\, x_q$

$x_{H_u} = -x_{H_d} = \frac{3}{5}(\alpha - 1)\, x_q$
$x_S = -x_{\bar{S}} = -2x_{T^c}$

Baryon and Lepton Violating Operators

$x_{QQQL} = -4x_{T^c}$
$x_{U^c U^c D^c E^c} = 2x_{T^c}$

$x_{QLD^c} = -x_{T^c}$
$x_{LLE^c} = -3x_{T^c}$
$x_{U^c D^c D^c} = 3x_{T^c}$
$x_{LH_u} = -x_{T^c}$

Lepton Sector

$W_e = \frac{h_e}{M_{pl}} L H_d E^c \bar{S}$
$\langle \bar{S} \rangle \sim 3 \times 10^{15}$ GeV

$\frac{\lambda_L}{M_{pl}^2} QQQL\bar{S}^2 \rightarrow \lambda_L \sim 10^{-2}$
$\frac{\lambda_R}{M_{pl}} U^c U^c D^c E^c \bar{S} \rightarrow \lambda_R \sim 10^{-4}$

Conclusions

- Implementation of seesaw mechanisms to give mass to neutrinos in the presence of an anomaly-free $U(1)_X$ gauge symmetry by expanding the lepton sector
- Both type-I and type-III seesaw mechanisms cannot be simultaneously implemented with an anomaly-free local $U(1)_X$, unless the symmetry is a replica of the well-known hypercharge or extra matter is added
- Combined type-I/II or type-III/II seesaw models it is always possible to find non-trivial anomaly-free X charge assignments
- The phenomenological consequences on X neutral gauge boson and, its decays into third-generation quarks and its mixing with the ordinary Z boson, tightly constrain the seesaw models presented - hopefully detectable at LHC
- SUSY type-III can ameliorate the proton decay problem due to Baryon and Lepton violating terms (in particular the Planck scale induced higher dimensional operators)


Ευχαριστώ!