Nearly-Kähler dimensional reduction of the heterotic string

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Overview

Motivation

Heterotic string theory: promising candidate for realistic low-energy phenomenology.

It includes chiral fermions, its gauge group is large enough to accommodate the gauge group of the standard model.

$\rightarrowtail$ search for vacua of the form $M_4 \times B$.

Determine four-dimensional theory by dimensionally reducing over B, find contact with low-energy phenomenology.

- Requirement of $\mathcal{N} = 1$ susy in four dimensions $\rightsquigarrow$ Calabi-Yau threefolds ($SU(3)$-holonomy)
  - complicated geometry
  - moduli stabilization problem
- Flux compactifications $\rightsquigarrow$ backgrounds other than CY ($SU(3)$-structure)
  - cases with simple geometry
  - fluxes can generate potentials which stabilize the moduli
- manifolds with $SU(3)$-structure
  - in type IIA compactifications (Lüst-Tsimpis '04, House-Palti '05, Kashani-Poor '07, Koerber-Lüst-Tsimpis + Caviezel et.al. '08 Cassani-Kashani-Poor '09...)
  - in heterotic string compactifications (Gurrieri-Lukas-Micu '07, Benmachiche-Louis-Martinez-Pedrera '08...)

- Simple examples of manifolds admitting an $SU(3)$-structure: non-symmetric coset spaces
- Supersymmetric compactifications of the heterotic string theory of the form $AdS_4 \times (S/R)$ exist when H-flux and fermion condensates are present (Manousselis-Prezas-Zoupanos '05).
- Perform reduction employing the Coset Space Dimensional Reduction scheme which provides
  - Gauge-Higgs-Yukawa unification
  - Interesting GUT models with chiral fermions in 4-dims
  - $\mathcal{N} = 1$ softly broken susy Lagrangians
  - Consistency

A manifold admits a G-structure when the structure group of its frame bundle can be reduced to G.

$\rightsquigarrow$ all tensors/spinors can be globally decomposed into reps of G.

The G-structure is classified by the intrinsic torsion $\hookrightarrow$ measures the failure of tensors/spinors to be covariantly constant w.r.t. the Levi-Civita connection.

In six-dimensions: $SU(3)$-structure $\hookrightarrow$ amounts to the reduction of $SO(6)$ to $SU(3)$.
Define:

- nowhere-vanishing, globally-defined spinor η, the singlet of the decomposition $\mathbf{4} = \mathbf{3} + \mathbf{1}$,
- structure forms: 2-form J and 3-form Ω,

all covariantly constant w.r.t. a connection with torsion.
J and Ω satisfy:

$$\begin{aligned} dJ &= \frac{3}{4} i(\mathcal{W}_1 \Omega^* - \mathcal{W}_1^* \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3, \\ d\Omega &= \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega. \end{aligned}$$

$\rightsquigarrow$ five intrinsic torsion classes $\mathcal{W}_i$

Torsion classes provide classification of manifolds, e.g.

- Complex: $\mathcal{W}_1 = \mathcal{W}_2 = 0$
- Half-flat: $Im\mathcal{W}_1 = Im\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
- Kähler: $\mathcal{W}_{1-4} = 0$
- Calabi-Yau: all torsion classes vanish
- nearly-Kähler: $\mathcal{W}_{2-5} = 0$

6-dim homogeneous nearly-Kähler manifolds:

- $G_2/SU(3)$
- $Sp_4/(SU(2) \times U(1))_{non-max}$
- $SU(3)/U(1) \times U(1)$
- $SU(2) \times SU(2)$

Note that:

- The first three manifolds are also the only non-symmetric coset spaces S/R in 6 dims with rankR=rankS.
- They admit 1,2 and 3 different radii respectively (the nearly-Kähler limit corresponds to equal radii).
- They admit S-invariant 2-forms ω_i and 3-forms ρ_1, ρ_2.
- Structure forms: $J = R_i^2 \omega_i$, $\Omega \propto (\rho_2 + i\rho_1)$

$\rightsquigarrow$ use the S-invariant forms to expand fields

Spectrum and Lagrangian

Heterotic Supergravity-Yang-Mills spectrum =

$\mathcal{N}=1$ sugra multiplet + $\mathcal{N}=1$ vector supermultiplet:

$$e_M^N, \psi_M, B_{MN}, \lambda, \phi \text{ and } A_M, \chi$$

Gauge group $E_8 \times E_8$.

Reduction of the bosonic part $\hookrightarrow$ obtain Kähler potential K and superpotential W $\hookrightarrow$ sufficient to find sugra description in 4 dims

Bosonic Lagrangian:

$$\hat{e}^{-1}\mathcal{L}_B = -\frac{1}{2\hat{\kappa}^2}\left(\hat{R}\hat{*}\mathbf{1}+\frac{1}{2}d\hat{\phi}\wedge\hat{*}d\hat{\phi}+\frac{1}{2}e^{-\hat{\phi}}\hat{H}\wedge\hat{*}\hat{H}+\frac{\alpha'}{2}e^{-\frac{1}{2}\hat{\phi}}Tr\hat{F}\wedge\hat{*}\hat{F}\right).$$

Metric & dilaton

Metric ansatz:

$$d\hat{s}^2 = e^{2\alpha\varphi(x)}\eta_{mn}e^m e^n + e^{2\beta\varphi(x)}\gamma_{ab}(x)e^a e^b.$$

Note:

- the metric is S-invariant
- in general there exist Kaluza-Klein gauge fields valued in $S \times N(R)/R$ (maximal isometry group) $\longrightarrow$ 4-dim theory inconsistent with the original (Coquereaux-Jadczyk '86). Consistency guaranteed when KK fields valued in $N(R)/R$. In our cases this group is trivial $\rightsquigarrow$ KK gauge fields vanish, only scalar fluctuations (A.C.-Manousselis-Prezas-Zoupanos '07)
- γ_{ab} is unimodular and generically contains extra scalars parametrizing the internal metric

Using this ansatz we obtain:

$$\mathcal{L} = -\frac{1}{2\kappa^2}(R * \mathbf{1} + P_{ab} \wedge * P_{ab} + \frac{1}{2} d\varphi \wedge * d\varphi) - V,$$

$\rightsquigarrow$ P_{ab} provide kinetic terms for the additional metric moduli.
The potential is:

$$V = \frac{1}{8\kappa^2} e^{2(\alpha-\beta)\varphi}(\gamma_{ab}\gamma^{cd}\gamma^{ef} f^a_{\ ce} f^b_{\ df} + 2\gamma^{ab} f^c_{\ da} f^d_{\ cb} + 4\gamma^{ab} f_{iac} f^{ic}_b)$$

i: R-index

a: coset index

Higher-dimensional dilaton: $\hat{\phi}(x, y) = \phi(x)$

Gauge fields

CSDR principle (Forgacs-Manton '79): $\mathcal{L}_{X^I}\hat{A} = DW_I$,
where $W_I \rightarrow$ gauge transformation parameter, $X^I \rightarrow$ Killing vectors.
(see Dolan's talk for a similar approach)

Ansatz for the gauge field: $\hat{A}^I = A^I + \phi^I_A e^A$
Constraints: $D\phi^I_i = F^I_{ai} = F^I_{ij} = 0$.
Then, in four dimensions:

$$\mathcal{L}_{gauge} = -\frac{\alpha'}{4\kappa^2} e^{-\frac{1}{2}\phi} \left[F^I \wedge *F^I + \gamma^{ab} D\phi^I_a \wedge *D\phi^I_b \right] - V_{gauge},$$

where the initial gauge group G is broken to $H = C_G(R)$. In the present framework $H = E_6$. The potential reads

$$V_{gauge} = \frac{\alpha'}{8\kappa^2} e^{-\frac{1}{2}\phi} \gamma^{ac}\gamma^{bd} F_{ab}F_{cd}.$$

Three-form

Multidimensional 3-form: $\hat{H} = d\hat{B} - \frac{\alpha'}{2}(\hat{\omega}_{YM} - \hat{\omega}_L)$.
where the abelian 2-form potential is expanded as:

$$\hat{B} = B(x) + b^i(x)\omega_i(y).$$

$\omega_i(y)$: the S-invariant 2-forms of the internal space.
Then in four dimensions:

$$\begin{aligned}\mathcal{L}_H = -\frac{1}{4\kappa^2}e^{-\phi}\Big[& \quad d\theta \wedge *d\theta - \theta F^I \wedge F^I + m db^i \wedge *db^i \\ & + \alpha' \epsilon_i^{ab} db^i \wedge Tr(\phi_a * D\phi_b) \\ & + \frac{\alpha'^2}{4} Tr(\phi_a \overleftrightarrow{D} \phi_b) \wedge Tr(\phi_a * \overleftrightarrow{D} \phi_b)\Big] - V_H,\end{aligned}$$

The potential has the form

$$V_H = \frac{1}{4\kappa^2} e^{-\phi} \Big[\quad b^i b^j (n_1 \delta_{ij} + n_2 \epsilon_{ij}) - \frac{2\alpha'}{3} \epsilon_i^{abc} b^i Tr(\phi_a \phi_b \phi_c) + \frac{\alpha'}{2} \epsilon_i^{abc} b^i Tr(f_{ab}^d \phi_c \phi_d) + \frac{2\alpha'^2}{3} Tr(\phi_a \phi_b \phi_c)^2 + \frac{\alpha'^2}{16} Tr(f_{ab}^d \phi_c \phi_d) Tr(f_{[ab}^d \phi_{c]} \phi_d) - \alpha'^2 Tr(\phi_a \phi_b \phi_c) Tr(f_{ab}^d \phi_c \phi_d) \Big],$$

- θ is the pseudoscalar obtained by duality transformation on dB.
- m, n_1 and n_2 are fixed constants for each manifold.

Counting scalar moduli:

- $G_2/SU(3)$
  - one radius + one G_2-invariant 2-form
  - four moduli: $\phi, \theta, \varphi, b_1$ + one **27** multiplet β^i in E_6 from the internal components of the gauge field.
- $Sp_4/(SU(2) \times U(1))_{non-max}$
  - two radii + two Sp_4-invariant 2-forms
  - six moduli: $\phi, \theta, \varphi, \chi, b_1, b_2$ + two multiplets β^i, γ^i.
- $SU(3)/U(1) \times U(1)$
  - three radii + three $SU(3)$-invariant 3-forms
  - eight moduli $\phi, \theta, \varphi, \chi, \psi, b_1, b_2, b_3$ + three multiplets $\alpha^i, \beta^i, \gamma^i$.

$G_2/SU(3)$ case

The three contributions to the effective potential have been determined in all cases in terms of the genuine Higgs fields, e.g. for $G_2/SU(3)$:

gravity: $V_{grav} = -\frac{15}{\kappa^2}\frac{1}{R_1^2}$,

gauge: $V_{gauge} = \frac{\alpha'}{8\kappa^2}e^{-\frac{1}{2}\phi}\Bigg(\frac{8}{R_1^4} - \frac{40}{3R_1^2}\beta^2 - \left[\frac{4}{R_1}d_{ijk}\beta^i\beta^j\beta^k + h.c\right] +$

$\beta^i\beta^j d_{ijk}d^{klm}\beta_l\beta_m + \frac{11}{4}\sum_\alpha \beta^i(G^\alpha)_i^j\beta_j\beta^k(G^\alpha)_k^l\beta_l\Bigg)$,

$H-flux$: $V_H = \frac{1}{\kappa^2}e^{-\phi}\Bigg[\frac{b^2}{R_1^6} + \frac{\sqrt{2}}{R_1^3}i\alpha' b(d_{ijk}\beta^i\beta^j\beta^k - h.c.) +$

$2\alpha'^2\beta^i\beta^j\beta^k d_{ijk}d^{lmn}\beta_l\beta_m\beta_n + \frac{3}{R_1^2}\alpha'^2(\beta^2)^2 - \frac{\sqrt{6}}{R_1}\alpha'^2\beta^2(d_{ijk}\beta^i\beta^j\beta^k + h.c.)\Bigg]$.

4-dim sugra description

Determine the superpotential by the Gukov-Vafa-Witten formula:
$W = \frac{1}{4} \int_{S/R} \Omega \wedge (\hat{H} + idJ)$

and the Kähler potential by special Kähler geometry:
$K = K_S + K_T$,
where $K_S = -ln(S + S^*)$ in terms of the superfield $S = e^{\phi} + i\theta$
and $K_T = -ln(\frac{1}{6} \int_{S/R} J \wedge J \wedge J)$

Note that there are no complex structure moduli.

Then:

- $G_2/SU(3)$
  - $W = 3T_1 - \sqrt{2}\alpha' d_{ijk} B^i B^j B^k$
  - $K = -ln(S+S^*)(T_1 + T_1^* - 2\alpha' B_i B^i)^3$
- $Sp_4/(SU(2)\times U(1))_{non-max}$
  - $W = 2T_1 + T_2 - \sqrt{2}\alpha' d_{ijk} B^i B^j \Gamma^k$
  - $K = -ln(S+S^*)(T_1 + T_1^* - 2\alpha' B_i B^i)^2 (T_2 + T_2^* - 2\alpha' \Gamma_i \Gamma^i)$
- $SU(3)/U(1)\times U(1)$
  - $W = T_1 + T_2 + T_3 - \sqrt{2}\alpha' d_{ijk} A^i B^j \Gamma^k$
  - $K = -ln(S+S^*)(T_1 + T_1^* - 2\alpha' A_i A^i)(T_2 + T_2^* - 2\alpha' B_i B^i)\times$
    $\times(T_3 + T_3^* - 2\alpha' \Gamma_i \Gamma^i)$

with the superfields $T_1 = R_1^2 + ib_1 + \alpha' \beta^i \beta_i \ \ldots$ and A, B, Γ the superfields of α, β, γ.

Conclusions

- The four-dimensional action resulting by dimensionally reducing the heterotic supergravity-Yang-Mills theory over nearly-Kähler manifolds has been derived.
- A detailed case by case analysis has been performed for all the homogeneous nearly-Kähler manifolds.
- Due to their simple geometry, nearly-Kähler manifolds provide interesting realizations of the general formalism of $SU(3)$-structure compactifications.