Stringy Instanton Effects in Models with Rigid Magnetised D-branes

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Summary

- Magnetised D-branes and the $T^6/Z_2 \times Z_2$ Orientifold
- E1 Instantons and Non-perturbative Superpotentials
- Explicit Models
- Conclusions

Magnetised D-branes and the $T^6/Z_2 \times Z_2$ Orientifold

- We consider a factorisable internal space $T^6 = T^2_1 \times T^2_2 \times T^2_3$, with complex coordinates

$$z^i = x^i + iy^i$$

- The action of $\mathbb{Z}_2 \times \mathbb{Z}_2$ on the three internal two-tori is

$$g : (+,-,-)\ , \qquad f : (-,+,-)\ , \qquad h : (-,-,+)\ .$$

  A sign $\pm$ in the i-th position means that the two coordinates (x^i, y^i) of the T^2_i are mapped into $\pm(x^i, y^i)$ under the orbifold action.

- There are 16 fixed points per orbifold operation.

Discrete Torsion

The $T^6/Z_2 \times Z_2$ comes in two versions

- with discrete torsion $\epsilon = -1$
- without discrete torsion $\epsilon = +1$

The twisted sectors are different, hence their spectra are different.

The presence or absence of discrete torsion determines the type of orientifold planes in the theory.

We are going to focus on the version with discrete torsion.

- We have the following sets of boundary conditions

	x	z_1	z_2	z_3
O9	$\times$	$\times$	$\times$	$\times$
$\mathrm{O5}_{1-}$	$\times$	$\times$	$\cdot$	$\cdot$
$\mathrm{O5}_{2-}$	$\times$	$\cdot$	$\times$	$\cdot$
$\mathrm{O5}_{3+}$	$\times$	$\cdot$	$\cdot$	$\times$
D9	$\times$	$\times$	$\times$	$\times$
$\mathrm{D5}_1$	$\times$	$\times$	$\cdot$	$\cdot$
$\mathrm{D5}_2$	$\times$	$\cdot$	$\times$	$\cdot$
$\mathrm{D5}_3$	$\times$	$\cdot$	$\cdot$	$\times$

where a cross (dot) indicates that the object wraps (is localised along) the corresponding directions.

- We introduce two “families” of D9-branes, labelled by the complex charges p_a, $\bar{p}_a$, q_α and $\bar{q}_\alpha$, so that the action of $\mathbb{Z}_2 \times \mathbb{Z}_2$ on the Chan-Paton labels is

$$\begin{aligned} N_{a,o} &= p_a + \bar{p}_a \,, & N_{\alpha,o} &= q_\alpha + \bar{q}_\alpha \,, \\ N_{a,g} &= i(p_a - \bar{p}_a)\,, & N_{\alpha,g} &= i(q_\alpha - \bar{q}_\alpha)\,, \\ N_{a,f} &= i(p_a - \bar{p}_a)\,, & N_{\alpha,f} &= -i(q_\alpha - \bar{q}_\alpha)\,, \\ N_{a,h} &= p_a + \bar{p}_a \,, & N_{\alpha,h} &= -q_\alpha - \bar{q}_\alpha \,, \end{aligned}$$

- the resulting gauge group is the product of unitary factors

$$G_{\rm CP} = \prod_a U(p_a) \times \prod_\alpha U(q_\alpha)\,.$$

- Dirac quantisation condition for the background magnetic fields

$$H_i^{(A)} \;=\; \frac{m_i^{(A)}}{n_i^{(A)} v_i} \;,$$

- With a T-duality along the horizontal directions of the three two-torii, D9-branes are transformed into D6 branes, and the magnetic fields $H_i^{(A)}$ are related to the angles $\theta_i^{(A)}$ the rotated D6-branes make with the horizontal axis of the i-th torus

$$H_i^{(A)} = \tan\,\theta_i^{(A)}$$

.

The integers $(m_i^{(A)}, n_i^{(A)})$, that in the magnetised picture denote the familiar degeneracies of the Landau levels from quantum mechanics, now count the number of times D6 branes wrap the fundamental cycles of the T_i^2.

- Consistent models must satisfy tadpole conditions, for both RR and NS-NS massless states.
- Untwisted tadpole conditions

$$\sum_a p_a\, n_1^{(a)} n_2^{(a)} n_3^{(a)} + \sum_\alpha q_\alpha\, n_1^{(\alpha)} n_2^{(\alpha)} n_3^{(\alpha)} = 16\,,$$

$$\sum_a p_a\, n_1^{(a)} m_2^{(a)} m_3^{(a)} + \sum_\alpha q_\alpha\, n_1^{(\alpha)} m_2^{(\alpha)} m_3^{(\alpha)} = -16\,\epsilon_1\,,$$

$$\sum_a p_a\, m_1^{(a)} n_2^{(a)} m_3^{(a)} + \sum_\alpha q_\alpha\, m_1^{(\alpha)} n_2^{(\alpha)} m_3^{(\alpha)} = -16\,\epsilon_2\,,$$

$$\sum_a p_a\, m_1^{(a)} m_2^{(a)} n_3^{(a)} + \sum_\alpha q_\alpha\, m_1^{(\alpha)} m_2^{(\alpha)} n_3^{(\alpha)} = -16\,\epsilon_3\,,$$

- Twisted tadpole conditions

$$\sum_a p_a\, m_1^{(a)} \epsilon_l^{(a),g} + \sum_\alpha q_\alpha\, m_1^{(\alpha)} \epsilon_l^{(\alpha),g} = 0\,,$$

$$\sum_a p_a\, m_2^{(a)} \epsilon_l^{(a),f} - \sum_\alpha q_\alpha\, m_2^{(\alpha)} \epsilon_l^{(\alpha),f} = 0\,,$$

$$\sum_a p_a\, n_3^{(a)} \epsilon_l^{(a),h} - \sum_\alpha q_\alpha\, n_3^{(\alpha)} \epsilon_l^{(\alpha),h} = 0\,,$$

- $\mathcal{N}=1$ supersymmetry is preserved in four dimensions

$$H_1^{(A)}+H_2^{(A)}+H_3^{(A)} \;=\; H_1^{(A)}H_2^{(A)}H_3^{(A)}\,.$$

or in terms of the rotation angles of the D6 branes

$$\theta_1^{(A)}+\theta_2^{(A)}+\theta_3^{(A)}=0\,.$$

The above condition guarantees that the A-th stack of magnetised branes preserves $\mathcal{N}=1$ supersymmetry.

- Demanding that all stacks preserve the same supersymmetry charges further constrains the magnetic fields to satisfy the inequality

$$H_1^{(A)}H_2^{(A)}+H_1^{(A)}H_3^{(A)}+H_2^{(A)}H_3^{(A)} \;\le 1\,.$$

The generic spectrum of models with magnetised D-branes is given by chiral multiplets in

- Bifundamental representations
- Antisymmetric representations
- Symmetric representations

General Spectrum

Multiplicity	Representation	Relevant Indices
$\frac{1}{8}(I^{aa'} + I^{aO} - 4I_1^{aa'} - 4I_2^{aa'} + 4I_3^{aa'})$	$\left(\frac{p_a(p_a-1)}{2}, 1\right)$	$\forall a$
$\frac{1}{8}(I^{\alpha\alpha'} + I^{\alpha O} - 4I_1^{\alpha\alpha'} - 4I_2^{\alpha\alpha'} + 4I_3^{\alpha\alpha'})$	$\left(1, \frac{q_\alpha(q_\alpha-1)}{2}\right)$	$\forall\alpha$
$\frac{1}{8}(I^{aa'} - I^{aO} - 4I_1^{aa'} - 4I_2^{aa'} + 4I_3^{aa'})$	$\left(\frac{p_a(p_a+1)}{2}, 1\right)$	$\forall a$
$\frac{1}{8}(I^{\alpha\alpha'} - I^{\alpha O} - 4I_1^{\alpha\alpha'} - 4I_2^{\alpha\alpha'} + 4I_3^{\alpha\alpha'})$	$\left(1, \frac{q_\alpha(q_\alpha+1)}{2}\right)$	$\forall\alpha$
$\frac{1}{4}(I^{a\alpha'} - S_g^{a\alpha} I_1^{a\alpha'} + S_f^{a\alpha} I_2^{a\alpha'} - S_h^{a\alpha} I_3^{a\alpha'})$	(p_a, q_α)	$\forall a, \forall\alpha$
$\frac{1}{4}(I^{a\alpha} + S_g^{a\alpha} I_1^{a\alpha} - S_f^{a\alpha} I_2^{a\alpha} - S_h^{a\alpha} I_3^{a\alpha})$	$(p_a, \bar{q}_\alpha)$	$\forall a, \forall\alpha$
$\frac{1}{4}(I^{ab'} - S_g^{ab} I_1^{ab'} - S_f^{ab} I_2^{ab'} + S_h^{ab} I_3^{ab'})$	(p_a, p_b)	$a < b$
$\frac{1}{4}(I^{ab} + S_g^{ab} I_1^{ab} + S_f^{ab} I_2^{ab} + S_h^{ab} I_3^{ab})$	$(p_a, \bar{p}_b)$	$a < b$
$\frac{1}{4}(I^{\alpha\beta'} - S_g^{\alpha\beta} I_1^{\alpha\beta'} - S_f^{\alpha\beta} I_2^{\alpha\beta'} + S_h^{\alpha\beta} I_3^{\alpha\beta'})$	(q_α, q_β)	$\alpha < \beta$
$\frac{1}{4}(I^{\alpha\beta} + S_g^{\alpha\beta} I_1^{\alpha\beta} + S_f^{\alpha\beta} I_2^{\alpha\beta} + S_h^{\alpha\beta} I_3^{\alpha\beta})$	$(q_\alpha, \bar{q}_\beta)$	$\alpha < \beta$
1	$(p_a, \bar{q}_\alpha) + (\bar{p}_a, q_\alpha)$	if $H_i^a = H_i^\alpha\ \forall i$

- The effective multiplicities of representations for this $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, depend on
- topological intersection numbers between branes of different types

$$I^{AB} = \prod_{i=1}^{3} I_i^{AB}\,, \qquad I_i^{AB} = m_i^{(A)} n_i^{(B)} - n_i^{(A)} m_i^{(B)}\,,$$

$$I^{AB'} = \prod_{i=1}^{3} I_i^{AB'}\,, \qquad I_i^{AB'} = m_i^{(A)} n_i^{(B)} + n_i^{(A)} m_i^{(B)}\,,$$

- topological intersection numbers between branes and O-planes

$$I^{AO} = 8\big(m_1^{(A)} m_2^{(A)} m_3^{(A)} - \epsilon_1 m_1^{(A)} n_2^{(A)} n_3^{(A)} - \epsilon_2 n_1^{(A)} m_2^{(A)} n_3^{(A)} - \epsilon_3 n_1^{(A)} n_2^{(A)} m_3^{(A)}\big)$$

- the action of the orbifold group on the open-string states and the Chan-Paton factors.
- Since the orbifold projection acts through the fixed points, it is clear that states at brane intersections feel the orbifold projection if and only if the intersection point coincides with one of the orbifold fixed points. This is encoded in the quantity

$$S^{AB}_{i=g,f,h} = \text{number of fixed points of the generator } i = g, f, h \\ \text{that both branes } A \text{ and } B \text{ intersect}\,,$$

Stringy Instantons and Non-perturbative Superpotentials

- One can consider the effect of E5 instantons and three types of E1 instantons on a configuration with O9 and O5 planes, and (possibly magnetised) D9 and D5 branes.

	x	z_1	z_2	z_3
E5	$\cdot$	$\times$	$\times$	$\times$
$\mathrm{E1}_1$	$\cdot$	$\times$	$\cdot$	$\cdot$
$\mathrm{E1}_2$	$\cdot$	$\cdot$	$\times$	$\cdot$
$\mathrm{E1}_3$	$\cdot$	$\cdot$	$\cdot$	$\times$

Uncharged Modes of the Brane Instantons

	G_{CP}	minimalz.m.
E5	$U(n)$	4
E1_1	$U(n)$	4
E1_2	$U(n)$	4
E1_3	$SO(n)$	2

- The so-called "$SO(1)''$ $E1_3$ instanton has only x^μ and θ_α as uncharged zero-modes and hence it can have a direct contribution to the superpotential

$$\mathcal{S} = \int d^4x d^2\theta W_{np}$$

- In addition to x^μ and θ_α, there are charged zero-modes η. One needs to integrate over them.

$$S_{inst} = S(vol) + S_{charged}(\eta, \Phi)$$

$$\int [d\eta] e^{S_{inst}} \qquad \rightarrow \; W_{np} \sim e^{-S(vol)} \prod_i \Phi_i$$

- Φ_i's are charged under the anomalous U(1)'s and

$$e^{-S(vol)} \rightarrow e^{-S(vol)} + \Lambda \sum_i Q_i$$

- Perturbatively forbidden couplings can be generated by stringy instantons.

Charged Modes of the $E1_3$ Instantons

Instanton	Multiplicity	Representation
$\mathrm{E1}_o$	$\frac{1}{4}\left(I^{ab} - S_g^{ab}I_1^{ab} - S_f^{ab}I_2^{ab} + S_h^{ab}I_3^{ab}\right)$	$(r_3, \bar{p}_b)$
	$\frac{1}{4}\left(I^{a\beta} - S_g^{a\beta}I_1^{a\beta} + S_f^{a\beta}I_2^{a\beta} - S_h^{a\beta}I_3^{a\beta}\right)$	$(r_3, \bar{q}_\beta)$
$\mathrm{E1}_g$	$\frac{1}{4}\left(I^{ab} - S_g^{ab}I_1^{ab} + S_f^{ab}I_2^{ab} - S_h^{ab}I_3^{ab}\right)$	$(r_3, \bar{p}_b)$
	$\frac{1}{4}\left(I^{a\beta} - S_g^{a\beta}I_1^{a\beta} - S_f^{a\beta}I_2^{a\beta} + S_h^{a\beta}I_3^{a\beta}\right)$	$(r_3, \bar{q}_\beta)$
$\mathrm{E1}_f$	$\frac{1}{4}\left(I^{ab} + S_g^{ab}I_1^{ab} - S_f^{ab}I_2^{ab} - S_h^{ab}I_3^{ab}\right)$	$(r_3, \bar{p}_b)$
	$\frac{1}{4}\left(I^{a\beta} + S_g^{a\beta}I_1^{a\beta} + S_f^{a\beta}I_2^{a\beta} + S_h^{a\beta}I_3^{a\beta}\right)$	$(r_3, \bar{q}_\beta)$
$\mathrm{E1}_h$	$\frac{1}{4}\left(I^{ab} + S_g^{ab}I_1^{ab} + S_f^{ab}I_2^{ab} + S_h^{ab}I_3^{ab}\right)$	$(r_3, \bar{p}_b)$
	$\frac{1}{4}\left(I^{a\beta} + S_g^{a\beta}I_1^{a\beta} - S_f^{a\beta}I_2^{a\beta} - S_h^{a\beta}I_3^{a\beta}\right)$	$(r_3, \bar{q}_\beta)$

Explicit Models

- A model with magnetised D9 branes, and non-magnetised D9 and D5 branes is given by the following magnetisation numbers

$$(m_i^{(1)}, n_i^{(1)}) = \{(2,1)\,,\,(1,1)\,,\,(-1,1)\}\,,$$
$$(m_i^{(2)}, n_i^{(2)}) = \{(-2,1)\,,\,(-1,1)\,,\,(1,1)\}\,,$$
$$(m_i^{(3)}, n_i^{(3)}) = \{(0,1)\,,\,(0,1)\,,\,(0,1)\}\,,$$
$$(m_i^{(4)}, n_i^{(4)}) = \{(0,1)\,,\,(1,0)\,,\,(-1,0)\}\,.$$

- Tadpole conditions then select the Chan-Paton gauge group

$$G_{\rm CP} = U(2)^2 \times U(2)^2 \times USp(4)^2 \times USp(4)^2\,,$$

where pairs of unitary groups live on each fractional brane, while the symplectic ones originate from non-magnetised D9 and D5 branes that are displaced in the bulk.

Multiplicity	Representation	field
1	$(2,\bar{2},1,1;1,1,1,1)$	$\Phi_{1\bar{2}}$
1	$(\bar{2},2,1,1;1,1,1,1)$	$\Phi_{\bar{1}2}$
12	$(\bar{1},1,1,1;1,1,1,1)$	A^1
12	$(1,\bar{1},1,1;1,1,1,1)$	A^2
4	$(\bar{2},\bar{2},1,1;1,1,1,1)$	$\Phi_{\bar{1}\bar{2}}$
1	$(1,1,2,\bar{2};1,1,1,1)$	$\Phi_{3\bar{4}}$
1	$(1,1,\bar{2},2,;1,1,1,1)$	$\Phi_{\bar{3}4}$
12	$(1,1,1,1;1,1,1,1)$	A^3
12	$(1,1,1,1;1,1,1,1)$	A^4
4	$(1,1,2,2;1,1,1,1)$	Φ_{34}
	....	

- In the presence of Wilson lines for the non-magnetised branes we obtain the following charged zero modes for the $E1_3$ instantons

Instanton	(k_1, k_2, k_3, k_4)	Representation	Zero mode
$\mathrm{E1}_o$	$(1,0,0,0)$	$(1,2,1,1,1,1,1,1)$	η_i^o
$\mathrm{E1}_g$	$(0,1,0,0)$	$(2,1,1,1,1,1,1,1)$	η_i^g
$\mathrm{E1}_f$	$(0,0,1,0)$	$(1,1,\bar{2},1,1,1,1,1)$	η_i^f
$\mathrm{E1}_h$	$(0,0,0,1)$	$(1,1,1,\bar{2},1,1,1,1)$	η_i^h

- The instanton action is

$$S_{\rm inst} = S_{{\rm E1}_o} + \sum_{i,j=1}^{2} \eta_i^o \, A^2_{ij} \, \eta_j^o + \sum_{i,j,k=1}^{2} \eta_i^o \, (\Phi_{1\bar{2}})_{ki} \, (\Phi_{\bar{1}\bar{2}})_{kj} \, \eta_j^o \, .$$

- Upon integration over the two charged zero modes η_i^o one gets the following non-perturbative contribution to the superpotential

$$\mathcal{W}_{\rm np} \sim e^{-S_{{\rm E1}_o}} \sum_{i,j=1}^{2} \epsilon_{ij} \left[A^2_{ij} + \sum_{k=1}^{2} (\Phi_{1\bar{2}})_{ki} \, (\Phi_{\bar{1}\bar{2}})_{kj} \right]$$

- Taking a closer look at the linear term

$$S_{\rm inst} = S_{\rm E1} + f(\xi, U) \sum_{i,j=1}^{2} \eta_i \, A_{ij} \, \eta_j$$

$$\mathcal{W}_{\rm np} \; = \; f(\xi, U) e^{-S_{\rm E1}} \; \sum_{i,j=1}^{2} \epsilon_{ij} \, A_{ij}$$

- the F-term associated to the antisymmetric field is proportional to

$$\langle F_A \rangle \sim f(\xi, U)$$

- the zeroes of $f(\xi, U)$ determine the true non-perturbative supersymmetric vacuum
- a potential is generated for the string moduli ξ, U

- A model with only magnetized D9-branes is given by the following magnetisation numbers

$$(m_i^{(1)}, n_i^{(1)}) = \{(1,1)\,,\,(1,1)\,,\,(-1,1)\}\,,$$
$$(m_i^{(2)}, n_i^{(2)}) = \{(-1,1)\,,\,(-1,1)\,,\,(1,1)\}\,.$$

  The gauge group is therefore

$$U(4)^2 \times U(4)^2$$

- All Abelian factors are anomalous, and therefore the low-energy group is $SU(4)^4$.

Multiplicity	Representation	field
1	$(\bar{4},4,1,1)_{(-1,1,0,0)}$	$\mathit{\Phi}_{\bar{1}2}$
1	$(4,\bar{4},1,1)_{(1,-1,0,0)}$	$\Phi_{1\bar{2}}$
1	$(1,1,\bar{4},4)_{(0,0,-1,1)}$	$\Phi_{\bar{3}4}$
1	$(1,1,4,\bar{4})_{(0,0,1,-1)}$	$\Phi_{3\bar{4}}$
8	$(\bar{6},1,1,1)_{(-2,0,0,0)}$	A^1
8	$(1,\bar{6},1,1)_{(0,-2,0,0)}$	A^2
8	$(1,1,6,1)_{(0,0,2,0)}$	A^3
8	$(1,1,1,6)_{(0,0,0,2)}$	A^4

- Charged zero-modes are

Instanton	(k_1, k_2, k_3, k_4)	Representation	Zero mode
$\mathrm{E1}_o$	$(1,0,0,0)$	$(1,4,1,1)$	η_i^o
$\mathrm{E1}_g$	$(0,1,0,0)$	$(4,1,1,1)$	η_i^g
$\mathrm{E1}_f$	$(0,0,1,0)$	$(1,1,\bar{4},1)$	η_i^f
$\mathrm{E1}_h$	$(0,0,0,1)$	$(1,1,1,\bar{4})$	η_i^h

- Let us analyse the case of a single $\mathrm{E1}_o$ instanton in detail. The gauge-invariant instantonic action including both neutral and charged zero modes is

$$S_{\text{inst}} = S_{\mathrm{E1}_o} + \sum_{i,j=1}^{4} \eta_i^o A_{ij}^2 \eta_j^o \,, \tag{1}$$

- integration over the charged instantonic zero-modes

$$\int \prod_{i=1}^{4} d\eta_i^o \, e^{-S_{\text{inst}}} \,, \tag{2}$$

- non-perturbative superpotential is generated

$$\mathcal{W}_{\text{non-pert}} = e^{-S_{\mathrm{E1}_o}} \sum_{i,j,k,l=1}^{4} \epsilon_{ijkl} A_{ij}^2 A_{kl}^2 \,, \tag{3}$$

- mass term for A_{ij}^2 is generated.

- The gauge theory divides into two non-interacting sectors
- beta functions for non-abelian couplings are vanishing at one loop
- speculate about the existence of an IR conformal fixed point
- conformal invariance is then broken by stringy instantons at a hierarchically small energy scale

Conclusions

- Stringy instantons can generate perturbatively forbidden couplings (e.g. Majorana neutrino masses, Higgs term in MSSM, Yukawa couplings)
- Moduli stabilisation
- Generating hierarchically small masses
- conformal symmetry breaking
- supersymmetry breaking?
- ....