Geometry and observables in (2+1)-gravity

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Department Mathematik
Universität Hamburg
Germany

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- apply this to get interesting physics from the theory

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References:

- C. Meusburger, Commun. Math. Phys. 273, 705-754
- C. Meusburger, Commun. Math. Phys. 266 735-775
- C. Meusburger, Class. Quantum Grav. 26 055006

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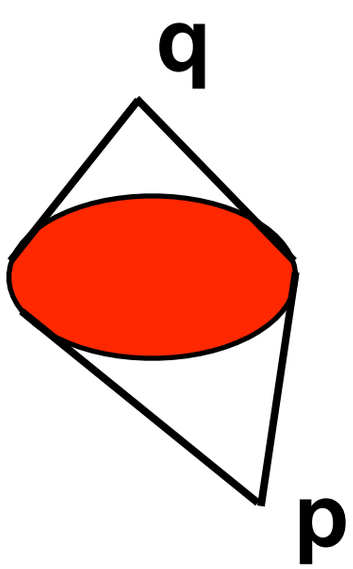

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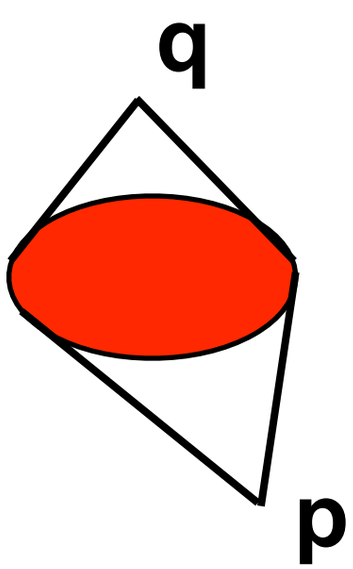

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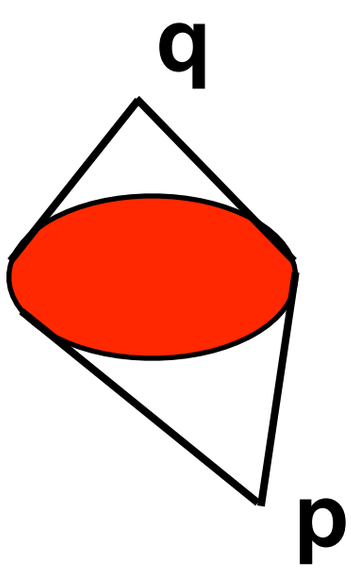

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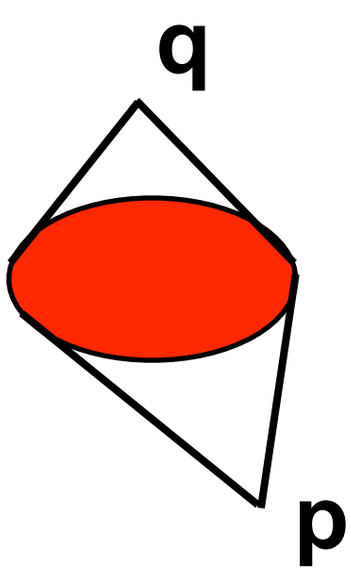

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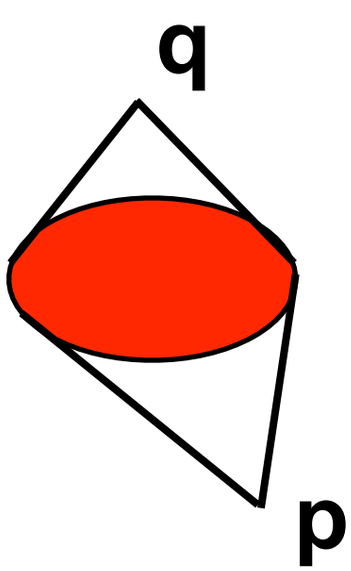

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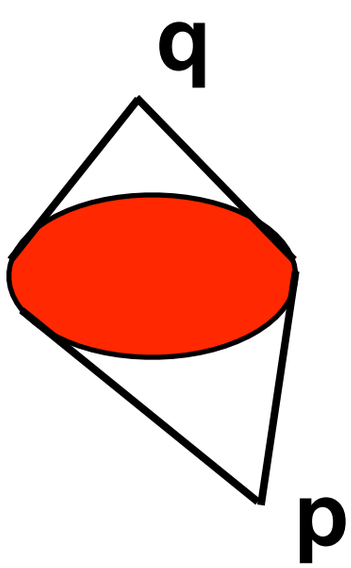

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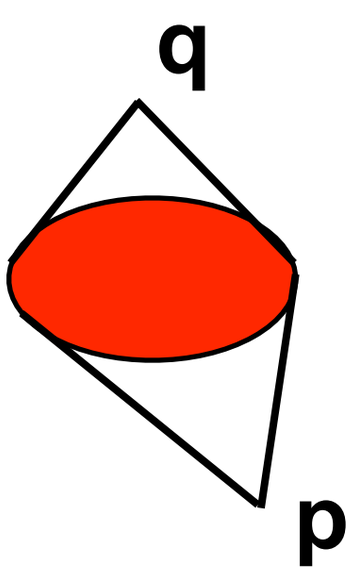

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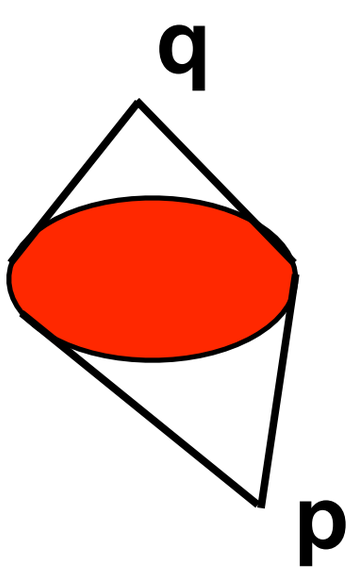

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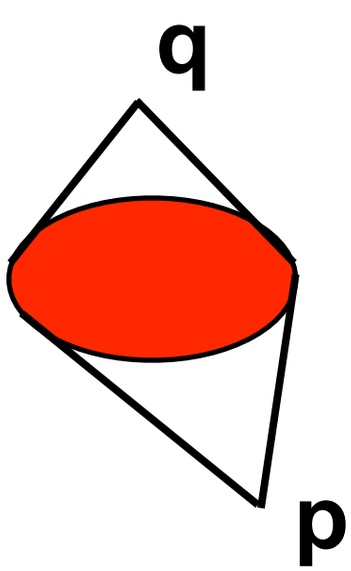

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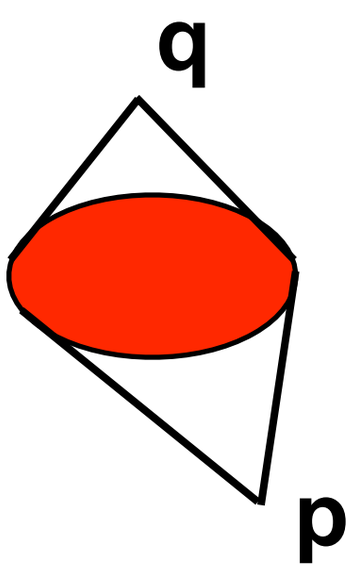

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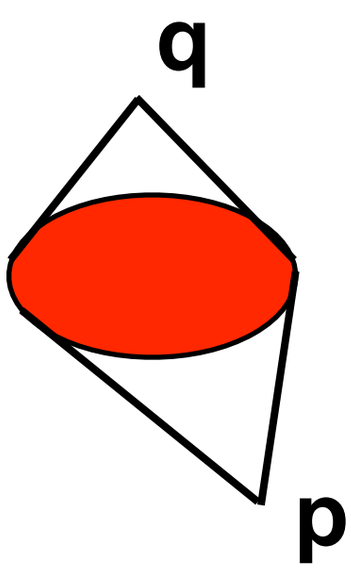

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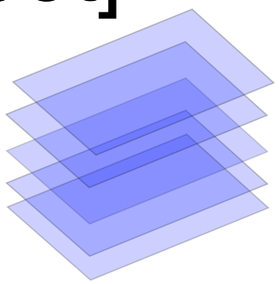

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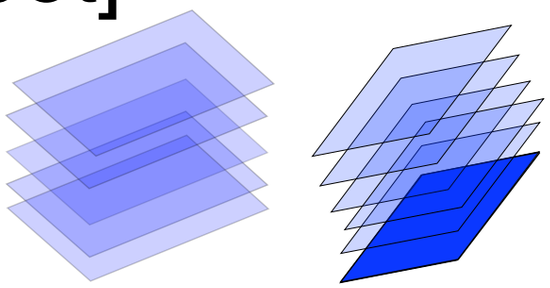

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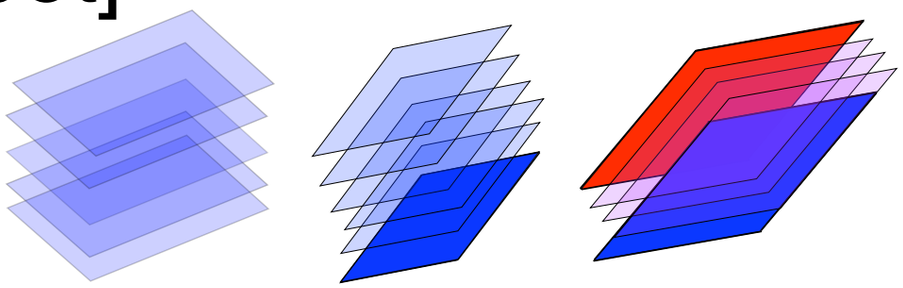

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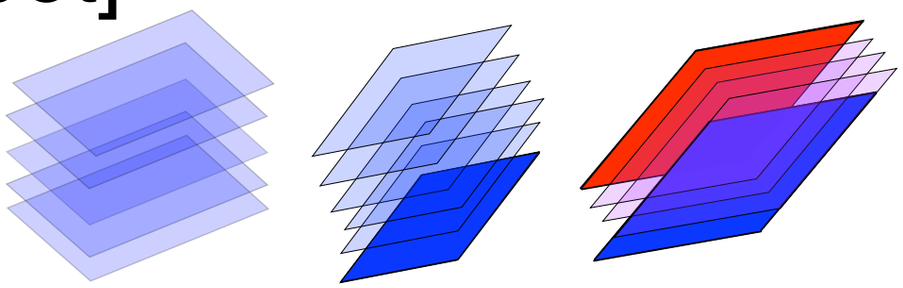

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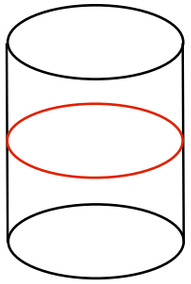

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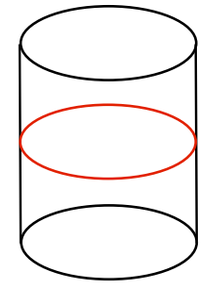

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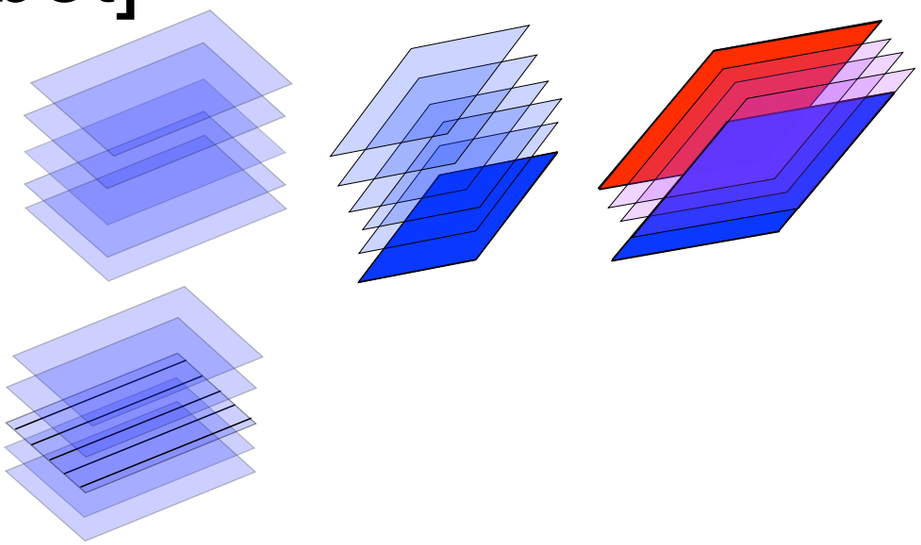

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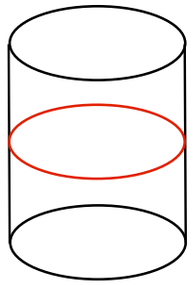

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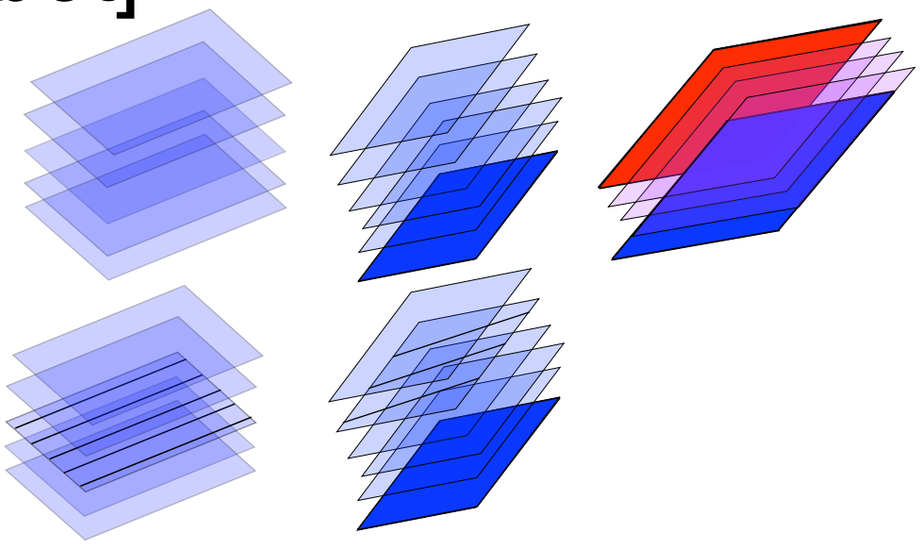

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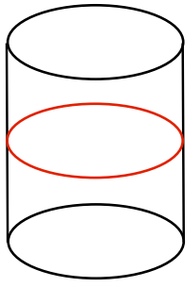

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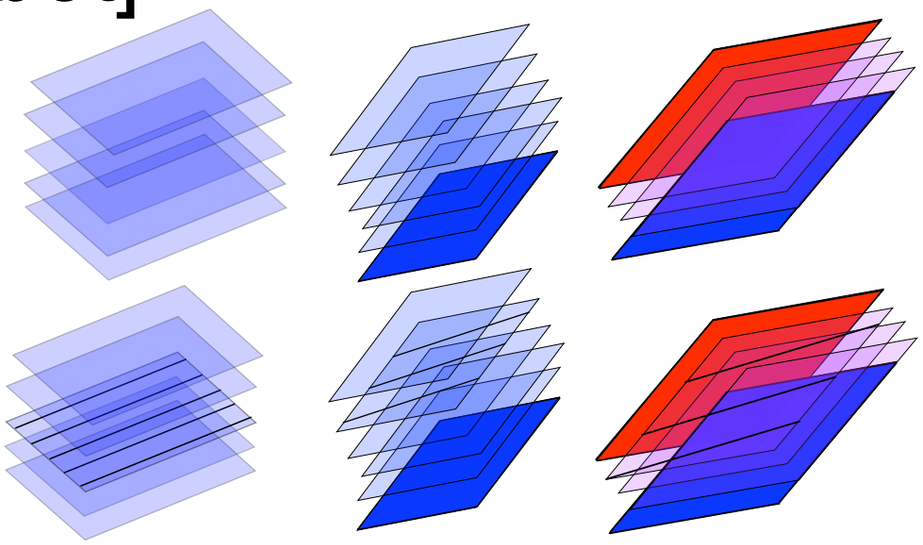

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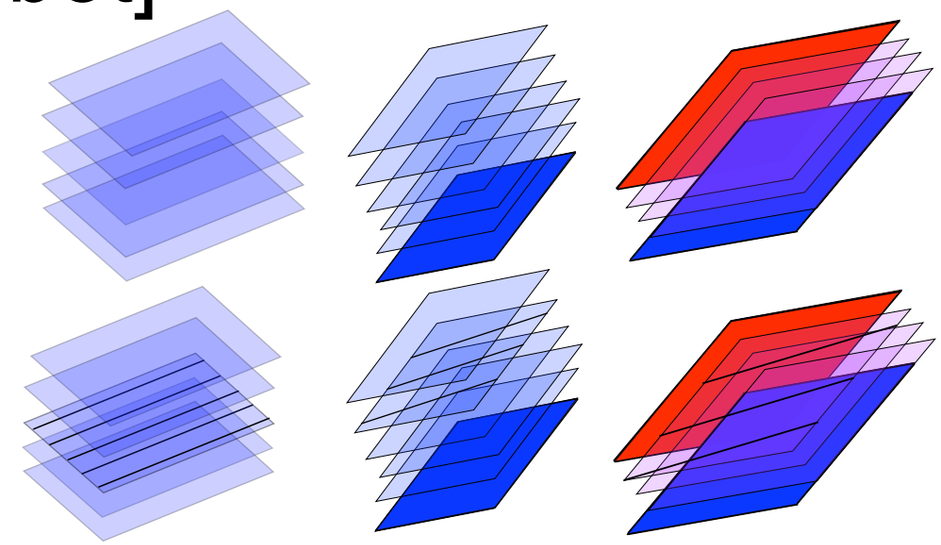

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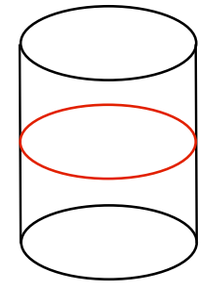

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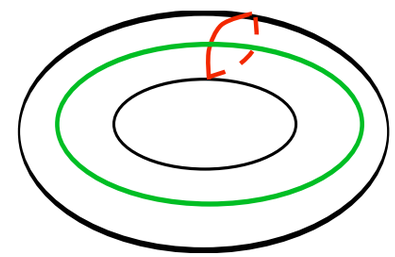

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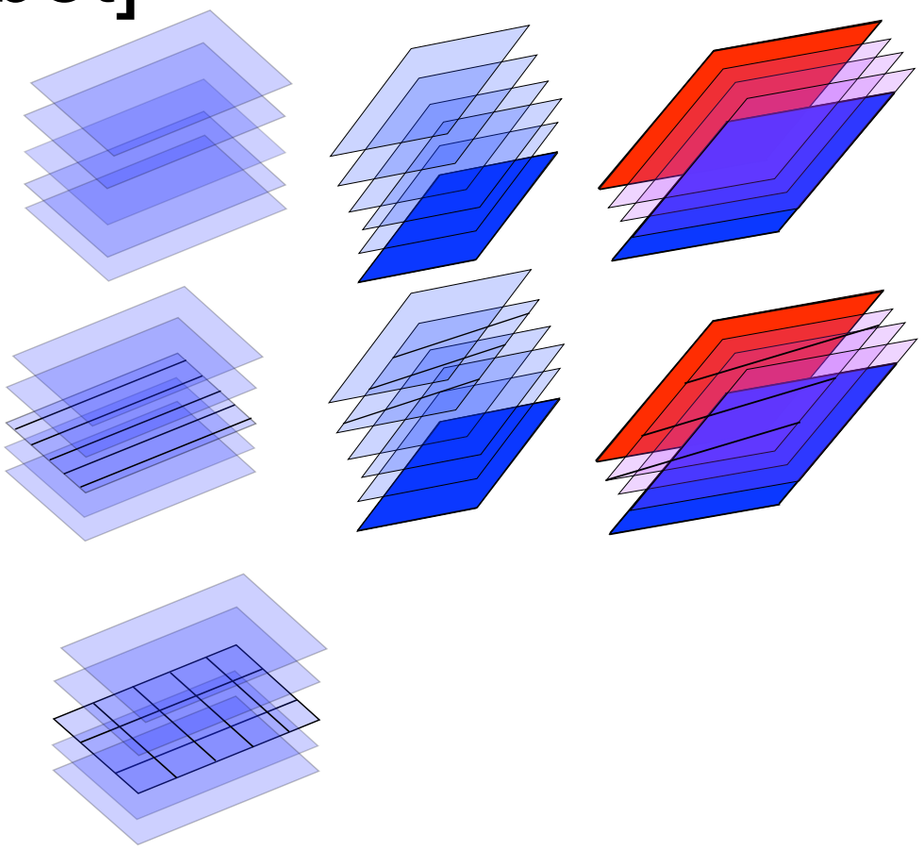

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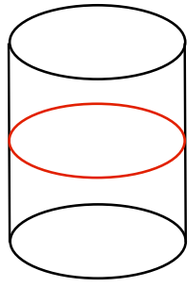

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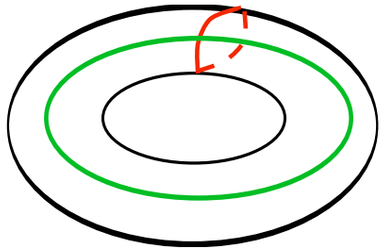

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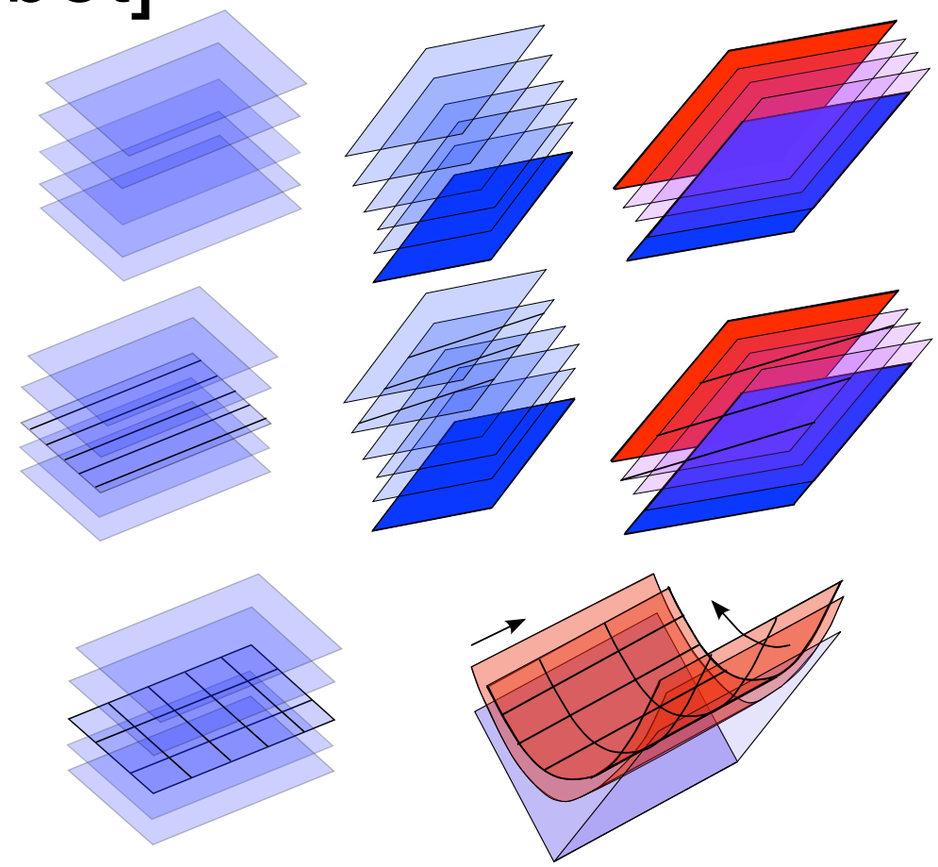

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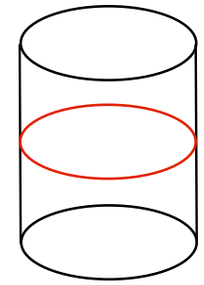

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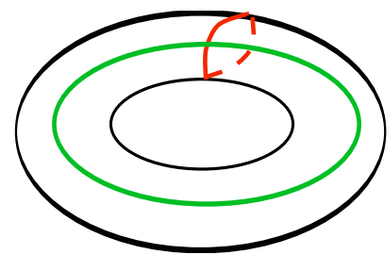

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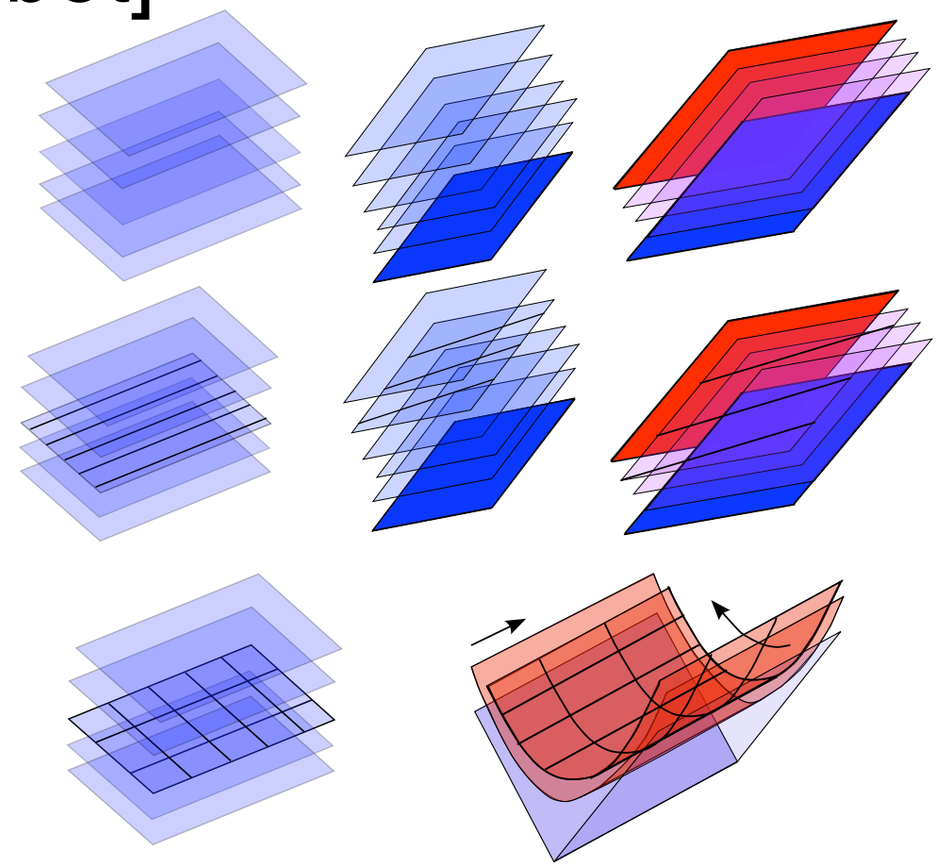

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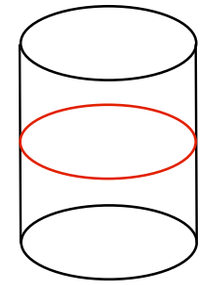

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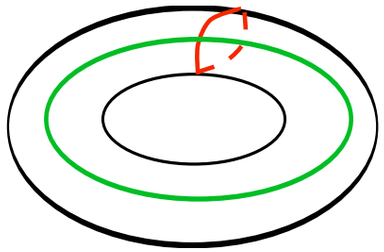

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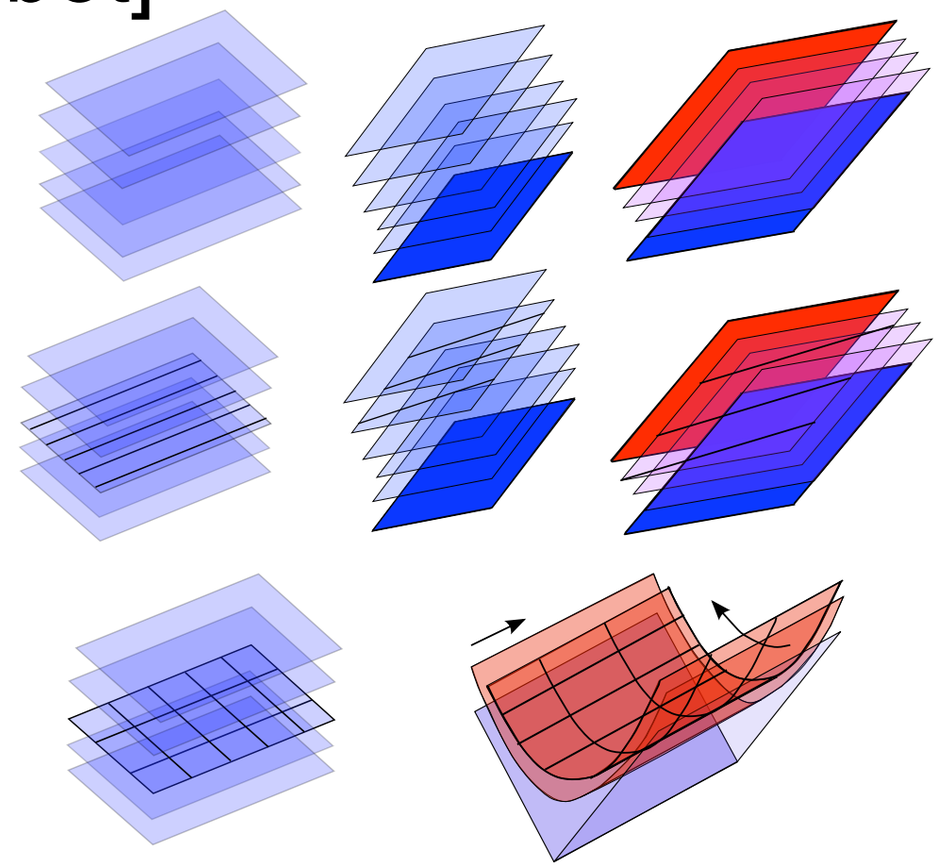

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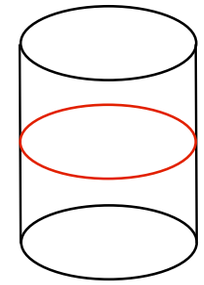

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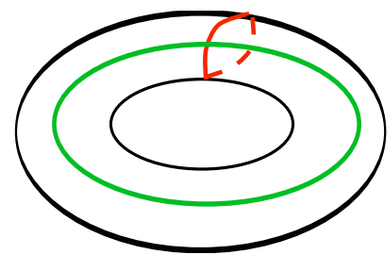

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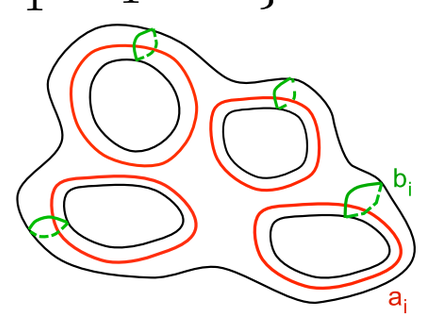

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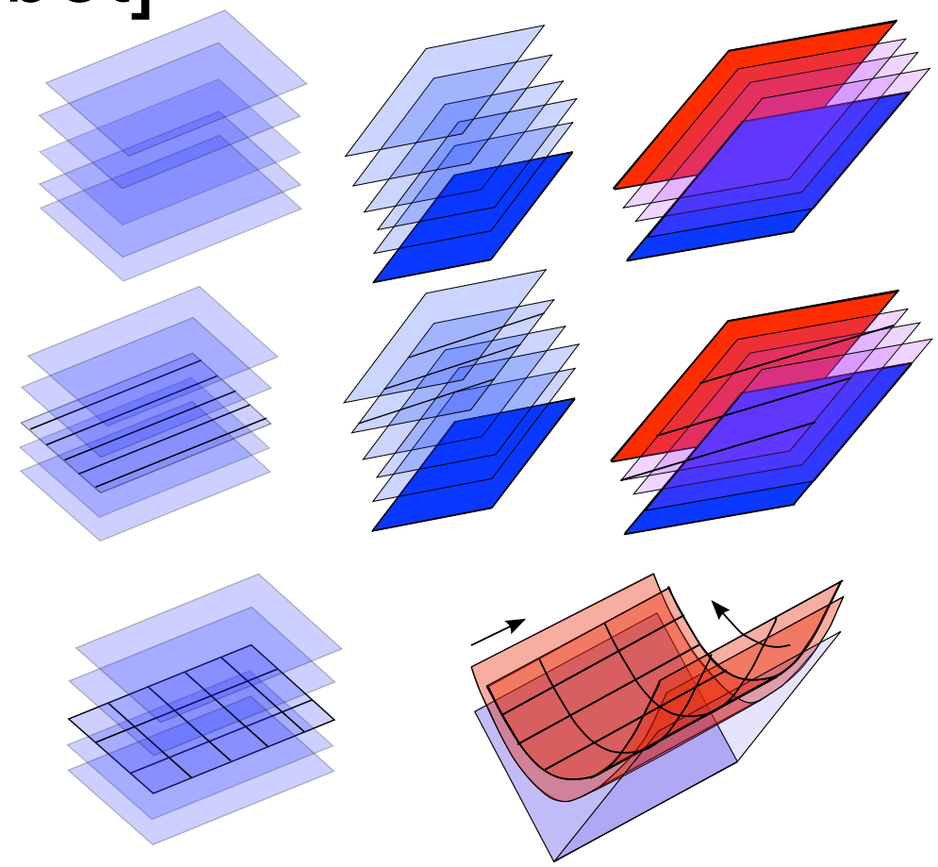

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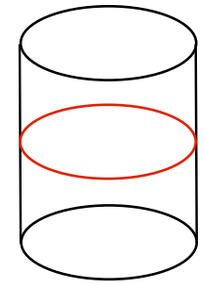

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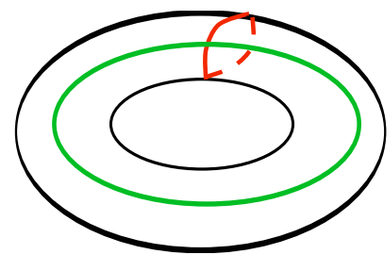

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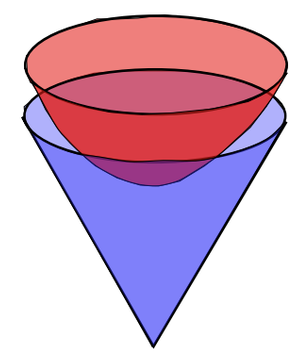

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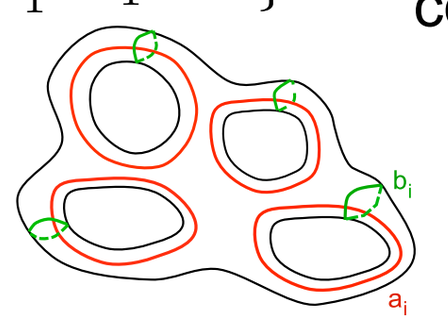

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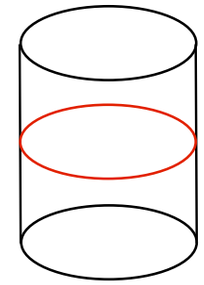

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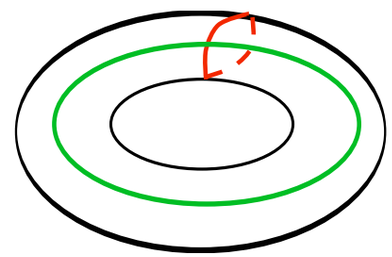

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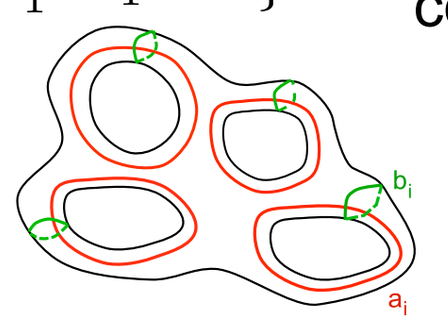

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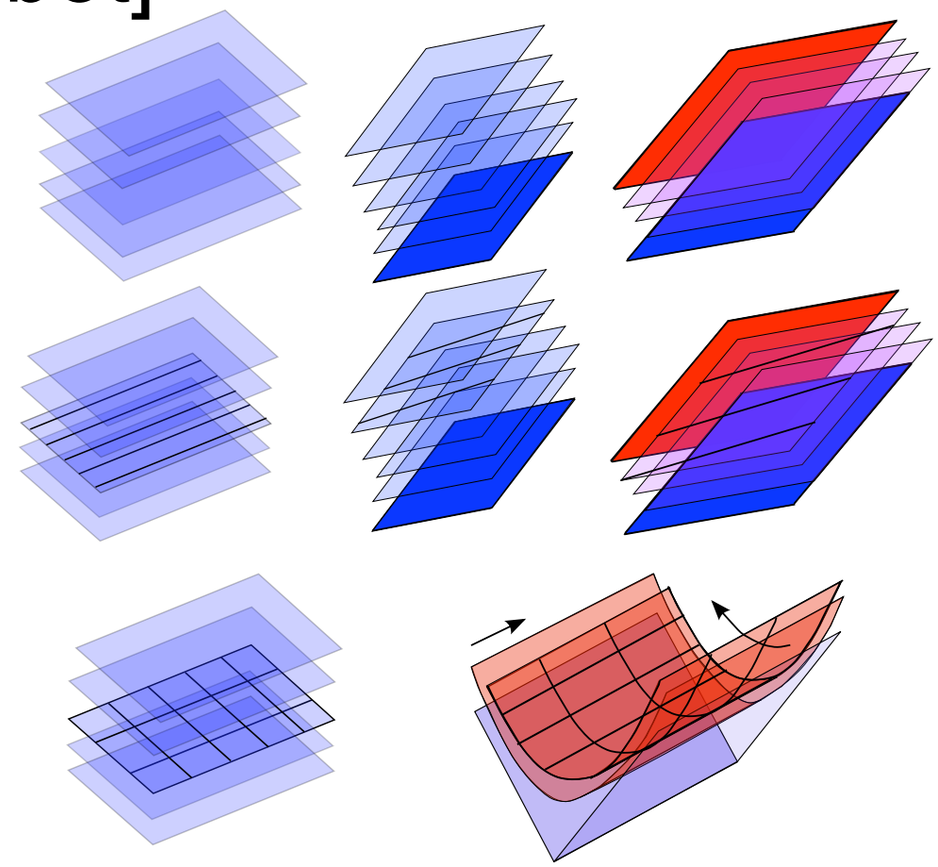

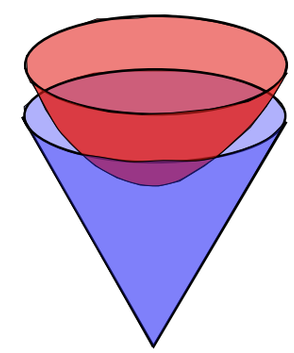

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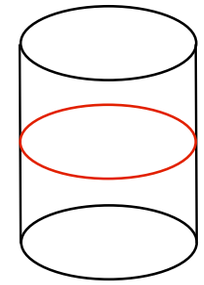

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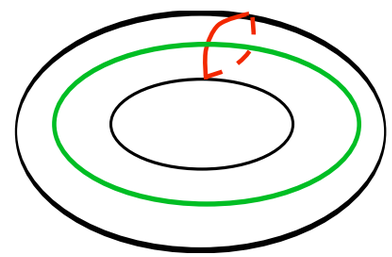

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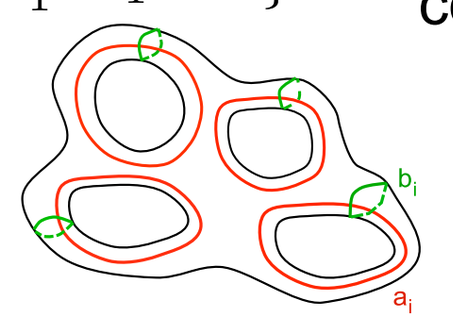

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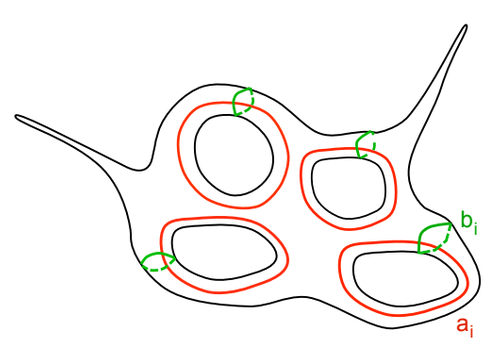

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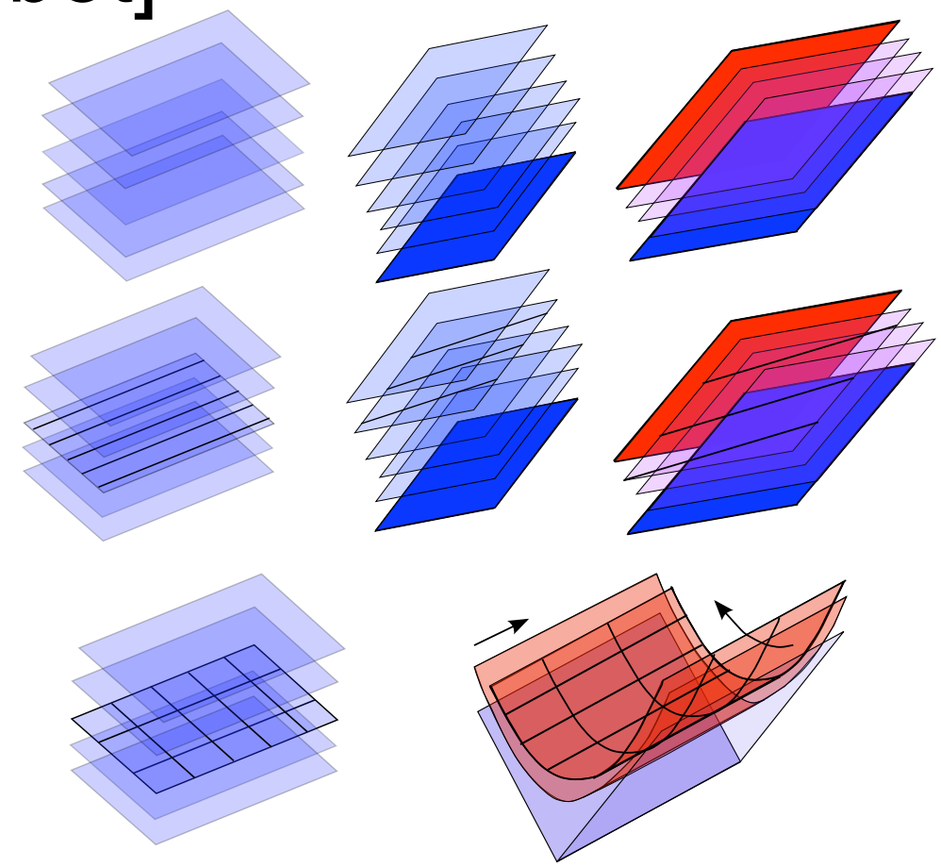

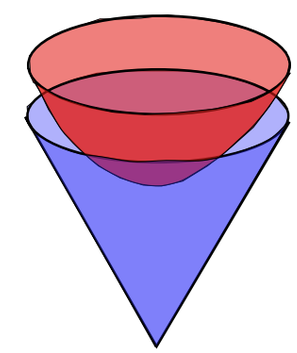

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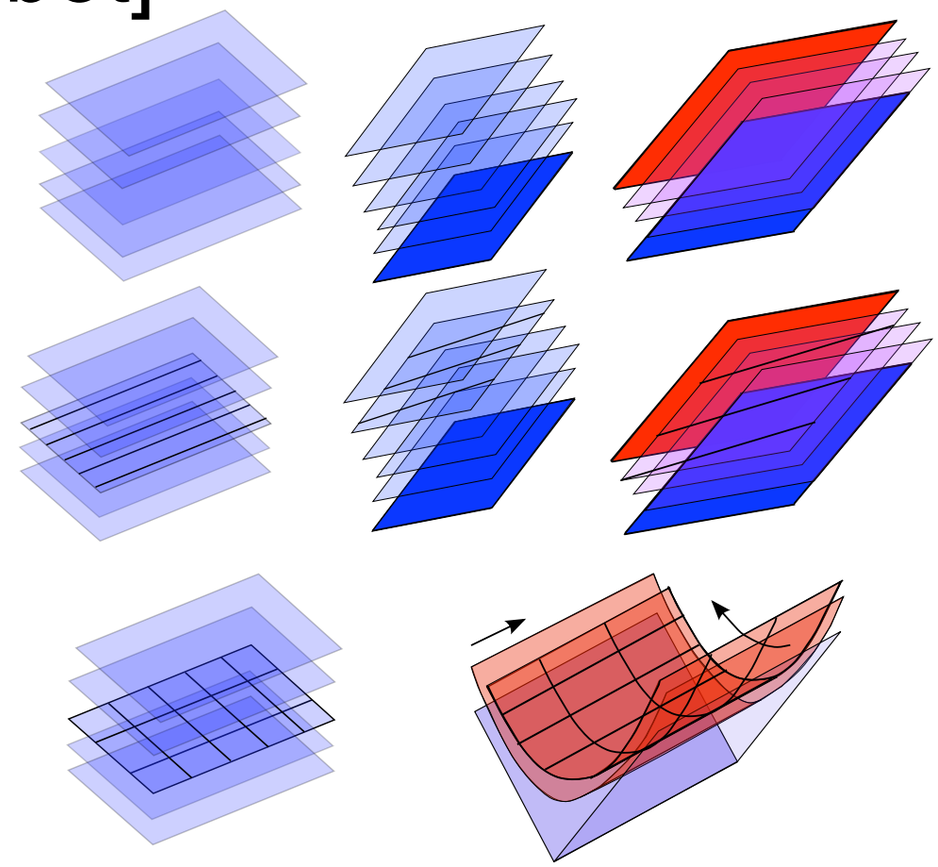

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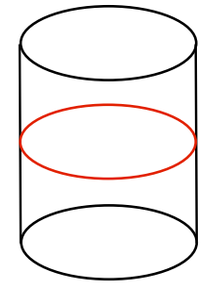

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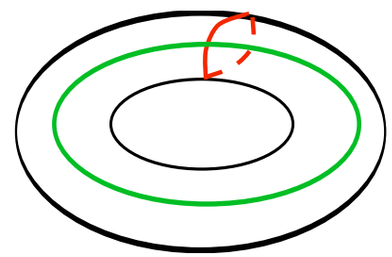

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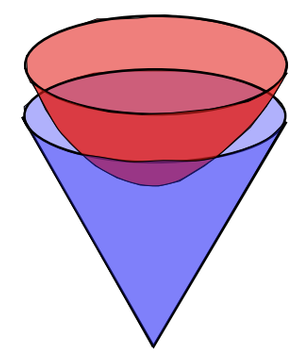

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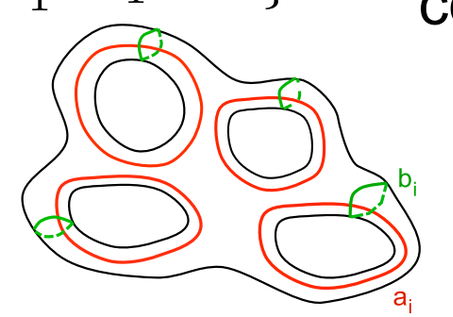

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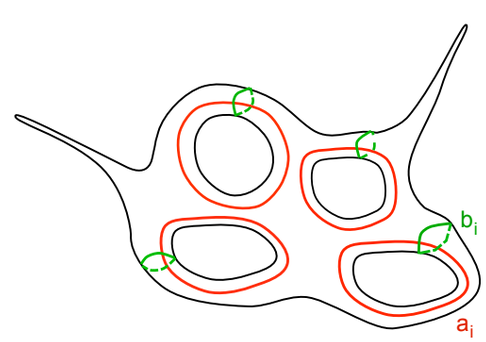

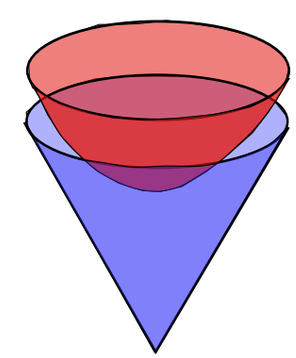

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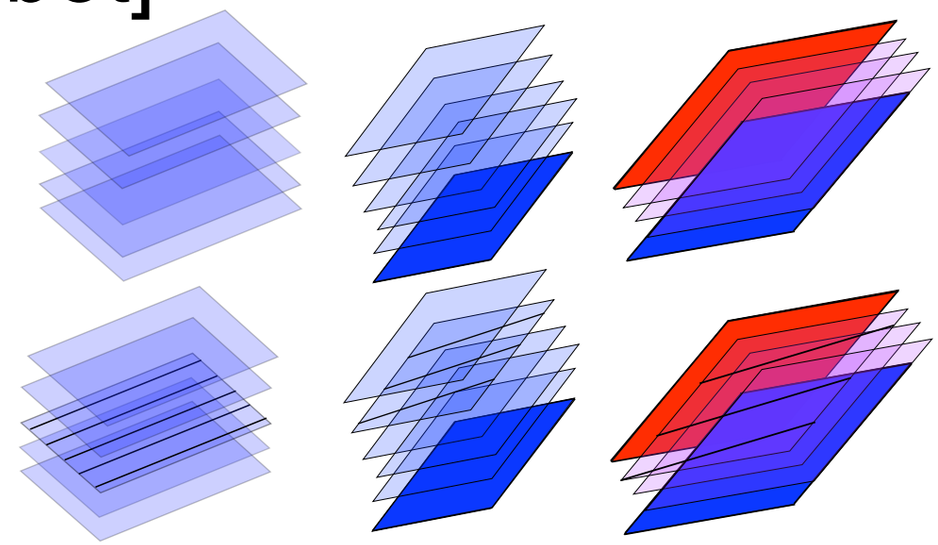

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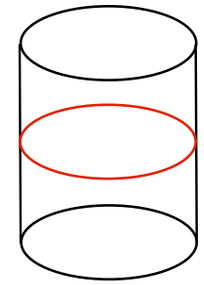

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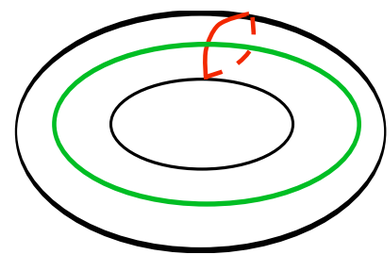

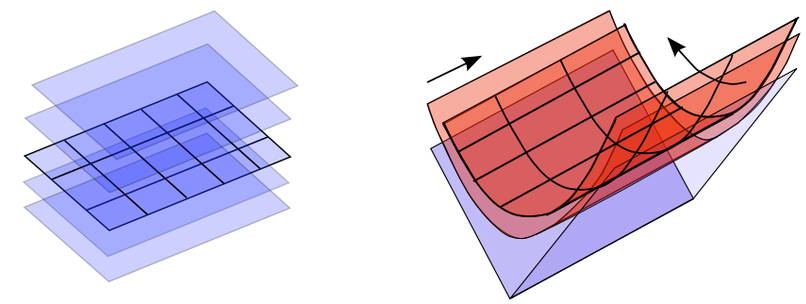

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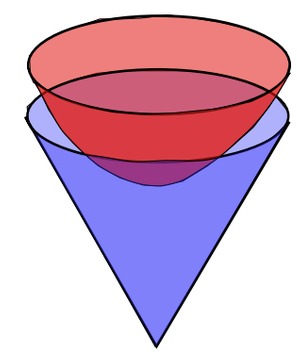

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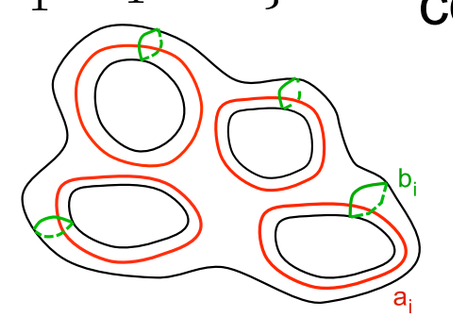

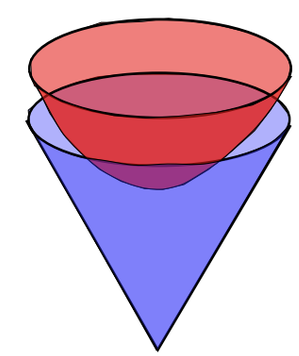

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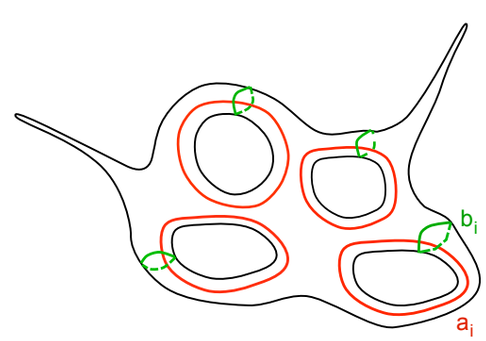

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➪ **phase space** $\mathcal{P} = \mathrm{Hom}_0(\pi_1(S), P_3)/P_3 = T^*\tau_g$

Dirac observables conjugation inv. functions of **holonomies**

➪ complete set of diffeomorphism invariant observables:

Wilson loops : 2 fundamental Wilson loops for $\lambda \in \pi_1(M)$

$$h(\lambda) = (\exp(n^a_\lambda J_a), \mathbf{a}_\lambda)$$

mass $m : \lambda \mapsto m_\lambda = |\mathbf{n}_\lambda|$

2. The phase space for genus g≥2 [Mess]

phase space [Mess]

- group homomorphism $h : \pi_1(S) \to P_3$ (holonomies) determines spacetime completely
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3. Geometry of (2+1)-spacetimes

[Mess, Barbot ,Benedetti, Bonsante, Schlenker,...]

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Conformally static spacetimes of genus g≥2

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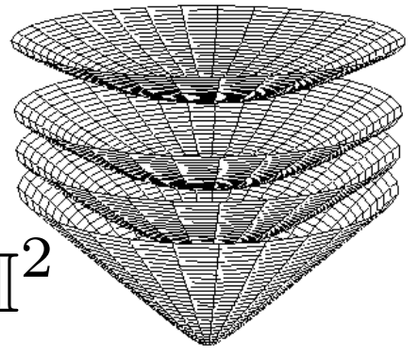

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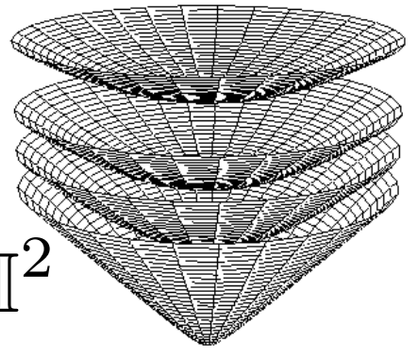

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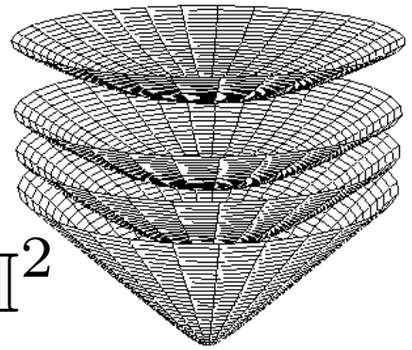

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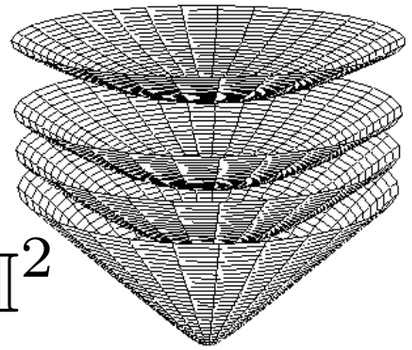

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$$\Gamma = \langle v_{a_1}, v_{b_1}, ...v_{a_g}, v_{b_g} \mid [v_{b_g}, v_{a_g}^{-1}] \cdots [v_{b_1}, v_{a_1}^{-1}] = 1\rangle \subset PSL(2, \mathbb{R}) \cong SO_0^+(2, 1)$$

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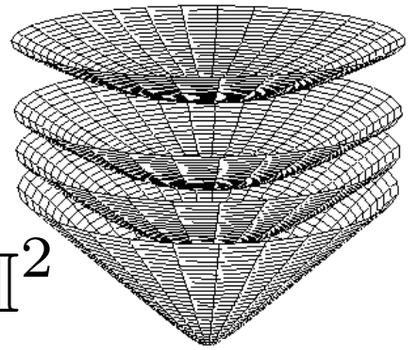

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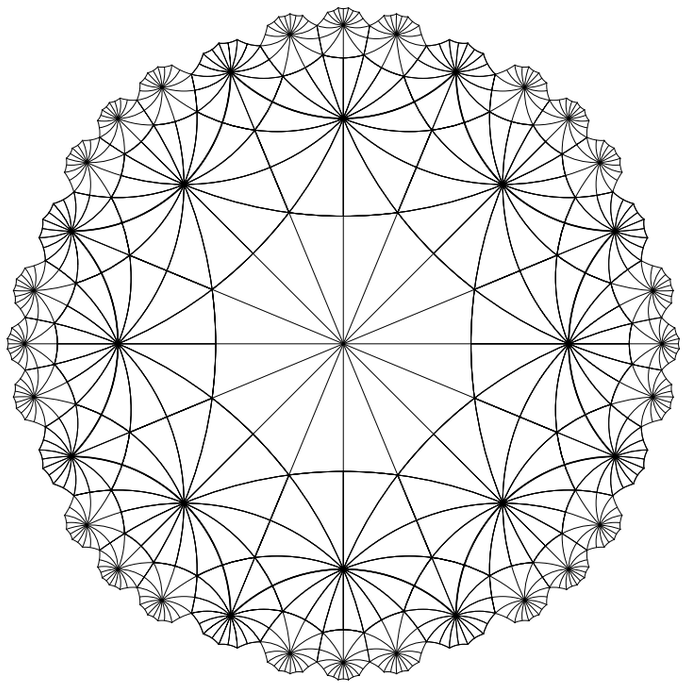

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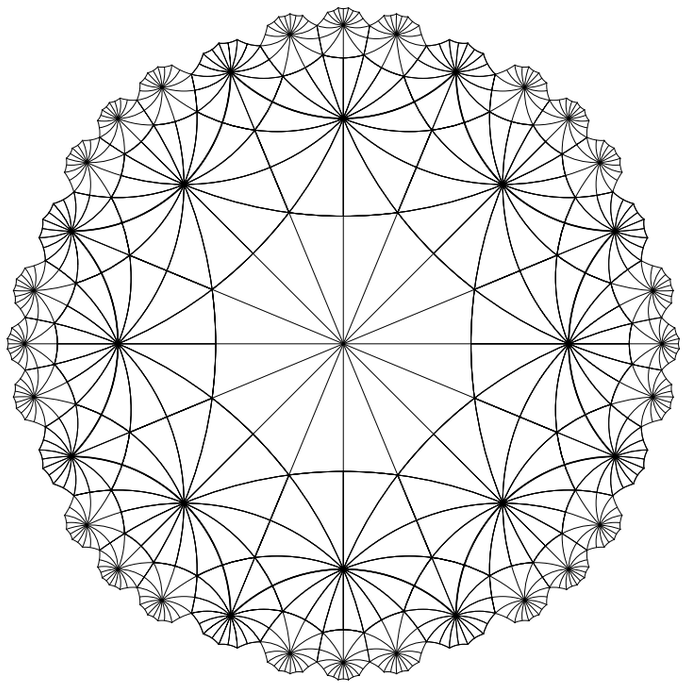

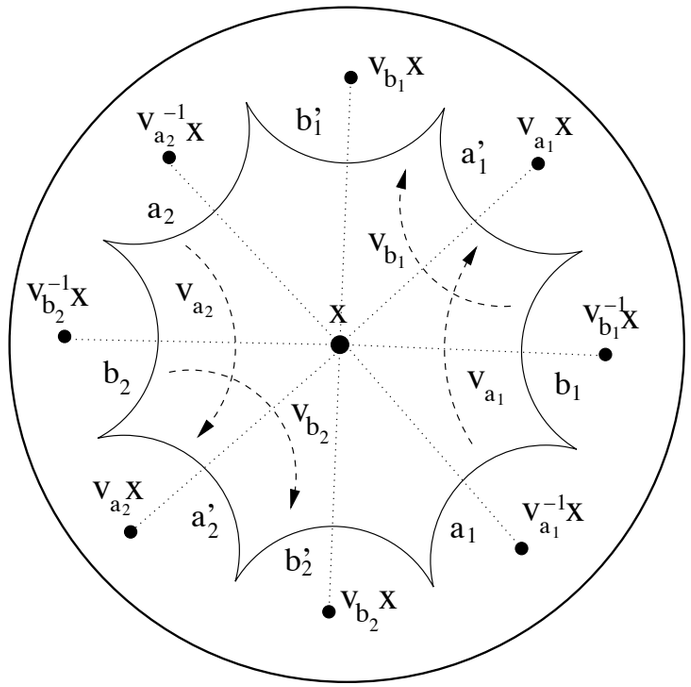

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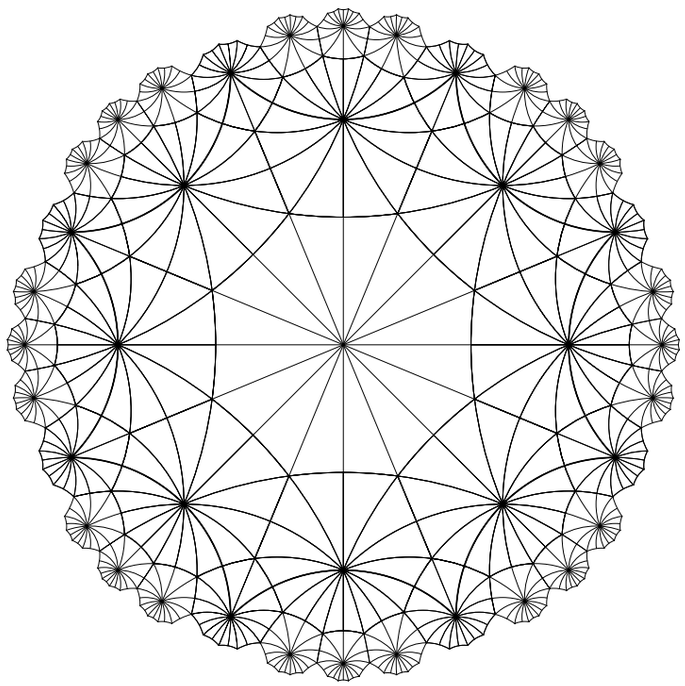

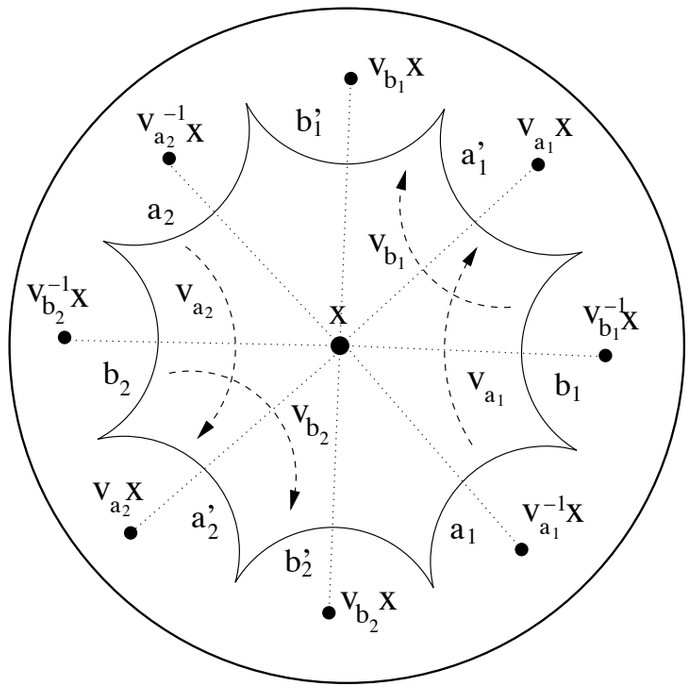

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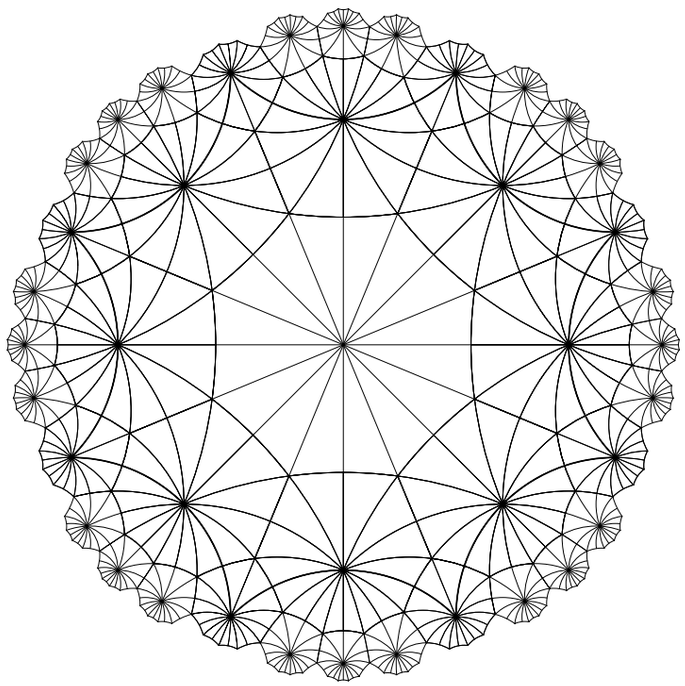

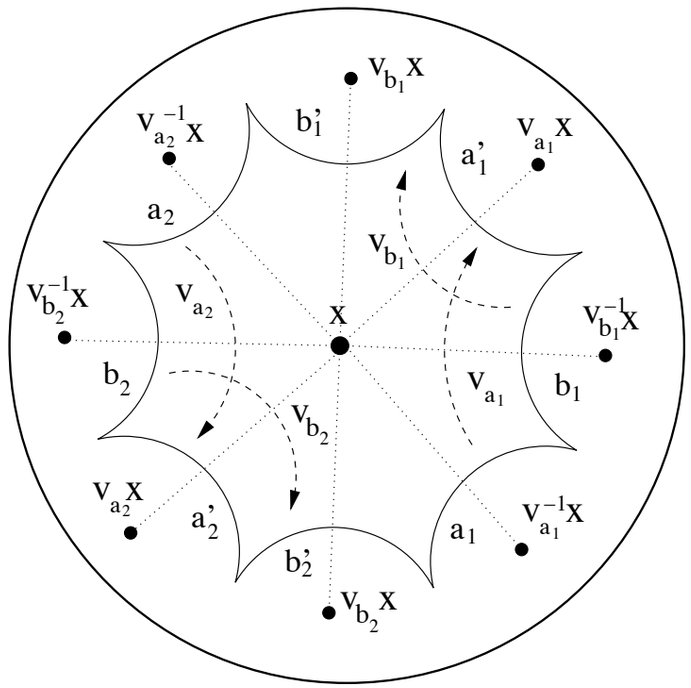

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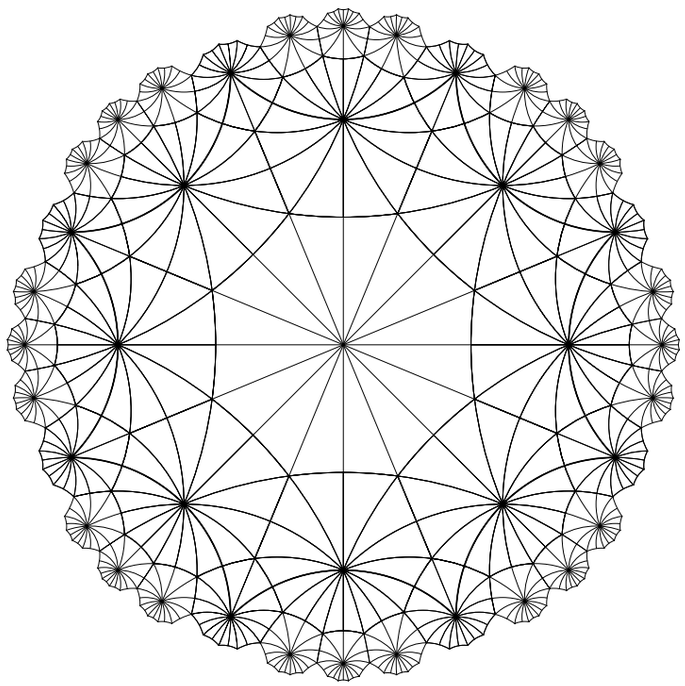

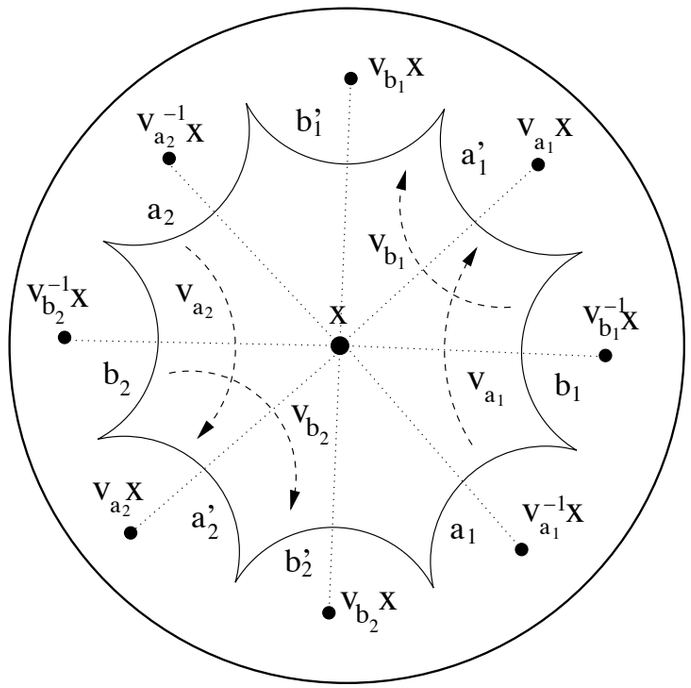

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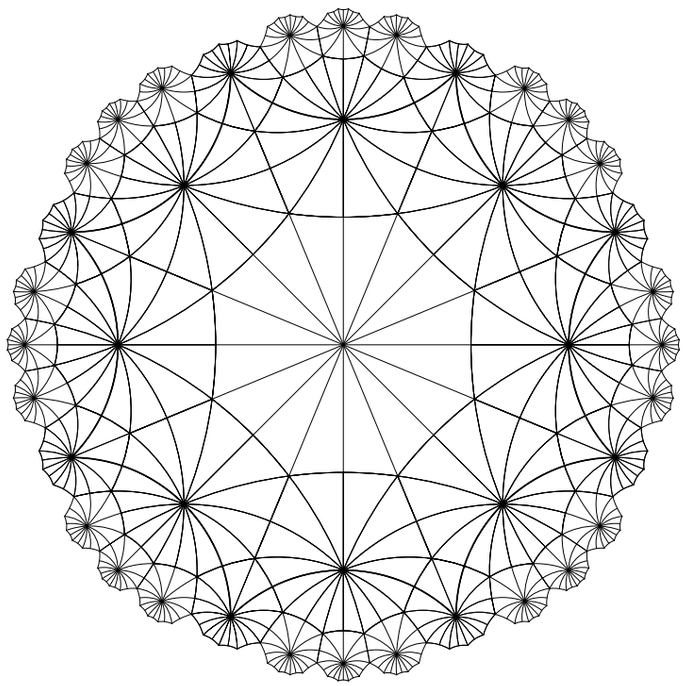

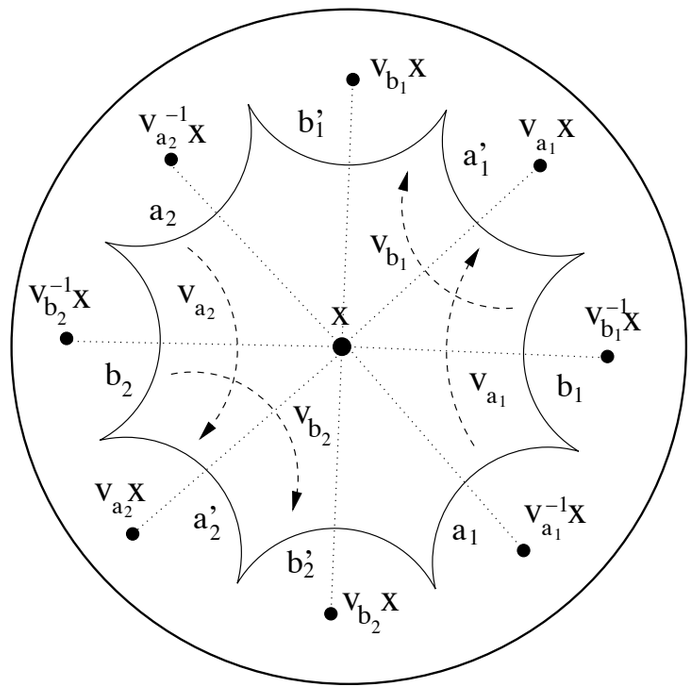

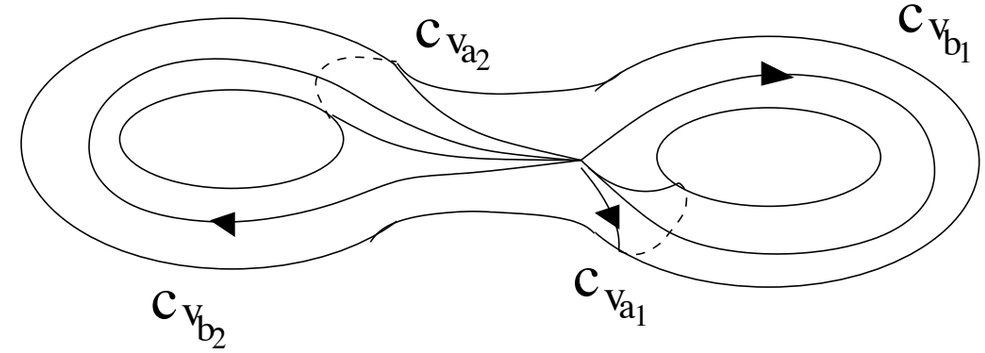

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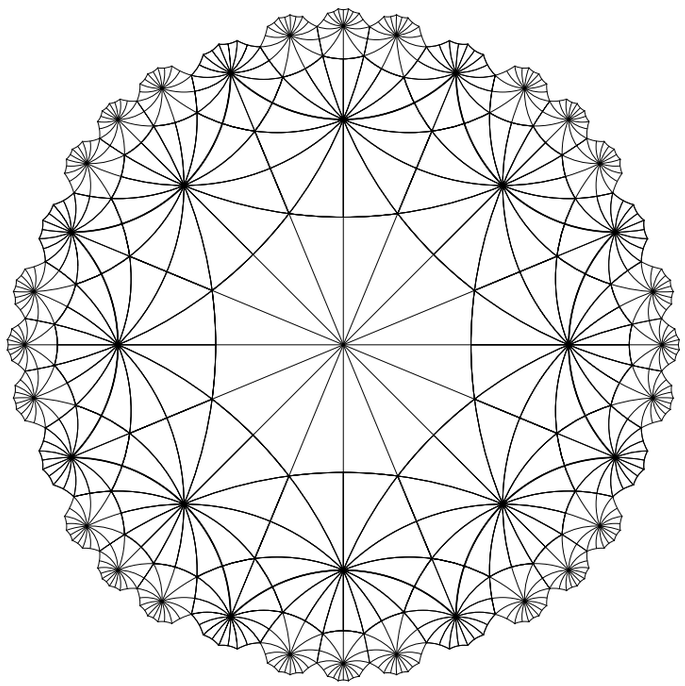

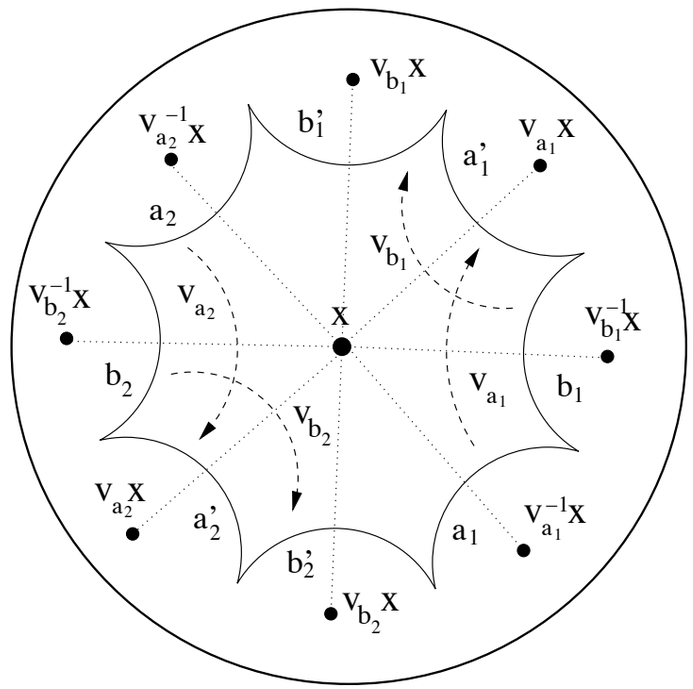

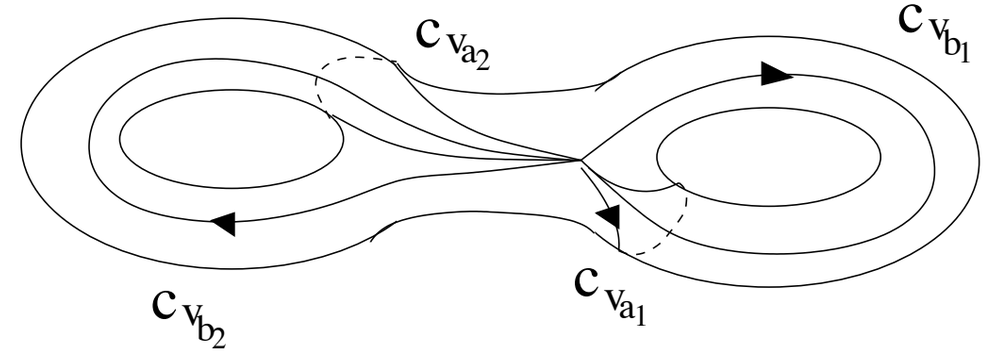

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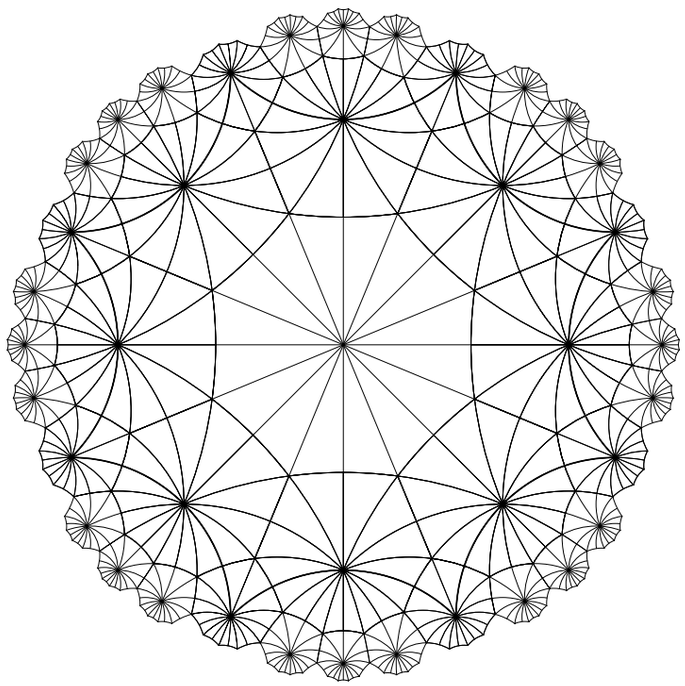

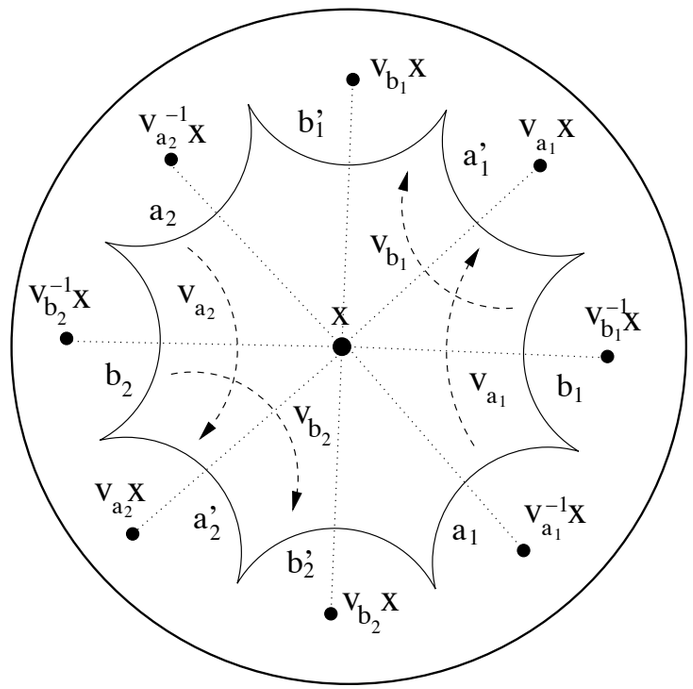

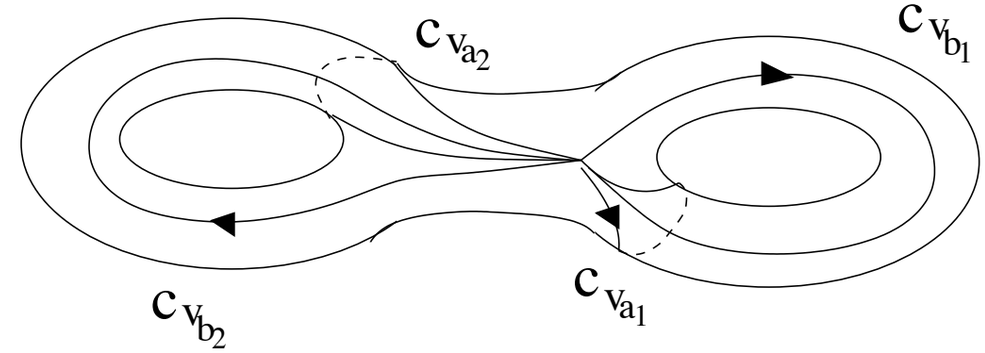

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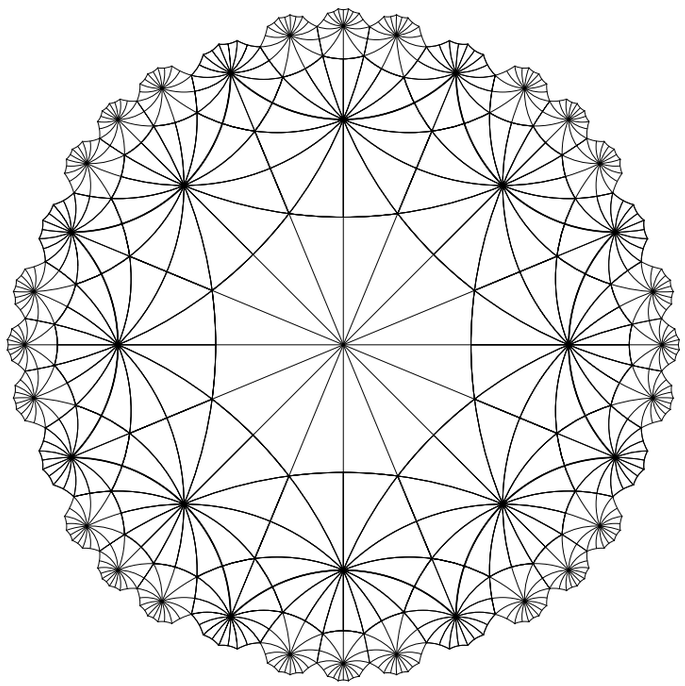

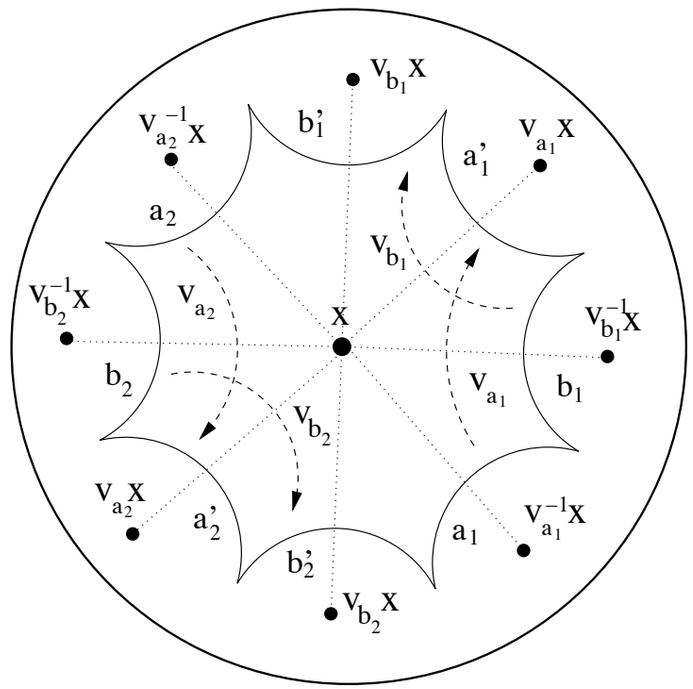

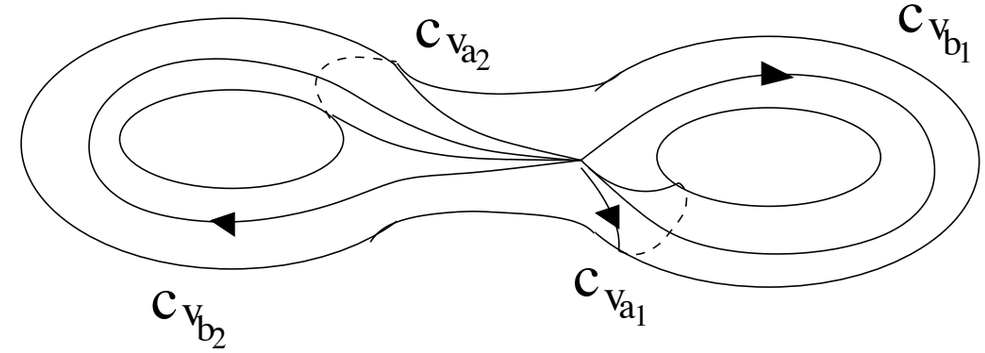

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Evolving spacetimes via grafting

[Mess], [Thurston]

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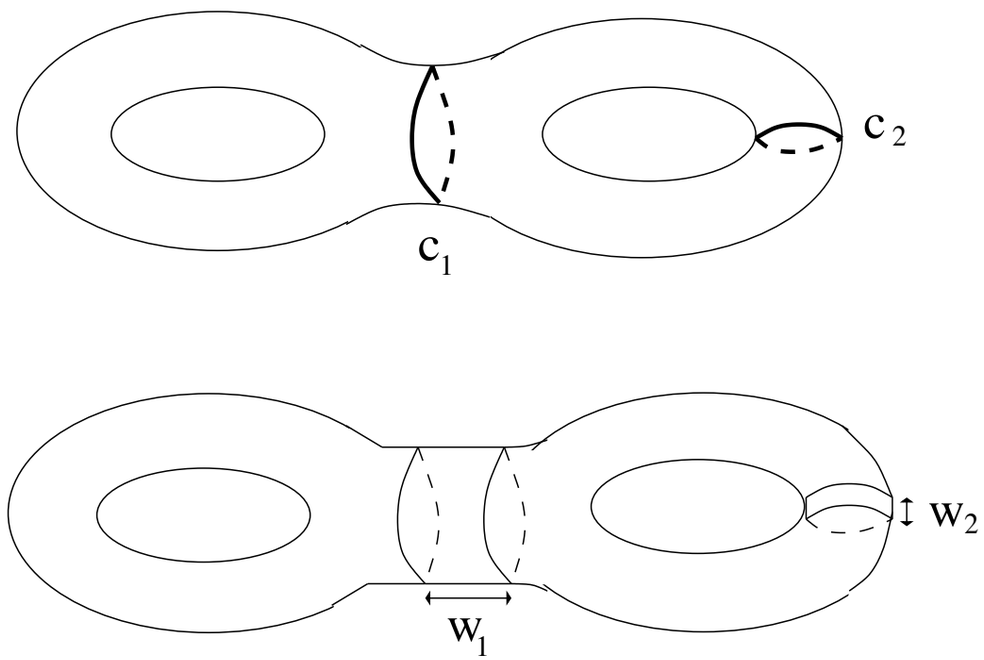

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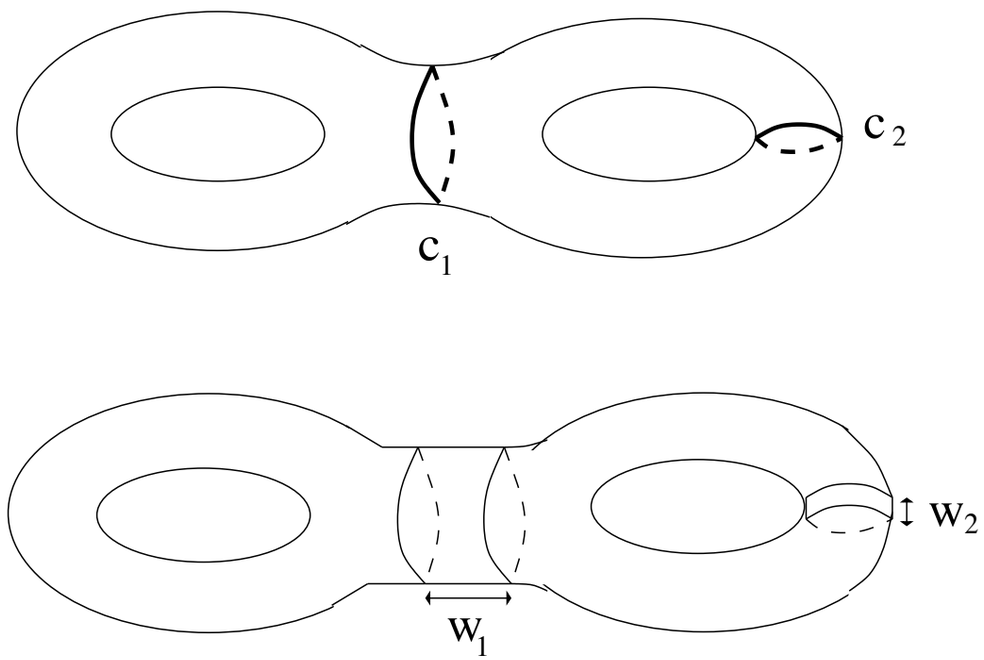

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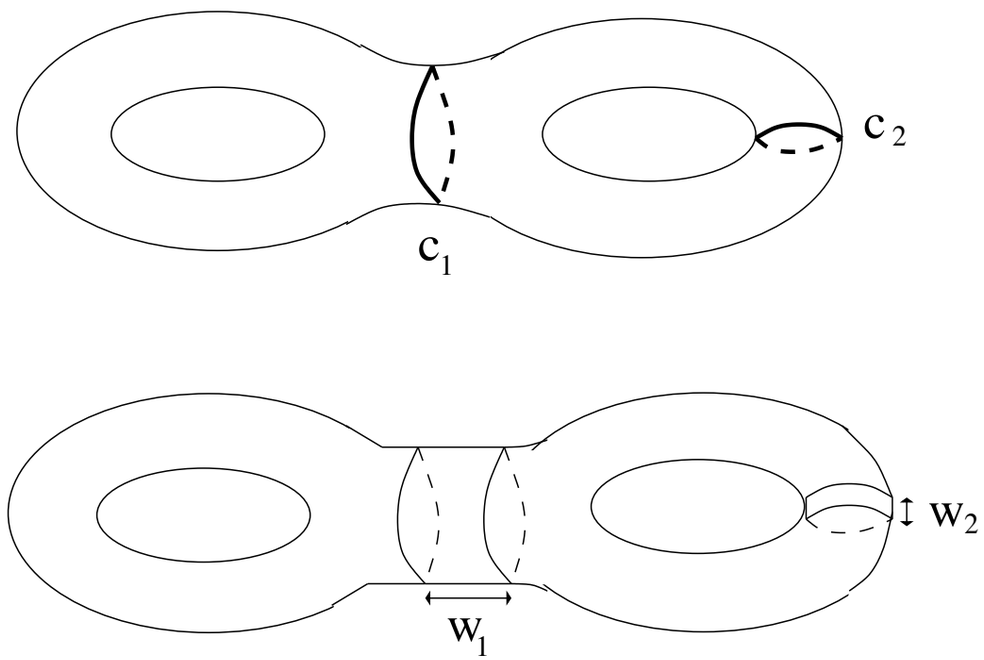

grafting (2+1)-spacetimes

- grafting simultaneously on each cct-surface of static spacetime

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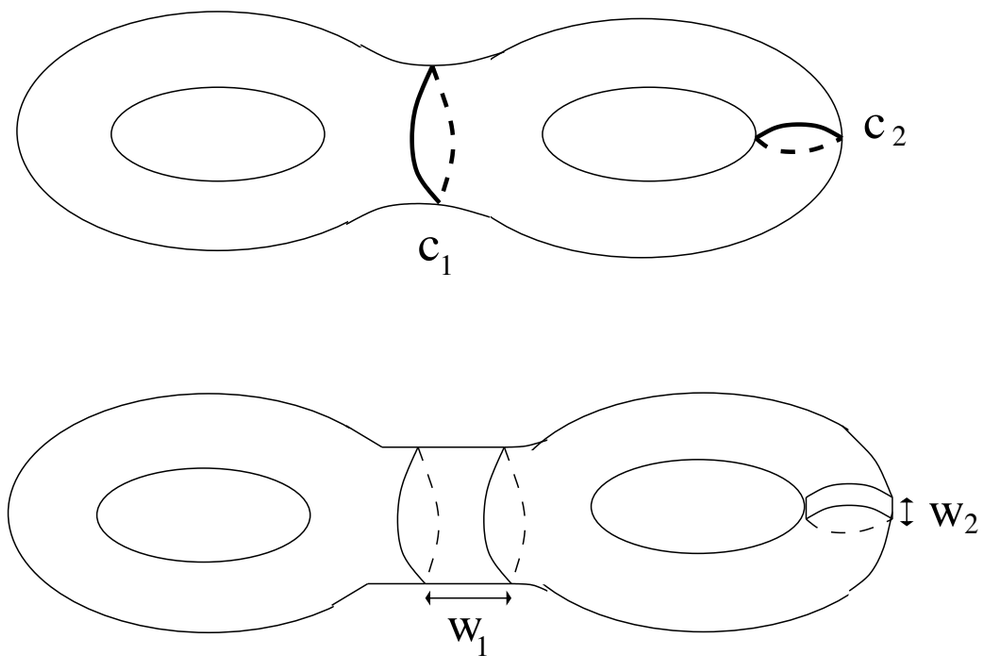

grafting (2+1)-spacetimes

- grafting simultaneously on each cct-surface of static spacetime
- width of the strips constant (given by weights)

Grafting in the domain

[Mess,Thurston, Benedetti, Bonsante,...]

Grafting in the domain construction

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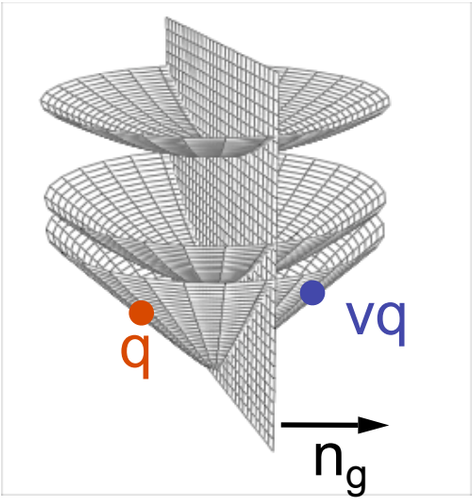

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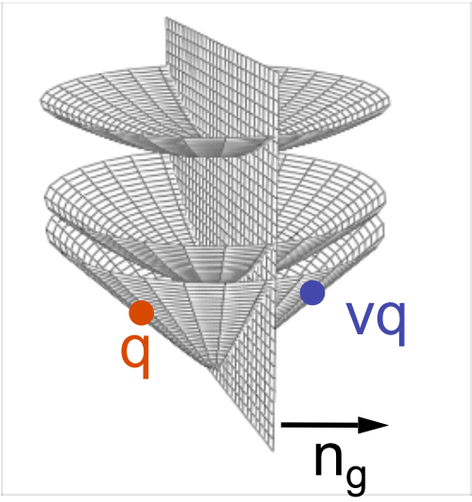

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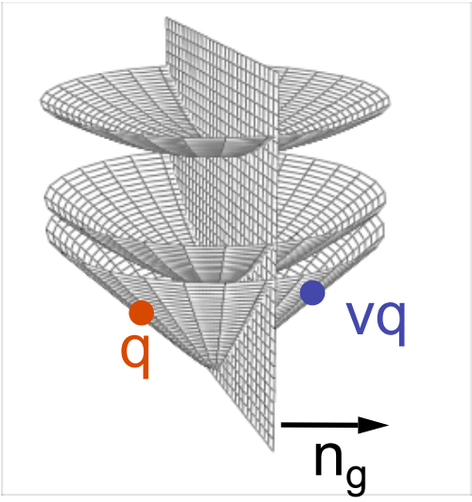

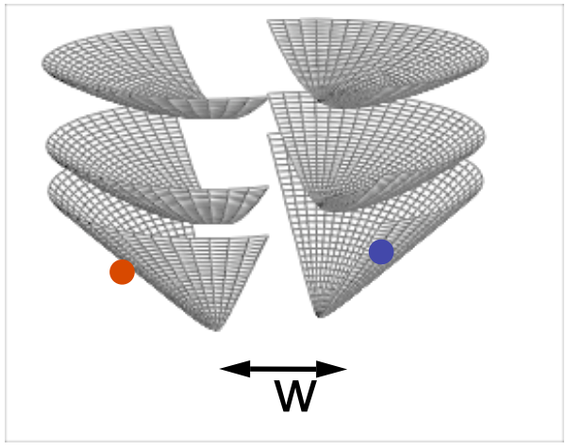

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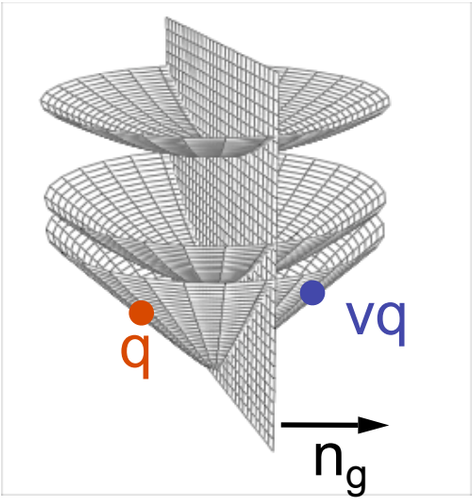

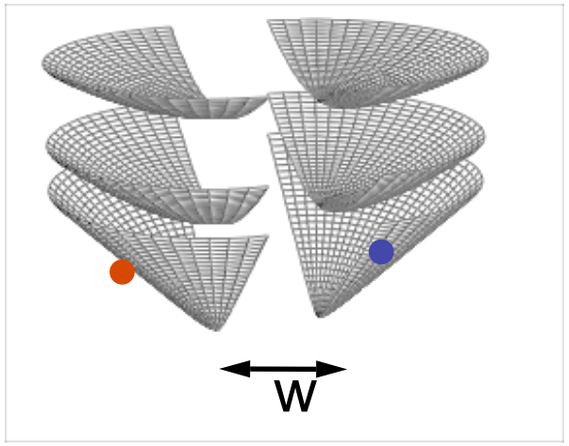

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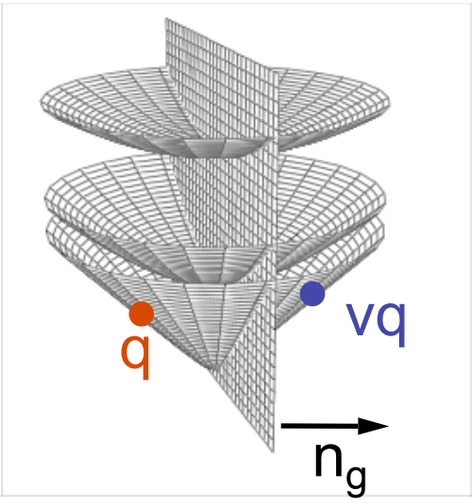

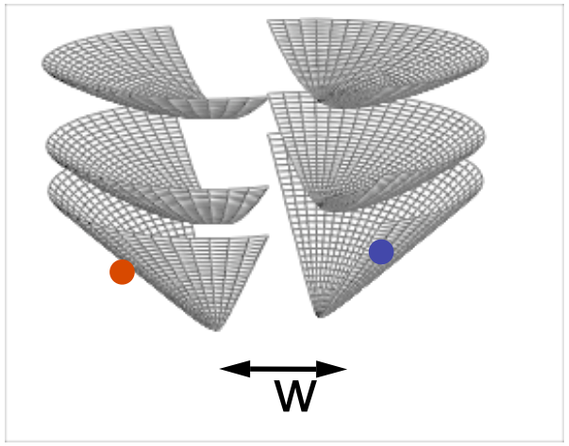

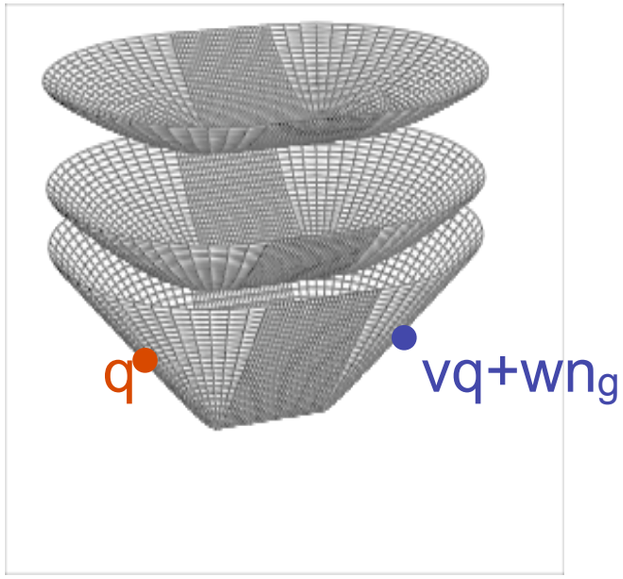

grafted domain

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- future of graph (infinite spacelike tree)

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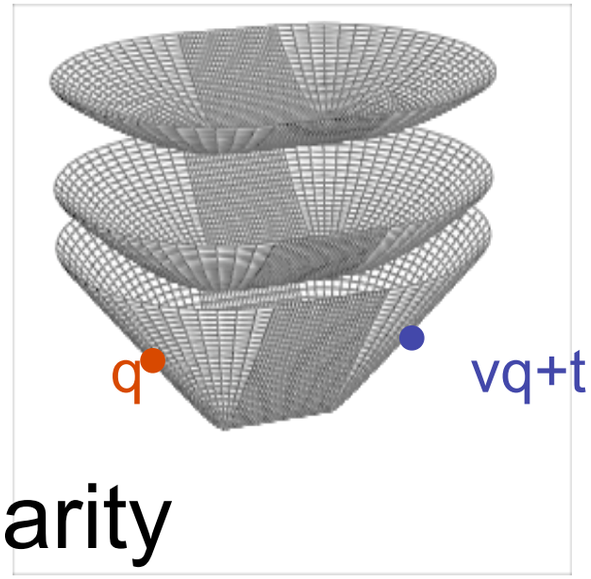

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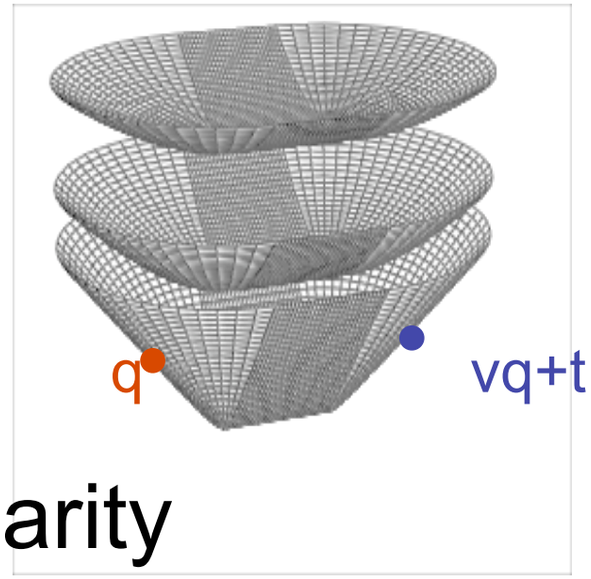

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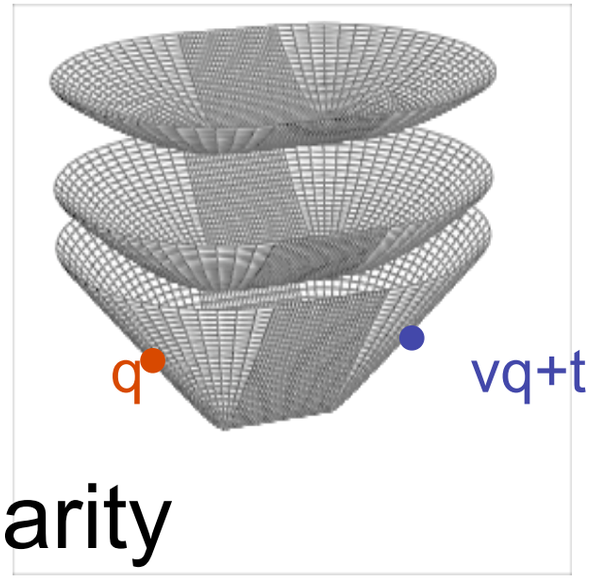

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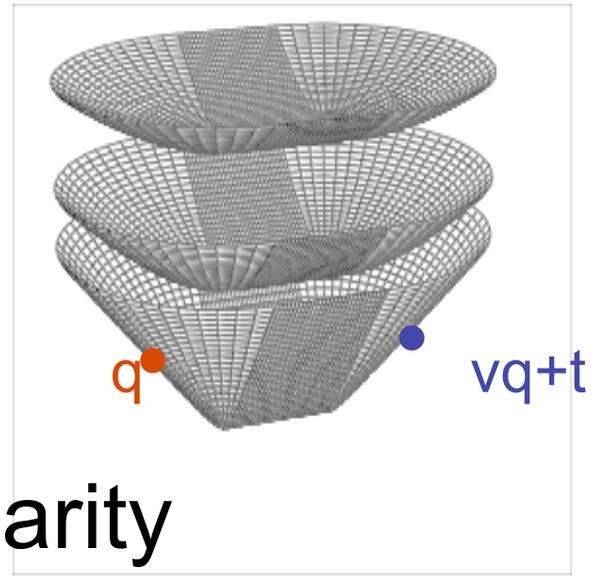

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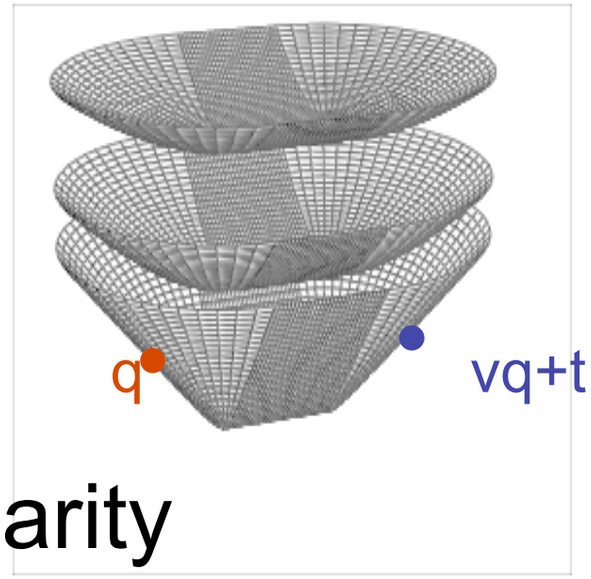

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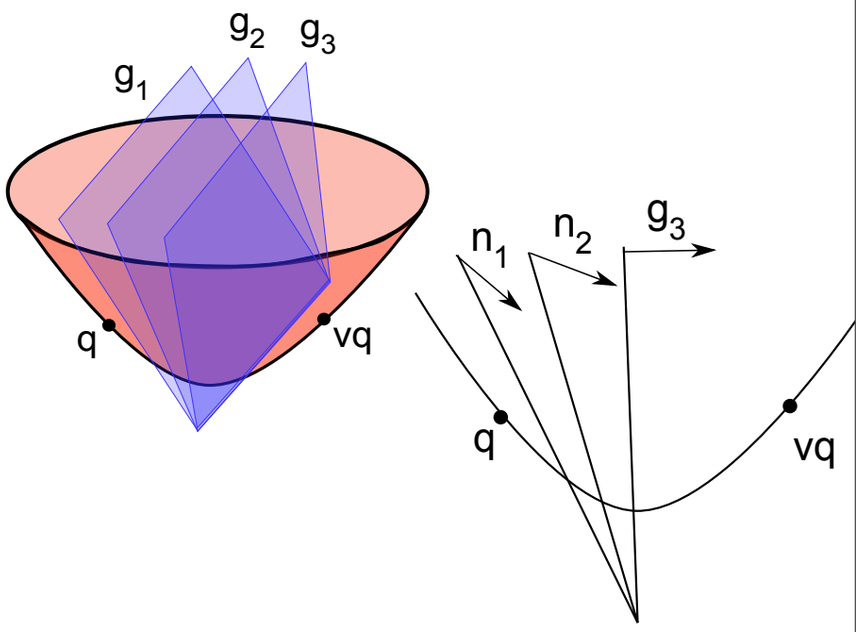

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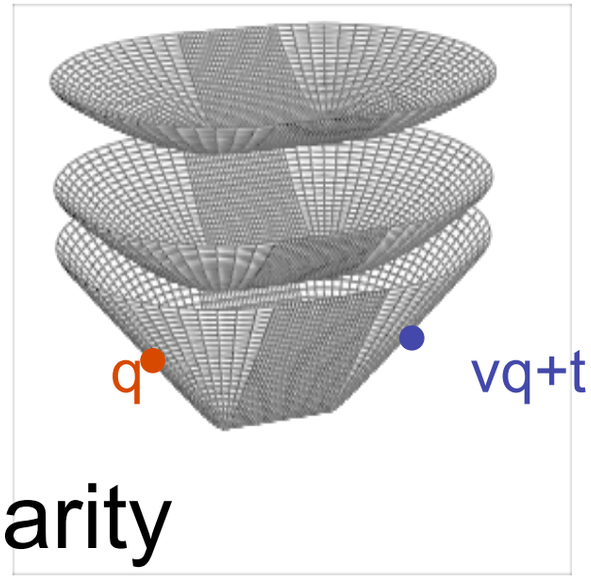

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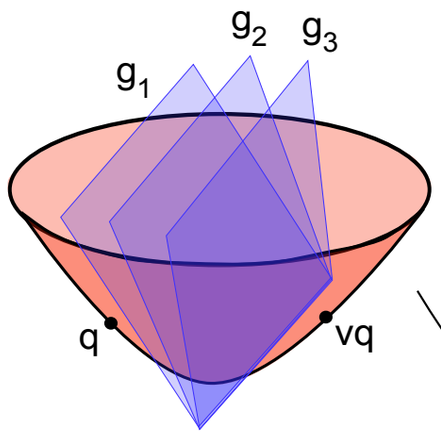

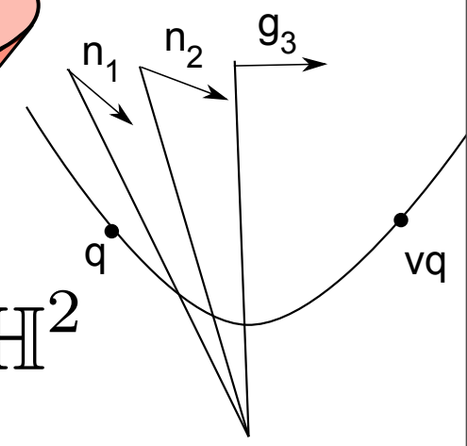

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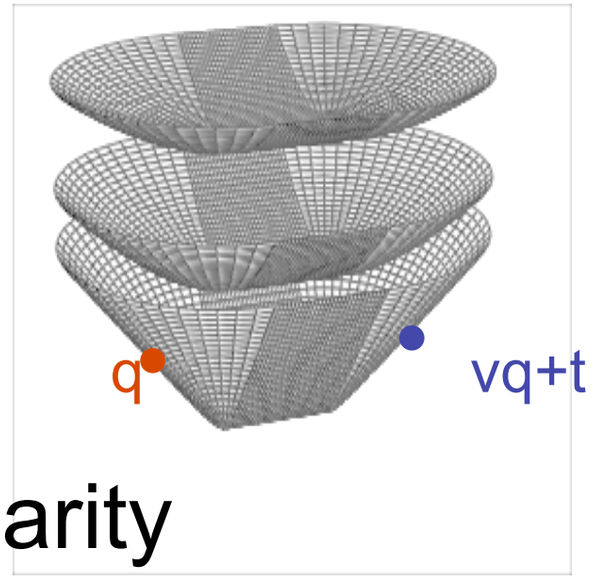

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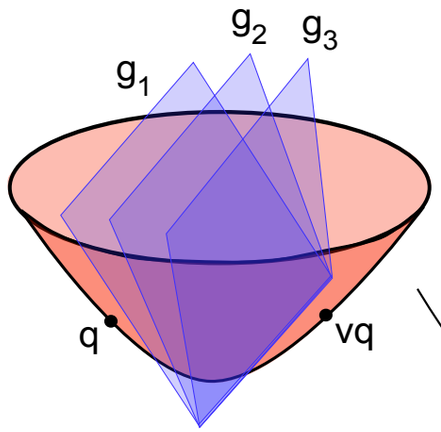

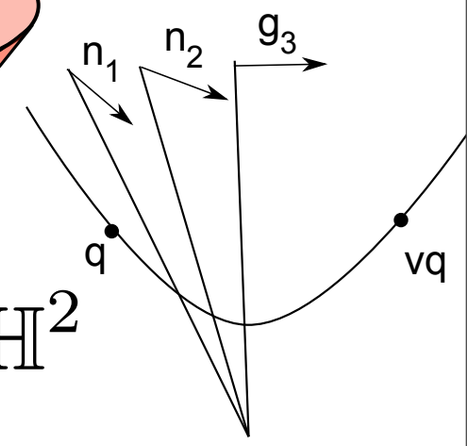

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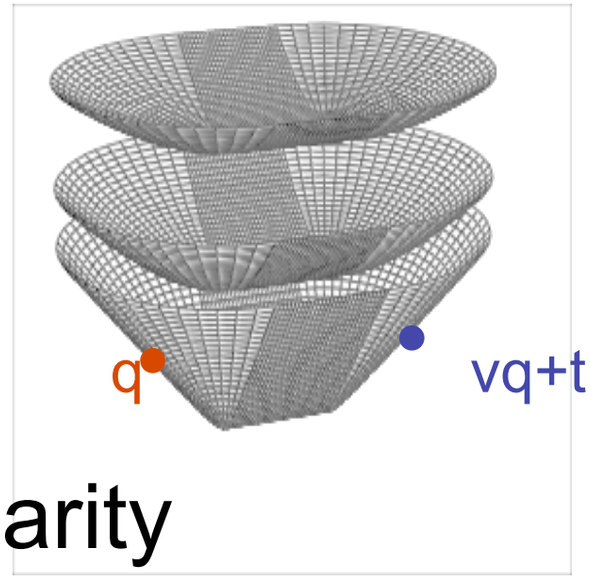

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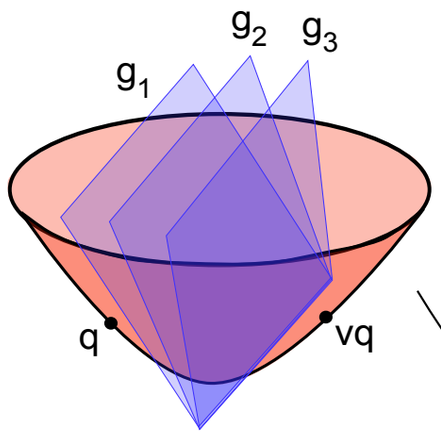

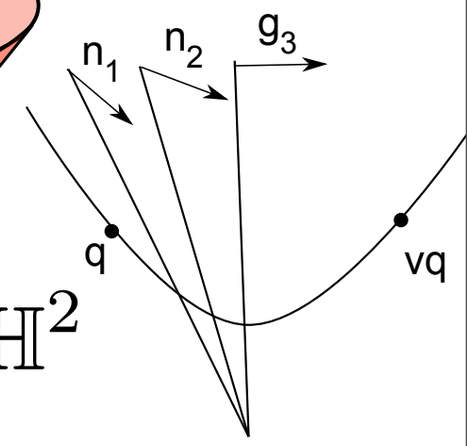

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quotient spacetime

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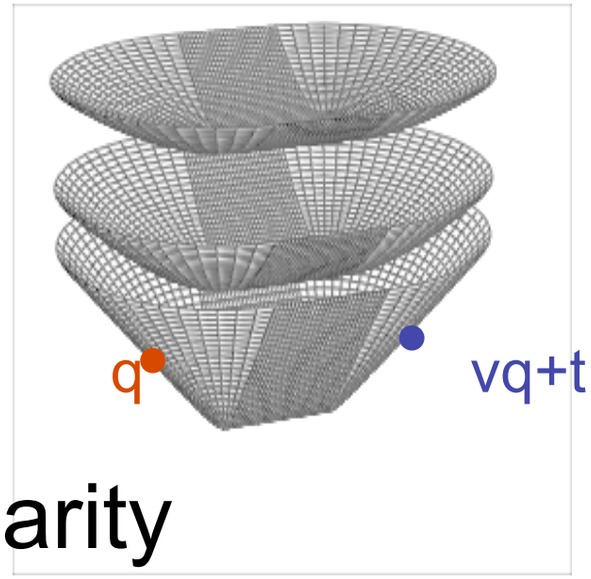

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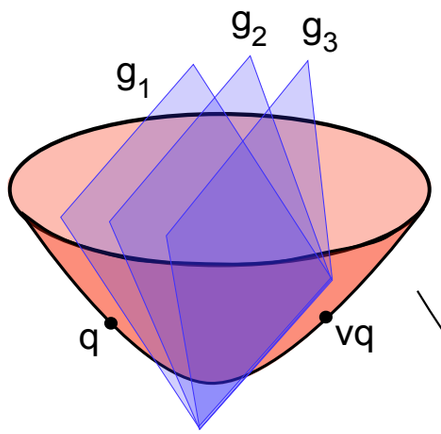

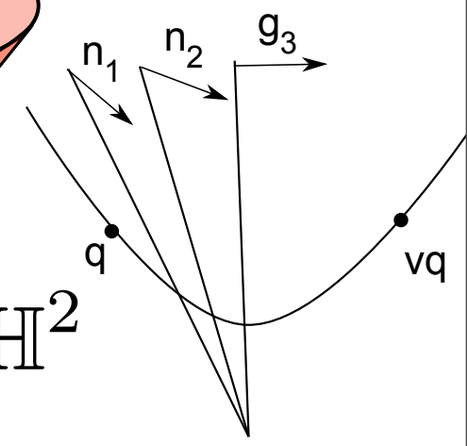

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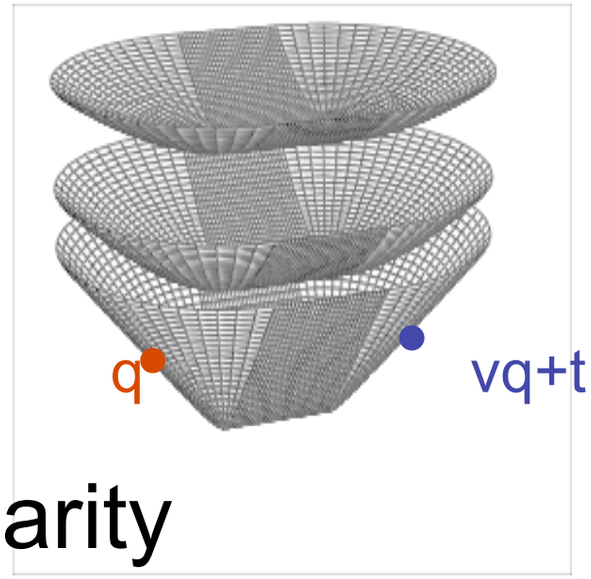

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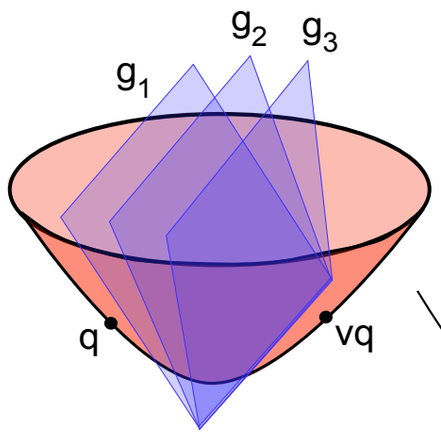

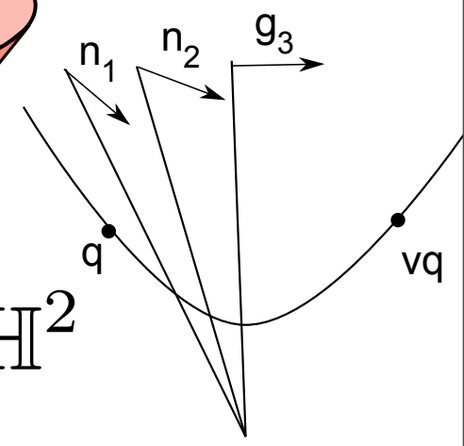

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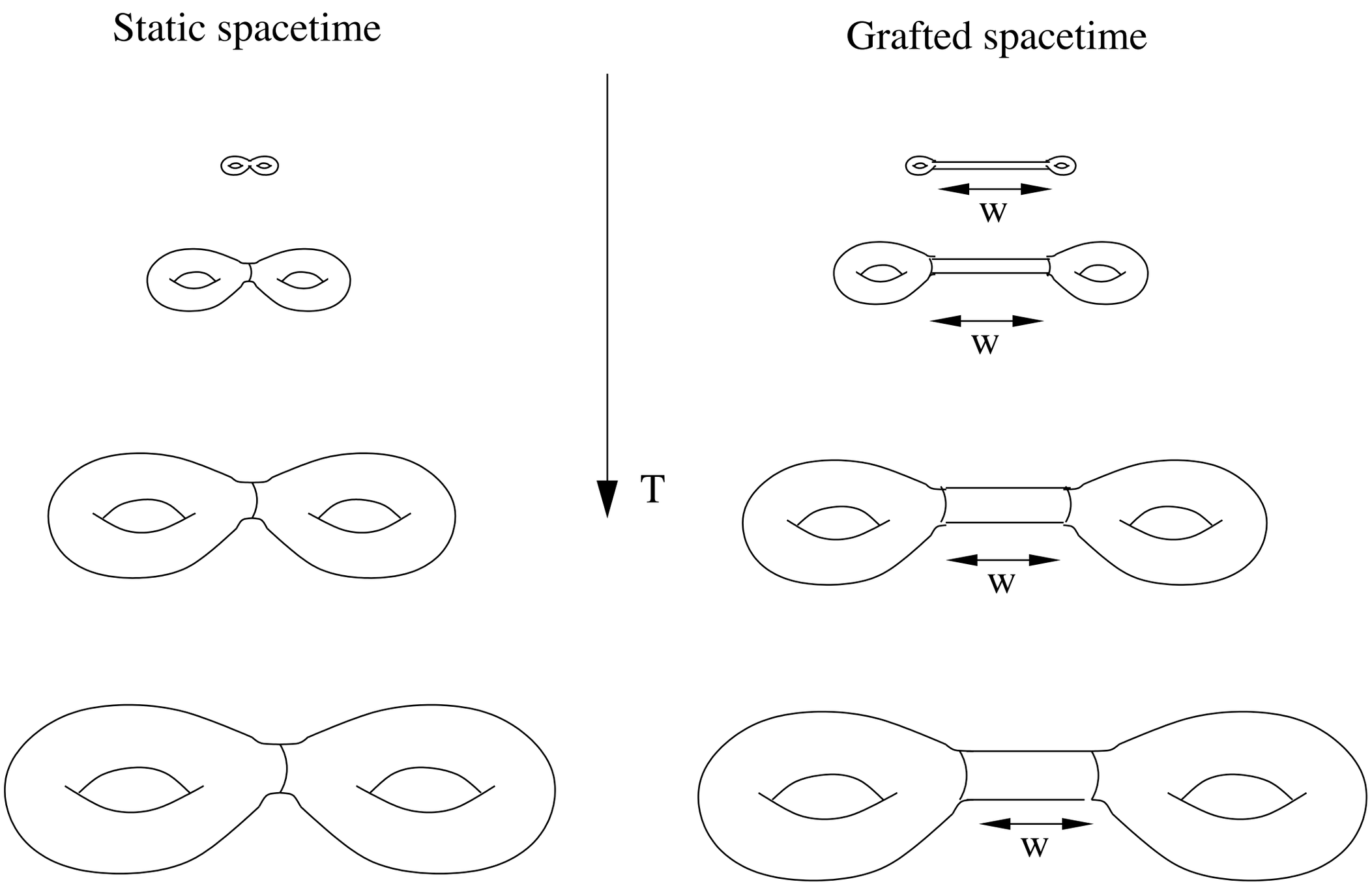
quotient spacetime

- quotient: deformed domain / deformed group action
- geometry of cct-surfaces changes with cosmological time

Evolution with the cosmological time



Geometry change via earthquake

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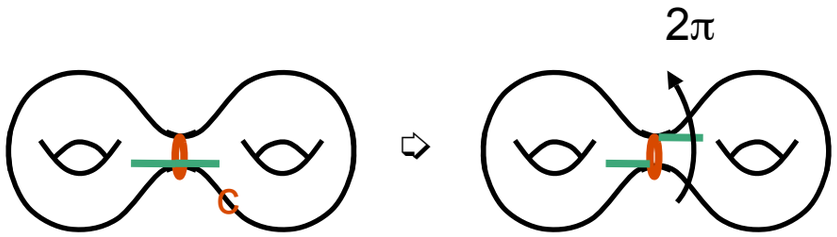

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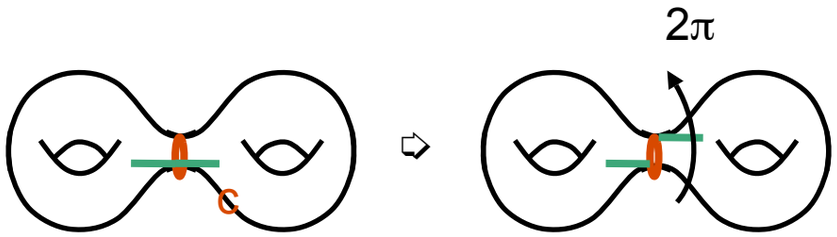

earthquake on (2+1)-spacetimes

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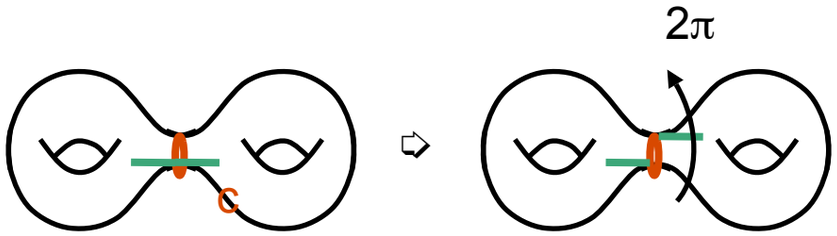

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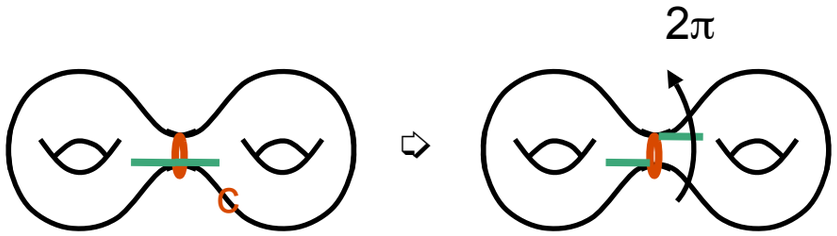

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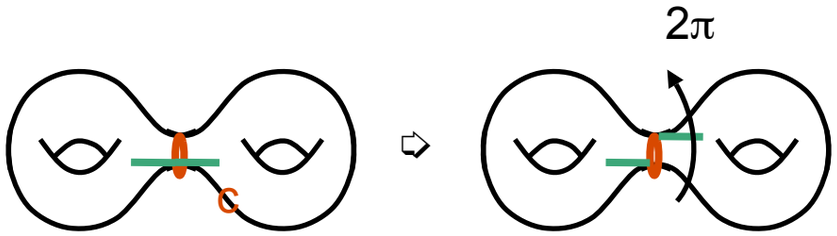

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earthquake simultaneously on each

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[Mess], [Thurston]

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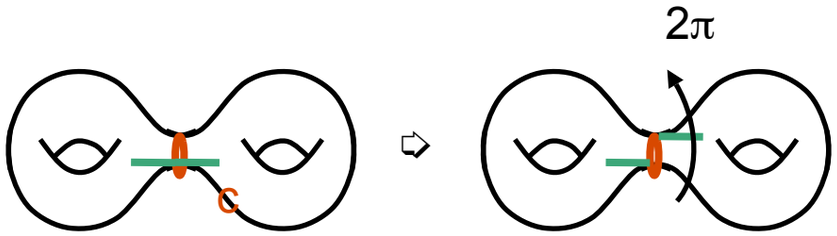

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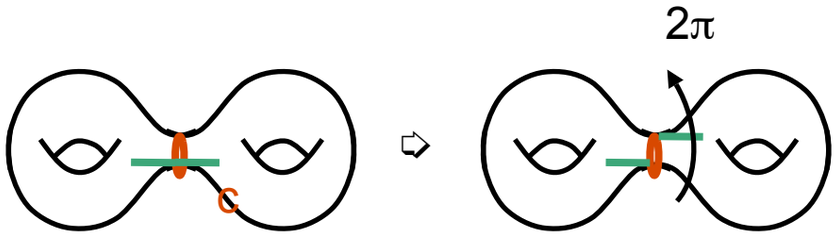

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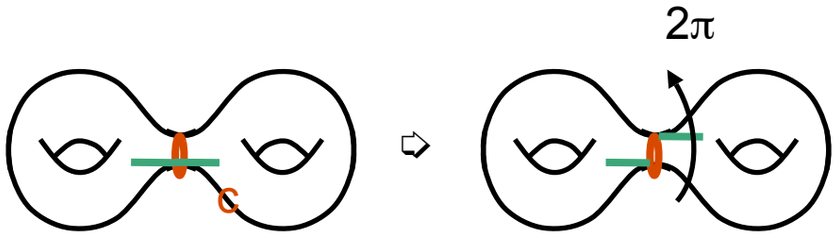

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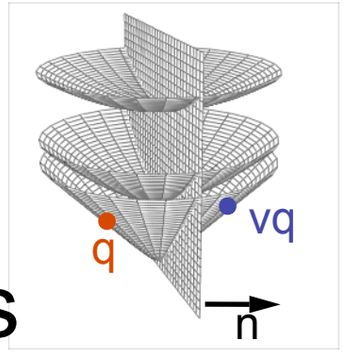

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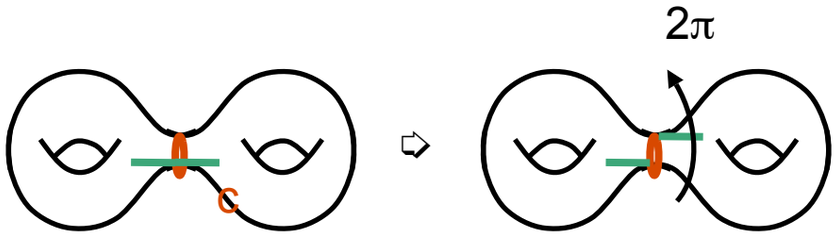

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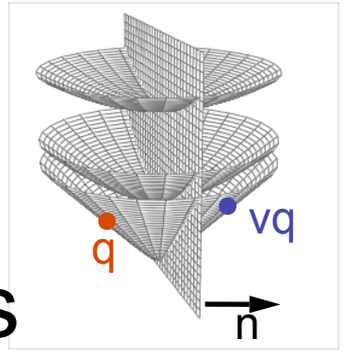

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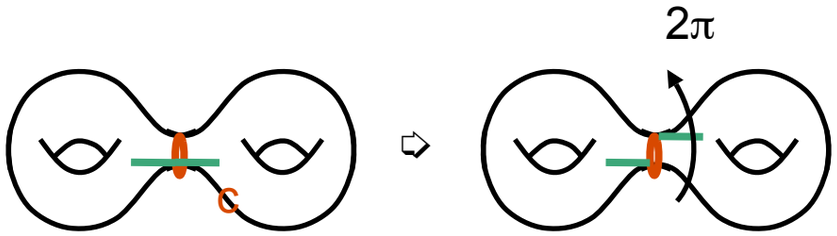

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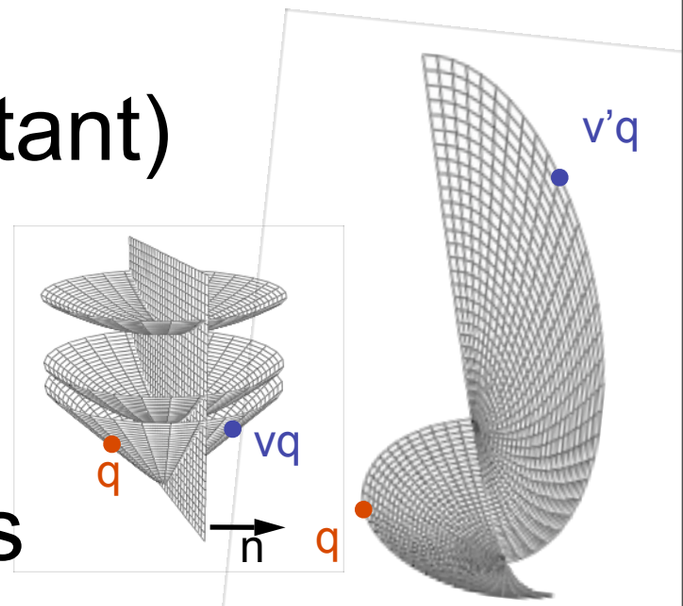

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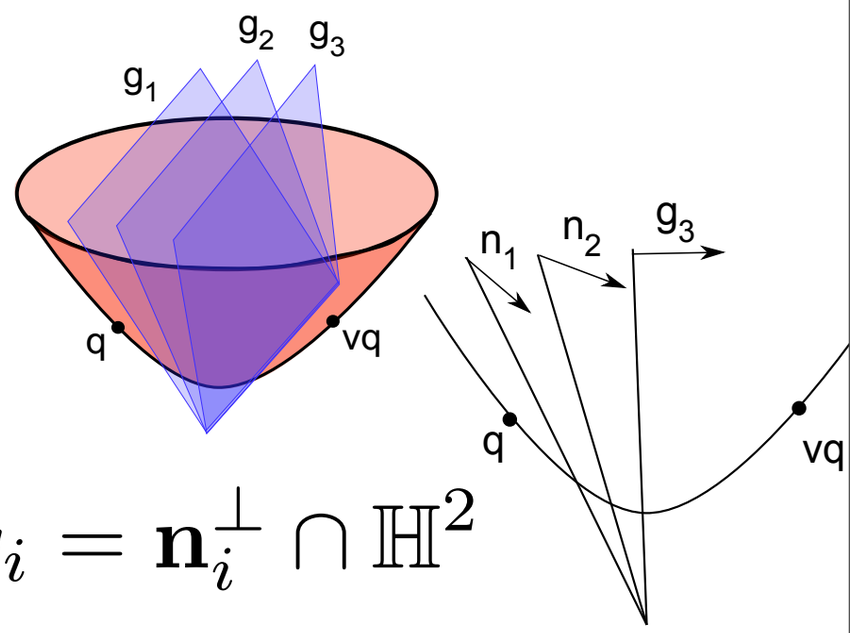

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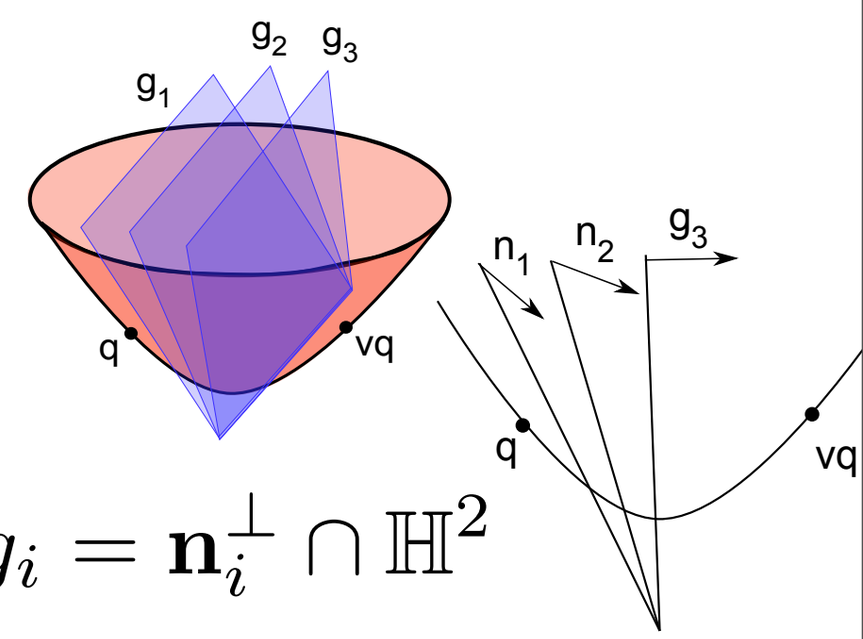

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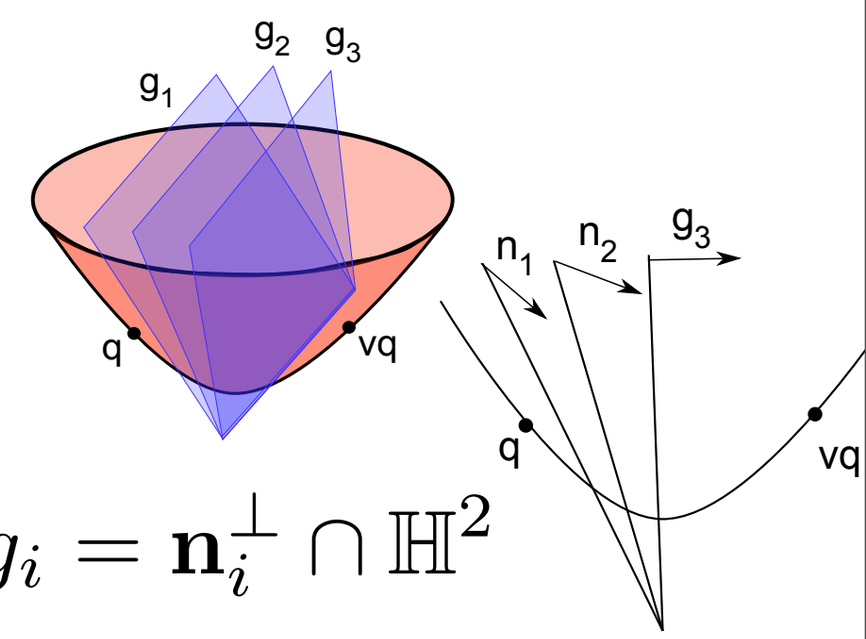

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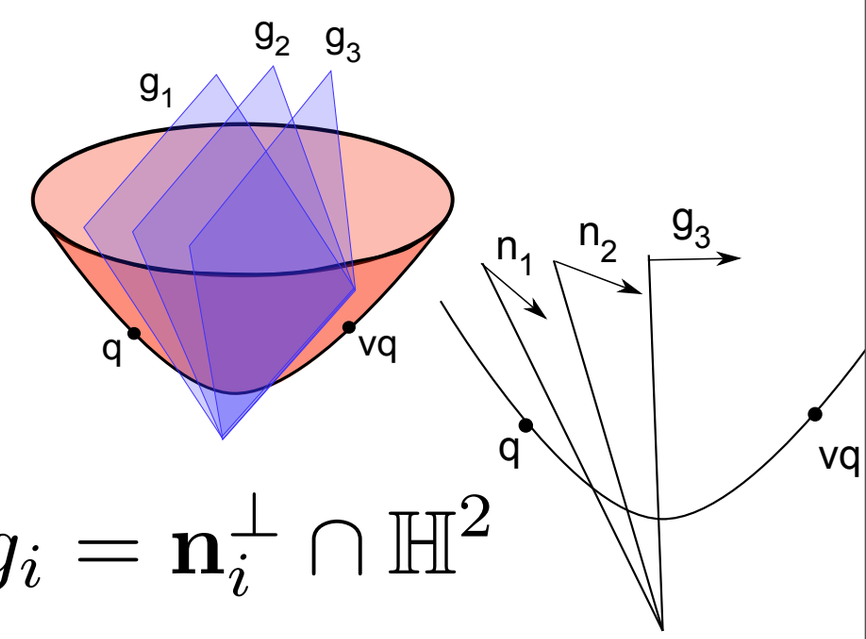

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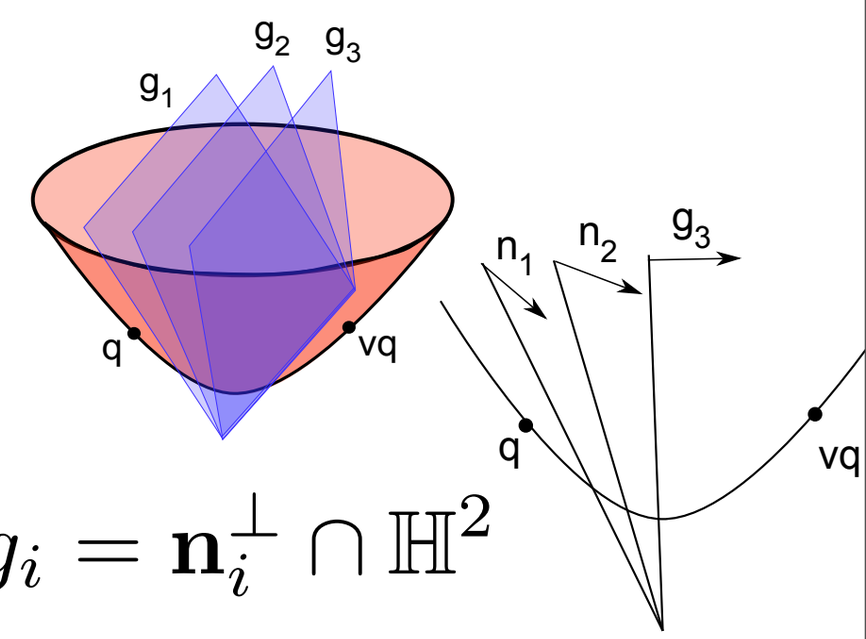

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[Goldman,C.M.]

The phase space transformations associated with grafting and earthquake along a closed geodesic λ on a cct-surface are generated via the Poisson bracket by its two Wilson loop observables: the mass m_λ generates grafting, the spin s_λ generates earthquakes

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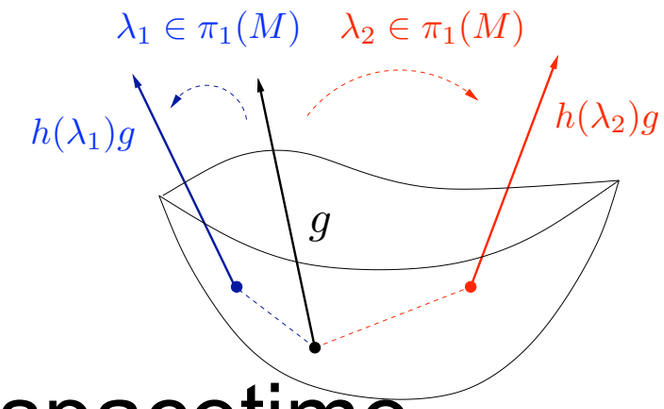

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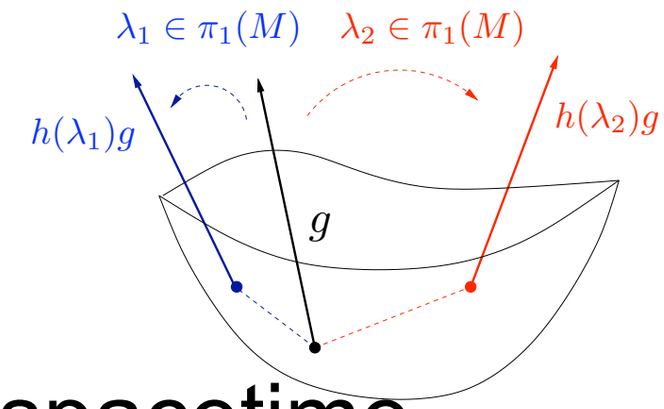

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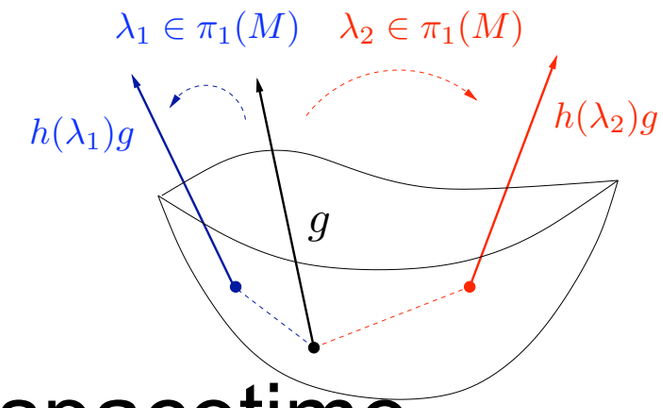

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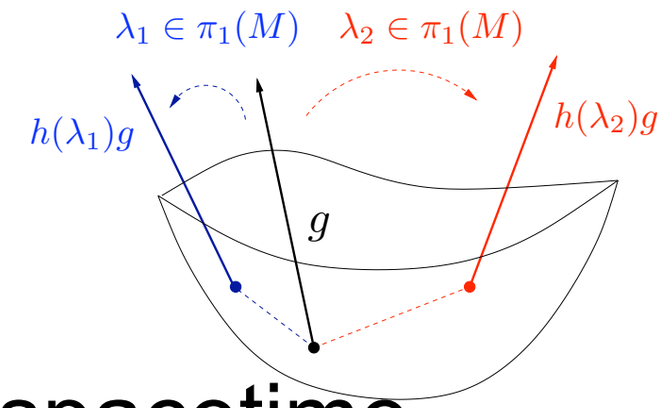

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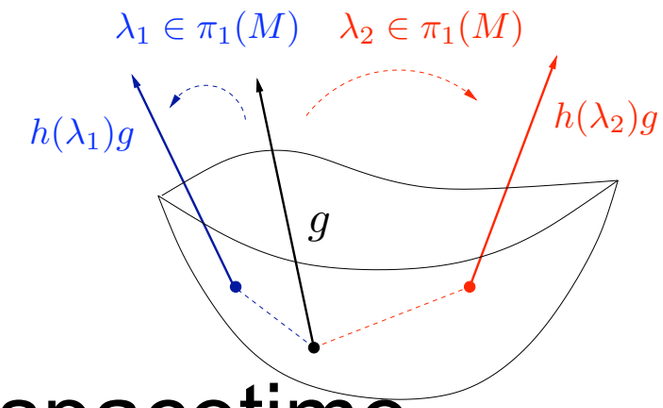

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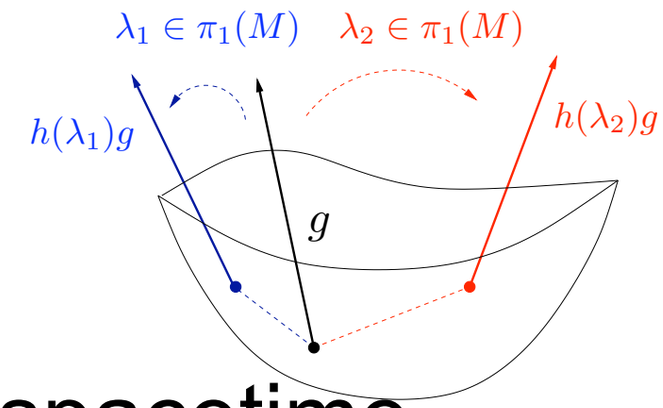

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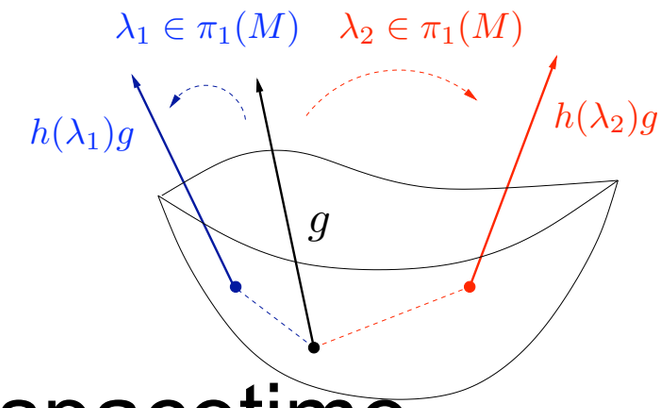

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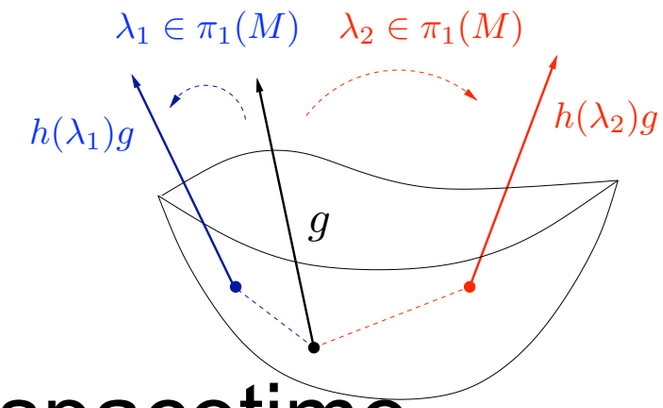

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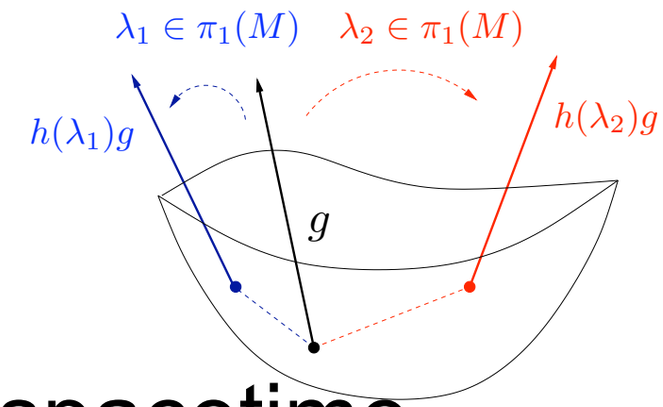

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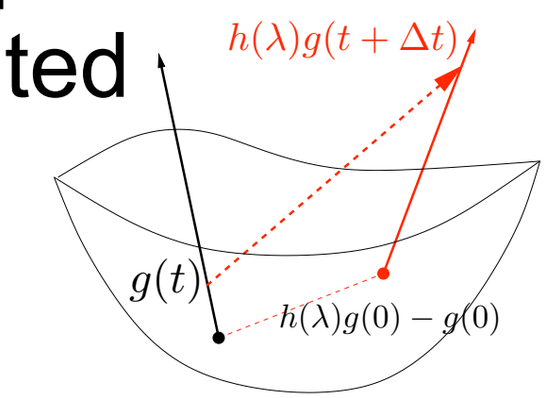

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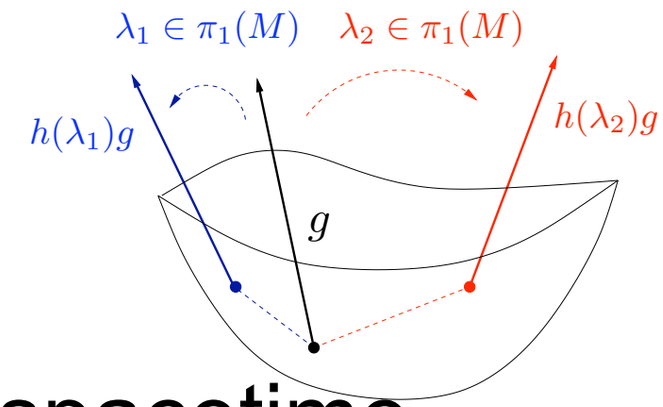

Qu: How to get physics from the theory ?

use universal cover (domains)

observer worldline of observer in free fall:

- timelike, future oriented geodesic in quotient spacetime
- $\pi_1(M)$-equiv. class of timelike, future oriented geod's in domain

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$$g(t) = t \cdot \mathbf{x} + \mathbf{x}_0 \quad \mathbf{x}^2 = -1, \mathbf{x}_0 \in D$$

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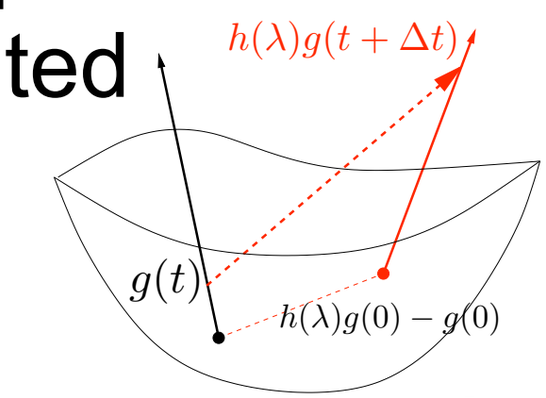

returning lightray

lightray emitted by observer at time t that returns at time t+Δt

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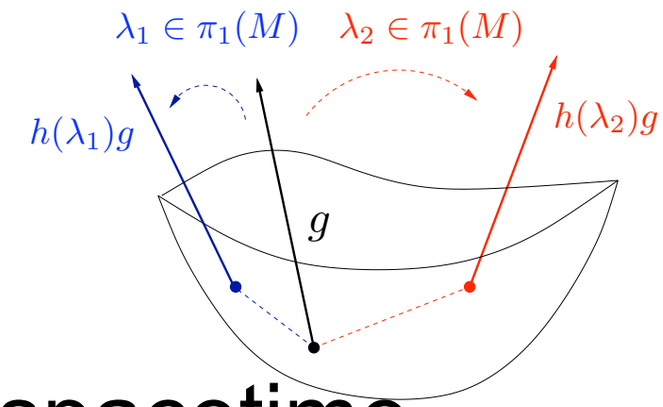

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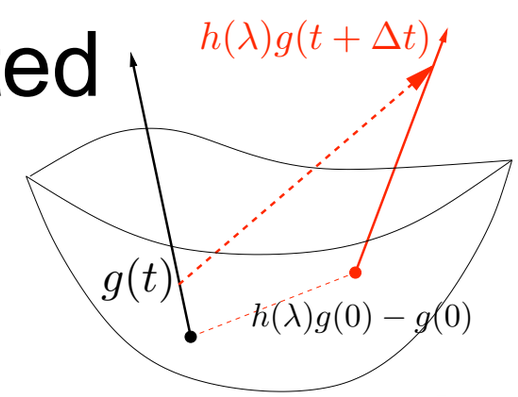

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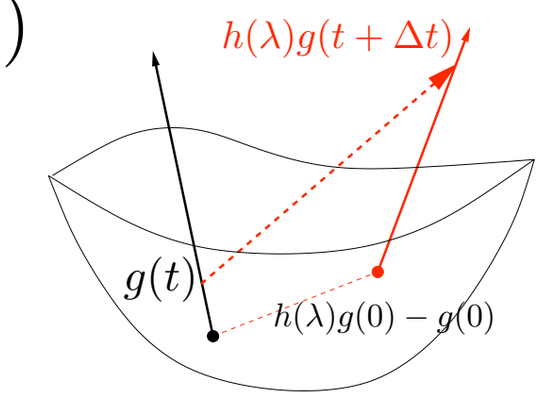

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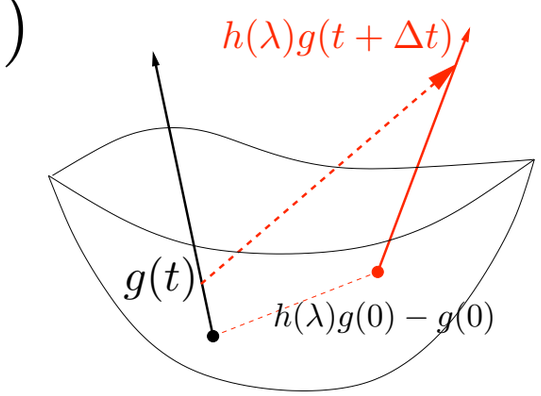

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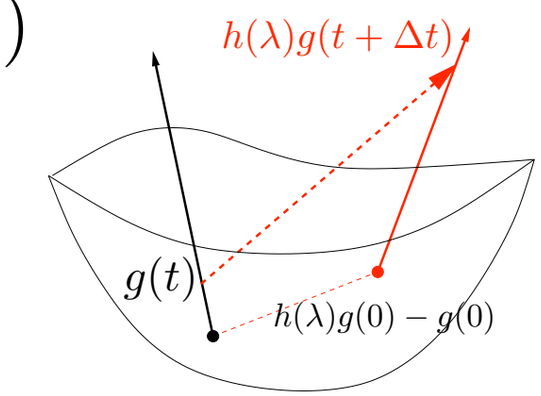

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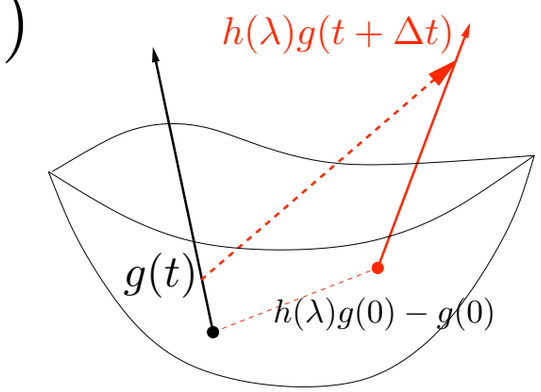

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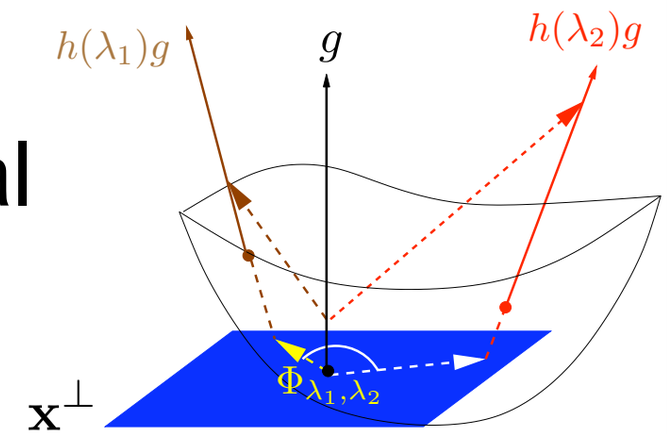

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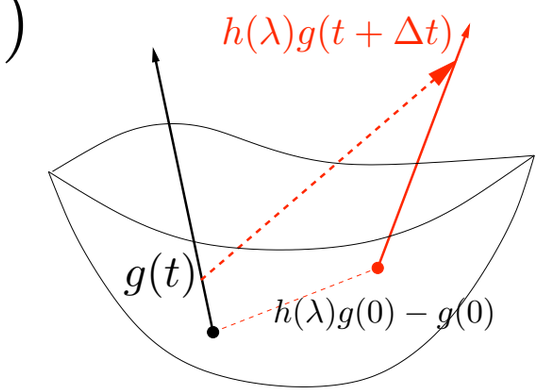

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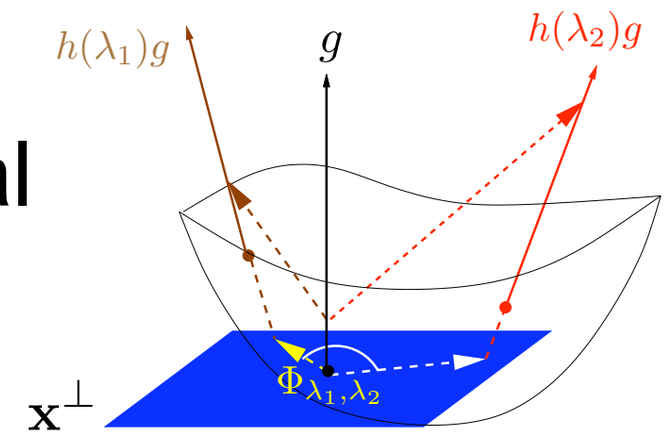

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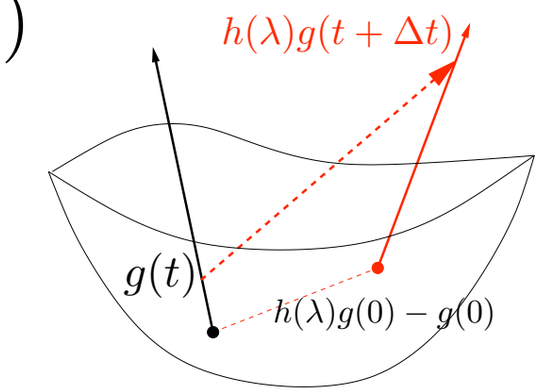

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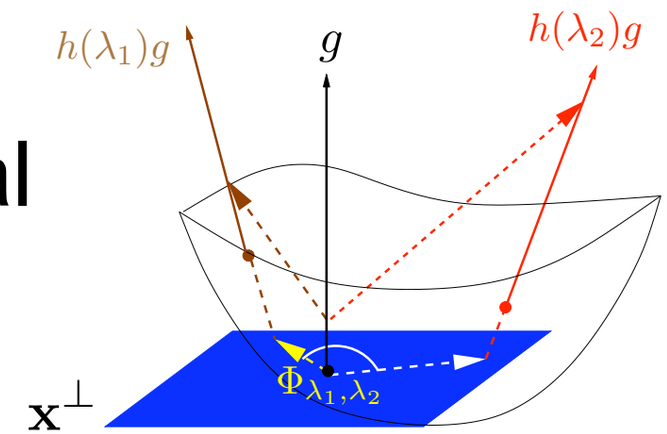

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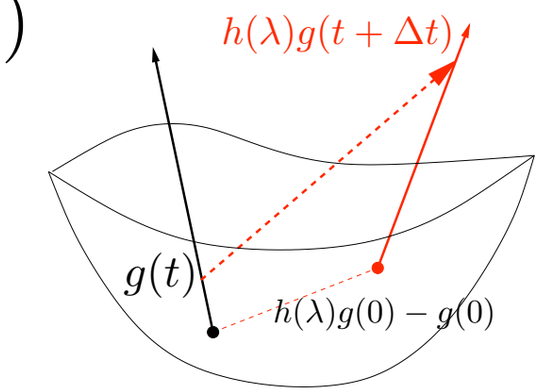

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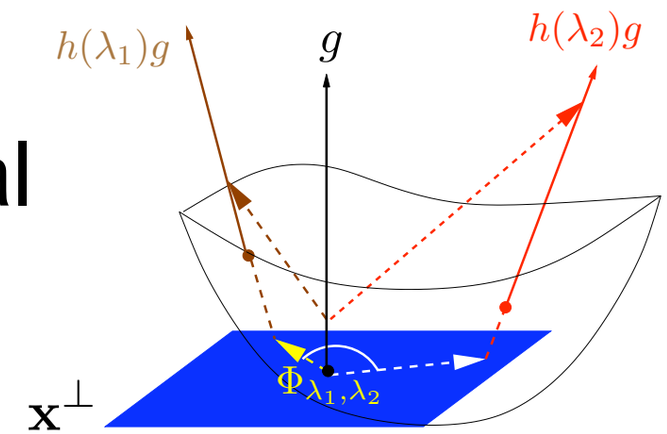

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explicit expressions as functions of

emission time t **observer** $\mathbf{x} \in \mathbb{H}^2, \mathbf{x}_0 \in D$ **holonomies** $h(\lambda)$

Results

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parameters

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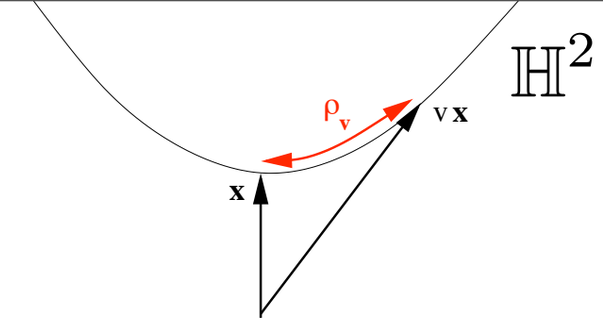

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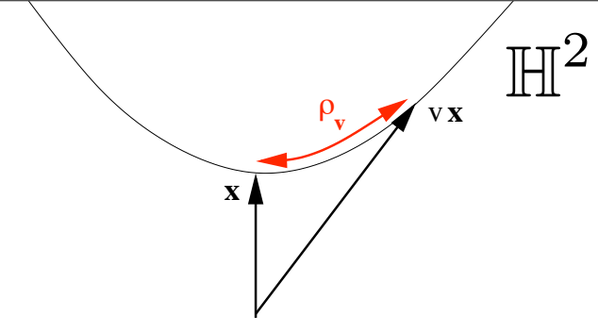

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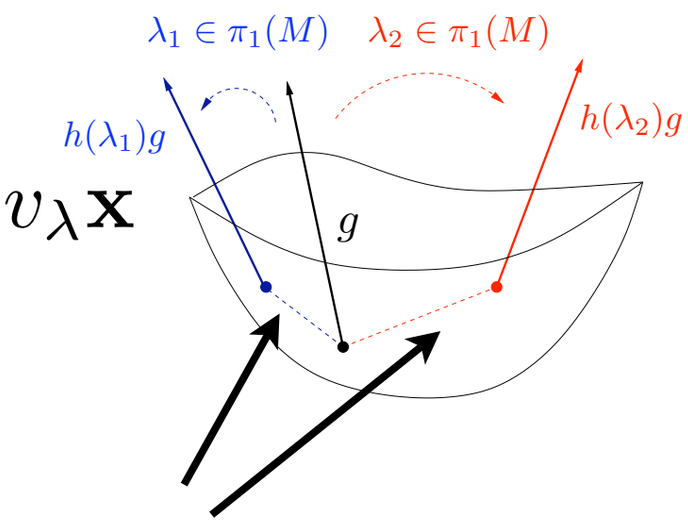

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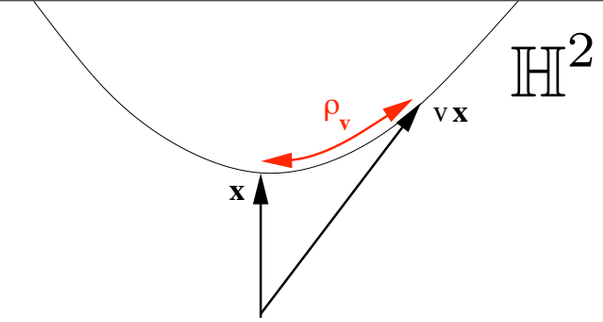

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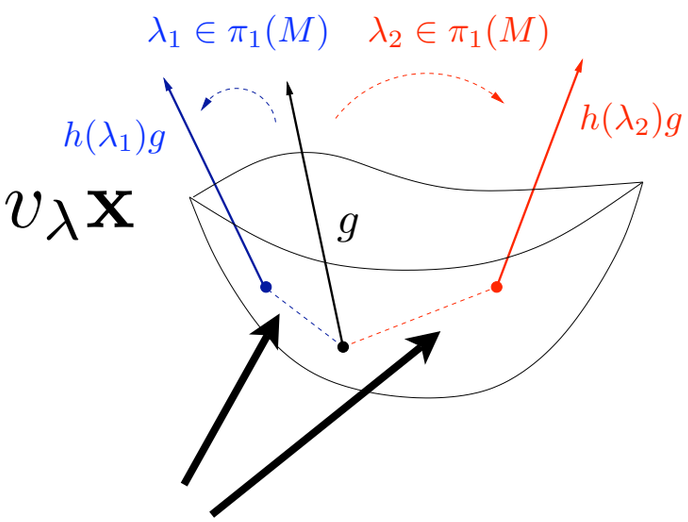

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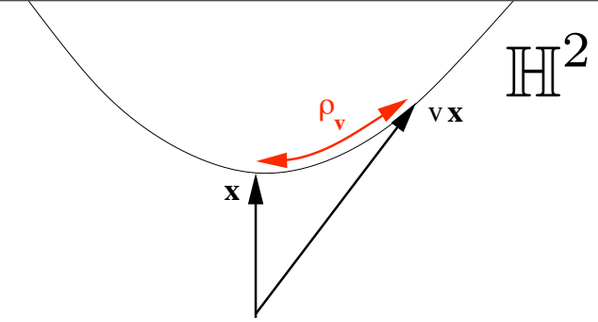

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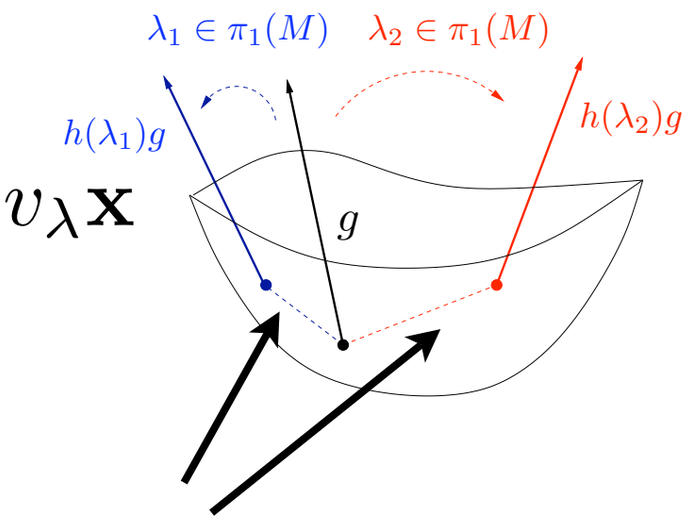

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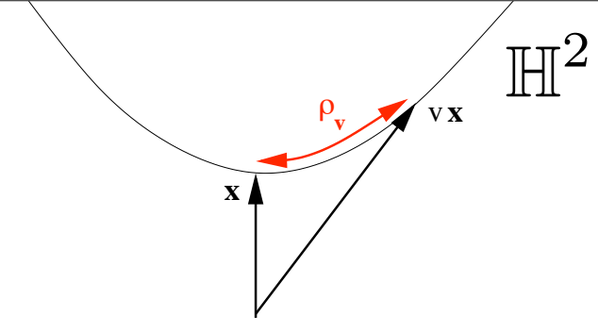

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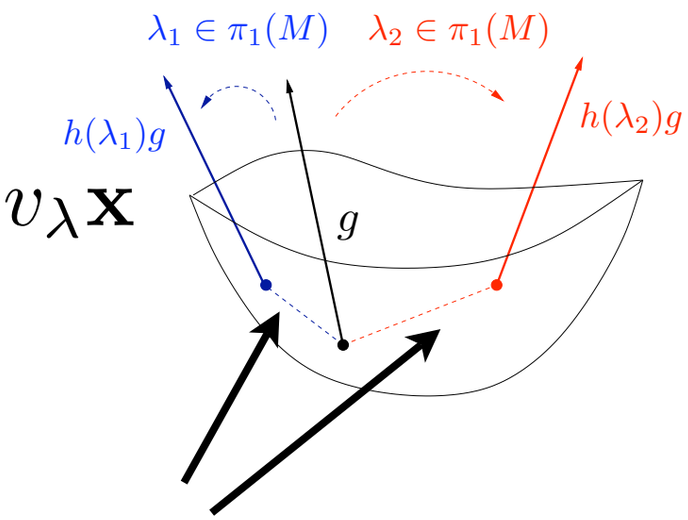

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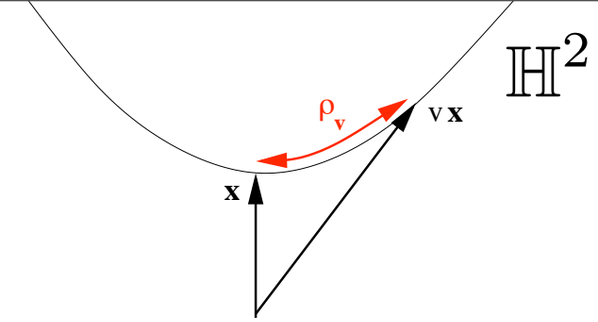

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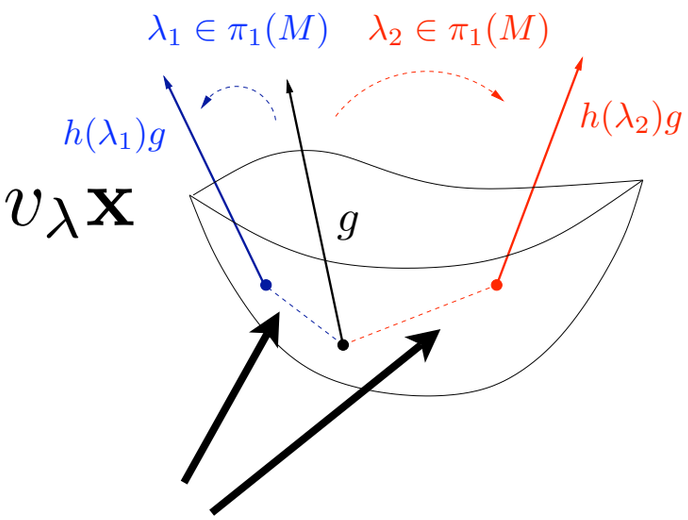

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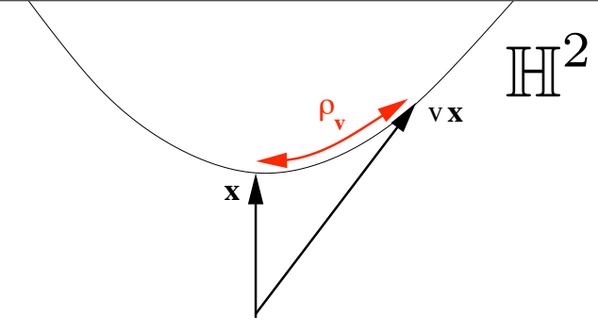

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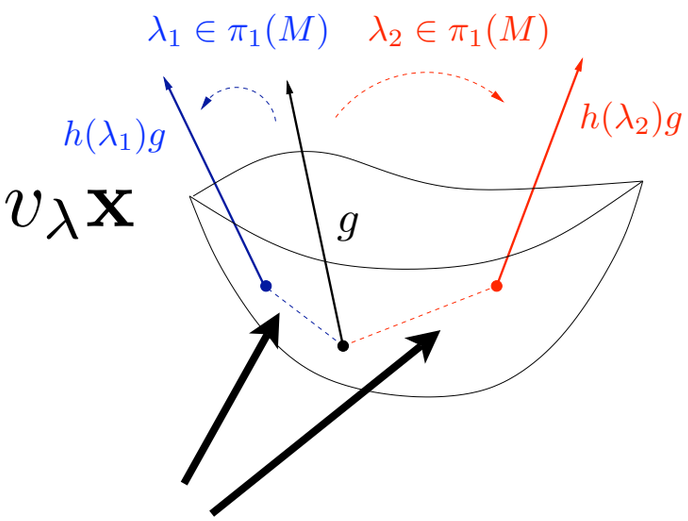

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5. Outlook and conclusions

vacuum spacetimes in Lorentzian (2+1)-gravity with Λ=0

- rich and subtle geometry
- description via universal cover: quotient of domain $D \subset \mathbb{M}^3$
- physical degrees of freedom:holonomies (up to conjugation)

relation geometry - observables

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open questions application to quantum theory !

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Setting
- Lorentzian (2+1)-gravity, Λ=0
- vacuum spacetimes: no matter