sugra breaking effects

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Kerkyra 06.08.09

summary

A review of supersymmetry breaking effects, in particular, generation of sparticle masses from spontaneous (F-type) breaking of global and local supersymmetry. After considering effective global susy theories, three basic models are introduced: gauge mediation, msugra, no-scale models

Kähler covariant expansion around SSB vacua

effects from renormalizable and D>4 couplings

gauge mediation from Kähler deformations

Kähler invariance, and msugra

no-scale models and $\Lambda_{cosm} = 0$

raw material: chiral superfields

N.B.: "slide-show units" (often, not always): $i = 1 = \sqrt{2} = c = M_P = \hbar = \pi = \ldots$

MATTER CHIRAL SUPERFIELD

$$\Phi = \varphi + \theta\psi + \theta^2 F$$

sfermion fermion auxiliary

GAUGE CHIRAL SUPERFIELD

$$W^\alpha = \lambda^\alpha + (\theta\sigma_{\mu\nu})^\alpha F^{\mu\nu} + (\varepsilon\theta)^\alpha D + (\theta^2\sigma^\mu\partial_\mu\bar{\lambda})^\alpha$$

gaugino gauge auxiliary

KAHLER POTENTIAL

SUPERPOTENTIAL

$$\mathcal{L} = K(\Phi^\dagger, \Phi)|_{\theta\theta\bar{\theta}\bar{\theta}} + \tfrac{1}{4} f(\Phi) W_\alpha W^\alpha|_{\theta\theta} + W(\Phi)|_{\theta\theta} + \text{h.c.}$$

matter kinetic lagrangian

gauge kinetic lagrangian

matter interaction lagrangian

gauge-matter interactions

super Yang-Mills

superYukawa WZ interactions

spontaneous global supersymmetry breaking

global symmetry	global susy
$[T^a,\ H_i] = it^a H_i$	$\left\{Q,\ \psi^i\right\} = \epsilon F^i$ $\left\{Q,\ \lambda^A\right\} = \epsilon D^A + \ldots$
GSB: Higgs field vev's	**SSB**: auxiliary field vev's
$\langle H_i \vert\, T^a\, \vert 0\rangle = it^a \langle H_i\rangle$	$\langle \psi^i \vert\, Q\, \vert 0\rangle = \langle F^i\rangle$ $\langle \lambda^A \vert\, Q\, \vert 0\rangle = \langle D^A\rangle$
$v = \sqrt{\sum_i \langle H_i\rangle^2}$	$\langle F\rangle = \sqrt{\sum_i \langle F^i\rangle^2 + \sum_A \langle D^A\rangle^2}$
Goldstone thm: **massless boson**	Goldstone thmino: **massless fermion** **GOLDSTINO**
$\vert G^a\rangle = \sum_i \langle H_i^*\rangle t^a \vert H_i\rangle$	$\vert \psi_G\rangle = \sum_i \langle F^i\rangle^* \vert \psi^i\rangle + \sum_A \langle D^A\rangle \vert \lambda^A\rangle$

spontaneous local supersymmetry breaking

YM

SSB: Higgs field vev's

$$v = \sqrt{\sum_i \langle H_i \rangle^2}$$

Higgs mechanism:

gauge + Goldstone boson

massive spin=1 boson

$$m_W = g_2 \frac{v}{\sqrt{2}}$$

gauge coupling

gauge symmetry breaking

SUGRA

SSSB: auxiliary field vev's

$$\langle F \rangle = \sqrt{\sum_i \langle F^i \rangle^2 + \sum_A \langle D^A \rangle^2}$$

superHiggs mechanism:

gravitino + Goldstino

massive spin=3/2 fermion

$$m_{3/2} = \frac{1}{M_{\mathrm{P}}} \frac{\langle F \rangle}{\sqrt{3}}$$

gravity coupling

sugra symmetry breaking

SSB and "soft terms": SM

renormalizable SM:

$$\mathcal{L}_{\rm SM}^{\rm soft} = \left(\langle H\rangle \frac{\partial}{\partial H} + \frac{1}{2}\langle H\rangle^2 \frac{\partial^2}{\partial H^2}\right)\mathcal{L}_{\rm SM}$$

$$= \lambda_t \langle H\rangle \bar{t}t + \ldots\ldots + \frac{1}{2} g_2^2 \langle H\rangle^2 W^{+\mu}W^-_\mu$$

$$= m_t \bar{t}t + m_b \bar{b}b + \ldots\ldots + m_W W^{+\mu}W^-_\mu$$

soft terms

coupling to Higgs vev's

matter and gauge masses

effective ν masses (NR):

coupling to Higgs vev's ⇒ neutrino masses

$$\mathcal{L}_\nu^{\rm soft} = \left(\langle H\rangle \frac{\partial}{\partial H} + \frac{1}{2}\langle H\rangle^2 \frac{\partial^2}{\partial H^2}\right)\frac{H^2\nu\nu}{\Lambda}$$

$$= \frac{2\langle H\rangle H\nu\nu}{\Lambda} + \frac{\langle H\rangle^2\nu\nu}{\Lambda} = \ldots + m_\nu^\nu \nu\nu$$

covariant derivatives and curvature

SUSY $\Rightarrow$ chiral (super) fields are coordinates on a **Kähler manifold**

KÄHLER POTENTIAL

$$\mathbf{K}(\mathbf{\Phi^a}\ \mathbf{\bar{\Phi}^{\bar{a}}})$$

KAHLER METRIC

$$K_{a\bar{b}} = \partial_a \partial_{\bar{b}} \mathbf{K}$$

$$\mathbf{\Phi}^a = (K^{-1}\mathbf{\Phi})^a = K^{\bar{b}a}\mathbf{\Phi}_b$$

COVARIANT DERIVATIVE

$$\nabla_a f_b = \partial_a f_b - \Gamma^c_{ab} f_c$$

$$\nabla = \partial - \Gamma$$

CONNECTION

$$\Gamma^c_{ab} = K^{\bar{d}c}\partial_a K_{b\bar{d}}$$

$$\Gamma = K^{-1}\,\partial\, K$$

RIEMANN CURVATURE

$$R_{a\bar{b}c\bar{d}} = \partial_a \partial_{\bar{b}} K_{c\bar{d}} - \Gamma^f_{ac} K_{f\bar{e}} \Gamma^{\bar{e}}_{\bar{b}\bar{d}}$$

susy breaking lagrangian

$$\mathcal{L} = K(\Phi^\dagger,\Phi)|_{\theta\bar\theta\bar\theta\theta} + \tfrac{1}{4} f(\Phi) W_\alpha W^\alpha|_{\theta\theta} + W(\Phi)|_{\theta\theta} + \text{h.c.}$$

$$\nabla = \partial - \Gamma$$

susy breaking terms $(\theta^2 F^k \nabla_k + \text{h.c.} + \bar\theta^2\theta^2 F^j \bar F^{\bar\imath} \nabla_j \nabla_{\bar\imath})$

KAHLER POTENTIAL $K(\Phi^\dagger, \Phi)$ $\quad F^k K_k|_{\bar\theta\bar\theta} + \text{h.c.} + F^j F^{\bar\imath} R_{j\bar\imath b\bar a} \Phi^{\bar a}\Phi^b$

scalar masses

GAUGE COUPLING $f(\Phi)$ $\quad F^k \nabla_k f(\phi)\, \lambda^\alpha \lambda_\alpha$

gaugino masses

SUPERPOTENTIAL $W(\Phi)$ $\quad F^k \nabla_k W(\phi)$

A-terms; analytic

effective global susy: Kähler

effective theory defined below a cutoff scale Λ by **{K,f,W}**

by running couplings, renormalized fields, v.e.v's & symmetries

$$\mathcal{L}_{\rm kin} = \mathbf{K}|_{\theta\theta\bar\theta\bar\theta} = K_{a\bar b}\left(\partial^\mu\bar\phi^{\bar b}\partial_\mu\phi^a + i\bar\psi^{\bar b}\not\partial\psi^a + \bar F^{\bar b}F^a\right)$$

Kähler potential/metric scale dependent:

$$\mathbf{K}(\mathbf{\Phi^a\,\bar\Phi^{\bar a}},\Lambda) \neq \mathbf{K}(\mathbf{\Phi^a\,\bar\Phi^{\bar a}},\mu)$$

@ cutoff Λ **@ scale μ**

metric deformations

(⇒scalar masses@μ)

encoded in Kähler

(K=logbook!)

- wave-function renormalization
- integration of heavy (>μ) states
- dependence on light (<μ) states

effective global susy: superpotential

effective theory defined below a cutoff scale Λ by {K,f,W}
by running couplings, renormalized fields, v.e.v's & symmetries

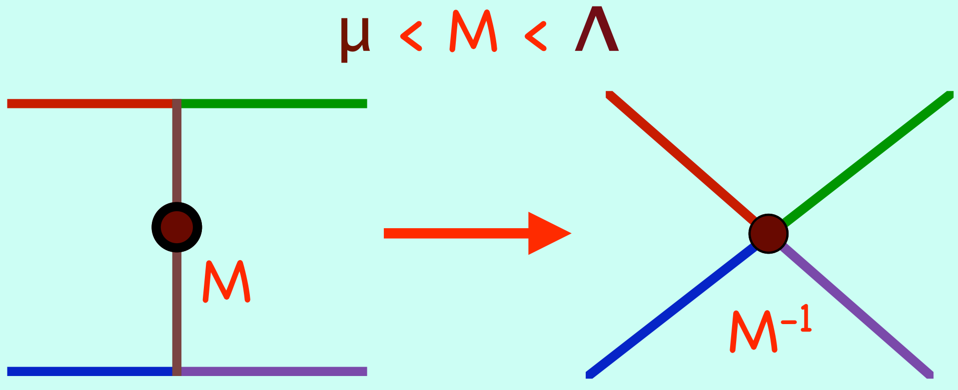
$$\mathcal{L}_{\rm int} = \mathbf{W}|_{\theta\theta} = F^a W_a + W_{ab}\psi^a\psi^b$$

superpotential non-renormalization thm

$$\mathbf{W}(\mathbf{\Phi^a}, \mu) = \mathbf{W}(\mathbf{\Phi^a}, \Lambda)$$!

up to (tree-level) integrating out heavy states ($M_i > \mu$)

thru: $W_i = 0$ ⇒



effective global susy: gauge coupling

effective theory defined below a cutoff scale Λ by {K, f, W}
by running couplings, renormalized fields, v.e.v's & symmetries

$$\mathcal{L}_{\text{gauge}} = \underset{4}{\mathbf{f}}(\mathbf{\Phi^a})\mathbf{W}^\alpha \mathbf{W}_\alpha|_{\theta\theta} = \underset{4}{f}(\phi^a)\left(F_{\mu\nu}F^{\mu\nu} + i\bar{\lambda}\partial\!\!\!/\lambda + D^2\right) + \text{h.c.}$$

gauge coupling renormalization

$$\mathbf{f}(\mathbf{\Phi^a}, \mu) \neq \mathbf{f}(\mathbf{\Phi^a}, \Lambda)$$

ANALYTIC GAUGE COUPLINGS

$$f(\langle\phi^a\rangle, \Lambda) = \frac{1}{g^2(\Lambda)}$$

$$b_i = -\left(\text{Tr}\, T_A^2\right)_i$$

$$f(\langle\phi^a\rangle, \mu) = \frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} + \frac{1}{8\pi^2}\sum_{<\mu} b_a \ln\left(\frac{\mu}{\Lambda}\right) + \frac{1}{8\pi^2}\sum_{>\mu} b_i \ln\left(\frac{M_i}{\Lambda}\right)$$

$\Lambda > M_i > \mu$ are the heavy state thresholds

IF $M_i = \langle\phi^i\rangle \Rightarrow \ln(\phi^i)$ DEPENDENCE IN $f(\phi)$

spontaneous susy breaking and "soft terms"

$$L = K(\Phi^\dagger, \Phi)|_{\theta\theta\theta\theta} + \tfrac{1}{4} f(\Phi) W_\alpha W^\alpha|_{\theta\theta} + W(\Phi)|_{\theta\theta} + \text{h.c.}$$

$F^j F^{\bar{\imath}} R_{j\bar{\imath}b\bar{a}}\, \phi^{\bar{a}} \phi^b$ $F^k \partial_k f(\phi)\, \lambda^\alpha \lambda_\alpha$ $F^k \partial_k W(\phi)$

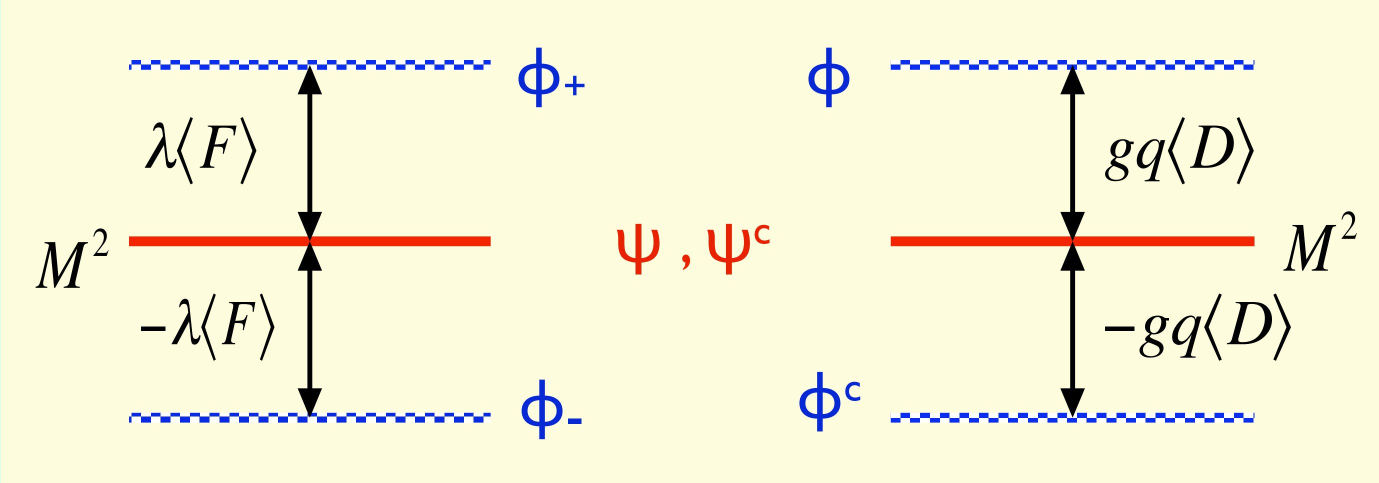
$\langle X \rangle = \theta^2 F$	renormalizable	effective interactions
$K = \Phi^\dagger \Phi + \frac{r}{\Lambda^2} X^\dagger X \Phi^\dagger \Phi + \frac{c_K}{\Lambda} X^\dagger \Phi^c \Phi + \ldots$	$g Q_\phi \langle D \rangle \phi^\dagger \phi$	$r \left\| \langle F/\Lambda \rangle \right\|^2 \phi^\dagger \phi + \ldots$ $c_K \langle F^*/\Lambda \rangle (\psi \psi^c + F_\phi \phi^c + \ldots)$
$f = \frac{1}{g^2} + \frac{X}{\Lambda} + \ldots$	(one-loop, see later)	$g^2 \frac{\langle F \rangle}{\Lambda} \lambda \lambda^c$
$W = X \phi^c \phi + \frac{c_W}{\Lambda} X \phi \phi' \phi''$	$\langle F \rangle \phi^c \phi$	$c_W \frac{\langle F \rangle}{\Lambda} \phi \phi' \phi''$

Everything defined @ Λ !

coupling of susy breaking to matter

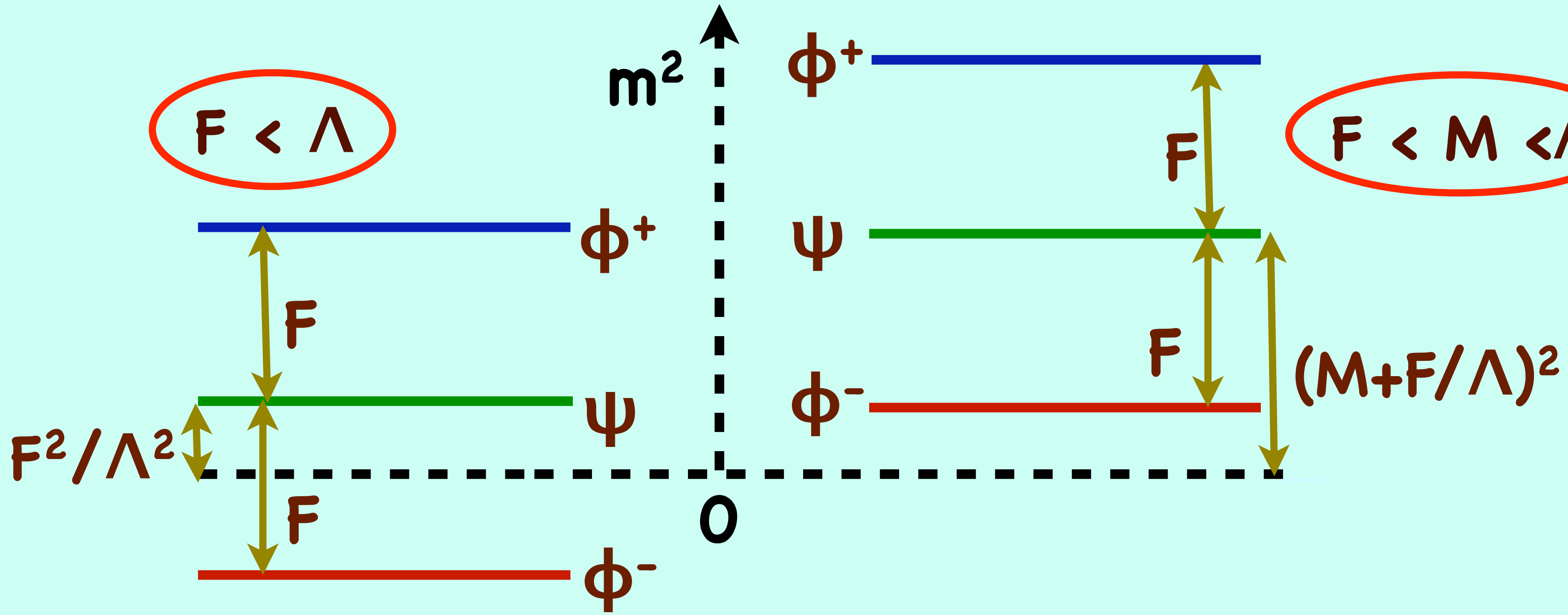
renormalizable broken susy theory

$$\mathcal{L}_{\psi,\phi} = M\psi\psi^c + M^2\left(|\phi|^2 + |\phi^c|^2\right) + \lambda\langle F\rangle\left(\phi^c\phi + \mathrm{h.c.}\right) + gQ_\phi\langle D\rangle\left(|\phi|^2 - |\phi^c|^2\right)$$



- renormalizable couplings are not suitable for squarks and sleptons
- coupling must be effective and involve large scale cutoff Λ

no (direct!) goldstino-matter coupling



- △ dim=4 coupling of F (goldstino) to chiral fields ⇒ $(\text{mass})^2$ splitting
- ▽ for light fermions ⇒ "tachyons" ○ OK for heavy states
- △ matter fields are chiral under SU(2)xU(1): only dim=4 coupling in W are HQU, HQD, HLE ⇒ would need goldstino = higgsino + ...
- ▽ exercise: show why the 2 Higgs are not good goldstino partners.

massses from "generic" susy breaking

SM

$$v = \sqrt{\textstyle\sum_i \langle H_i \rangle^2}$$

$$m_W = \frac{1}{\sqrt{2}} g v$$

$$m_f = Y_f \, v$$

$$m_H^2 = \lambda_H \, v^2$$

$$\lambda_H \, v \, \left(H^* H^2 + \mathrm{h.c.}\right)$$

$$v_{\mathrm{exp}} = 174 \, \mathrm{GeV}$$

from D=4 couplings only

...SSM

$$\langle F \rangle = \sqrt{\textstyle\sum_i \langle F^i \rangle^2 + \sum_A \langle D^A \rangle^2}$$ susy brkg scale

$$m_{3/2} = \frac{1}{\sqrt{3} M_{\mathrm{P}}} \langle F \rangle$$ gravitino mass

$$m_\lambda \sim g^2 \, \frac{F}{\Lambda}$$ gaugino masses

$$m_{\tilde{f}}^2 - m_f^2 \sim \left(\frac{F}{\Lambda}\right)^2$$ sfermion masses

$$\frac{F}{\Lambda} (HUQ + \mathrm{h.c.})$$ cubic "A-terms"

$$F \ll \Lambda \ll M_{\mathrm{P}}$$

from effective D>4 couplings only

summary I: generic global susy

(As expected) non-renormalizable couplings to the goldstino generate all possible "soft-terms" from SSB, with effective susy brkg scale F/Λ

a) scalars associated to light fermions (squarks, sleptons, ...) get masses from SSB in a generic global susy effetive theory

b) supersymmetric masses are generated from SSB if allowed by symmetries that forbid the tree-level ones ("Giudice-Masiero")

c) gaugino masses are generated with a factor g^2 due to canonical renormalization of gauginos

d) A-terms are generated as well

In an effective theory these terms allowed by symmetries are always there and contribute to $O(F/\Lambda)$!

N.B. 1 - For the example above, the connections vanish for $\phi^i = 0$!

exercise: find a symmetry that forbid the μ-term in W but not Giudice-Masiero

susy flavour problems

squarks and slepton mass² are matrices in flavour (family) space → 5.9+3.18=99 parameters in a generic effective theory

$$\tilde{m}^2_Q \quad \tilde{m}^2_{U^c} \quad \tilde{m}^2_{D^c} \quad \tilde{m}^2_L \quad \tilde{m}^2_{E^c}$$
$$A_U \quad A_D \quad A_E$$

Generically, sfermion masses (from SSB) and fermion masses (from EWSB) would not be diagonal/real in the same flavour/CP basis

$$\left[\tilde{m}^2_{\tilde{f}}, m^\dagger_f m_f\right] \neq 0$$

SUSY FLAVOUR PROBLEM :

squark/sleptons + gauginos/higgsinos loops exceed FCNC exptl. bounds

squarks and slepton mass² (almost) $\propto \mathrm{I}$

FLAVOUR "BLIND" SSB

FCNC, LFV⇒ 5+3 = 8 parameters + "very small corrections"

(*) Aligning solutions seem a very trying task !

mediation of susy breaking

hidden sector : no direct goldstino couplings to matter

1. Postulate (F-type) susy breaking in a hidden (=heavy) sector of states
   $\rightarrow \langle F \rangle$ & M_{hidden}

2. develop in $\langle F \rangle$: susy breaking lagrangian

3. "integrate out" the hidden states $\rightarrow$ effective dim>4 couplings:
   susy brkg & new cutoff $O(M_{hidden})$

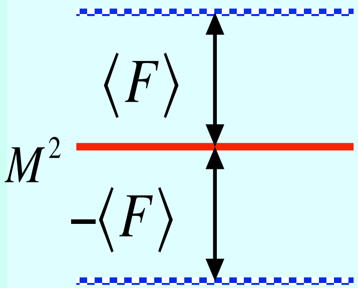
[4. decouple dim > 4 by limit $M_{hidden} \rightarrow \infty$ $\rightarrow$ renormalizable eff. theory
– BUT keep FIXED eff. low energy susy scale $m_{susy} = F/M_{hidden}$]

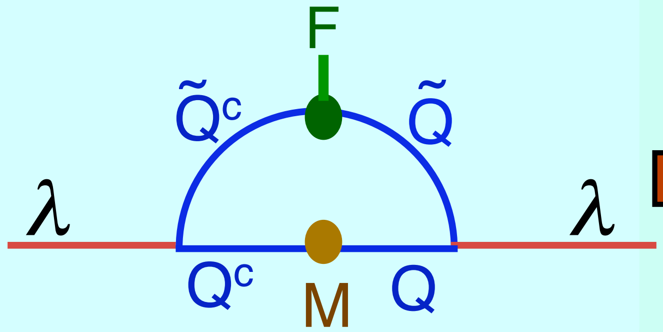
susy matter — mediation — hidden sector

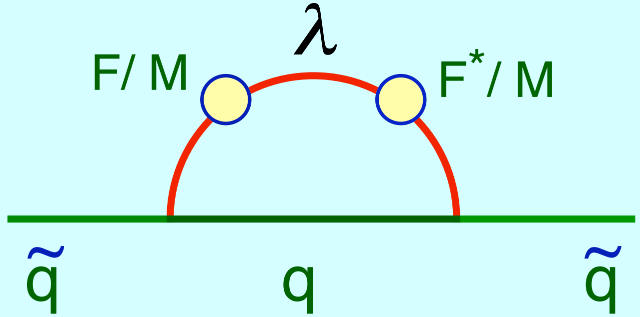
F/M — M, F

needs MEDIATION = fields coupling to both the hidden and matter sectors: gravity, gauge, scalars

GMSB: gauge mediation







$$M^2_{1,2} = M^2 \pm F$$

$$m_\lambda = \frac{b_Q\, g^2}{(4\pi)^2} \frac{F}{M}$$

$$m^2_{\tilde{q}} = \frac{2C_f}{b_Q} m^2_\lambda$$

specific spectrum: e.g., squarks much heavier than sleptons

$$m_{\tilde{q}} \sim \frac{\alpha_s}{\alpha_1} m_{\tilde{\ell}_R}$$

LSP= gravitino

$$m_{3/2} \sim \frac{16\pi^2 M}{g^2 M_{\rm P}} m_\lambda$$

$\Rightarrow$ bound on M from Ω_{DM}

gauge mediation: messengers

STEP 1: HIDDEN SECTOR

a) only GAUGE interactions with matter+Higgs

b) SUSY BREAKING and mass generation, e.g.,

$$\langle \mathbf{X} \rangle = \langle X \rangle + \theta^2 \langle F_X \rangle = M + \theta^2 F \qquad (F \ll M)$$

c) MESSENGERS: massive splitted chiral multiplets in a real representation of the gauge group, (Q, Q^c), e.g.,

$$\mathbf{W}_{\text{hidden}} \ni \mathbf{XQQ} \quad \Rightarrow \quad m_Q = M \ , \quad M_{1,2}^2 = M^2 \pm F$$

exercise: generalize to more fields, is M=<Y>, F=<X>$_{\theta\theta}$ possible?

gauge mediation: gaugino masses

$\langle \mathbf{X} \rangle = \langle X \rangle + \theta^2 \langle F_X \rangle = M + \theta^2 F \quad (F \ll M)$

$\mathbf{W}_{\text{hidden}} \ni \mathbf{XQQ} \quad \Rightarrow \quad m_Q = M \,, \quad M_{1,2}^2 = M^2 \pm F$

STEP 2: GAUGINO MASSES

a) messengers Q, Q^c contribute to the running of $g^2(\mu)$ above their threshold M= <X> which gives the X dependence **(one-loop)**:

$$f(\langle X \rangle , \mu) = \frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} + \frac{1}{8\pi^2} \sum_{<\mu} b_a \ln\left(\frac{\mu}{\Lambda}\right) + \frac{1}{4\pi^2} b_Q \ln\left(\frac{\langle X \rangle}{\Lambda}\right)$$

b) gaugino masses defined by the general relation:

$b_Q = -\left(\mathrm{Tr}\, T_A^2\right)_Q$

$$m_\lambda(M) = g^2 F \nabla_X f(X, \mu)|_{X=M=\mu} = \frac{b_Q\, g^2}{(4\pi)^2} \frac{F}{M}$$

exercise: compare to the previous dim=5 coupling of X to gauge fields

gauge mediation: scalar masses (I)

$$\mathrm{Re} f(\langle X\rangle, \mu) = ... + \frac{1}{8\pi^2} b_Q \ln\left(\frac{\langle X\bar{X}\rangle}{\Lambda^2}\right)$$

$$m_\lambda = \frac{b_Q\, g^2}{(4\pi)^2}\frac{F}{M}$$

STEP 2: SCALAR MASSES

- only GAUGE interactions with matter+Higgs fields:

$$\partial_{\mathrm{x}} K_{a\bar{b}}(\Lambda) = 0 \qquad \partial_{\mathrm{x}} W(\Lambda) = 0 \qquad \partial_{\mathrm{x}} f(\Lambda) = 0$$

- the metrics of matter fields K(X, μ) depend on X only thru g^2(X, μ)

RGE for wave fctn renormalisation at one loop:

$$\frac{\partial \ln K}{\partial \ln g^{-2}} = \frac{C}{2b}$$

$$C = \sum_A T_A^2$$

$$b \to b + b_Q \;\; (\mu > M)$$

gauge invariance of **K** and K (multiplicative renormalisation):

$$\left[K^{-1}, \partial_{\mathrm{x}} K\right] = 0 \qquad \Gamma_{\mathrm{x}} = K^{-1}\partial_{\mathrm{x}} K = \partial_{\mathrm{x}} \ln K \qquad R_{\mathrm{x}\bar{\mathrm{x}}a\bar{b}} = \left(K\partial_{\bar{\mathrm{x}}}\partial_{\mathrm{x}} \ln K\right)_{a\bar{b}}$$

exercise: understand it.

gauge mediation: scalar masses (II)

$$\mathrm{Re}f(\langle X\rangle,\mu) = ... + \frac{1}{8\pi^2} b_Q \ln\left(\frac{\langle X\bar{X}\rangle}{\Lambda^2}\right)$$

$$m_\lambda = \frac{b_Q\, g^2}{(4\pi)^2}\frac{F}{M}$$

$$\Gamma_{\mathrm{x}} = \partial_{\mathrm{x}} \ln K$$

$$R_{\mathrm{x}\bar{\mathrm{x}}a\bar{b}} = (K\partial_{\bar{\mathrm{x}}}\partial_{\mathrm{x}} \ln K)_{a\bar{b}}$$

RGE $$\frac{\partial \ln K}{\partial \ln g^{-2}} = \frac{C}{2b}$$

STEP 2: SCALAR MASSES

$$m^2(\mu) = F^{\bar{\mathrm{x}}}F^{\mathrm{x}}\partial_{\bar{\mathrm{x}}}\partial_{\mathrm{x}} \ln K(\mu)|_{X=\langle X\rangle}$$

the RGE for K gives the dependence of K on X thru $g^2(X)$ and $g^2(\mu)$

$$\partial_{\mathrm{x}} K_{a\bar{b}}(X, X^\dagger, \mu) \propto \partial_{\mathrm{x}}\left[\frac{g^2(\Lambda)}{g^2(X)}\right]^{\frac{2C_a}{b+b_Q}}\left[\frac{g^2(X)}{g^2(\mu)}\right]^{\frac{2C_a}{b}}$$

$$\left(b = \sum_{<\mu} b_i\right)$$

$$\rightarrow \quad m_a^2(M) = \frac{2C_a b_Q}{(4\pi)^4} g^4(M)\frac{F^2}{M^2} = \frac{2C_a}{b_Q} m_\lambda^2(M)$$

N.B. - RGE for $m^2(\mu)$ also given!

exercise: find $dm^2/d\mu$

exercise: find the mass spectrum of squarks, sleptons, gauginos @μ

gauge mediation: A-terms

$$\Gamma_{\rm x} = \partial_{\rm x} \ln K$$

$$\partial_{\rm x} K_{a\bar b}\left(X, X^\dagger, \mu\right) \propto \partial_{\rm x} \left[\frac{g^2(\Lambda)}{g^2(X)}\right]^{\frac{2C_a}{b+b_Q}} \left[\frac{g^2(X)}{g^2(\mu)}\right]^{\frac{2C_a}{b}}$$

STEP 2: "A-terms"

non-renormalization thm for W(Φ) → $\partial_{\rm x} W(\mu) = 0 \quad \forall \mu$

only the Γ term in the covariant derivative contributes and one gets:

$$F^{\rm x} \Gamma_{{\rm x}\,a}^{\;\,a}(\mu) = \frac{2C_a}{b}\left(m_\lambda(\langle X\rangle) - m_\lambda(\mu)\right)$$

to be applied to each field in the superpotential

$$W = \phi^a \phi^b \ldots. \quad \Rightarrow \quad A_{ab\ldots} = \frac{2(C_a + C_b + \ldots)}{b}\left(m_\lambda(\langle X\rangle) - m_\lambda(\mu)\right)$$

exercise: find the GMSB A-terms for the MSSM superpotential

gauge mediation: summary

$$m_\lambda = \frac{b_Q\, g^2}{(4\pi)^2} \frac{F}{M}$$

$$m_a^2(M) = \frac{2C_a b_Q}{(4\pi)^4}\, g^4(M)\, \frac{F^2}{M^2} = \frac{2C_a}{b_Q} m_\lambda^2(M)$$

$$A_{ab\ldots} = \frac{2(C_a + C_b + \ldots)}{b} \left(m_\lambda(\langle X\rangle) - m_\lambda(\mu)\right)$$

GOOD

gaugino masses fixed by representation of messengers and F/M

MSSM sfermion masses given in terms of 3 gaugino masses

suppress FCNC: squark and sleptons masses are family independant

NOT SO GOOD

μ-term and Bμ-term need relaxing hiddenness condition (additional mediation) and asks for relatively laborious models reducing predictivity

exercise: generalize for several group factors, messengers,

N.B. - GLOBAL SUSY BREAKING IS NOT AN EASY GAME: see literature

susy is sugra or nothing!

susy superalgebra

$$\{ Q^\alpha , Q^\beta \} = \sigma_\mu{}^{\alpha\beta} P^\mu$$

translations are local symmetries

local ⟵ local

SUGRA = local susy defined by gravity supermultiplet:

graviton (S=2) & gravitino (S=3/2)

= gauge fermion of local susy

coupling M_P^{-1} defines the Planck mass

⟶ non-renormalizable EFFECTIVE field theory below M_P

sugra mediation

- **gauge mediation** investigated since '81 in many explicit models, **revamped '96**
- Cremmer et al. found the **coupling of chiral matter and YM to sugra** and discussed the superHiggs effect
- this openned the way for **sugra mediation** in the limit $M_P \to \infty$ with $m_{3/2}$ fixed.

sugra: Kähler transformations

Sugra Lagrangian invariant under Kähler transforms

	KAHLER TRANFORM	
$\mathbf{K}(\phi,\phi^\dagger)$	$\mathbf{K}(\phi,\phi^\dagger)+g(\phi)+g^\dagger(\phi^\dagger)$	KÄHLER POTENTIAL
$K_{a\bar{b}}(\phi,\phi^\dagger)$	INVARIANT	KAHLER METRIC
$W(\phi)$	$e^{-g(\phi)}W(\phi)$	SUPERPOTENTIAL
$V=F^aF_a-3\|M\|^2$	INVARIANT	POTENTIAL
$M=e^{K/2}W$	$e^{-i\mathrm{Img}}e^{K/2}W=e^{-i\mathrm{Img}}M$	AUXILIARY GRAVITY (/3)
$F_A=(\partial_a+\frac{1}{2}\mathbf{K}_a)e^{K/2}W$	$e^{-i\mathrm{Img}}(\partial_a+\frac{1}{2}\mathbf{K}_a)e^{K/2}W=e^{-i\mathrm{Img}}F_a$	AUXILIARY CHIRAL
$\partial_a+\frac{1}{2}\mathbf{K}_a$	$\partial_a+\frac{1}{2}\mathbf{K}_a+\frac{1}{2}g_a$	KÄHLER DERIVATIVE

$$\nabla_a = \partial_a+\frac{1}{2}\mathbf{K}_a+\Gamma_a$$

SUGRA COVARIANT DERIVATIVE

N.B.- factor M_P^{-2} in K_a w.r.t. ∂_a and $\Gamma_a \Rightarrow$ irrelevant in the global susy limit $M_P \to \infty$

gravitino mass

$$V = F^a F_a - 3|M|^2$$

$$M = e^{K/2} W$$

$$F_A = (\partial_a + \tfrac{1}{2}\mathbf{K}_a) e^{K/2} W$$

interaction sugra-chiral fields → $m_{3/2} = \langle M \rangle = \langle e^{K/2} W \rangle$

COSMOLOGICAL CONSTANT

$$\Lambda_{\rm cosm} \lll\lll\lll M_{\rm P}$$

$M_P = 1$!

$$\Lambda_{\rm cosm} = \langle V \rangle = \langle F^a F_a \rangle + \langle (D_A)^2 \rangle - 3\langle |M|^2 \rangle = 0$$

→ $3\, m_{3/2}^2 = \langle F^a F_a \rangle + \langle (D_A)^2 \rangle$

GOLDSTINO

goldstino projector: $\psi^b_{\rm GOLD} = \dfrac{1}{3m_{3/2}^2} \langle F^b F_a \rangle \psi^a$ for $\langle D_A \rangle = 0$

what a difference a K makes

$$\nabla_a = \partial_a + \tfrac{1}{2}\mathbf{K}_a + \Gamma_a$$

$$M = e^{K/2}W$$

$$F_A = (\partial_a + \tfrac{1}{2}\mathbf{K}_a)e^{K/2}W$$

$$V = F^a F_a - 3|M|^2$$

$$\Lambda_{\rm cosm} = \langle V\rangle = \langle F^a F_a\rangle + \langle (D_A)^2\rangle - 3\langle |M|^2\rangle = 0$$

$$m_{3/2} = \langle M\rangle = \langle e^{K/2}W\rangle$$

$$3\, m_{3/2}^2 = \langle F^a F_a\rangle + \langle (D_A)^2\rangle$$

---

the K_a term in ∇_a modifies the susy breaking scalar mass formula: (for $\langle D_A\rangle = 0$)

$$\tilde m^2_{a\bar b} = F^{\bar j}F^{i}\left(\tfrac{1}{3}K_{i\bar j}K_{a\bar b} + R_{i\bar j a\bar b} + \tfrac{1}{3}K_{i\bar b}K_{a\bar j}\right)$$

$$= m^2_{3/2}K_{a\bar b} + F^{\bar j}F^{i}R_{i\bar j a\bar b} + F_a F_{\bar b}$$

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GOLDSTINO PROJECTOR

msugra (strict sense)

$$\nabla_a = \partial_a + \tfrac{1}{2}\mathbf{K}_a + \Gamma_a$$

$$\tilde{m}^2_{a\bar{b}} = m^2_{3/2} K_{a\bar{b}} + F^{\bar{\mathrm{j}}} F^{\mathrm{i}} R_{\mathrm{i}\bar{\mathrm{j}}a\bar{b}} + F_{\mathrm{a}} F_{\bar{\mathrm{b}}}$$

susy breaking in hidden sector ⟷ sugra mediation ⟷ matter/Higgs sector (MSSM, NMSSM

K = K$_{\text{hidden}}$ + K$_{\text{matter}}$ W = W$_{\text{hidden}}$ + W$_{\text{matter}}$

x = goldstino direction

$$\partial_{\mathrm{x}} K_{a\bar{\mathrm{j}}} = 0 \quad \partial_{\mathrm{x}} W_{\mathrm{matter}} = 0 \quad \Gamma^b_{xa} = 0 \quad R_{\mathrm{x}\bar{\mathrm{x}}a\bar{b}} = 0$$

SCALAR MASSES

$$\tilde{m}^2_{a\bar{b}} = m^2_{3/2} K_{a\bar{b}}$$

→ $m_{\text{scalar}} = m_{3/2}$ universal

A-TERMS

$$F^{\mathrm{x}} \nabla_{\mathrm{x}} e^{K/2} (W_{\mathrm{matter}}) = e^{K/2} K_{\mathrm{x}} F^{\mathrm{x}} \underbrace{(W_{\mathrm{matter}})}_{\mathrm{dim}=N}$$

$$A_{(N)} = N e^{K/2} K_{\mathrm{x}} m_{3/2}$$

$A_{(N)}/N$ = universal, free

msugra (@ work)

K = K$_{hidden}$ + K$_{matter}$ W = W$_{hidden}$ + W$_{matter}$

$$\tilde{m}^2_{a\bar{b}} = m^2_{3/2} K_{a\bar{b}}$$

$$A_{(N)} = N e^{K/2} K_x m_{3/2}$$

FLAT LIMIT

limit $M_P \to \infty$ with $m_{3/2}$ fixed

→ renormalizable broken susy theory

msugra spectrum @ M_P ← !

SCALAR MASSES	A-TERMS	GAUGINO MASSES	HIGGSINO MASS
$m_0^2 = m_{3/2}^2$	$A_0 = 3k m_{3/2} \quad B = \dfrac{2A_0}{3}$	$M_{1/2} = k' m_{3/2}$	$\mu = k'' m_{3/2}$

$(k,\, k',\, k'' = 0 \ldots O(1))$

O(100) → 4 real parameters
(+ 2 phases) !

msugra parameter space almost excluded
by HEP + cosmology (DM) data

summary msugra

In msugra, with complete factorization as defined:
- all the scalars get an universal mass equal to the gravitino mass
- all interactions in W → A-term = (degree)×universal

These terms are independent of geometry (metrics) and W, they follow only from Kähler invariance (only present in sugra)

gaugino masses can generated by a (gravitational) coupling to the goldstino in the gauge function $f(\phi)$ or radiative corrections

the μ-term is obtained by relaxing hiddeness through a couping of the goldstino to two chiral supermultiplets in K_{matter} (Giudice-Masiero)

However, these parameters are defined at M_P and are sensitive to the physics down to LE, which can violate universality.

no-scale models

$$V = F^a F_a - 3|M|^2$$

$$M = e^{K/2} W$$

$$F_A = (\partial_a + \tfrac{1}{2}\mathbf{K}_a) e^{K/2} W$$

$$m_{3/2} = \langle M \rangle = \langle e^{K/2} W \rangle$$

$$\tilde{m}^2_{a\bar{b}} = m^2_{3/2} K_{i\bar{\jmath}} + F^{\bar{t}} F^t R_{t\bar{t}i\bar{\jmath}}$$

SIMPLEST NO-SCALE MODEL

$$\mathbf{K} = -\lambda \ln \left(\mathbf{t} + \mathbf{t}^\dagger - \sum_i \phi^{\mathbf{i}\dagger} \phi^{\mathbf{i}} \right)$$

$$\mathbf{W} = \mathbf{W}(\phi^{\mathbf{i}})$$

very good exercise: find the metrics matrix K, K^{-1}, F^t, F^i, e^K, $m_{3/2}$, study V,...

Λ_{cosm} =0 for λ=3 from geometry:

1) maximally symmetric **K**; 2)R=-1/3

$$V = e^K \left((\lambda - 3) |W|^2 + W^i W_i \right)$$

V=0 along the t-direction, W^i=0

$$R_{t\bar{t}i\bar{\jmath}} = -\lambda^{-1} K_{t\bar{t}} K_{i\bar{\jmath}}$$

Einstein space

→

$$\tilde{m}^2_{i\bar{\jmath}} = F^t F_t K_{i\bar{\jmath}} \left(\frac{1}{3} - \frac{1}{\lambda} \right)$$

all ϕ^i are massless for λ = 3

no-scale models (II)

$$\mathbf{K} = -\lambda \ln \left(\mathbf{t} + \mathbf{t}^{\dagger} - \sum_i \phi^{\mathbf{i}\dagger}\phi^{\mathbf{i}}\right)$$

$$\mathbf{W} = \mathbf{W}(\phi^{\mathbf{i}})$$

$$V = e^K \left((\lambda - 3)\,|W|^2 + W^i W_i\right)$$

$$\tilde{m}^2_{i\bar{\jmath}} = F^t F_t K_{i\bar{\jmath}} \left(\frac{1}{3} - \frac{1}{\lambda}\right)$$

exercise: generalize for the case where t is a nxn matrix and the ϕ^i are n-vectors. hint: project with a hermitian matrix basis

N.B. - many other Kähler manifolds share the no-scale property

these are tree-level results modified radiative corrections

exercise: calculate the Coleman-Weinberg potential and compare with the tree-level ones (don't forget the gravitino contribution!)

the e^K efactor in V vanishes at infinity = unstable: requires stabilizing the modulus t !