Recent developments in automated NLO calculations

Giuseppe Bevilacqua

NCSR “Demokritos”, Athens

Workshop on Standard Model and Beyond
and Standard Cosmology

Corfu - August 31, 2009

In collaboration with M. Czakon, P. Draggiotis, M. Garzelli, I. Malamos,
G. Ossola, R. Pittau, C. Papadopoulos, A. van Hameren, M. Worek

Outline

Introduction

- motivations for a *multi-purpose, automatic* NLO generator

A sketch of the OPP method

- reconstructing one-loop amplitudes from the *integrand*

The project `HELAC-NLO`

- an overview of the framework

A proof of concept

- complete NLO QCD corrections to $pp \rightarrow t\bar{t}b\bar{b}$

Introduction and motivations

What do we expect from LHC?

The ”theory space” of possible new physics scenarios is highly populated...

Must be prepared for the unexpected!

Achieving a good theoretical control over SM backgrounds is fundamental for New Physics searches at LHC

Introduction and motivations

The physics program of the upcoming LHC (and future colliders) requires high-precision predictions for multi-particle processes

A leading-order (LO) calculation typically predicts only the order of magnitude of a given cross section and the rough features of a given observable (especially in perturbative QCD)

At present, several interesting SM processes are described with an accuracy lower than what is required by the experimental precision expected at LHC

A theoretical accuracy of *at least* next-to-leading order (NLO) is desirable (and demanded) for most physics analyses

The Les Houches NLO Wishlist

Priority list of LHC processes experimentalists wish to know at NLO

NLO Multi-leg - Wiki Les Houches 2009

http://www.lpthe.jussieu.fr/LesHouches09Wiki/index.php/Programme

NLO Wishlist 2007

- $pp \to W + j$
- $pp \to H + 2j$
- $pp \to VVV$
- $pp \to t\bar{t} + 2j$
- $pp \to VVb\bar{b}$
- $pp \to VV + 2j$
- $pp \to V + 3j$
- $pp \to t\bar{t}b\bar{b}$
- $pp \to b\bar{b}b\bar{b}$

Proposed update 2009

- $pp \to t\bar{t}t\bar{t}$
- $pp \to Z + 3j$
- $pp \to 4j$
- $pp \to Wbbj$
- $pp \to W + 4j$

$\Rightarrow$ Need for automatization

Some recent calculations for LHC

Non-exhaustive list of recent NLO QCD results:

$2 \to 3$

- $pp \to H + 2j$ [Campbell *et al*; Andersen *et al*]
- $pp \to t\bar{t}Z$ [Lazopoulos, Melnikov, Petriello '07]
- $pp \to t\bar{t} + j$ [Dittmaier, Uwer, Weinzierl '07]
- $pp \to t\bar{t} + \gamma$ [Peng-Fei *et al* '09]
- $pp \to VV + j$ [Dittmaier, Kallweit, Uwer]
- $pp \to ZZZ$ [Lazopoulos, Melnikov, Petriello '07]
- $pp \to VVV$ [Binoth *et al*; Campanario *et al* '08]
- $pp \to b\bar{b}V$ [Febres Cordero, Reina, Wackeroth '08]

$2 \to 4$

- $pp \to VV + 2j$ (via VBF) [Bozzi, Jäger, Oleari, Zeppenfeld '07]
- $pp \to W + 3j$ [Ellis *et al*; Berger *et al* '09]
- $pp \to t\bar{t}b\bar{b}$ [Bredenstein, Denner, Dittmaier, Pozzorini '09; G.B., Czakon, Papadopoulos, Pittau, Worek '09]

Most complete calculations involve $2 \to 2$ or $2 \to 3$, very few $2 \to 4$ processes

The road to automatization

In the last decade, a remarkable progress has been achieved in the description of multi-particle processes at the leading order

$\hookrightarrow$ efficient recursive algorithms for tree-level amplitudes

Many general-purpose MC tools for LO calculations have been developed

GRACE	Sherpa/AmegiC++
HELAS/MadGraph/MadEvent	HELAC–PHEGAS
Whizard+Omega	...

At NLO there is still room for progress

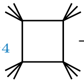
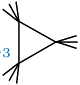
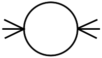
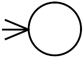
- libraries of *specific* scattering processes $(2 \to 2, 2 \to 3)$ based on analytic calculations, *e.g.* MCFM [Campbell *et al*]
  - ✓ good efficiency
  - ✗ limited applicability
- *automatic* tools based on Passarino-Veltman (PV) reduction of Feynman diagrams, *e.g.* FeynCalc [Mertig *et al*], FormCalc [Hahn]
  - ✓ well-established, general applicability
  - ✗ stronger than factorial increase of terms while reducing tensor integrals

Recent years have seen an impressive progress in developing *numerical* methods for *efficient* and *fully automated* one-loop calculations

OPP reduction

Basic idea:

- decompose one-loop amplitudes into a basis of scalar integrals

$\mathcal{A} = \sum d_{i_1 i_2 i_3 i_4}$  $+ \sum c_{i_1 i_2 i_3}$  $+ \sum b_{i_1 i_2}$  $+ \sum a_{i_1}$  $+ R$

- evaluate the coefficients of the expansion at the *integrand* level

Systematic framework for the computation of the coefficients of scalar integrals (a, b, c, d) and the rational term (R)

A lot of work has been done since the OPP algorithm has been proposed

The first *unitarity-based* programs for automated NLO calculations have started to appear

- BlackHat/Sherpa [Berger *et al*]
- Rocket/MCFM [Ellis *et al*]
- HELAC−NLO [Papadopoulos *et al*]

Also based on traditional methods

- GOLEM [Binoth]

This talk focuses on the HELAC−NLO project

A sketch of the OPP method

A generic one-loop amplitude has the form

$$\mathcal{A} \quad = \sum_{I\subset\{0,1,\cdots,m-1\}} \int \frac{\mu^{(4-d)}d^d q}{(2\pi)^d} \frac{\bar{N}_I(\bar{q})}{\prod_{i\in I} \bar{D}_i(\bar{q})}$$

where the bar denotes objects living in $d = 4 + \epsilon$ dimensions:

$$\bar{q}^2 = q^2 + \tilde{q}^2 \qquad \bar{D}_i = D_i + \tilde{q}^2 \qquad \bar{N}_I(\bar{q}) = N(q) + \tilde{N}(q, \tilde{q}, \epsilon)$$

Using known basis of scalar one-loop integrals

$$\mathcal{A} = \sum_i d_i \,\text{Box}_i + \sum_i c_i \,\text{Triangle}_i + \sum_i b_i \,\text{Bubble}_i + \sum_i a_i \,\text{Tadpole}_i + R$$

Can we apply a similar decomposition at the *integrand* level?

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} [d(i_0,i_1,i_2,i_3) + \tilde{d}(q;i_0,i_1,i_2,i_3)] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} [c(i_0,i_1,i_2) + \tilde{c}(q;i_0,i_1,i_2)] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} [b(i_0,i_1) + \tilde{b}(q;i_0,i_1)] \prod_{i\neq i_0,i_1}^{m-1} D_i \\
&+ \sum_{i_0}^{m-1} [a(i_0) + \tilde{a}(q;i_0)] \prod_{i\neq i_0}^{m-1} D_i \\
&+ \tilde{P}(q) \prod_{i}^{m-1} D_i
\end{aligned}
$$

$\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}$ are ”spurious” terms (vanish upon integration). Their q-dependence is known

Ossola, Papadopoulos and Pittau, Nucl. Phys. B 763, 147 (2007)

Calculation is now reduced to an algebraic problem:
sample $N(q)$ *numerically* to extract the coefficients

Rational terms

Note: the OPP expansion of $N(q)$ is written in terms of 4-dimensional denominators. Rational terms (R) originate taking the additional $(n-4)$-dimensional structure of the numerator into account

Two sources ($R = R_1 + R_2$):

- $R_1 \rightarrow$ rewrite all D_i's in the $N(q)$ expansion in terms of $\bar{D}_i$'s

$$D_i \rightarrow \bar{D}_i - \tilde{q}^2$$

$\hookrightarrow$ computable within the framework of OPP reduction

[Ossola, Papadopoulos and Pittau, JHEP 0805 (2008) 004]

- $R_2 \rightarrow \epsilon$-dimensional part of the numerator function

$$\bar{q} = q + \tilde{q} \qquad \bar{\gamma}_\mu = \gamma_\mu + \tilde{\gamma}_\mu \qquad \bar{g}^{\mu\nu} = g^{\mu\nu} + \tilde{g}^{\mu\nu}$$

$\hookrightarrow$ computable with effective tree-level Feynman rules

[Draggiotis, Garzelli, Papadopoulos and Pittau, arXiv:0903:0356 [hep-ph]]

$$\mathcal{A} = \int d^n\bar{q} \frac{N(q)}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{m-1}} + \int d^n\bar{q} \frac{f(\tilde{q}^2,q)}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{m-1}} + \int d^n\bar{q} \frac{\tilde{N}(q,\tilde{q},\epsilon)}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{m-1}}$$

The HELAC-NLO project

People involved: G. Bevilacqua, M. Czakon, P. Draggiotis, M. Garzelli, I. Malamos, G. Ossola, R. Pittau, C. Papadopoulos, A. van Hameren, M. Worek

Virtual corrections

- coefficients of the OPP expansion and rational term R_1 with CutTools [Ossola, Papadopoulos, Pittau]
- scalar integrals with OneLOop [van Hameren]
- rational term R_2 via tree-level counterterms

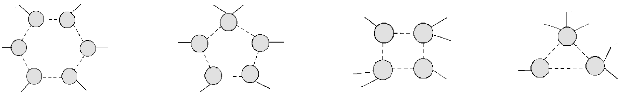
Real corrections

- Catani-Seymour dipole subtraction with HELAC-DIPOLES [Czakon, Papadopoulos, Worek]

Calculational framework - virtual corrections

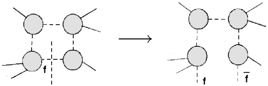
Example: $2 \to 4$ process. The integrand has the form

$$\mathcal{A}(q) = \sum \underbrace{\frac{N_i^{(6)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_5}}} + \underbrace{\frac{N_i^{(5)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_4}}} + \underbrace{\frac{N_i^{(4)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_3}}} + \underbrace{\frac{N_i^{(3)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}}} + \cdots$$



In order to apply the OPP reduction, `HELAC-NLO` must provide a numerical evaluation of the numerators $N_i^6(q), N_i^5(q), ...$ with the values of the loop momentum q provided by `CutTools`

- generate all inequivalent partitions of 6,5,4,3... blobs attached to the loop, and check all possible flavours (and colours) that can be consistently running inside
- *hard-cut* the loop (q is fixed) to get a $n+2$ tree-like process $\to$ use efficient *tree-level* machinery (DS equations) to compute *one-loop* integrands



The R_2 contributions (rational terms) are calculated in the same way as the tree-order amplitude, taking into account *extra vertices*

Calculational framework - real corrections

Insert subtraction terms encoding IR/collinear divergences

$$\sigma^{NLO} = \int_m d\sigma^B + \int_{m+1} d\sigma^R - \int_{m+1} d\sigma^A + \int_{m+1} d\sigma^A + \int_m d\sigma^V$$

$$\hookrightarrow \quad \int d\sigma^B + \int_{m+1} \left[d\sigma^R - d\sigma^D \right] + \int_m \left[d\sigma^V + d\sigma^I + d\sigma^{KP} \right]$$

Catani-Seymour dipole formalism

Catani, Seymour, Nucl. Phys. B485, 291 (1997)
Catani, Dittmaier, Seymour, Trocsanyi, Nucl. Phys. B627, 189 (2002)

extended to arbitrary helicity eigenstates of the external partons

Czakon, Papadopoulos, Worek, arXiv:0905.0883 [hep-ph]

- allows replacement of exact summation over external-state polarizations by a probabilistic approach (Monte Carlo sampling)

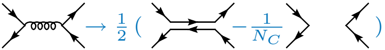
Real corrections calculated with `HELAC-DIPOLES`

$$d\sigma^D, d\sigma^I, d\sigma^{KP}$$

Colour organization of virtual matrix elements

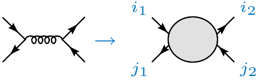
HELAC-NLO merges two complementary representations:

Colour flow decomposition



$|\text{Amp}|^2 = \sum_{\sigma,\sigma'} A_\sigma A^*_{\sigma'} C_{\sigma\sigma'}$

Colour assignment



$|\text{Amp}|^2 = \sum_{\{i\},\{j\}} |\mathcal{M}^{i_1,i_2,\cdots,i_k}_{j_1,j_2,\cdots,j_k}|^2$

✓ Simple colour factors / Feynman rules for A_σ

✗ Factorial growth of colour connections

✓ Possiblity of MC sampling

✗ More complex Feynman rules

$$\mathcal{M}^{i_1,i_2,\cdots,i_k}_{j_1,j_2,\cdots,j_k} = \sum_\sigma \delta_{i_{\sigma(1)}j_1} \delta_{i_{\sigma(2)}j_2} \cdots \delta_{i_{\sigma(k)}j_k} A_\sigma$$

$\hookrightarrow$ Monte Carlo sampling over colours:

- generate random color assignment for each external particle (colour must be conserved)
- find which colour connections (σ) are compatible with the given assignment
- restrict calculation to the eligible A_σ's

Exact treatment of colour sum $\leftrightarrow$ Improvement in speed

Virtual corrections via re-weighting

$$\sigma^{LO+V} = \int dx_1 dx_2 d\Phi_m pdf_a(x_1) pdf_b(x_2) (|\mathcal{M}|^2 + \mathcal{M}\mathcal{L}^* + \mathcal{M}^*\mathcal{L})$$

[$\mathcal{M}$ = LO amp; $\mathcal{L}$ = virtual one-loop amp (time consuming)]

Factorizing $|\mathcal{M}|^2$ and dividing by σ^{LO} we recover a *tree-order* probability density:

$$\frac{\sigma^{LO+V}}{\sigma^{LO}} = \int dx_1 dx_2 d\Phi_m \, g(x_1, x_2, \Phi_m) \left(1 + \frac{\mathcal{M}\mathcal{L}^* + \mathcal{M}^*\mathcal{L}}{|\mathcal{M}|^2}\right)$$

where $$g(x_1, x_2, \Phi_m) \equiv \frac{1}{\sigma^{LO}} pdf_a(x_1) pdf_b(x_2) |\mathcal{M}|^2 = \frac{1}{\sigma^{LO}} \frac{d\sigma^{LO}}{dx_1 dx_2 d\Phi_m}$$

$\hookrightarrow$ We get the LO+V result by *re-weighting* a sample of *tree-level unweighted* events

- generate a sample S of tree-level unweighted events (with running couplings and pdf's set up at one-loop level) including all relevant information for the evaluation of the one-loop matrix element ($\mathcal{L}$): colour/helicity assignment, x_1, x_2, $\mathcal{M}$
- compute $\mathcal{L}$ event by event and get the re-weighting factor
- get the LO+V cross section: $\sigma^{LO+V} = \frac{\sigma^{LO}}{N_S} \sum_{i \in S} \left(1 + \frac{\mathcal{M}\mathcal{L}^* + \mathcal{M}^*\mathcal{L}}{|\mathcal{M}|^2}\right)$

Speed-up in the calculation of virtual corrections

Lazopoulos, Melnikov, Petriello, Phys. Rev. D76 (2007) 014001

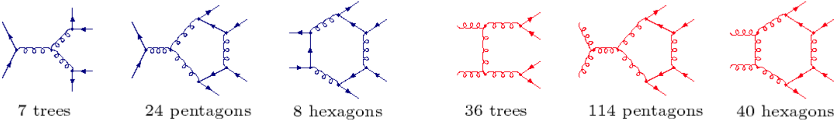
Binoth, Ossola, Papadopoulos, R. Pittau, JHEP 0806 (2008) 082

A proof of concept: QCD corrections to $pp \to t\bar{t}b\bar{b}$

Irreducible background for light-Higgs searches in the $t\bar{t}H$ channel ($H \to b\bar{b}$)

Benchmark process at the current frontier of NLO

$\hookrightarrow$ $\mathcal{O}(10^3)$ diagrams involving hexagons up to rank 4; six coloured legs with massless and massive particles



(S. Pozzorini, Les Houches 2009)

Our goals:

- reproduce recently appeared complete results with an independent approach

  Bredenstein, Denner, Dittmaier and Pozzorini, arXiv:0905.0110 [hep-ph]

- show the potential of `HELAC-NLO` in a realistic multi-leg computation

Comparison of total cross sections

Total cross sections for $pp \to t\bar{t}b\bar{b} + X$ at the LHC for the scale choice $\mu_F = \mu_R = m_t$: comparison between the results of Refs. [1,2] and `HELAC-NLO`

[1] Bredenstein, Denner, Dittmaier and Pozzorini, JHEP 0808 (2008) 108

[2] Bredenstein, Denner, Dittmaier and Pozzorini, arXiv:0905.0110 [hep-ph]

Process	$\sigma^{\text{LO}}_{[1,2]}$ [fb]	σ^{LO} [fb]	$\sigma^{\text{NLO}}_{[1,2]}$ [fb]	$\sigma^{\text{NLO}}_{\alpha_{max}=1}$ [fb]	$\sigma^{\text{NLO}}_{\alpha_{max}=0.01}$ [fb]
$q\bar{q} \to t\bar{t}b\bar{b}$	85.522(26)	85.489(46)	87.698(56)	87.545(91)	87.581(134)
$pp \to t\bar{t}b\bar{b}$	1488.8(1.2)	1489.2(0.9)	2638(6)	2642(3)	2636(3)

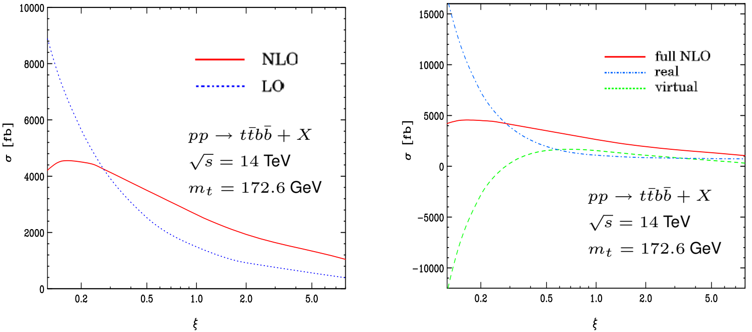
$p_T(b/\bar{b}) > 20$ GeV $\quad \Delta R(b\bar{b}) > 0.8$

Sampling colour/helicity and re-weighting works and pays off

Scale dependence

Scale dependence at LHC for $\mu_R = \mu_F = \xi m_t$

G.B, Czakon, Papadopoulos, Pittau, Worek, arXiv:0907.4723 [hep-ph]



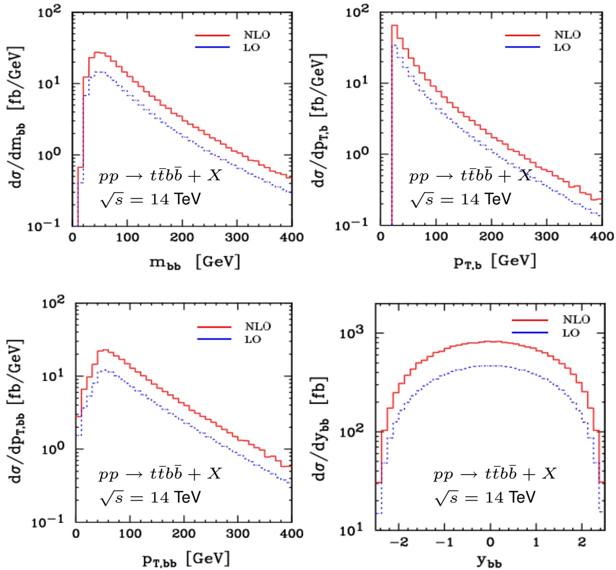
- At central scale ($\xi = 1$) the full cross section receives a NLO correction of 77%
- Varying the scale up and down by a factor 2 we find:

$$\sigma^{LO}_{t\bar{t}b\bar{b}}(\text{LHC}, m_t = 176.2 \text{ GeV}, \text{CTEQ6L1}) = 1489.2 \begin{array}{l} +1036.8(70\%) \\ -\ \ 565.8(38\%) \end{array} \text{ fb}$$

$$\sigma^{NLO}_{t\bar{t}b\bar{b}}(\text{LHC}, m_t = 176.2 \text{ GeV}, \text{CTEQ6M}) = 2636 \begin{array}{l} +862(33\%) \\ -703(27\%) \end{array} \text{ fb}$$

A collection of differential distributions

G.B, Czakon, Papadopoulos, Pittau, Worek, arXiv:0907.4723 [hep-ph]



Summary and perspective

The technology of unitarity-based methods is mature and promising

- several projects of automatic multi-leg NLO generators in advanced status of development
- the first *unitarity-based* complete results to $2 \to 4$ processes have started to appear: $pp \to W + 3j$, $pp \to t\bar{t}b\bar{b}$

HELAC-NLO is now ready to tackle complete NLO QCD calculations

- several optmizations implemented and successfully tested
- features and performances competitive with other approaches

Future projects

- realistic calculations and phenomenological analyses

$$pp \to t\bar{t} + j \qquad pp \to b\bar{b}W^+W^- \qquad pp \to t\bar{t} + 2j \quad \ldots$$

- further optimizations under study