Neutrino Properties from $E6 \times SO(3)$

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Corfu, September 2009

GUT: Extention of $SU(2)_L \times U(1)_Y \times SU(3)_C$

i) Quarks and Leptons are united in in one representation formed of (left handed) 2-component Weyl spinor fields

ii) Quantum numbers are no more ad hoc (positron charge equal to proton charge)

iii) unification of gauge couplings

Because of the existence of 3 generations the symmetry of GUT's should be extented to include a

generation (flavor) symmetry

Flavor symmerties are least understood sofar.
In the simplest extention one has a direct product

GUT $\times$ Flavor

The invariant Yukawa interaction relates now all fermions with each other.
In particular, the masses and mixings of heavy and light neutrinos are strongly correlated with the masses and mixings of the charged fermions.
It is my aim to describe this connection in a specific E6 model, which allows a quantitative fit to all known fermion masses and mixings. This model then predicts the sofar unknown neutrino properties such as the neutrino hierarchy and neutrino CP violation.

To write a flavor invariant Yukawa interaction one has essentially only two alternatives:

- To give Higgs fields generation indices f.i.

$$(\psi^{\alpha T}\ H_{\alpha\beta}\ \psi^{\beta})$$

- or to identify the coupling matrices as VEVs of new scalar fields "flavons".

$$\frac{1}{M}\langle\Phi_{\alpha\beta}\rangle\ (\psi^{\alpha T} H \psi^{\beta})$$

In this talk I will restrict myself to this second possibility.

Achievements of E6 GUT's

Phys. Rev. D77, 076009 (2008) Z. Tavartkiladze, B. S.

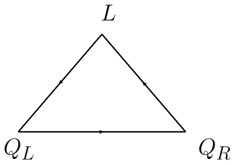
- economic: few parameters (VEV's of scalar fields) determine the properties of all fermions.
- simple structure of the effective Lagrangian which is obtainable from a renormalizable model
- the small couplings involved allow perturbative calculations up to M_{Planck}
- it is easy to incorporate a local non abelian generation symmetry. We take $SO(3)_g \times P_g$ and thus use

$$E6 \times SO(3)_g \times P_g$$

$E6 \supset SU(3)_L \times SU(3)_R \times SU(3)_c$

Single generation for fermions $\psi(27)$

$$\mathbf{27} = Q_L(x) + L(x) + Q_R(x)$$



$$(Q_L)^a_i = \begin{pmatrix} u^a \\ d^a \\ D^a \end{pmatrix}, \quad L^i_k = \begin{pmatrix} L^1_1 & E^- & e^- \\ E^+ & L^2_2 & \nu \\ e^+ & \hat{\nu} & L^3_3 \end{pmatrix}, \quad (Q_R)^k_a = \begin{pmatrix} \hat{u}_a, & \hat{d}_a, & \hat{D}_a \end{pmatrix},$$

$$Q_L(x) = \left(3, \quad 1, \quad \bar{3}\right), \quad L(x) = \left(\bar{3}, \quad 3, \quad 1\right), \quad Q_R(x) = \left(1, \quad \bar{3}, \quad 3\right)$$

- mixing: $d \leftrightarrow D$ $\mathcal{U}_L$-spin, $\hat{d} \leftrightarrow \hat{D}$ $\mathcal{U}_R$-spin

Flavor $SO(3)_g \times P_g$

- All fermions transform as 3-vectors under this group The 3×3 coupling matrices in front of the Higgs fields are then obtained from the VEV's of $3 \times 3 = 9$ real scalar flavons which can be represented by the hermitian matrix field $\phi_{\alpha\beta}(x)$:
- $\phi_{\alpha\beta}(x) = \chi_{\alpha\beta}$ (symmetric) $+i\, \xi_{\alpha\beta}$ (antisymmetric)

$$\mathcal{L}_Y^{eff} = \frac{\phi_{\alpha\beta}}{M}(\psi^{\alpha T} H \psi^{\beta}) + \ldots$$

$G_{\alpha\beta} = \frac{\langle \chi_{\alpha\beta} \rangle}{M}$ defines a real symmetric and

$A_{\alpha\beta} = i\frac{\langle \xi_{\alpha\beta} \rangle}{M'}$ a purely imaginary and antisymmetric generation matrix

The coupling matrix G determines the mass hierarchy.
It can be taken diagonal

$$G = \begin{pmatrix} m_u & 0 & \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \frac{1}{m_t} = \begin{pmatrix} \sigma^4 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\text{at } \mu = M_I}$$

$\sigma = 0.050 \Rightarrow$ correct up quark masses

The coupling matrix A describes all particle mixings.
It is antisymmetric and hermitian, 1 real parameter:

$$A = i\frac{\langle \xi \rangle}{M'} = i \begin{pmatrix} 0 & \sigma & -\sigma \\ -\sigma & 0 & 1/2 \\ \sigma & -1/2 & 0 \end{pmatrix}$$

After complete spontaneous symmetry breaking of $SO(3)_g \times P_g$ the generation matrices $G_{\alpha\beta}$ and $A_{\alpha\beta}$ appear in the effective Yukawa interaction

$$\mathcal{L}_Y^{\text{eff}} = G_{\alpha\beta}(\psi^{\alpha T} H \psi^{\beta}) + A_{\alpha\beta}(\psi^{\alpha T} H_A \psi^{\beta}) + \frac{(G^2)_{\alpha\beta}}{M}\left((\psi^{\alpha T} H^{+})_1 (\tilde{H}^{+} \psi^{\beta})_1\right) + h.c.$$

- hierarchy of masses and the mixings of all fermions are now fully determined
- The A term is CP odd → unique ~~CP~~

The Masses and Mixings of all 3×27 fermions are obtained from the mass matrix

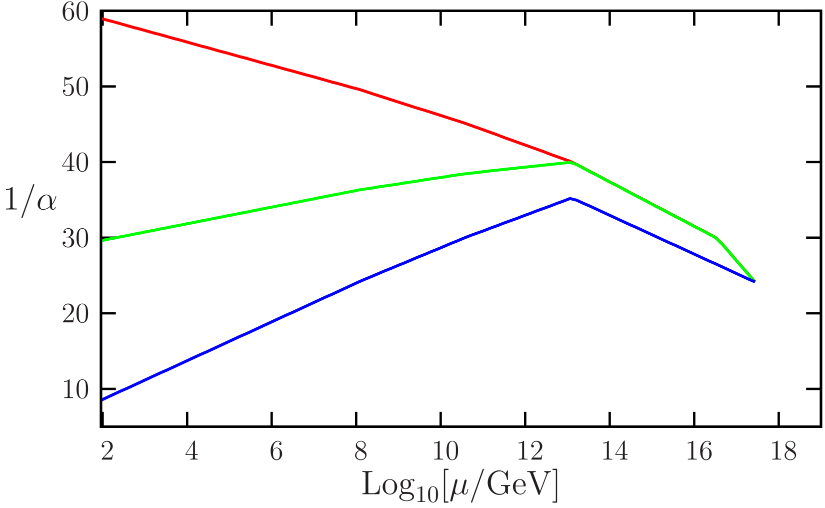
$$M_{ij}^{\alpha\beta} = G_{\alpha\beta}\langle H\rangle_{ij} + A_{\alpha\beta}\langle H_A\rangle_{ij} + (G^2)_{\alpha\beta}\langle H^\dagger\rangle_i \frac{1}{M} \langle \tilde{H}^\dagger\rangle_j$$

$$\langle H\rangle = \underbrace{\begin{pmatrix} e_1^1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -ze_3^3 & e_3^3 \end{pmatrix}}_{\langle H_u\rangle} + z \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_2^2 & 0 \\ 0 & \epsilon_2^3 & 0 \end{pmatrix}}_{\langle H_d\rangle} = \underbrace{\begin{pmatrix} e_1^1 & 0 & 0 \\ 0 & z\epsilon_2^2 & 0 \\ 0 & 0 & e_3^3 \end{pmatrix}}_{\text{diagonal}}$$

$$\langle H_A\rangle_{\text{quarks}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & f_2^2 & f_3^2 \\ 0 & f_2^3 & f_3^3 \end{pmatrix}, \quad f \to g \text{ for leptons}$$

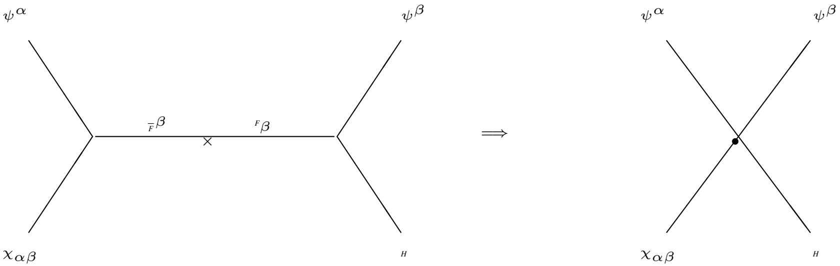
$$e_1^1 = m_t, \quad z\,\epsilon_2^2 = m_b, \quad e_3^3 \simeq \epsilon_2^3 \simeq M_I = m_D$$

Concorde


60
50
40
30
20
10
$1/\alpha$
2
4
6
8
10
12
14
16
18
$\mathrm{Log}_{10}[\mu/\mathrm{GeV}]$

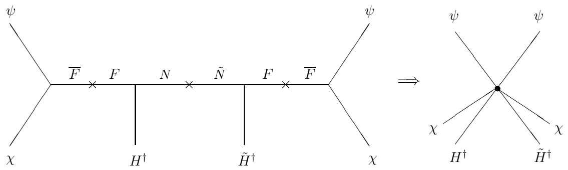
- Renormalizable interaction: introduce massive spinor fields

$$\text{Vertices :}\qquad \chi_{\alpha\beta}\psi^{\alpha}\bar{F}^{\beta}+\psi^{\alpha}HF_{\alpha}+MF_{\alpha}\bar{F}^{\alpha}$$



$$\frac{\langle\chi_{\alpha\beta}\rangle}{M}\left(\psi^{\alpha\mathrm{T}}H\psi^{\beta}\right)=G_{\alpha\beta}\left(\psi^{\alpha\mathrm{T}}H\psi^{\beta}\right)$$

- Masses for the 'right handed' neutrinos $\hat{\nu}$ are not obtained from $\langle H \rangle$.
- Coupling with two H needed: $H(-), \tilde{H}(+)$
  via a <u>massive</u> spinor singlet field
  $N^\alpha(1,3,+), \bar{N}^\alpha(1,3,-)$
  Vertices: $F^\alpha H^\dagger N^\alpha + F^\alpha \tilde{H}^\dagger \bar{N}^\alpha + M_N N^\alpha \bar{N}^\alpha$



$$\Rightarrow \frac{\langle \chi_{\alpha\beta} \rangle}{M} \frac{1}{M_N} \left(\psi^\alpha H^\dagger \tilde{H} \psi^\gamma \right) \frac{\langle \chi_{\gamma\beta} \rangle}{M}$$

- results (qualitative)

$$m_d \approx G\, m_b + A\, f_2^2 \qquad \text{small mixing and } \not{CP}$$

$$m_\nu \approx \frac{m_t^2}{M}\left(c_1 \mathbb{1} + c_2(AG^{-1} - G^{-1}A)\right) \qquad \text{large mixing and } \not{CP}$$

- heavy neutrino masses $M_L \sim G^2\, M_I \quad 1 : \sigma^4 : \sigma^8$
  super hierarchy 2×10^{13}, 1×10^8, 8×10^2 GeV!
- 8 parameters for qualitative results:
  $m_t, m_b, f_2^2, g_2^2, c_1, c_2, G \to 1, A \to 1$
- observables:
  quarks $\to$ 10, charged leptons $\to$ 3, light neutrinos $\to$ 9, 22
  heavy fermions $3\,\hat{\nu} + 9\,L + 3\,D$ +15
  =37

Mass matrix for quarks

- $M_u = G\, m_t$ at $\mu = M_I$ $\sigma = 0.050$
- $M_{d,D} = 6 \times 6$ matrix
- $\rightarrow 3 \times 3$ matrix $M_d = m_b^0\, G + f_2^2 A - \frac{f_3^2 f_2^3}{e_3^3}\, A\, G^{-1}\, A$

$$A = i\frac{\langle \xi \rangle}{M'} = i \begin{pmatrix} 0 & \sigma & -\sigma \\ -\sigma & 0 & 1/2 \\ \sigma & -1/2 & 0 \end{pmatrix}$$

- This gives an excellent fit for CKM unitarity triangle

Results: Quarks

- Fit: $m_b^0 = 2.95\ GeV$, $f_2^2 = -0.23$ GeV, $f_0 = 1.30$ GeV

$$m_b = 2.89 \text{ GeV}, \qquad m_s = 50 \text{ MeV}, \qquad m_d = 2.6 \text{ MeV}$$

- exp: 2.89 ± 0.09 $\qquad 55 \pm 15$ $\qquad 2.9 \pm 1.2$

$$|V_{cd}| = 0.228, \qquad |V_{cb}| = 0.042, \qquad |V_{ub}| = 0.0039$$

$$\alpha_q = 97^o, \qquad \beta_q = 23^o, \qquad \gamma_q = 60^o.$$

- all results are within experimental errors
- Light D quark for $e_3^3 = M_I = 2 \cdot 10^{13}$ GeV
  $m_{D_1} \simeq 1 \cdot 10^8$ GeV

Neutral Lepton Mass Matrix

- Neutral Lepton Mass Matrix 15×15

$$M_L = \begin{array}{c} \\ L_3^2 \\ L_2^3 \\ L_3^3 \\ L_1^1 \\ L_2^2 \end{array} \begin{array}{c} \begin{array}{ccccc} L_3^2 & L_2^3 & L_3^3 & L_1^1 & L_2^2 \end{array} \\ \begin{pmatrix} 0 & -e_1^1 G & 0 & -g_2^3 A & 0 \\ -e_1^1 G & 0 & e_3^3 G^2 + F_A A & 0 & 0 \\ 0 & e_3^3 G^2 + F_A A & -z e_3^3 G^2 & 0 & e_1^1 G \\ -g_2^3 A^T & 0 & 0 & 0 & e_3^3 G \\ 0 & 0 & e_1^1 G & e_3^3 G & 0 \end{pmatrix} \end{array} .$$

- New element: $F_A = \langle H_A \rangle_{33,\,1}$
  $15 \times 15 \quad \Rightarrow \quad 3 \times 3$ *multiple* see-saw

- $m_\nu = \frac{(e_1^1)^2}{e_3^3}\left\{ -z\,\mathbf{1} + \frac{g_2^3}{e_3^3}(A\frac{1}{G} - \frac{1}{G}A) - \frac{F_A}{e_3^3}\frac{g_2^3}{e_3^3}(A\frac{1}{G^2}A\frac{1}{G} + \frac{1}{G}A\frac{1}{G^2}A)\right\}$

  to simplify we set $F_A = x_A \sigma^5 e_3^3\ ,\quad g_2^3 = x_g \sigma^3 e_3^3$
  and obtain - taking only linear terms in σ -

$$m_\nu \Rightarrow \quad m\begin{pmatrix} \rho(1+r_1) & -\mathrm{i} & \mathrm{i} \\ -\mathrm{i} & \rho(1+r_2) - 2x_A & -\mathrm{i}\frac{\sigma}{2}r_{23} + x_A \\ \mathrm{i} & -\mathrm{i}\frac{\sigma}{2}r_{23} + x_A & \rho \end{pmatrix}$$

  with the abbreviations $\rho = \frac{-z}{x_g}\ ,\ m = x_g\frac{(e_1^1)^2}{e_3^3}.$
- Expectation: $m \simeq 0.3,\ \rho \simeq 1\ ,\ x_A \simeq 0$

Results: Neutrinos

- Without renormalization group effects, i.e. for $r_1 = r_2 = 0$, $r_{23} = 1$ and for $x_A = 0$ one has

$$m_\nu \Rightarrow \quad m \begin{pmatrix} \rho & -\mathrm{i} & \mathrm{i} \\ -\mathrm{i} & \rho & -\mathrm{i}\frac{\sigma}{2} \\ \mathrm{i} & -\mathrm{i}\frac{\sigma}{2} & \rho \end{pmatrix}$$

- Eigenvalues of $m_\nu m_\nu^\dagger$

$$(m_2)^2 \simeq (\rho^2 + 2 + \frac{\sigma}{\sqrt{2}})\, m^2 \;,$$
$$(m_1)^2 \simeq (\rho^2 + 2 - \frac{\sigma}{\sqrt{2}})\, m^2 \;,$$
$$(m_3)^2 \simeq \rho^2 m^2$$

i) Inverted hierarchy

ii) degeneracy in the no mixing limit $x_g \to 0$

iii) $R = (m_2^2 - m_1^2)/(m_2^2 - m_3^2) \simeq \sigma/\sqrt{2} \simeq 0.035$

iv) m and e_3^3 fixed from $\Delta m^2_{\mathrm{atm}}$

$$m \simeq \frac{1}{\sqrt{2}}\sqrt{\Delta m^2_{\mathrm{atm}}} \simeq 0.34 \quad e_3^3 \simeq \frac{x_g\sqrt{2}(e_1^1)^2}{\sqrt{\Delta m^2_{\mathrm{atm}}}} \simeq 2 \cdot 10^{13}\ \text{GeV !!}$$

v) $e_3^3 \simeq M_N \simeq M \simeq M_I \quad e_3^3 \simeq e_3^3 \epsilon_3^3 / M_N$
fixes masses of all heavy fermions

vi) In the limit of no RG effects and $x_A \to 0$ one gets bimaximal neutrino mixing.

Neutrinos final Results

- RG effects and $x_A \neq 0$ gives $\quad m = 0.0234, \quad x_g = 0.033,$
  $\rho = 1.132, \quad x_A = -0.06$
- mixing angles:
  $\Theta_{12} = 34^\circ(35^\circ), \quad \Theta_{23} = 43^\circ(39^\circ), \quad \Theta_{13} = 6^\circ(3^\circ)$
  $F^{23} \simeq 2 * 10^{13}$ GeV

  Predictions:

- $m_2 = 0.0624$ eV, $m_1 = 0.0616$ eV, $m_3 = 0.0374$ eV
- $\delta_\nu = 67^\circ$
- Neutrinoless double β decay parameter:
  $|m_{\beta\beta}| = 0.046$ eV

  very satisfactory ! little freedom for changes

Heavy Neutrino Spectrum

- 2 states ≈ 0.7 TeV, $(L_2^3)_1$, $(L_3^3)_1$,
  4 states $\approx 1 \cdot 10^8$ GeV
  2 states $\approx 6 \cdot 10^{10}$ GeV
  4 states $\approx 1 \cdot 10^{13}$ GeV
- Light neutrinos mix with L_1^1 and L_2^2 states $\approx 2 \cdot 10^{-2}$ and with the lightest heavy neutrionos $\approx 8 \cdot 10^{-7}$
- Decay properties of the lightest heavy neutrinos:

$$\rightarrow H^u + \nu, \quad \rightarrow W^+ + e^-, \quad \rightarrow Z + \nu$$

Summary

- Non-SUSY GUT's allow for the construction of models renormalizable up to the Planck scale
- Effective Yukawa interaction at the weak scale has a very simple form with only few parameters, the VEV's of Higgs and generation fields.
- $E_6 \times SO(3)_g \times P_g$: the few symmetry breaking parameters allow for a quantitative fit of all fermion masses and mixings !
- $M_I \simeq 10^{13}$ GeV, the meeting point of g_1 and g_2 fixes the mass scales of light and heavy neutrinos.

- The known quark masses and CKM matrix determine to a large extent the neutrino properties,
  Neutrino hierarchy, ~~CP~~ and Majorana phases as well as the mass parameter for $O\nu\beta\beta$ decays.
- Spontaneous breaking of generation symmetry from suitable potentials $\sim \phi^4$.
- No understanding of the dynamics of E_6 breaking and Higgs masses.

Spontaneous symmetry breaking of $SO(3)_g \times P_g$

$$\mathcal{L}_\Phi = \frac{1}{2}\mathrm{Tr}\,|\partial_\mu \Phi - \mathrm{i}e[B_\mu, \Phi]|^2 - V_{SO(3)}(\chi, \xi)$$

$$B_\mu = B_\mu^i t^i \qquad t^i : \; SO(3)_g \;\text{ generators} \quad i = 1, 2, 3$$

The potential $V_{SO(3)} = V_0 + V_{CW}$ can be constructed such that spontaneous symmetry breaking of $SO(3)_g \times P_g$ occurs which breaks this symmetry completely. Moreover, the coefficients of the potential can be tuned to give for the absolute minimum the Vev's we need for the particle hierarchies and mixings :

$$\langle \Phi \rangle = M \langle \chi \rangle + iM^{'} \langle \xi \rangle$$

with the numerical value $\sigma = 0.050$

- $SO(3)_g$ invariance allows to choose χ diagonal

$$\Phi = \begin{pmatrix} \chi_1 & 0 & 0 \\ 0 & \chi_2 & 0 \\ 0 & 0 & \chi_3 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & \xi_3 & -\xi_2 \\ -\xi_3 & 0 & \xi_1 \\ \xi_2 & -\xi_1 & 0 \end{pmatrix}$$

- $V_{SO(3)}$ can be chosen such that it has a minimum for the wanted values of the invariants

$$\mathrm{Tr}(\chi)\ , \quad \mathrm{Tr}\left(\chi - \frac{1}{3}\mathrm{Tr}\chi\right)^2 \ \text{and} \ \mathrm{Tr}(\xi^2)$$

- remaining symmetry $\Longrightarrow$ subgroup of $SO(3)_g$
- Subgroup must keep χ diagonal
- Discrete symmetry

- $\Longrightarrow A_4$ symmetry $\psi \sim$ "**3**" $\quad F, F' \sim$ "**3**"
  $\chi_1, \chi_2, \chi_3 \sim$ "**3**", $\xi \sim$ "**3**"
- Further $SO(3)$ invariant potentials which include Coleman-Weinberg terms
  break $SO(3)$ completely !
- $V = V_{SO(3)} + \tilde{V}_{SO(3)}$
- $\tilde{V} =$
  $c_{\chi\xi 0} Tr[\chi.(i\xi)^2] Tr[\chi] + c_{\chi\xi 1} Tr[\chi.i\xi.\chi.i\xi] + c_W P_{CW}(\mu 1, \mu 2, \mu 3)$

- coefficients can be chosen (tuned) such that
  $\langle\Phi\rangle = \langle\chi\rangle + \mathrm{i}\langle\xi\rangle$
  and has the structure we need phenomenological !
- $V_{SO(3)} + \tilde{V}_{SO(3)}$ has an absolute minimum
  at
  $\frac{\chi_1}{M} = \sigma^4,\ \frac{\chi_2}{M} = \sigma^2,\ \frac{\chi_3}{M} = 1,\ \frac{\xi_1}{M'} = \frac{1}{2},\ \frac{\xi_2}{M'} = \sigma,\ \frac{\xi_3}{M'} = \sigma$
- for unification: $M \simeq M_I$ and $M' \simeq 10^3\ M_I$

Gauge coupling unification

- non-susy GUT's need intermediate symmetry with scale M_I

$$M_Z \ll M_I \ll M_{\text{GUT}}$$

- This is welcome! Scale of heavy neutrino's $M_\nu \ll M_{\text{GUT}}$ electroweak unification:
- gauge couplings $g_1(\mu)$ and $g_2(\mu)$ meet at $M_I \approx 2 \times 10^{13}\ GeV$
  - just right!
- new physics starts at $\mu = 10^{13}\ GeV \quad g_1 = g_2 \quad \mu \geq M_I$

$$SU(2)_L \times U(1) \times SU(3)_C \rightarrow \begin{cases} SU(2)_L \times SU(2)_R \times SU(4) & \rightarrow & SO(10) \\ SU(3)_L \times SU(3)_R \times SU(3)_C & \rightarrow & E_6 \end{cases}$$