Introduction to Loop Quantum Gravity

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A broad perspective on the challenges, structure and successes of loop quantum gravity.

Focus on conceptual issues with emphasis on cosmology.

Organization:

1. Historical & Conceptual Setting
2. Structure of Loop Quantum Gravity
3. The Big Bang: Loop Quantum Cosmology

1. Historical and Conceptual Setting

Einstein's resistance to accept quantum mechanics as a fundamental theory is well known. However, he had a deep respect for quantum mechanics and was the first to raise the problem of unifying general relativity with quantum theory.

"Nevertheless, due to the inner-atomic movement of electrons, atoms would have to radiate not only electro-magnetic but also gravitational energy, if only in tiny amounts. As this is hardly true in Nature, it appears that quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation."

(Albert Einstein, Preussische Akademie Sitzungsberichte, 1916)

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• Physics has advanced tremendously in the last nine decades but the the problem of unification of general relativity and quantum physics is still open. Why?

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⋆ No experimental data with direct ramifications on quantum Gravity.
⋆ But then this should be a theorist's haven! Why isn't there a plethora of theories?

• In general relativity, gravity is coded in space-time geometry. Most spectacular predictions —e.g., the Big-Bang, Black Holes & Gravitational Waves— emerge from this encoding. Suggests: Geometry itself must become quantum mechanical. How do you do physics without a space-time continuum in the background?

• Several approaches: Causal sets, twistors, AdS/CFT conjecture of string theory. Loop Quantum Gravity grew out of the Hamiltonian approach pioneered by Bergmann, Dirac, and developed by Wheeler, DeWitt and others.

Contrasting LQG with String theory

Because there are no direct experimental checks, approaches are driven by intellectual prejudices about what the core issues are and what will “take care of itself” once the core issues are resolved.

Particle Physics: ‘Unification’ Central: Extend Perturbative, flat space QFTs; Gravity just another force.

- Higher derivative theories; • Supergravity
- String theory incarnations:

$\star$ Perturbative strings; $\star$ M theory
$\star$ Matrix Models; $\star$ AdS/CFT Correspondence.

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• Particle Physics: ‘Unification’ Central: Extend Perturbative, flat space QFTs; Gravity just another force.

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• String theory incarnations:

⋆ Perturbative strings; ⋆ M theory/F theory;

⋆ Matrix Models; ⋆ AdS/CFT Correspondence.

• General Relativity: ‘Background independence’ Central: LQG

⋆ Quantum Geometry (Hamiltonian Theory used for cosmology & BHs),

⋆ Spin-foams (Path integrals used to bridge low energy physics.)

Issues:

• Unification: Ideas proposed in LQG but strong limitations;

Recall however, QCD versus Grand Unified Theories.

• Background Independence: Progress through AdS/CFT; but a ‘small corner’ of QG; Physics beyond singularities and S-matrices?

A. Ashtekar: LQG: Four Recent Advances and a dozen FAQs;
arXiv:0705.2222

2. Loop Quantum Gravity: Quantum Geometry

- Geometry: Physical entity, as real as tables and chairs.

Riemann 1854: Göttingen Address; Einstein 1915: General Relativity

- Matter has constituents. GEOMETRY??

'Atoms of Geometry'? Why then does the continuum picture work so well? Are there physical processes which convert Quanta of Geometry to Quanta of Matter and vice versa?

2. Loop Quantum Gravity: Quantum Geometry

- A Paradigm shift to address these issues

"The major question for anyone doing research in this field is: Of which mathematical type *are the variables . . . which permit the expression of physical properties of space. . . Only after that, which* equations *are satisfied by these variables?" Albert Einstein (1946); Autobiographical Notes.*

- Choice in General Relativity: Metric, $g_{\mu\nu}$. Directly determines Riemannian geometry; Geometrodynamics.

In all other interactions, by contrast, the basic variable is a Connection, i.e., a matrix valued vector potential A_a^i;
Gauge theories: Connection-dynamics

- Key new idea: 'Kinematic unification.' Cast GR also as a theory of connections. Import into GR techniques from gauge theories.

Holonomies/Wilson Lines and Frames/Triads

- Connections: Vehicles for parallel transport.

In QED: parallel transport of the state of an electron

Recall: $\partial\psi \rightarrow (\partial - ieA)\psi$

In QCD: parallel transport of the state of a quark

In gravity: parallel transport a spinor

$$p \bullet - - - > - - - \bullet\ q \quad \psi(q) = [\mathcal{P}\ \exp \int_p^q A \cdot dS]\, \psi(p)$$

- In Gravity: the (canonically conjugate) non-Abelian electric fields E_i^a interpreted as *orthonormal frames/triads*. They determine the physical, curved geometry. Structure group: Rotations of triads SO(3) or, in presence of spinors, its double covering SU(2).

Uniqueness of Canonical Quantization?

- von Neumann's uniqueness theorem:

There is a unique IRR of the Weyl operators $\hat{U}(\lambda), \hat{V}(\mu)$ by 1-parameter unitary groups on a Hilbert space satisfying:

i) $\hat{U}(\lambda)\,\hat{V}(\mu) = e^{i\lambda\mu}\,\hat{V}(\mu)\,\hat{U}(\lambda)$; and ii) Weak continuity in λ, μ.

This is the standard Schrödinger representation: $\mathcal{H} = L^2(\mathbb{R}, \mathrm{d}x)$;

$\hat{x}\Psi(x) = x\Psi(x)$; $\hat{p}\Psi(x) = -i\hbar \mathrm{d}\Psi(x)/\mathrm{d}x$, and $U(\lambda) = e^{i\lambda\hat{x}}$, $V(\mu) = e^{i\mu\hat{p}}$

- Strategy for more general systems: Consider the analog $\mathfrak{a}$ of the Weyl algebra. Look for cyclic representations. The 'VEVs' i.e. expectation values in the cyclic state determine the representation through an explicit (GNS) construction. If the VEVs are invariant under a group, the group is unitarily implemented in the representation.

- Uniqueness does not hold for systems with an infinite number of degrees of freedom even after imposing additional symmetry requirements such as Poincaré invariance.

Uniqueness of LQG Kinematics!

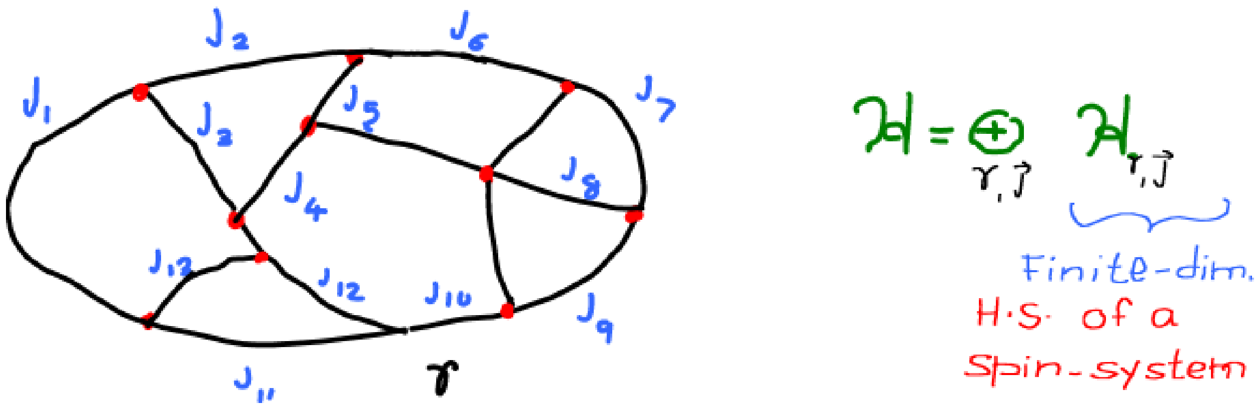
- Surprise: Quantum algebra $\mathfrak{a}$ generated by holonomies and triad-fluxes. It admits a unique *diff invariant* state $\Rightarrow$. Thanks to background independence, quantum kinematics is unique in LQG! (Lewandowski, Okolow, Sahlmann, Thiemann; Fleischhack)

Surprisingly powerful role of diff invariance! (AA)

Polymer Geometry

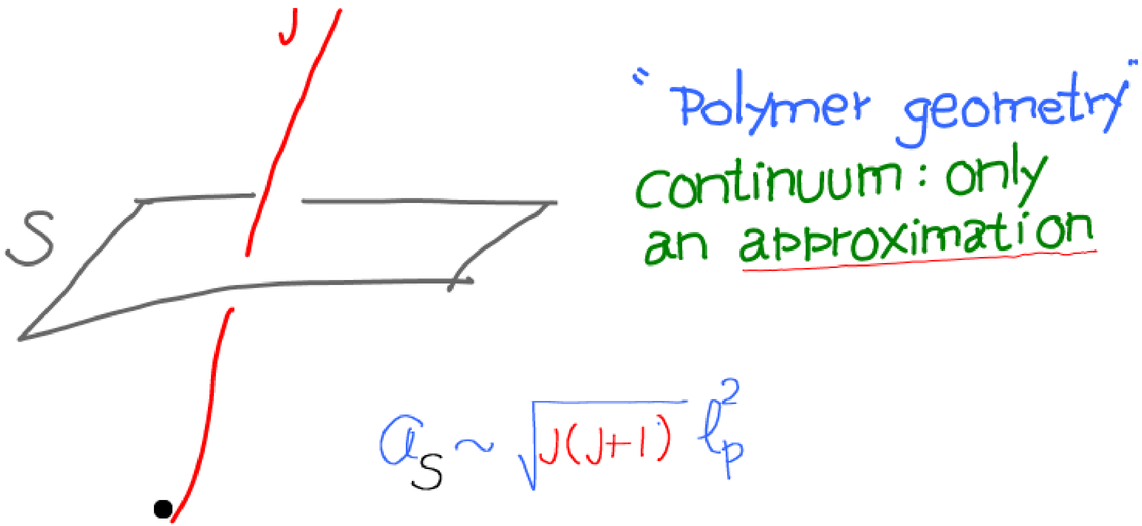
- This unique kinematics was first constructed explicitly in the early nineties. High mathematical precision. Provides a Quantum Geometry which replaces the Riemannian geometry used in classical gravity theories. (AA, Baez, Lewandowski, Marolf, Mourão, Rovelli, Smolin, Thiemann,...)
Details: Review by AA & Lewandowski; monographs by Rovelli; Thiemann.

- Quantum States: $\Psi \in \mathcal{H} = L^2(\bar{\mathcal{A}}, d\mu_o)$
μ_o a diffeomorphism invariant, regular measure on the space $\bar{\mathcal{A}}$ of (generalized) connections.



- Fundamental excitations of geometry 1-dimensional. Polymer geometry at the Planck scale. Continuum arises only in the coarse rained approximation.

- Flux lines of area. Background independence!



- Examples of Novel features:

⋆ All eigenvalues of geometric operators discrete. Area gap. Eigenvalues not just equally-spaced but crowd in a rather sophisticated way. Geometry quantized in a very specific way. (Recall Hydrogen atom.)

⋆ Inherent non-commutativity: Areas of intersecting surfaces don't commute. Inequivalent to the Wheeler-DeWitt theory (quantum geometrodynamics).

Summary: AA & Lewandowski, Encyclopedia of Mathematical Physics

Applications of this Quantum Geometry

- Quantum geometry used crucially in calculating the statistical mechanical entropy of isolated horizons. Encompass physically realistic black holes as well as cosmological horizons in one go.

- This unique kinematical arena provides a Quantum Riemannian Geometry to formulate dynamics, i.e. quantum Einstein's equations. Main challenge of LQG. Progress in the Hamiltonian theory (Giesel, Thiemann,...) and path integrals/spin foams (Baez, Barrett, Reisenberger, Rovelli, ...) which address issues of propagators, low energy limit, ...

- As lectures in this workshop will show there have been impressive advances in the last couple of years in bridging the two approaches. In my lectures, I will use cosmology to illustrate how: i) both programs can be completed; and ii) the Big Bang Singularity can be resolved; and iii) New Planck scale physics emerges.

LQC started with a seminal paper by Bojowald in 2001. Over 500 papers by now. Will focus on the basic conceptual and mathematical, rather than phenomenological issues.

3. Big-Bang: Loop Quantum Cosmology

• Deepest Feature of General Relativity: Gravity encoded in Geometry. Space-time geometry became a physical and dynamical entity. Spectacular consequences: Cosmology, Black holes, Gravitational Waves. Impressive mathematical applications: Geometric Analysis.

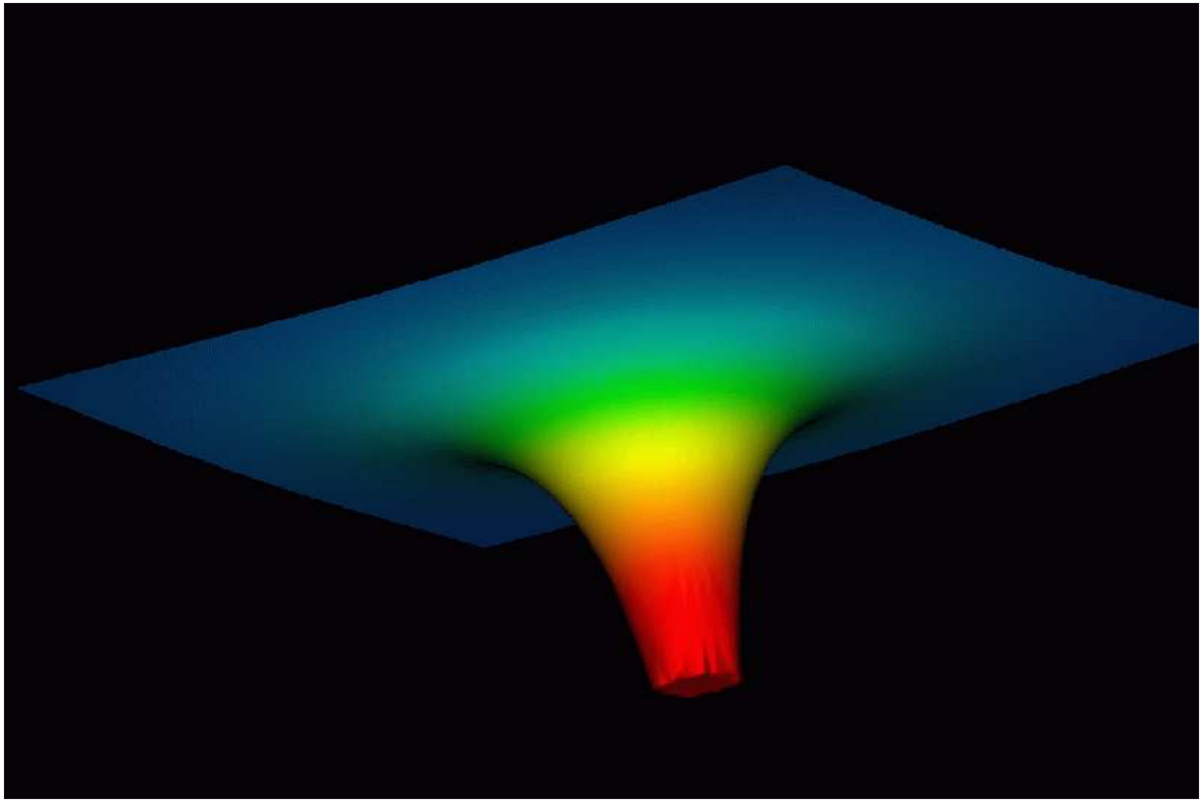
• But this fusion comes with a price: Now space-time itself ends at singularities (also in inflationary scenarios). Big Bang thought of as the Beginning and black hole singularities as the End.

• In particular, the assumption of spatial homogeneity & isotropy implies that the metric has the FLRW form: $ds^2 = -\mathrm{d}t^2 + a^2(t)\ \mathrm{d}\vec{x}^2$

$a(t)$: Scale Factor; Volume $\sim [a(t)]^3$; Curvature $\sim [a(t)]^{-n}$

Einstein Equations $\Rightarrow$ volume $\to 0$ and Curvature $\to \infty$: BIG BANG!!
Classically: PHYSICS STOPS!!

The Big Bang in classical GR



Artist's conception of the Big-Bang. Credits: Pablo Laguna.

In classical general relativity the fabric of space-time is violently torn apart at the Big Bang singularity.

• Expectation: Just an indication that the theory is pushed beyond its domain of validity. Example: H-atom. Energy unbounded below in the classical theory; instability. Quantum theory: $E_o = -\frac{me^4}{2\hbar^2}$

• Is this the case? If so, what is the true physics near the Big Bang? Need a theory which can handle both strong gravity/curvature and quantum physics, i.e., Quantum Gravity.
Classical singularities are gates to Physics Beyond Einstein.

• Serious Challenge to LQG since the Gravity-Geometry duality lies at the heart of this approach. UV-IR Challenge: Do Quantum Geometry effects resolve the big bang singularity? If they are so strong as to overwhelm classical gravitational pull near the singularity, why aren't there observable deviations from GR today. The UV-IR tension!

• In cosmological models Quantum Physics does not stop at singularities. Quantum Riemannian geometry extends its life. Rather startling perspectives on the nature of space-time in LQG. Models simple but, in contrast to string theory, encompass physically most interesting singularities.

• Some Long-Standing Questions expected to be answered by Quantum Gravity from first principles:

⋆ How close to the big-bang does a smooth space-time of GR make sense? (Onset of inflation?)
⋆ Is the Big-Bang singularity naturally resolved by quantum gravity? Or, Is a new principle/ boundary condition at the Big Bang essential?
⋆ Is the quantum evolution across the 'singularity' deterministic?
(answer 'No' e.g. in the Pre-Big-Bang and Ekpyrotic scenarios)
⋆ What is on the other side? A quantum foam? Another large, classical universe? ...
(Fascinating history within classical GR: de Sitter, Tolman, Gamow, Dicke, Sakharov, Weinberg ...)

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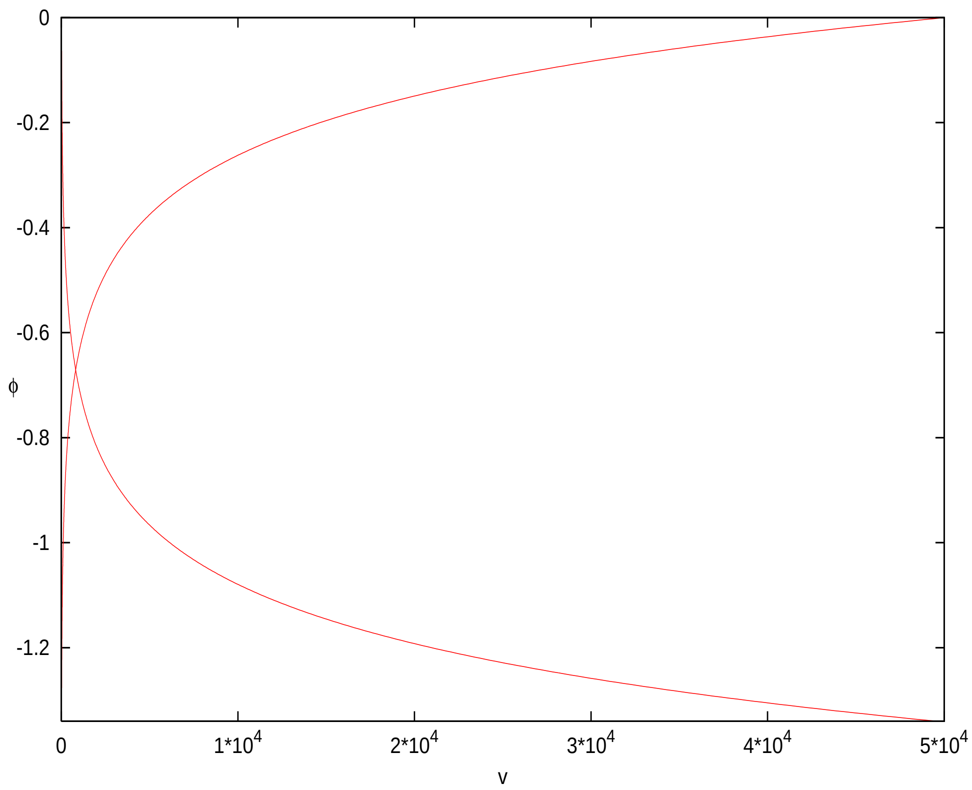
• Emerging Scenario: vast classical regions bridged deterministically by quantum geometry. No new principle needed. (AA, Bojowald, Chiou, Corichi, Pawlowski, Singh, Vandersloot, Wilson-Ewing,...)

• In the classical theory, don't need full Einstein equations in all their complexity. Almost all work in physical cosmology based on homogeneous isotropic models and perturbations thereon. At least in a first step, can use the same strategy in the quantum theory: mini and midi-superspaces.

A Simple Model

The k=0, $\Lambda = 0$ FRW Model coupled to a massless scalar field ϕ. Instructive because every classical solution is singular. Provides a foundation for more complicated models.



Classical trajectories

Older Quantum Cosmology (DeWitt, Misner, ... 70's)

- Since only finite number of DOF $a(t), \phi(t)$, field theoretical difficulties bypassed; analysis reduced to standard quantum mechanics.

- Quantum States: $\Psi(a,\phi)$; $\hat{a}\Psi(a,\phi) = a\Psi(a,\phi)$ etc.
Quantum evolution governed by the Wheeler-DeWitt equation on $\Psi(a,\phi)$
Without additional assumptions, singularity is not resolved by the equation.

- General belief since late seventies: This situation can not be remedied because of von-Neumann's uniqueness theorem for quantum mechanics of systems with a finite number of DOF. How could LQC escape this conclusion?

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- In WDW theory one did not have access to well-defined kinematics. In LQG we do. The procedure that led us to unique kinematics in LQG now leads us to a specific representation of the quantum algebra of LQC. The analog of the continuity assumption of von Neumann fails in LQC $\Rightarrow$ von-Neumann's uniqueness result naturally bypassed (AA, Bojowald, Lewandowski). New Quantum Mechanics! ($\mathcal{H} \neq L^2(\mathbb{R})$ but rather $\mathcal{H} = L^2(\bar{\mathbb{R}}_{\text{Bohr}})$.) Novel features precisely in the deep Planck regime.

LQC versus thew WDW Dynamics

• Because of these key differences, the LQC kinematics does not support the WDW dynamics. The WDW differential equation is replaced by a difference equation. (AA, Bojowald, Lewandowski, Pawlowski, Singh)

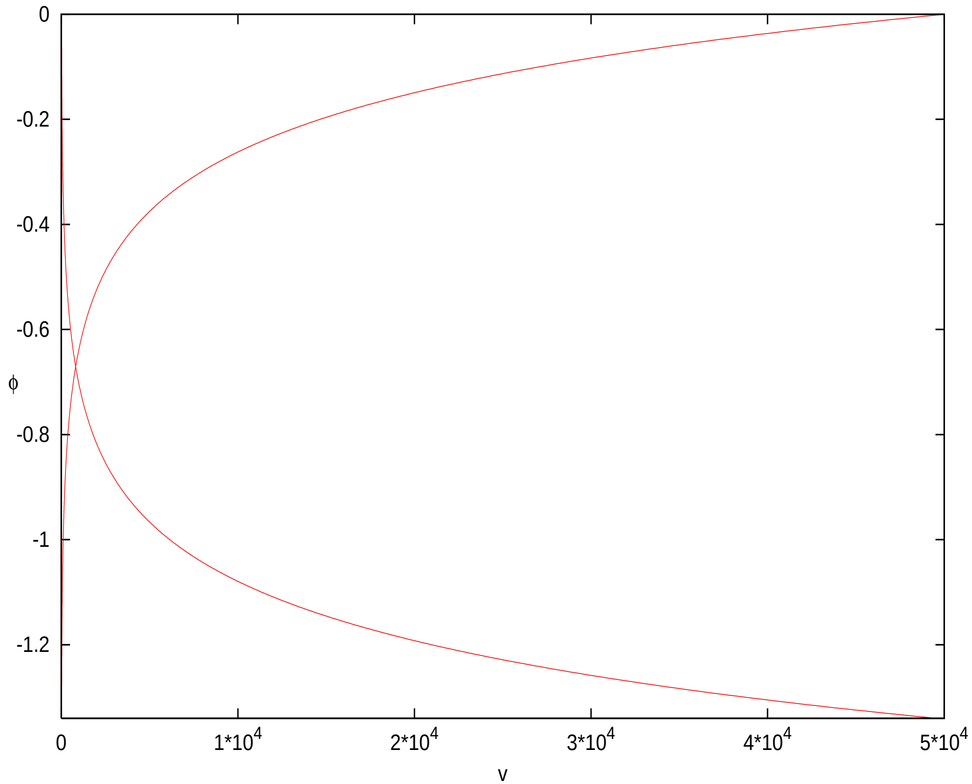
$$C^+(v)\,\Psi(v+4,\,\phi)+C^o(v)\,\Psi(v,\phi)+C^-(v)\Psi(v-4,\,\phi) \;=\; \gamma\ell_P^2\;\hat{H}_\phi\Psi(v,\phi) \qquad (\star)$$

• In LQG, basic geometrical observables such as areas and volumes are quantized. The area operator has a smallest eigenvalue, the area gap Δ.

We will see that the step size in $(\star)$ is governed by the area gap Δ. Good agreement with the WDW equation at low curvatures but drastic departures in the Planck regime precisely because the WDW theory ignores quantum geometry.

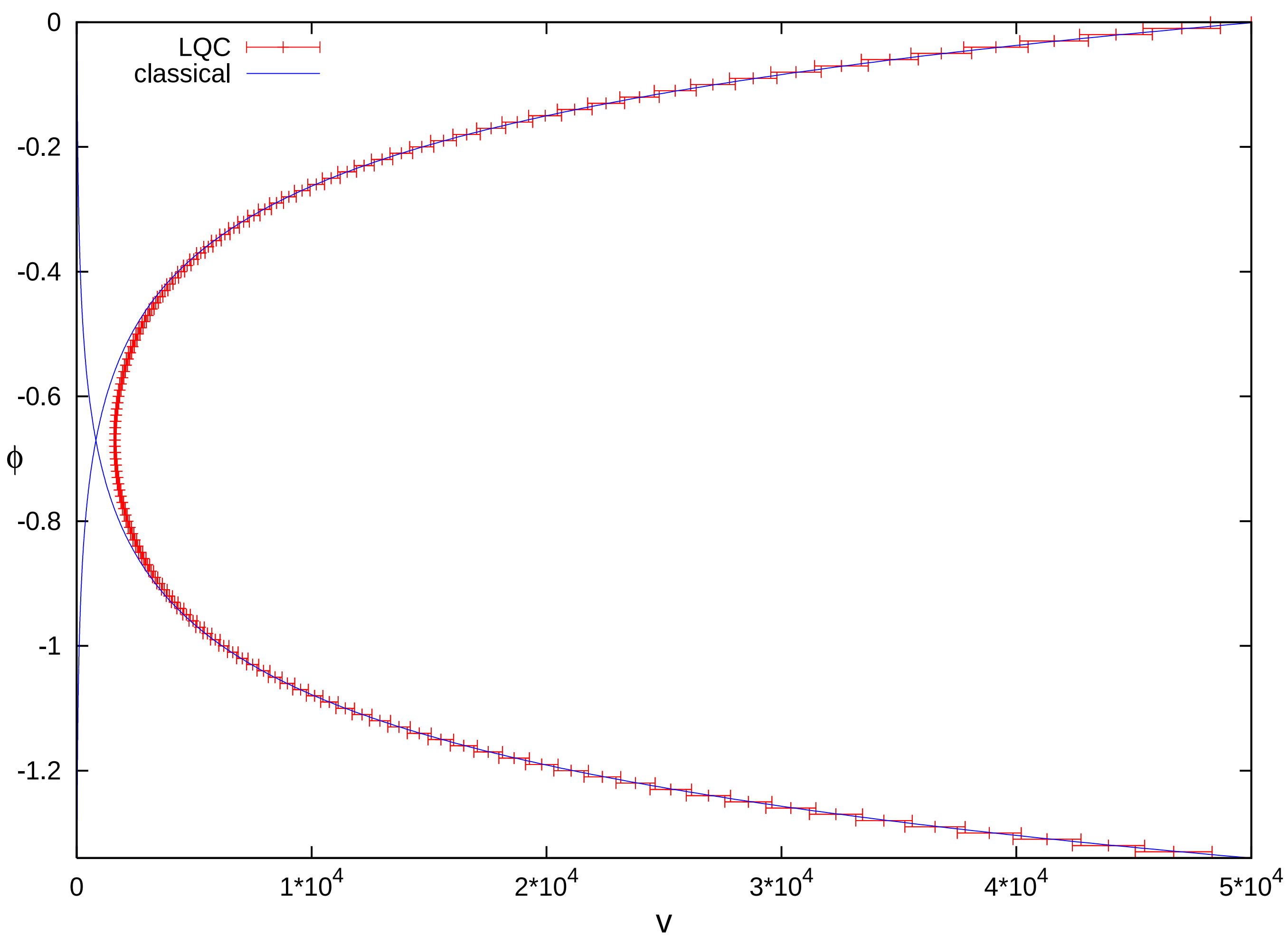
LQC: Return to the k=0 FLRW Model

FLRW, k=0 Model coupled to a massless scalar field ϕ. Instructive because every classical solution is singular. Provides a foundation for more complicated models.



Classical Solutions

k=0 LQC

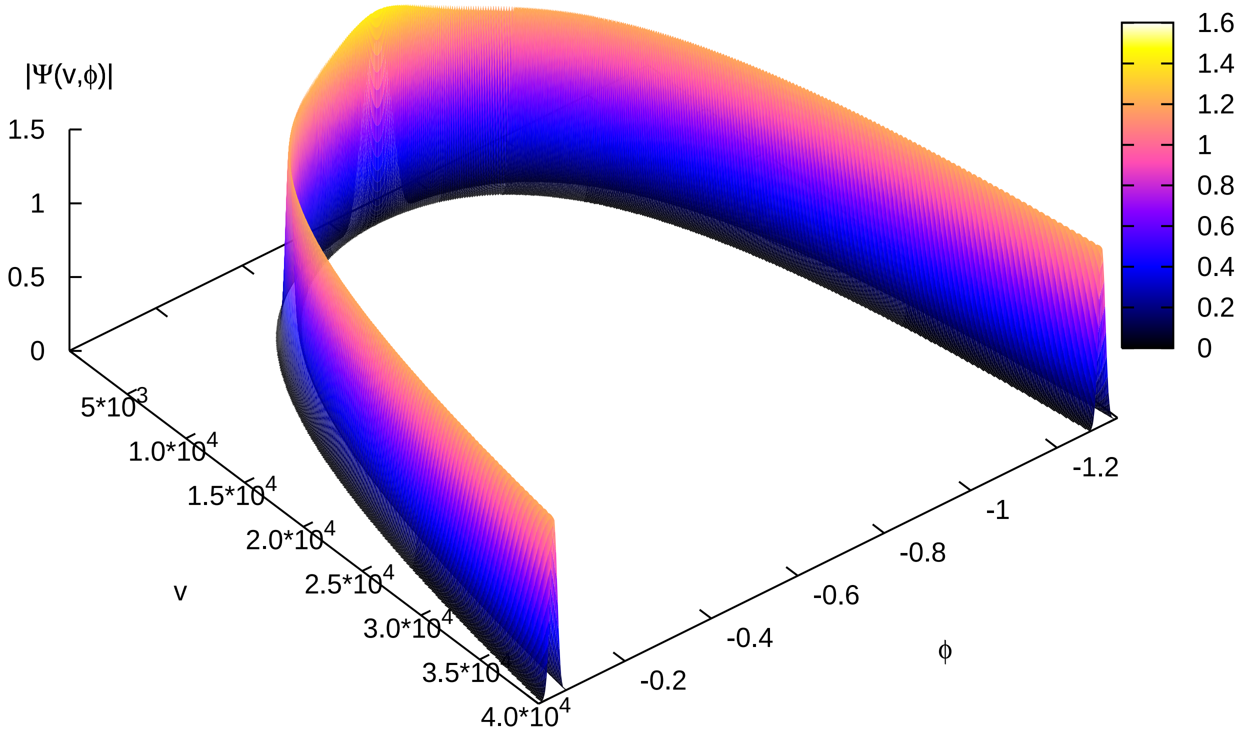


(AA, Pawlowski, Singh)

Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

Gamow's favorite paradigm realized.

k=0 LQC



Absolute value of the physical state $\Psi(v, \phi)$

(AA, Pawlowski, Singh)

k=0 Results

Assume that the quantum state is semi-classical at a late time and evolve backwards and forward. Then: (AA, Pawlowski, Singh)

• The state remains semi-classical till *very* early and *very* late times, i.e., till $R \approx 1/lp^2$ or $\rho \approx 0.01\rho_{\mathrm{Pl}}$. $\Rightarrow$ We know 'from first principles' that space-time can be taken to be classical during the inflationary era (since $\rho \sim 10^{-12}\rho_{\mathrm{Pl}}$ at the onset of inflation).

• In the deep Planck regime, semi-classicality fails. But quantum evolution is well-defined through the Planck regime, and remains deterministic unlike in other approaches. No new principle needed.

• No unphysical matter. All energy conditions satisfied. But the left side of Einstein's equations modified because of quantum geometry effects (discreteness of eigenvalues of geometric operators.): Main difference from WDW theory.

k=0 Results

• To compare with the standard Friedmann equation, convenient to do an algebraic manipulation and move the quantum geometry effect to the right side. Then:

$$(\dot{a}/a)^2 = (8\pi G\rho/3)[1 - \rho/\rho_{\rm crit}] \quad \text{where } \rho_{\rm crit} \sim 0.41\rho_{\rm Pl}.$$

Big Bang replaced by a quantum bounce.

• The matter density operator $\hat{\rho} = \frac{1}{2}\,(\hat{V}_\phi)^{-1}\,\hat{p}_{(\phi)}^2\,(\hat{V}_\phi)^{-1}$ has an absolute upper bound on the physical Hilbert space (AA, Corichi, Singh):

$$\rho_{\rm sup} = \sqrt{3}/16\pi^2\gamma^3 G^2\hbar \approx 0.41\rho_{\rm Pl}!$$

Provides a precise sense in which the singularity is resolved.
(Brunnemann & Thiemann)

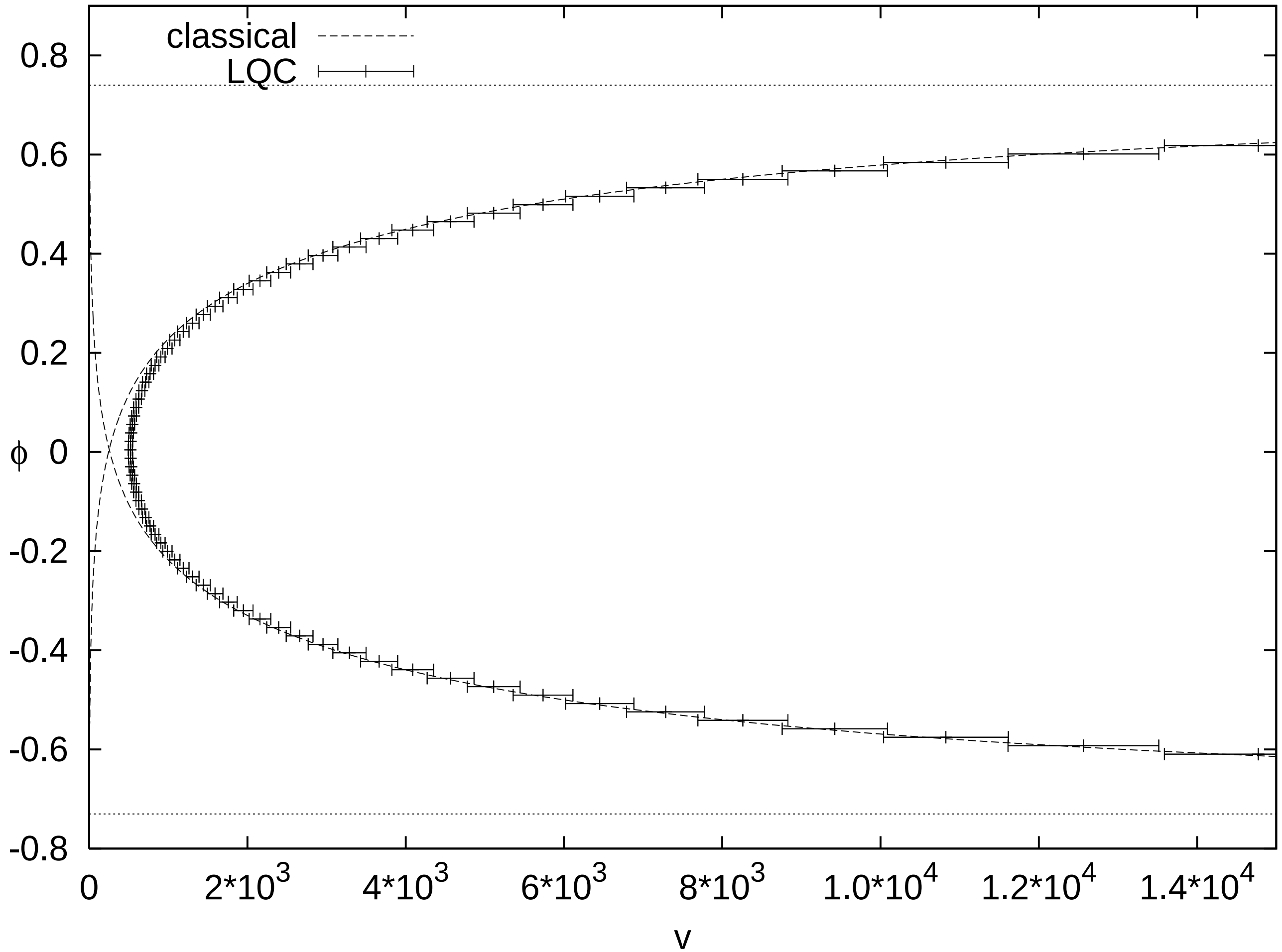
• Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Physics does not end at singularities. A robust super-inflation phase immediately after the bounce.

Generalizations

- More general singularities: At finite proper time, scale factor may blow up, along with similar behavior of density or pressure (Big rip) or curvature or their derivatives diverge at finite values of scale factor (sudden). Quantum geometry resolves all strong singularities in homogeneous isotropic models with $p = p(\rho)$ matter (Singh).

- Inclusion of a cosmological constant and the standard $m^2\phi^2$ inflationary potential. Inclusion of anisotropies. $k = 1$ closed cosmologies. Singularities resolved and Planck scale physics explored in all these cases. (AA, Bentevigna, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ...)

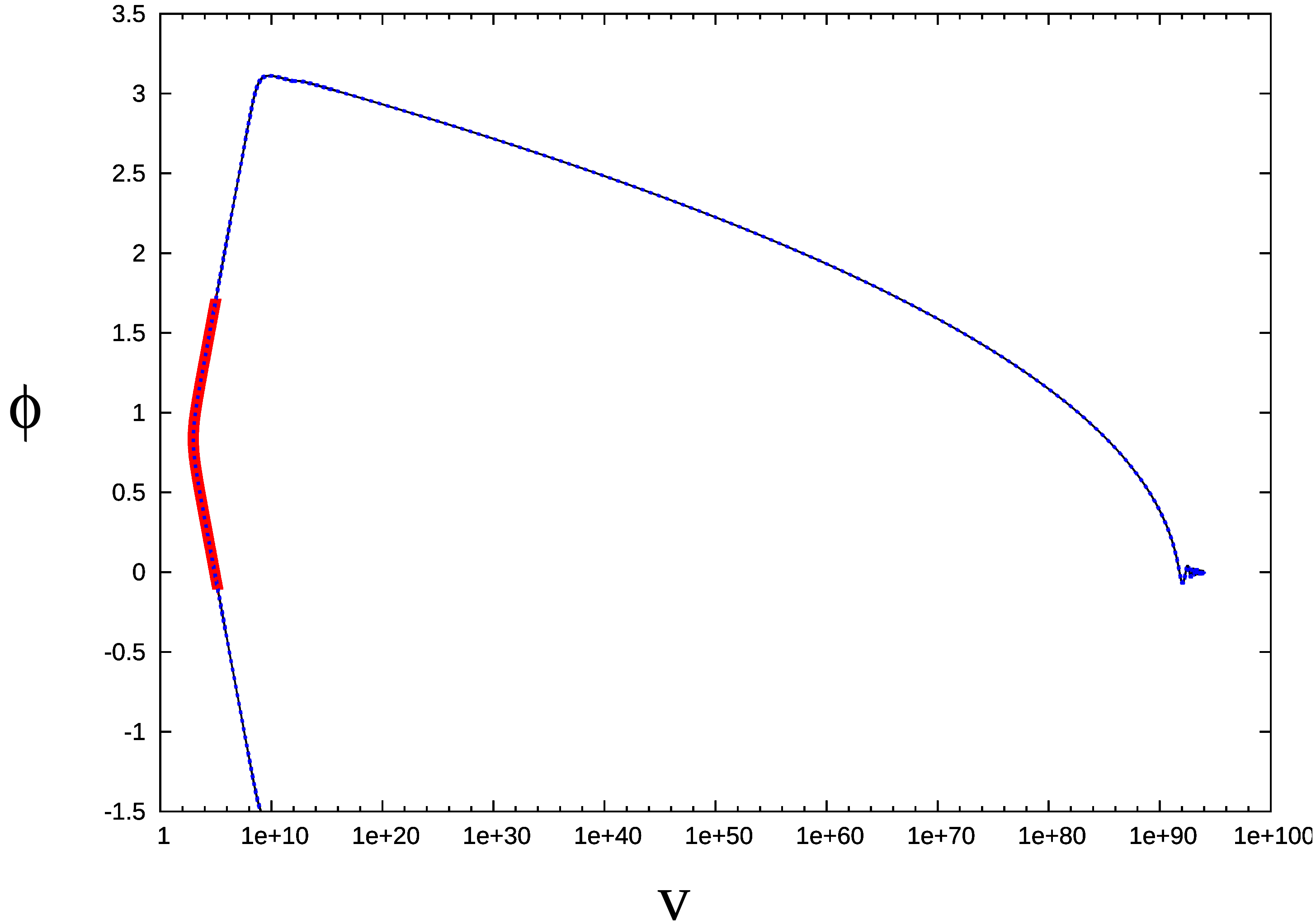
k=0 Model with Positive Λ



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski)

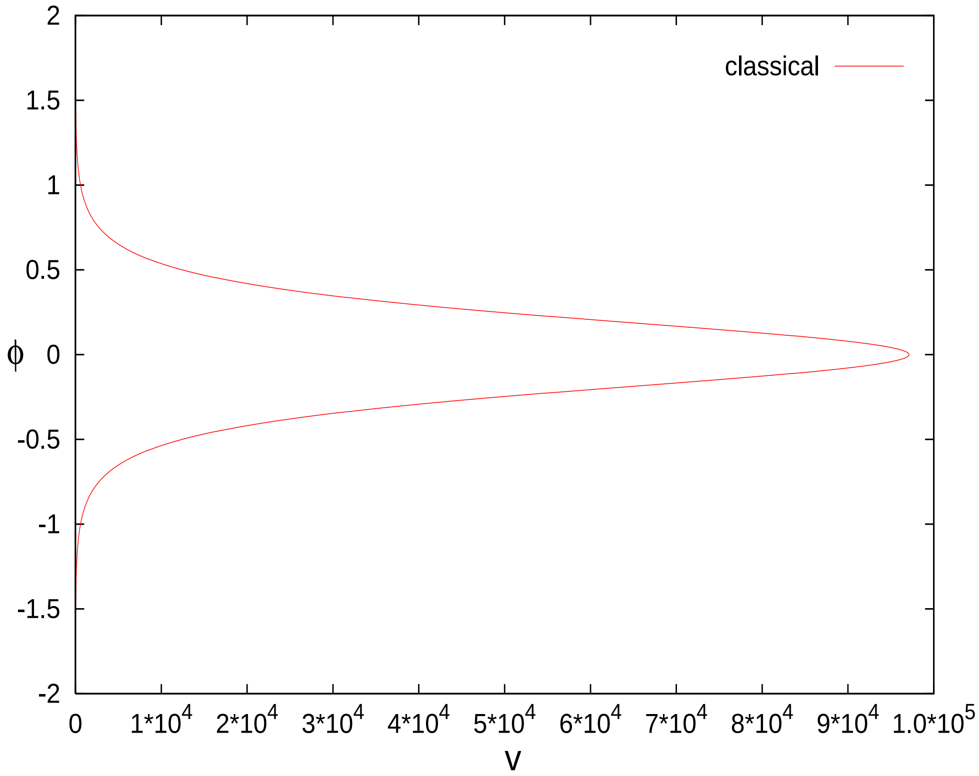
Inflation



Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh).

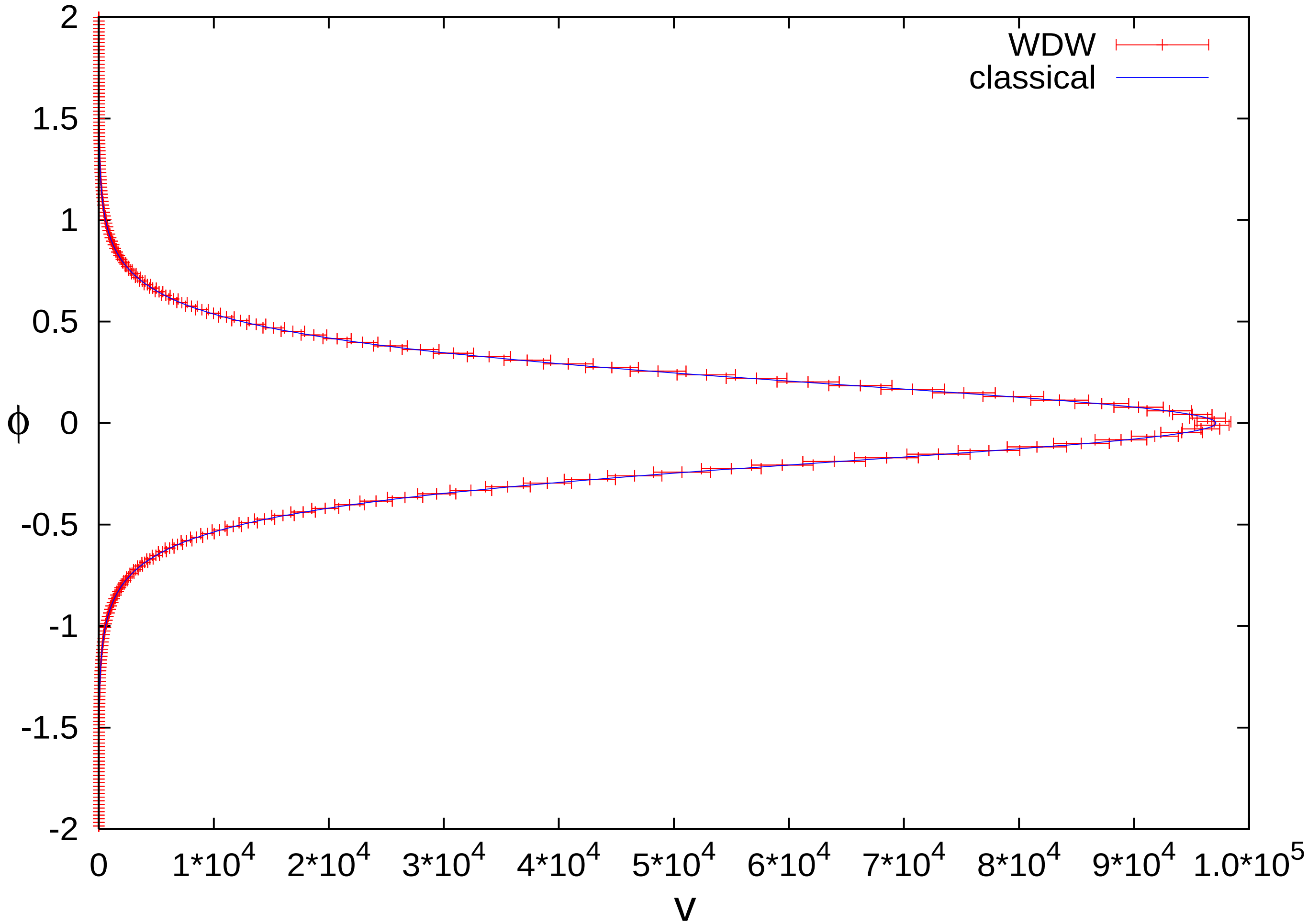
The k=1 Closed Model: Bouncing/Phoenix Universes.

Another Example: $k = 1$ FLRW model with a massless scalar field ϕ. Instructive because again every classical solution is singular; scale factor not a good global clock; More stringent tests because of the classical re-collapse. (Le Maître, Tolman, Sakharov, Dicke,...)



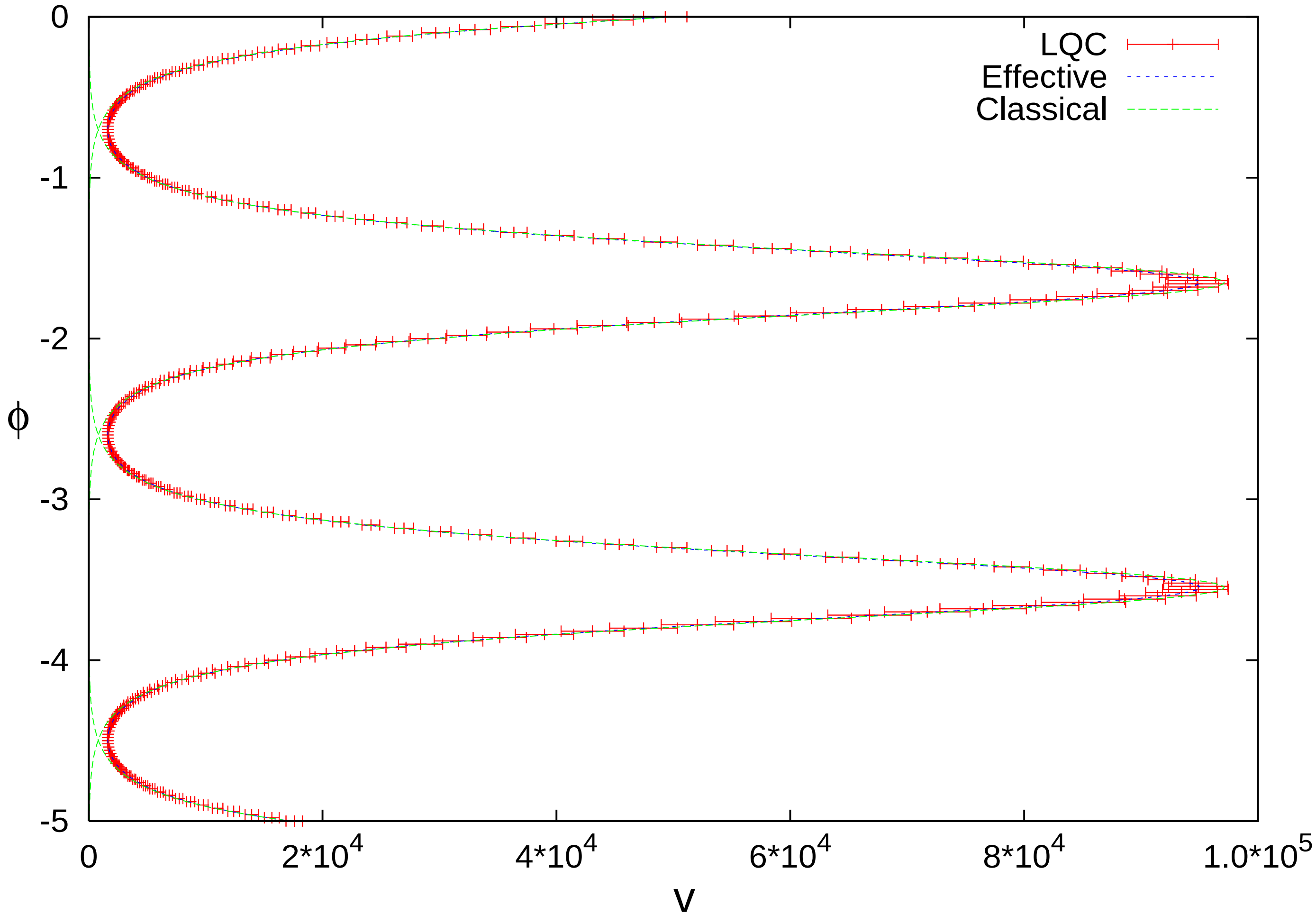
Classical Solutions

k=1 Model: WDW Theory



Expectations values and dispersions of $\hat{V}|_\phi$.

k=1 Model: LQC



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Infinite number of degrees of freedom: Gowdy Model. Again the singularity resolved by quantum geometry effects.

(Martin-Benito, Mena, Pawlowski).

• Together with the BKL conjecture on the nature of generic singularities in general relativity, the accumulated evidence suggests that in Loop Quantum Gravity there may well be a general singularity resolution theorem. If so, we will have come a full circle from the singularity theorems of Penrose, Hawking, and others which marked the beginning of the modern era of general relativity.

4. Summary and Plan

- The interplay between geometry and physics is the deepest feature of general relativity. Loop Quantum Gravity elevates it to the quantum level. Just as classical GR is based on Riemannian geometry, LQG is based on a specific quantum theory of Riemannian geometry.

- Thanks to the LOST-F theorems, background independence leads to a unique quantum kinematics. In it, the basic excitations of Riemannian geometry are 1-dimensional and eigenvalues of geometrical operators are discrete. Continuum Riemannian geometry arises as a coarse grained approximation.

- Quantum geometry has several important physical ramifications. Using the isolated horizon framework to describe black holes in equilibrium, one can account for entropy of physically relevant black holes as well as cosmological horizons in one go. Singularities in homogeneous classical GR are resolved because of unforeseen effects of quantum geometry. Quantum space-times can be vastly larger than what GR had us believe. Many long standing issues in QC have been resolved.

- Challenge to background independent theories: Detailed recovery of classical GR at low curvatures/densities? Met in homogeneous models.

4. Summary and Plan (contd)

• What was there before the big bang? Broadly, “Phoenix Universe”. Since the advent of GR, leading thinkers (Le Maître, Tolman, Gamow, Dicke, Sakharov, Weinberg, ...) have expressed hopes and philosophical preferences. Now, because the singularity is resolved and LQC equations provide a deterministic evolution from the pre-bing to the post-big-bang branch, what happens is not a philosophical preference but depends on the cosmological parameters of today’s universe.

• In the remaining 4 lectures, I will focus on LQC and use it to illustrate ideas, constructions and results of LQC. Specifically, I will
i) Discuss in detail the new Quantum Mechanics underlying LQC;
ii) Show how the singularity resolution arises in the FLRW model I discussed (exactly soluble!);
iii) Discuss key features of more complicated models;
iv) Construct a path integral description to illustrate spin foam constructions; and
v) Discuss elements of QFT on quantum cosmological space-times and current work on phenomenology if time permits.