Supergravity

A. B. Lahanas

University of Athens
Nuclear and Particle Physics Section Athens - Greece

Outline

Outline

Outline

Outline

Outline

Why Supergravity ?

- The fermion - boson symmetry extends the space - time symmetries and hence relevant to the theory of gravity.
- Supergravities are natural generalizations of global Supersymmetries when these are promoted to local symmetries.
- Supergravities connect the weak and the Planck scales and seem to play a key role in unification scenarios.
- Arise in particular compactification schemes in the string theory.
- Why not ?

Why Supergravity ?

- The fermion - boson symmetry extends the space - time symmetries and hence relevant to the theory of gravity.
- Supergravities are natural generalizations of global Supersymmetries when these are promoted to local symmetries.
- Supergravities connect the weak and the Planck scales and seem to play a key role in unification scenarios.
- Arise in particular compactification schemes in the string theory.
- Why not ?

Gauged Supersymmetry

- **Gauge principle:**
  - Gauge group with generators $\left[T^a\ T^b\right] = i\, f^{abc}\, T^c$
  - Fields transform $f \rightarrow exp(-i\,\lambda^a\, T^a)\, f$
  - Gauge fields $A_\mu \equiv A^a_\mu\, T^a$ transform as $\delta A_\mu = \partial_\mu \lambda - i\,[\lambda,\, A_\mu]$
  - Covariant derivatives $D_\mu f = \partial_\mu f + i\, g\, A_\mu\, f$

Curvature (gauge field strength !)

$$R_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\,[A_\mu\, A_\nu]$$

In components $R_{\mu\nu} = R^a_{\mu\nu}\, T^a$: $R^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - f^{abc}\, A^b_\mu\, A^c_\nu$

- Gauge invariant Lagrangian :

$$Trace(R_{\mu\nu}\, R^{\mu\nu}) + |D_\mu \Phi|^2 + i\,\overline{\Psi}\,\gamma^\mu\, D_\mu \Psi + \ldots$$

Gauged Supersymmetry

- **Gauge principle:**
  - Gauge group with generators $[T^a\ T^b] = i\, f^{abc}\, T^c$
  - Fields transform $f \to exp(-i\,\lambda^a\, T^a)\, f$
  - Gauge fields $A_\mu \equiv A_\mu^a\, T^a$ transform as $\delta\, A_\mu = \partial_\mu \lambda - i\, [\lambda\,,\ A_\mu]$
  - Covariant derivatives $D_\mu\, f = \partial_\mu\, f + i\, g\, A_\mu\, f$

Curvature (gauge field strength !)

$$R_{\mu\,\nu} = \partial_\mu\, A_\nu - \partial_\nu\, A_\mu + i\,[\, A_\mu\ A_\nu\,]$$

In components $R_{\mu\,\nu} = R^a_{\mu\,\nu}\, T^a$: $R^a_{\mu\,\nu} = \partial_\mu\, A^a_\nu - \partial_\nu\, A^a_\mu - f^{abc}\, A^b_\mu\, A^c_\nu$

- Gauge invariant Lagrangian :

$$Trace\,(\, R_{\mu\,\nu}\, R^{\mu\,\nu}\,) + |\, D_\mu\, \Phi\,|^2 + i\,\overline{\Psi}\,\gamma^\mu\, D_\mu\, \Psi + \ldots$$

Gauging the Poincare symmetry :

- Gauge group = Poincare with generators P^m , M^{mn} inducing **local translations** and **local Lorentz**.
- Gauge fields e_μ^m for translations, ω_μ^{mn} for Lorentz,

$$A_\mu \equiv e_\mu^m P_m - \frac{1}{2} \omega_\mu^{mn} M_{mn}$$

- Curvature : $$R_{\mu\nu} = R_{\mu\nu}^m P_m - \frac{1}{2} R_{\mu\nu}^{mn} M_{mn}$$

A Lagrangian invariant under local translations and local Lorentz is not invariant under general coordinate transformations (g.c.t.)

Unless a curvature constraint is imposed !

$$R_{\mu\nu}^m = 0$$

Gauging the Poincare symmetry :

- Gauge group = Poincare with generators P^m , $M^{m\,n}$ inducing **local translations** and **local Lorentz**.
- Gauge fields e_μ^m for translations, $\omega_\mu^{m\,n}$ for Lorentz,

$$A_\mu \equiv e_\mu^m\, P_m - \frac{1}{2}\,\omega_\mu^{m\,n}\, M_{m\,n}$$

- Curvature : $$R_{\mu\,\nu} = R_{\mu\,\nu}^m\, P_m - \frac{1}{2}\, R_{\mu\,\nu}^{m\,n}\, M_{m\,n}$$

A Lagrangian invariant under local translations and local Lorentz is not invariant under general coordinate transformations (g.c.t.)

Unless a curvature constraint is imposed !

$$R_{\mu\,\nu}^m = 0$$

Gauging the Poincare symmetry :

- Gauge group = Poincare with generators P^m , M^{mn} inducing **local translations** and **local Lorentz**.
- Gauge fields e_μ^m for translations, ω_μ^{mn} for Lorentz,

$$A_\mu \equiv e_\mu^m P_m - \frac{1}{2}\omega_\mu^{mn} M_{mn}$$

- Curvature : $$R_{\mu\nu} = R_{\mu\nu}^m P_m - \frac{1}{2} R_{\mu\nu}^{mn} M_{mn}$$

A Lagrangian invariant under local translations and local Lorentz is not invariant under general coordinate transformations (g.c.t.)

Unless a curvature constraint is imposed !

$$R_{\mu\nu}^m = 0$$

- With the constraint

$$R^m_{\mu\nu} = D_\mu e^n_\nu - D_\nu e^m_\mu = 0$$

"gct" are combinations of "gauge transformations" !

- Then spin-connection is not independent $\omega^{mn}_\mu = \omega^{mn}_\mu(e)$
- If the vierbein is covariantly constant, **"Tetrad Postulate"** , (= length is preserved under parallel transports !)

$$\mathcal{D}_\mu e^m_\nu \equiv \partial_\mu e^m_\nu + \omega^{mn}_\mu e_{n\nu} - \Gamma^\alpha_{\mu\nu} e^m_\alpha = 0$$

then "gct" connection $\Gamma^k_{\mu\nu}$ is the "Christoffle connection".

- The Einstein - Hilbert Lagrangian (with $R^\lambda_{\mu\nu\rho} = e^\lambda_m e_{n\mu} R^{mn}_{\nu\lambda}$)

$$\mathcal{L}_{EH} = -\frac{1}{2k^2} e R$$

can be viewed as a gauge invariant Lagrangian !

- With the constraint

$$R^m_{\mu\nu} = D_\mu e^n_\nu - D_\nu e^m_\mu = 0$$

"gct" are combinations of "gauge transformations" !

- Then spin-connection is not independent $\omega^{mn}_\mu = \omega^{mn}_\mu(e)$
- If the vierbein is covariantly constant, **"Tetrad Postulate"** , (= length is preserved under parallel transports !)

$$\mathcal{D}_\mu e^m_\nu \equiv \partial_\mu e^m_\nu + \omega^{mn}_\mu e_{n\,\nu} - \Gamma^\alpha_{\mu\,\nu} e^m_\alpha = 0$$

then "gct" connection $\Gamma^k_{\mu\,\nu}$ is the "Christoffle connection".

■ The Einstein - Hilbert Lagrangian (with $R^\lambda_{\mu\nu\rho} = e^\lambda_m e_{n\,\mu} R^{m\,n}_{\nu\,\lambda}$)

$$\mathcal{L}_{EH} = -\frac{1}{2\,k^2}\, e\, R$$

can be viewed as a gauge invariant Lagrangian !

Gauging the N = 1 Supersymmetry :

- Gauge group : N = 1 SUSY with generators P^m M^{mn} Q_α $\overline{Q}^{\dot\alpha}$
- In addition to e_μ^m , ω_μ^{mn} need gauge fields for the SUSY generators $Q\,,\,\bar{Q} \Longrightarrow \psi_\mu\,,\,\bar\psi_\mu$ both vector and fermion fields !

$$A_\mu \equiv e_\mu^m P_m - \frac{1}{2}\omega_\mu^{mn} M_{mn} + \frac{k}{2}\overline{\mathbf{Q}}\Psi_\mu$$

- Ψ_μ = **Gravitino** , Majorana spin - $\frac{3}{2}$ massless fermion made of ψ_μ , $\bar\psi_\mu$, superpartner of the graviton.
  $\mathbf{Q}$ = Majorana SUSY generator.
- $R_{\mu\nu}^m = 0 \Longrightarrow$ Spin-connection depends also on the gravitino
  $\mathcal{D}_\mu e_\nu^m = 0 \Longrightarrow$ "gct" connection $\Gamma_{\mu\nu}^k$ receives "torsion" terms !
- From $\delta A_\mu = \partial_\mu\lambda - i\,[\lambda,\,A_\mu]$ can read SUSY transformations

$$\delta_Q e_\mu^m = -i\,k\,\bar\epsilon\gamma^m\Psi_\mu \quad , \quad \delta_Q\Psi_\mu = \frac{2}{k}D_\mu\epsilon + \ldots$$

Gauging the N = 1 Supersymmetry :

- Gauge group : N = 1 SUSY with generators P^m M^{mn} Q_α $\overline{Q}^{\dot{\alpha}}$
- In addition to e_μ^m , ω_μ^{mn} need gauge fields for the SUSY generators Q , $\bar{Q}$ $\Longrightarrow$ ψ_μ , $\bar{\psi}_\mu$ both vector and fermion fields !

$$A_\mu \equiv e_\mu^m P_m - \frac{1}{2}\omega_\mu^{m\,n} M_{m\,n} + \frac{k}{2}\overline{\mathbf{Q}}\Psi_\mu$$

- Ψ_μ = **Gravitino** , Majorana spin - $\frac{3}{2}$ massless fermion made of ψ_μ , $\bar{\psi}_\mu$, superpartner of the graviton.
  $\mathbf{Q}$ = Majorana SUSY generator.
- $R_{\mu\nu}^m = 0 \Longrightarrow$ Spin-connection depends also on the gravitino
  $\mathcal{D}_\mu e_\nu^m = 0 \Longrightarrow$ "gct" connection $\Gamma_{\mu\,\nu}^k$ receives "torsion" terms !
- From $\delta A_\mu = \partial_\mu\lambda - i\,[\lambda\,,\,A_\mu]$ can read SUSY transformations

$$\delta_Q e_\mu^m = -i\,k\,\bar{\epsilon}\gamma^m\Psi_\mu \quad , \quad \delta_Q\Psi_\mu = \frac{2}{k}D_\mu\epsilon + \ldots$$

Gauging the N = 1 Supersymmetry :

- Gauge group : N = 1 SUSY with generators P^m M^{mn} Q_α $\overline{Q}^{\dot\alpha}$
- In addition to e_μ^m , ω_μ^{mn} need gauge fields for the SUSY generators Q , $\bar{Q}$ $\Longrightarrow$ ψ_μ , $\bar{\psi}_\mu$ both vector and fermion fields !

$$A_\mu \equiv e_\mu^m P_m - \frac{1}{2}\omega_\mu^{mn} M_{mn} + \frac{k}{2}\overline{\mathbf{Q}}\Psi_\mu$$

- Ψ_μ = **Gravitino** , Majorana spin - $\frac{3}{2}$ massless fermion made of ψ_μ , $\bar{\psi}_\mu$, superpartner of the graviton.
  $\mathbf{Q}$ = Majorana SUSY generator.
- $R_{\mu\nu}^m = 0 \Longrightarrow$ Spin-connection depends also on the gravitino
  $\mathcal{D}_\mu e_\nu^m = 0 \Longrightarrow$ "gct" connection $\Gamma_{\mu\nu}^k$ receives "torsion" terms !
- From $\delta A_\mu = \partial_\mu\lambda - i\,[\lambda, A_\mu]$ can read SUSY transformations

$$\delta_Q e_\mu^m = -i\,k\,\bar{\epsilon}\gamma^m\Psi_\mu \quad , \quad \delta_Q\Psi_\mu = \frac{2}{k}D_\mu\epsilon + \ldots$$

The Lagrangian $\mathcal{L}_{SUGRA}$ is invariant undel local SUSY transformations

$$\mathcal{L}_{SUGRA} = -\frac{1}{2k^2}\, e\, R - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\, \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_\rho \Psi_\sigma + \mathcal{L}_{aux}$$

$D_\mu = \partial_\mu - \frac{i}{2}\omega_\mu^{mn}\sigma_{mn}$ is the spin covariant derivative.

Describes the interactions of a **graviton**, with helicities ± 2, and its superpartner **gravitino** , massless Majorana fermion with helicities $\pm\frac{3}{2}$.

- The **graviton** and the **gravitino** are the physical d.o.f of the **graviton multiplet** .
- To build realistic theories need construct **Chiral** and **Vector** multiplets, in curved space-time, and couple them to gravity !

The easiest (?) way is through the **Superconformal Symmetry** !

The Lagrangian $\mathcal{L}_{SUGRA}$ is invariant undel local SUSY transformations

$$\mathcal{L}_{SUGRA} = -\frac{1}{2k^2} e R - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_\rho \Psi_\sigma + \mathcal{L}_{aux}$$

$D_\mu = \partial_\mu - \frac{i}{2}\omega_\mu^{mn}\sigma_{mn}$ is the spin covariant derivative.

Describes the interactions of a **graviton**, with helicities ± 2, and its superpartner **gravitino** , massless Majorana fermion with helicities $\pm\frac{3}{2}$.

- The **graviton** and the **gravitino** are the physical d.o.f of the **graviton multiplet** .
- To build realistic theories need construct **Chiral** and **Vector** multiplets, in curved space-time, and couple them to gravity !

The easiest (?) way is through the **Superconformal Symmetry** !

The Lagrangian $\mathcal{L}_{SUGRA}$ is invariant undel local SUSY transformations

$$\mathcal{L}_{SUGRA} = -\frac{1}{2\,k^2}\, e\, R - \frac{1}{2}\, \epsilon^{\mu\nu\rho\sigma}\, \bar{\Psi}_\mu\, \gamma_5\, \gamma_\nu\, D_\rho\, \Psi_\sigma + \mathcal{L}_{aux}$$

$D_\mu = \partial_\mu - \frac{i}{2}\, \omega_\mu^{mn}\, \sigma_{mn}$ is the spin covariant derivative.

Describes the interactions of a **graviton**, with helicities ± 2, and its superpartner **gravitino** , massless Majorana fermion with helicities $\pm\frac{3}{2}$.

- The **graviton** and the **gravitino** are the physical d.o.f of the **graviton multiplet** .
- To build realistic theories need construct **Chiral** and **Vector** multiplets, in curved space-time, and couple them to gravity !

The easiest (?) way is through the **Superconformal Symmetry** !

Superconformal group :

Superset of the Poincare group including (in addition)

- Conformal transformations
- Dilatations
- 2 Supersymmetries (Yes two !)
- $U_A(1)$ axial symmetry

Easy to construct SC (= Superconformal) multiplets and build invariant actions !

Introducing a "compensating" chiral multiplet, with Weyl-weight = 1

$$S = a + \theta \chi_S + \theta \theta F_S$$

and imposing the gauge fixing

$$S = 1 + \theta \theta F_S$$

Superconformal breaks to **Superpoincare** !

Superconformal group :

Superset of the Poincare group including (in addition)

- Conformal transformations
- Dilatations
- 2 Supersymmetries (Yes two !)
- $U_A(1)$ axial symmetry

Easy to construct SC (= Superconformal) multiplets and build invariant actions !

Introducing a "compensating" chiral multiplet, with Weyl-weight = 1

$$S = a + \theta \chi_S + \theta\theta F_S$$

and imposing the gauge fixing

$$S = 1 + \theta\theta F_S$$

Superconformal breaks to **Superpoincare** !

- **Superconformal group :**

  Superset of the Poincare group including (in addition)

  - Conformal transformations
  - Dilatations
  - 2 Supersymmetries (Yes two !)
  - $U_A(1)$ axial symmetry

- Easy to construct SC (= Superconformal) multiplets and build invariant actions !

- Introducing a "compensating" chiral multiplet, with Weyl-weight = 1

$$S = a + \theta\chi_S + \theta\theta F_S$$

and imposing the gauge fixing

$$S = 1 + \theta\theta F_S$$

Superconformal breaks to **Superpoincare** !

- The one supersymmetry surviving the gauge fixing is a combination of the two **Superconformal** symmetries !
- **SC** multiplets turn to multiplets of the N = 1 local SUSY !

The building blocks (= multiplets) of the N = 1 local SUSY :

Chiral multiplet	Σ
Vector multiplet	V
Gravity multiplet	E

From any chiral multiplet $\boldsymbol{\Sigma = z + \theta\,\chi_L + \theta\,\theta\,h'}$ its kinetic multiplet $\boldsymbol{T(\Sigma)}$ can be constructed which includes the derivatives $\not{\partial}\,\chi_L$, $\Box z$

$$\Sigma\ T(\Sigma)\,|_{\theta^2}\ =\ \textbf{kinetic terms}$$

- The off - shell gravity multiplet ***E*** includes six auxiliary fields, in addition to the vierbein and the gravitino, so that the number of fermionic and bosonic components match,

$$E = (e_\mu^m \quad \Psi_\mu \quad A_\mu \quad S \quad P)$$

- Any chiral multiplet $\Sigma = (z , \chi_L , h')$, which can be also product of other chiral multiplets, couples to the gravitational multiplet in an invariant way :

$$S \sim \int d^4 x \, e \left[h' + k (S - i P) z - i k \overline{\Psi}_{\mu_L} \gamma^\mu \chi_L + i k^2 \overline{\Psi}_{\mu_L} \sigma^{\mu\nu} \Psi_R \, z \right]$$

- The general N = 1 Supergravity Lagrangian, in D = 4, has been derived long ago,

Cremmer, Julia, Scherk, Ferrara, Girardello, Nieuwenhuizen 1979
Cremmer, Ferrara, Girardello, van Proyen, 1982

- The off - shell gravity multiplet E includes six auxiliary fields, in addition to the vierbein and the gravitino, so that the number of fermionic and bosonic components match,

$$E = (e_\mu^m \quad \Psi_\mu \quad A_\mu \quad S \quad P)$$

- Any chiral multiplet $\Sigma = (z , \chi_L , h')$, which can be also product of other chiral multiplets, couples to the gravitational multiplet in an invariant way :

$$S \sim \int d^4 x\, e \left[h' + k (S - i P) z - i k \overline{\Psi}_{\mu_L} \gamma^\mu \chi_L + i k^2 \overline{\Psi}_{\mu_L} \sigma^{\mu\nu} \Psi_R\, z \right]$$

- The general N = 1 Supergravity Lagrangian, in D = 4, has been derived long ago,

Cremmer, Julia, Scherk, Ferrara, Girardello, Nieuwenhuizen 1979
Cremmer, Ferrara, Girardello, van Proyen, 1982

- The off - shell gravity multiplet $\boldsymbol{E}$ includes six auxiliary fields, in addition to the vierbein and the gravitino, so that the number of fermionic and bosonic components match,

$$E = (e_\mu^m \quad \Psi_\mu \quad A_\mu \quad S \quad P)$$

- Any chiral multiplet $\boldsymbol{\Sigma = (z , \chi_L , h')}$, which can be also product of other chiral multiplets, couples to the gravitational multiplet in an invariant way :

$$S \sim \int d^4 x \, e \left[h' + k (S - i P) z - i k \overline{\Psi}_{\mu_L} \gamma^\mu \chi_L + i k^2 \overline{\Psi}_{\mu_L} \sigma^{\mu\nu} \Psi_R \, z \right]$$

- The general N = 1 Supergravity Lagrangian, in D = 4, has been derived long ago,

Cremmer, Julia, Scherk, Ferrara, Girardello, Nieuwenhuizen 1979
Cremmer, Ferrara, Girardello, van Proyen, 1982

- The Lagrangians can be finally expressed in terms of three arbitrary functions which depend on the scalar fields involved z_i , $z^{j\,*}$

Superpotential	$W(z)$
Kähler function	$K(z, z^*)$
Gauge kinetic function	$f_{ab}(z)$

- N = 1 SUGRA depends on the combination

$$\mathcal{G}(z, z^*) = K(z, z^*) - \ln |W(z)|^2 / m_P{}^6$$

and its derivatives !

$$(k^{-1} = m_P \equiv M_{Planck} / \sqrt{8\pi} \simeq 2.4 \times 10^{18}\ GeV)$$

- The Lagrangians can be finally expressed in terms of three arbitrary functions which depend on the scalar fields involved z_i , $z^{j\,*}$

Superpotential	$W(z)$
Kähler function	$K(z, z^*)$
Gauge kinetic function	$f_{ab}(z)$

- N = 1 SUGRA depends on the combination

$$\mathcal{G}(z, z^*) = K(z, z^*) - \ln |W(z)|^2 / m_P{}^6$$

and its derivatives !

$(k^{-1} = m_P \equiv M_{Planck} / \sqrt{8\pi} \simeq 2.4 \times 10^{18}\ GeV)$

- The bosonic part of the Lagrangian :

$$e^{-1}\mathcal{L}_B = -\frac{m_P{}^2}{2}R - \mathcal{G}_i^j D_\mu z_j D^\mu z^{*i} + m_P{}^4 e^{\mathcal{G}}(3 + \mathcal{G}^i \mathcal{G}_i^{-1j}\mathcal{G}_j)$$
$$-\frac{1}{4}(Re\, f_{ab})\, G^{\mu\nu,(a)} G_{\mu\nu}^{(b)} - \frac{1}{2}(Re\, f^{-1}_{ab})\, D^a D^b$$
$$+\frac{i}{2}(Im\, f_{ab})\, G^{\mu\nu,(a)} \tilde{G}_{\mu\nu}^{(b)}$$

with : $\mathcal{G}^i \equiv m_P \frac{\partial \mathcal{G}}{\partial z_i}$, $\mathcal{G}_j^i \equiv m_P{}^2 \frac{\partial^2 \mathcal{G}}{\partial z_i \partial z^{*j}}$, $D^a \equiv g\,\mathcal{G}^i T_i^{ja} z_j$ etc.

- For canonical kinetic terms :

$$\mathcal{G} = -\frac{z_i z^{*i}}{m_P{}^2} \quad , \quad f_{ab} = \delta_{ab}$$

but not mandatory in a theory of Gravity !

- The bosonic part of the Lagrangian :

$$e^{-1}\mathcal{L}_B = -\frac{{m_P}^2}{2}R - \mathcal{G}_i^j D_\mu z_j D^\mu z^{*i} + {m_P}^4 e^{\mathcal{G}}(3 + \mathcal{G}^i \mathcal{G}_i^{-1j}\mathcal{G}_j) - \frac{1}{4}(Re\, f_{ab})\, G^{\mu\nu,(a)} G_{\mu\nu}^{(b)} - \frac{1}{2}(Re\, f^{-1}_{ab})\, D^a D^b + \frac{i}{2}(Im\, f_{ab})\, G^{\mu\nu,(a)} \tilde{G}_{\mu\nu}^{(b)}$$

with : $\mathcal{G}^i \equiv m_P \frac{\partial \mathcal{G}}{\partial z_i}$, $\mathcal{G}_j^i \equiv {m_P}^2 \frac{\partial^2 \mathcal{G}}{\partial z_i \partial z^{*j}}$, $D^a \equiv g\, \mathcal{G}^i T_i^{j\,a} z_j$ etc.

- For canonical kinetic terms :

$$\mathcal{G} = -\frac{z_i z^{*i}}{{m_P}^2} \quad , \quad f_{ab} = \delta_{ab}$$

but not mandatory in a theory of Gravity !

Scalar Potential

- The scalar potential :

$$V \;=\; -\,m_P{}^4\, e^{\mathcal{G}}\,(\,3 + \mathcal{G}^{\,i}\,\mathcal{G}_i^{-1j}\,\mathcal{G}_j\,) + \frac{1}{2}\,(\,Re\,f^{-1}_{a\,b}\,)\,D^a\,D^b$$

The potential is not positive definite and SUSY can be broken with vanishing vacuum energy unlike in global SUSY !

- In the minimal case : $\mathcal{G} \;=\; -\,\frac{z_i\,z^{*\,i}}{M^2}$, $f_{a\,b} \;=\; \delta_{a\,b}$

$$V \;=\; e^{\,|z_i|^2/\,m_P{}^2}\,\left[\,\left|\frac{\partial\,W}{\partial z_i} + \frac{W}{m_P{}^2}\,z^{*\,i}\right|^2 - \frac{|W|^2}{m_P{}^2}\right] + \frac{1}{2}\,(\,D^a\,)^2$$

In the "flat limit" $m_P \to \infty$ the result $V \;=\; \left|\frac{\partial\,W}{\partial z_i}\right|^2 + \frac{1}{2}\,(\,D^a\,)^2$ of the rigid N = 1 SUSY is recovered.

Super - Higgs effect !

The vacuum energy is not an order parameter for SUSY breaking in local Supersymmetry and one can impose vanishing cosmological constant and break Supersymmetry at the same time !

If the vacuum energy $V_0 = 0$ vanishes at the minimum $z_i^0 = 0$ of the potential and $W_0 \neq 0$ then local SUSY is spontaneously broken !

The reason is that $V_0 = 0$ and $W_0 \neq 0$ result to some $\langle \mathcal{G}^i \rangle_0 \neq 0$ and some auxiliary fields (F - terms) develop non-vanishing vevs

$$\langle h_i' \rangle \sim W_0 \langle \mathcal{G}^i \rangle_0 \neq 0$$

If local SUSY is broken, a bilinear mass term arises :

$$m_P \, e^{-\mathcal{G}_0/2} \; \overline{\Psi}_{\mu_L} \gamma^\mu \eta_L$$

with η defined by $\eta_L = \langle \mathcal{G}^i \rangle_0 \chi_{Li}$

Super - Higgs effect !

The vacuum energy is not an order parameter for SUSY breaking in local Supersymmetry and one can impose vanishing cosmological constant and break Supersymmetry at the same time !

If the vacuum energy $V_0 = 0$ vanishes at the minimum $z_i^0 = 0$ of the potential and $W_0 \neq 0$ then local SUSY is spontaneously broken !

The reason is that $V_0 = 0$ and $W_0 \neq 0$ result to some $\langle \mathcal{G}^i \rangle_0 \neq 0$ and some auxiliary fields (F - terms) develop non-vanishing vevs

$$\langle h_i' \rangle \sim W_0 \langle \mathcal{G}^i \rangle_0 \neq 0$$

If local SUSY is broken, a bilinear mass term arises :

$$m_P \, e^{-\mathcal{G}_0/2} \; \overline{\Psi}_{\mu_L} \gamma^\mu \eta_L$$

with η defined by $\quad \eta_L = \langle \mathcal{G}^i \rangle_0 \chi_{Li}$

Super - Higgs effect !

The vacuum energy is not an order parameter for SUSY breaking in local Supersymmetry and one can impose vanishing cosmological constant and break Supersymmetry at the same time !

If the vacuum energy $V_0 = 0$ vanishes at the minimum $z_i^0 = 0$ of the potential and $W_0 \neq 0$ then local SUSY is spontaneously broken !

The reason is that $V_0 = 0$ and $W_0 \neq 0$ result to some $\langle \mathcal{G}^i \rangle_0 \neq 0$ and some auxiliary fields (F - terms) develop non-vanishing vevs

$$\langle h_i' \rangle \sim W_0 \langle \mathcal{G}^i \rangle_0 \neq 0$$

If local SUSY is broken, a bilinear mass term arises :

$$m_P \, e^{-\mathcal{G}_0/2} \, \overline{\Psi}_{\mu_L} \gamma^\mu \eta_L$$

with η defined by $\eta_L = \langle \mathcal{G}^i \rangle_0 \, \chi_{L i}$

Super - Higgs effect !

The vacuum energy is not an order parameter for SUSY breaking in local Supersymmetry and one can impose vanishing cosmological constant and break Supersymmetry at the same time !

If the vacuum energy $V_0 = 0$ vanishes at the minimum $z_i^0 = 0$ of the potential and $W_0 \neq 0$ then local SUSY is spontaneously broken !

The reason is that $V_0 = 0$ and $W_0 \neq 0$ result to some $\langle \mathcal{G}^i \rangle_0 \neq 0$ and some auxiliary fields (F - terms) develop non-vanishing vevs

$$\langle h_i' \rangle \sim W_0 \langle \mathcal{G}^i \rangle_0 \neq 0$$

If local SUSY is broken, a bilinear mass term arises :

$$m_P \, e^{-\mathcal{G}_0/2} \, \overline{\Psi}_{\mu_L} \gamma^\mu \eta_L$$

with η defined by $\quad \eta_L = \langle \mathcal{G}^i \rangle_0 \, \chi_{L i}$

This term is the analog of

$$g\,\upsilon\,A^{\mu}\,\partial_{\mu}\,G$$

in spontaneously broken gauge theories, with G the Goldstone mode !

- The fermion η is the Goldstone fermion (Goldstino) of the broken supersymmetry and has zero mass !
- By use of $V_0 = 0$ and by defining

$$\Psi'_{\mu L} = \Psi_{\mu L} - \frac{1}{3}\gamma_{\mu}\,\eta_L - m_P\,e^{-\mathcal{G}_0/2}\,\partial_{\mu}\,\eta_L$$

the Goldstino is eliminated and Ψ'_{μ} acquires a mass!

$$m_{3/2} = m_P\,e^{-\mathcal{G}_0/2}$$

Super - HIGGS effect !

The Goldstino mode is absorbed by the gravitino that gets a mass describing 4 - spin d.o.f.

This term is the analog of

$$g\, v\, A^{\mu}\, \partial_{\mu}\, G$$

in spontaneously broken gauge theories, with G the Goldstone mode !

- The fermion η is the Goldstone fermion (Goldstino) of the broken supersymmetry and has zero mass !
- By use of $V_0 = 0$ and by defining

$$\Psi'_{\mu L} = \Psi_{\mu L} - \frac{1}{3}\gamma_{\mu}\, \eta_L - m_P\, e^{-\mathcal{G}_0/2}\, \partial_{\mu}\, \eta_L$$

the Goldstino is eliminated and Ψ'_{μ} acquires a mass!

$$m_{3/2} = m_P\, e^{-\mathcal{G}_0/2}$$

Super - HIGGS effect !

The Goldstino mode is absorbed by the gravitino that gets a mass describing 4 - spin d.o.f.

This term is the analog of

$$g\, \upsilon\, A^{\mu}\, \partial_{\mu}\, G$$

in spontaneously broken gauge theories, with G the Goldstone mode !

- The fermion η is the Goldstone fermion (Goldstino) of the broken supersymmetry and has zero mass !
- By use of $V_0 = 0$ and by defining

$$\Psi'_{\mu L} = \Psi_{\mu L} - \frac{1}{3}\gamma_{\mu}\, \eta_L - m_P\, e^{-\mathcal{G}_0/2}\, \partial_{\mu}\, \eta_L$$

the Goldstino is eliminated and Ψ'_{μ} acquires a mass!

$$m_{3/2} = m_P\, e^{-\mathcal{G}_0/2}$$

Super - HIGGS effect !
The Goldstino mode is absorbed by the gravitino that gets a mass describing 4 - spin d.o.f.

This term is the analog of

$$g\,\upsilon\,A^{\mu}\,\partial_{\mu}\,G$$

in spontaneously broken gauge theories, with G the Goldstone mode !

- The fermion η is the Goldstone fermion (Goldstino) of the broken supersymmetry and has zero mass !
- By use of $V_0 = 0$ and by defining

$$\Psi'_{\mu L} = \Psi_{\mu L} - \frac{1}{3}\gamma_{\mu}\,\eta_L - m_P\,e^{-\mathcal{G}_0/2}\,\partial_{\mu}\,\eta_L$$

the Goldstino is eliminated and Ψ'_{μ} acquires a mass!

$$m_{3/2} = m_P\,e^{-\mathcal{G}_0/2}$$

Super - HIGGS effect !

The Goldstino mode is absorbed by the gravitino that gets a mass describing 4 - spin d.o.f.

Analogies with SB Gauge Theories striking !

Higgs effect	Super - Higgs effect
$G \quad =$ Goldstone	$\eta \quad =$ Goldstino
$g\, \upsilon\; A^\mu\, \partial_\mu\, G$	$m_P\, e^{-\mathcal{G}_0/2}\; \overline{\Psi}_{\mu_L}\, \gamma^\mu\, \eta_L$
$\delta\, G = \upsilon\omega + \dots$	$\delta\, \eta = M_S^2\, \epsilon + \dots$
$M_A = g\, \upsilon$	$m_{3/2} = M_S^2 / m_P$

- Is gravitino Cosmologically relevant ?
- How is SUSY broken and how this is fed to the Weak scale ?

Analogies with SB Gauge Theories striking !

Higgs effect	Super - Higgs effect
G = Goldstone	η = Goldstino
$g\,v\;A^{\mu}\,\partial_{\mu}\,G$	$m_P\;e^{-\mathcal{G}_0/2}\;\overline{\Psi}_{\mu_L}\,\gamma^{\mu}\,\eta_L$
$\delta\,G = v\,\omega + \ldots$	$\delta\,\eta = M_S^2\,\epsilon + \ldots$
$M_A = g\,v$	$m_{3/2} = M_S^2\,/\,m_P$

- Is gravitino Cosmologically relevant ?
- How is SUSY broken and how this is fed to the Weak scale ?

Analogies with SB Gauge Theories striking !

Higgs effect	**Super - Higgs effect**
G = **Goldstone**	η = **Goldstino**
$g\, v\ A^\mu \partial_\mu G$	$m_P\, e^{-\mathcal{G}_0/2}\ \overline{\Psi}_{\mu L} \gamma^\mu \eta_L$
$\delta G = v\omega + \ldots$	$\delta \eta = M_S^2 \epsilon + \ldots$
$M_A = g\, v$	$m_{3/2} = M_S^2 / m_P$

- Is gravitino Cosmologically relevant ?
- How is SUSY broken and how this is fed to the Weak scale ?

Comments :

- N=1, D=4 Supergravity may be conceived as effective description of the supestring theory valid at Planckian energies, well below the string scale, upon compactification on particular manifolds.

- The arbitrariness encoded within $W(z)$, $K(z, z^*)$, $f_{ab}(z)$ will be lifted once physics beyond the Planck scale is better known !
  - Supergravities at highier dimensions ($D > 4$) seem to play a key role in String theory and Branes.
  - $N = 1$, $D = 11$ is utilized for a unified view of certain types of String Theories.
  - ...

Study of supergravities in extra dimensions offers alternatives towards understanding fundamental mechanisms, important for phenomenological studies, such as the SUSY breaking

Comments :

■ N=1, D=4 Supergravity may be conceived as effective description of the supestring theory valid at Planckian energies, well below the string scale, upon compactification on particular manifolds.

■ The arbitrariness encoded within $W(z), K(z, z^*), f_{ab}(z)$ will be lifted once physics beyond the Planck scale is better known !

- Supergravities at highier dimensions ($D > 4$) seem to play a key role in String theory and Branes.
- $N = 1\,,\ D = 11$ is utilized for a unified view of certain types of String Theories.
- ...

Study of supergravities in extra dimensions offers alternatives towards understanding fundamental mechanisms, important for phenomenological studies, such as the SUSY breaking

Gravitino

- The spin-3/2 gravitino field, partner of graviton. acquires a mass $m_{3/2}$ after SSB of local SUSY (superHiggs effect)

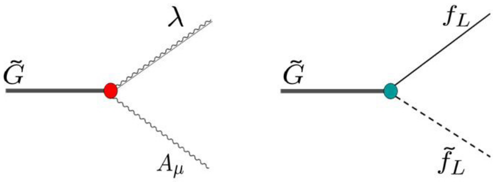
$$m_{3/2} \simeq \frac{M_S^2}{M_{Planck}}$$

- The gravitino couples to matter with Planck suppressed couplings. Its couplins to matter are

$$\begin{aligned} e^{-1}\mathcal{L}_{3/2} = & -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\,\overline{\Psi}_\mu\,\gamma_5\,\gamma_\nu\,\partial_\rho\Psi_\sigma - \frac{m_{3/2}}{2}\,\overline{\Psi}_\mu\,\sigma^{\mu\nu}\,\Psi_\nu \\ & -\frac{i}{4\,m_P}\,\overline{\Psi}_\mu\,\sigma^{\nu\rho}\,\gamma^\mu\,\lambda^a\,G^a_{\nu\rho} \\ & -\frac{i}{\sqrt{2}\,m_P}\left(\,(D_\mu\,\tilde{f}_L^*)\,\overline{\Psi}_\nu\,\gamma^\mu\,\gamma^\nu\,f_L - h.c.\right) \end{aligned}$$

The kinetic and the mass term are shown and $\sigma^{\mu\nu} = \frac{1}{2}\,[\gamma^\mu,\gamma^\nu]$

The couplings to matter are $\sim {m_{Planck}}^{-1}$ (here $\tilde{G}$ denotes the gravitino)

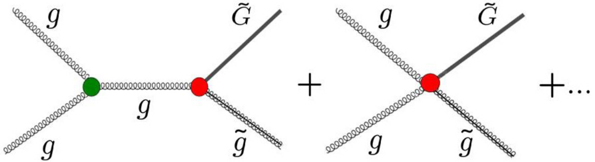


A_μ , λ gauge-boson and its partner (gaugino)

$f_L, \tilde{f}_L$ fermion and its partner (sfermion)

The gravitino couplings are small and they are not in thermal equilibrium in the Early Universe !

$\tilde{G}$'s are produced by inelastic scattering processes during the reheating of the Universe after inflation (Bolz, Brandenburg and Buchmuller 2001, Pradler and Steffen 2006)



gluon+gluon → gluino + Gravitino, $g\,g \rightarrow \tilde{g}\,\tilde{G}$

Solve Boltzmann

$$\frac{d\,n_{\tilde{G}}}{d\,t} + 3\,H\,n_{\tilde{G}} = C_{\tilde{G}} \quad , \quad C_{\tilde{G}} = \text{collision terms}$$

to obtain the gravitino "yield" in the regime $T \leq T_R$,

$$Y_{\tilde{G}} \equiv \frac{n_{\tilde{G}}}{n_\gamma} \simeq 1.9 \times 10^{-12} \left(1 + \frac{m_{\tilde{g}}^2}{3\,m_{3/2}^2}\right) \frac{T}{10^{10}\ GeV}$$

Unstable Gravitino

Decays to radiation $\tilde{G} \rightarrow \gamma + \tilde{\gamma}$

$$\tau \simeq 4 \times 10^8 \left(\frac{100\ GeV}{m_{3/2}}\right)^3 sec$$

Decays to hadrons $\tilde{G} \rightarrow g + \tilde{g}\ ,\ q + \tilde{q}$

$$\tau \simeq 6 \times 10^7 \left(\frac{100\ GeV}{m_{3/2}}\right)^3 sec$$

- For $m_{3/2} = 10^2\ GeV\ -\ 10\ TeV$ gravitino decays during and after primordial Nucleosynthesis with disastrous effects for BBN !
- Their overproduction may dissociate light nuclei $\gamma +^3 He \rightarrow D + p$, $\gamma + D \rightarrow n + p$

Gravitino Problem !

Unstable Gravitino

Decays to radiation $\tilde{G} \rightarrow \gamma + \tilde{\gamma}$

$$\tau \simeq 4 \times 10^8 \left(\frac{100\ GeV}{m_{3/2}} \right)^3 sec$$

Decays to hadrons $\tilde{G} \rightarrow g + \tilde{g}\ ,\ q + \tilde{q}$

$$\tau \simeq 6 \times 10^7 \left(\frac{100\ GeV}{m_{3/2}} \right)^3 sec$$

- For $m_{3/2} = 10^2\ GeV\ -\ 10\ TeV$ gravitino decays during and after primordial Nucleosynthesis with disastrous effects for BBN !
- Their overproduction may dissociate light nuclei $\gamma +^3 He \rightarrow D + p$, $\gamma + D \rightarrow n + p$

Gravitino Problem !

Unstable Gravitino

Decays to radiation $\tilde{G} \to \gamma + \tilde{\gamma}$

$$\tau \simeq 4 \times 10^8 \left(\frac{100\ GeV}{m_{3/2}} \right)^3 sec$$

Decays to hadrons $\tilde{G} \to g + \tilde{g}\ ,\ q + \tilde{q}$

$$\tau \simeq 6 \times 10^7 \left(\frac{100\ GeV}{m_{3/2}} \right)^3 sec$$

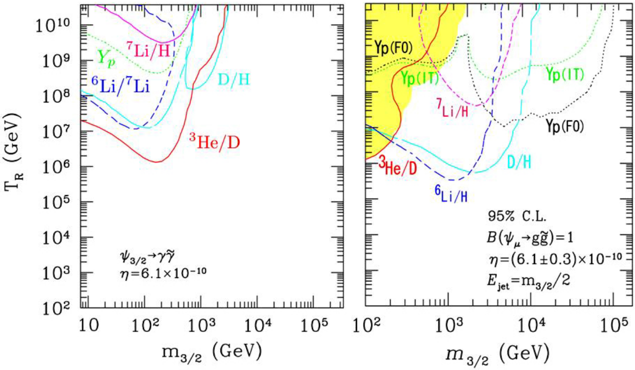
- For $m_{3/2} = 10^2\ GeV\ -\ 10\ TeV$ gravitino decays during and after primordial Nucleosynthesis with disastrous effects for BBN !
- Their overproduction may dissociate light nuclei $\gamma +^3 He \to D + p$, $\gamma + D \to n + p$

Gravitino Problem !

For late decaying particles X bounds on their lifetimes τ_X and abundances Y_X are imposed by BBN. For gravitino these translate to bounds on T_R and its mass $m_{3/2}$!

$$T_R = 10^5 - 10^7\ GeV \quad \text{for} \quad m_{3/2} = 10^2\ GeV - 3\ TeV$$

In contradiction with thermal Leptogenesis scenarios which require $T_R \simeq 10^9\ GeV$ and Inflation models with $T_R > 10^7\ GeV$!



Kawasaki M, Kohri K and Moroi T, 2005

Gravitino DM

If the gravitino is the LSP the gravitino problem may be avoided but the next to LSP particles (NLSP), neutralino $\tilde{\chi}$ or stau $\tilde{\tau}$, decay late !

- $\tilde{\chi}$ as NLSP is disfavoured by BBN bounds (allowed only for unaturally small gravitino masses $m_{3/2} << m_{1/2}$)
- BBN bounds are weaker if $\tilde{\tau}$ is the NLSP

Catalyzed BBN, Pospelov M, 2006

Bound-state formation of long-lived negatively charged particles with the primordial nuclei enhances ^6Li production, by almost seven orders of magnitude, putting severe limits on $\tilde{\tau}$ abundances, prior to their decays !

Gravitino DM

If the gravitino is the LSP the gravitino problem may be avoided but the next to LSP particles (NLSP), neutralino $\tilde{\chi}$ or stau $\tilde{\tau}$, decay late !

- $\tilde{\chi}$ as NLSP is disfavoured by BBN bounds (allowed only for unaturally small gravitino masses $m_{3/2} << m_{1/2}$)
- BBN bounds are weaker if $\tilde{\tau}$ is the NLSP

Catalyzed BBN, Pospelov M, 2006

Bound-state formation of long-lived negatively charged particles with the primordial nuclei enhances ^{6}Li production, by almost seven orders of magnitude, putting severe limits on $\tilde{\tau}$ abundances, prior to their decays !

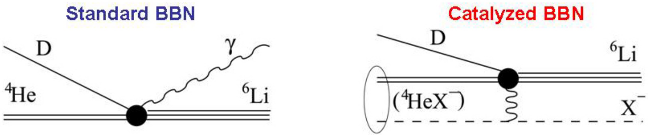
Gravitino DM

If the gravitino is the LSP the gravitino problem may be avoided but the next to LSP particles (NLSP), neutralino $\tilde{\chi}$ or stau $\tilde{\tau}$, decay late !

- $\tilde{\chi}$ as NLSP is disfavoured by BBN bounds (allowed only for unaturally small gravitino masses $m_{3/2} << m_{1/2}$)
- BBN bounds are weaker if $\tilde{\tau}$ is the NLSP

Catalyzed BBN, Pospelov M, 2006

Bound-state formation of long-lived negatively charged particles with the primordial nuclei enhances ^6Li production, by almost seven orders of magnitude, putting severe limits on $\tilde{\tau}$ abundances, prior to their decays !



In CBBN cross-section is enhanced by 7 orders of magnitude !

The NLSP decays $\tilde{\chi} \rightarrow \tilde{G}\gamma$ or $\tilde{\tau} \rightarrow \tilde{G}\gamma$ produce gravitinos non-thermally. This adds up to their thermal production density !

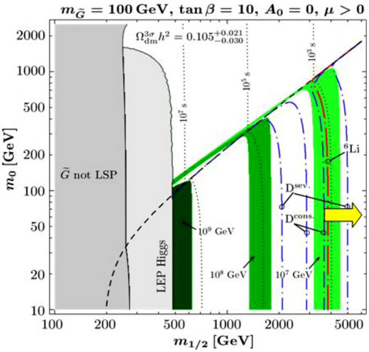
$$\Omega^{\tilde{G}} = \Omega^{\tilde{G}}_{(thermal)} + \frac{m_{3/2}}{m_{NLSP}}\,\Omega^{NLSP}$$

For fixed T_R the WMAP bounds on $\Omega^{\tilde{G}} h_0^2$, and all other available data, shrink the available SUSY parameter space considerably !

The NLSP decays $\tilde{\chi} \to \tilde{G}\gamma$ or $\tilde{\tau} \to \tilde{G}\gamma$ produce gravitinos non-thermally. This adds up to their thermal production density !

$$\Omega^{\tilde{G}} = \Omega^{\tilde{G}}_{(thermal)} + \frac{m_{3/2}}{m_{NLSP}}\, \Omega^{NLSP}$$

For fixed T_R the WMAP bounds on $\Omega^{\tilde{G}} h_0^2$, and all other available data, shrink the available SUSY parameter space considerably !



WMAP bounds

- $T_R = 10^7\ GeV$
- $T_R = 10^8\ GeV$
- $T_R = 10^9\ GeV$

CBBN ^{6}Li bounds

$T_R < 10^7\ GeV$

Pradler J & Steffen F (2007)

Gravitino Dark Matter, with stau NLSP, incosistent with $T_R > 10^7$ *GeV* in the popular supersymmetric models.

Ways to avoid it :

- **Depart from the simple supersymmetric schemes**
- **SUSY violates R-parity**
  $\tilde{\tau}$ decays to SM particles before Nucleosynthesis as its lifetime shortens.
- **Late entropy production mechanisms**
  Gravitino abundances are diluted.
- **Stau abundance $Y_{\tilde{\tau}}$, before its decay, is depleted**
  Can occur by enhancing stau-Higgs coupling or staus annihilate fast via Higgs resonance.
- ...

Gravitino Dark Matter, with stau NLSP, incosistent with $T_R > 10^7\ GeV$ in the popular supersymmetric models.

Ways to avoid it :

- **Depart from the simple supersymmetric schemes**
- **SUSY violates R-parity**
  $\tilde{\tau}$ decays to SM particles before Nucleosynthesis as its lifetime shortens.
- **Late entropy production mechanisms**
  Gravitino abundances are diluted.
- **Stau abundance $Y_{\tilde{\tau}}$, before its decay, is depleted**
  Can occur by enhancing stau-Higgs coupling or staus annihilate fast via Higgs resonance.
- ...

Supersymmetry Breaking

Supersymmetry must be broken at some scale, M_S, to lift up masses of sparticles !

- The potential of discovering SUSY at the LHC depends on the magnitude of M_S .
- The pattern of the low energy parameters and experimental signatures depend on the mechanism of $\not{S}USY$.

Spontaneous breaking of SUSY is an elegant mechanism.

- F - term breaking models (O' Raifertaight)
  F - type auxiliary fields of certain chiral multiplets get non-vanishing vev's, $\langle F_i \rangle \neq 0$
- D - term breaking models (Fayet - Iliopoulos)
  D - type auxiliary fields of U(1) vector multiplets get non-vanishing vev's, $\langle D^a \rangle \neq 0$

Supersymmetry Breaking

A Goldstone fermion (Goldstino) appears, due to the fermionic character of the SUSY generators. In global SUSY

$$\psi = \frac{1}{f} \left[- \left\langle F_i^{\dagger} \right\rangle \psi_i + \frac{i}{\sqrt{2}} \left\langle D^a \right\rangle \lambda^a \right]$$

$f \equiv \sqrt{V_{min}} \sim M_S^2$ sets the order parameter. In SUGRA the picture changes and the Goldstino is absorbed by a massive gravitino !

Dynamical breaking may also a possibility (through condensation of gauginos for instance)

$$M_S = M_P \, exp \left\{ - \frac{8\pi^2}{b_0 \, g^2(M_P)} \right\}$$

g = gauge coupling of a UV free theory at a high scale M_P

Supersymmetry Breaking

Supergravities, String Theories, D - branes, have more to offer about the possible breaking schemes.

Supersymmetry Breaking

- The Weak scale supersymmetry must be broken softly to preserve its good properties in the UV regime. Absence of quadratic divergences protects Higgses from getting large masses.
- Supersymmetry breaking terms must be flavor blind not to conflict with FCNC constraints.

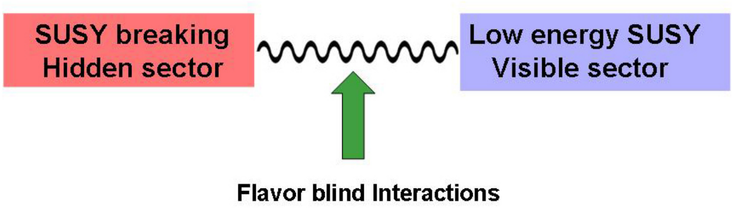
Any viable model must address the origin of $S\!\!\!/USY$!

In most scenarios there are three sectors :

1. The **visible** (or observable) sector
2. The **hidden** sector, where SUSY breaking occurs
3. The **messenger** sector, which communicates SUSY breaking from the hidden to the visible sector

A schematic picture of this scenario $\Longrightarrow$

Supersymmetry Breaking



Supersymmetry Breaking

Messenger sector options :

1. Non-renormalizable operators coupling hidden (H) to visible (V) fields (like in Gravity mediation models)

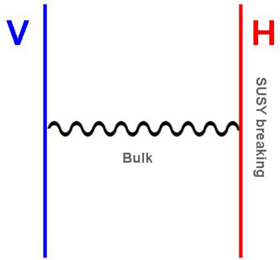
$$\frac{H \times V}{M^n} \quad \Longrightarrow \quad \frac{\langle F_{hidden} \rangle}{M^n} \, V$$

2. Non - singlet, non - SM heavy fields, that feel directly the SUSY breaking and transmit it to the visible fields through loop interactions (like in GMSB models).
3. Anomalies of Scaling (like in AMSB models).
4. ...

Supersymmetry Breaking

In class 1 the hidden-visible interactions may not be direct but induced. This subclass defines the **sequestered** models.

Naturally arise in ED theories where bulk fields mediate the SUSY breaking from the hidden to the visible brane

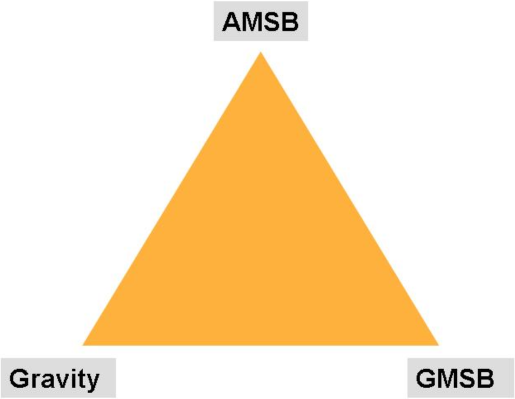


The $\not{S}USY$ mechanisms

Three main mechanisms for SUSY breaking :

- **Gravity** or **Modulus** mediation
  - **Nilles 1984**
  - **Kaplunovsky and Louis 1993**
- **Gauge Mediation** , GMSB
  - **Giudice and Rattazzi 1999**
- **Anomaly Mediation** , AMSB
  - **Giudice, Luty, Murayama and Rattazzi 1998**
  - **Randall and Sundrum 1998**

The $\not{SUSY}$ mechanisms



The $\not{SUSY}$ mechanisms

Two mechanisms may combine :

- **Deflected Anomaly Mediation** (AMSB and GMSB)

- Rattazzi, Strumia and Wells 2000

- Katz, Shadmi and Shirman 1999

- **Mirage Mediation** (AMSB and Gravity/Modulus)

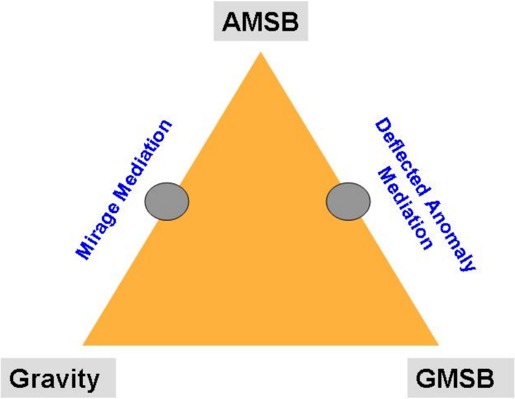
Comparable contributions of the two mechanisms in certain classes of heterotic orbifold models received attention after KKLT - type moduli stabilization in D - brane models **- Kachru, Kallosh, Linde and Trivedi 2003**

Particular class of Type IIB string compactifications with fluxes gives rise to "Mirage Mediation"

- Choi, Falkowski, Nilles, Olechowski and Pokorski 2003
- Choi, Falkowski, Nilles and Olechowski 2005
- Choi, Jeong and Okumura 2005

- Choi and Nilles 2007

The $\not{SUSY}$ mechanisms



The $\not{SUSY}$ mechanisms

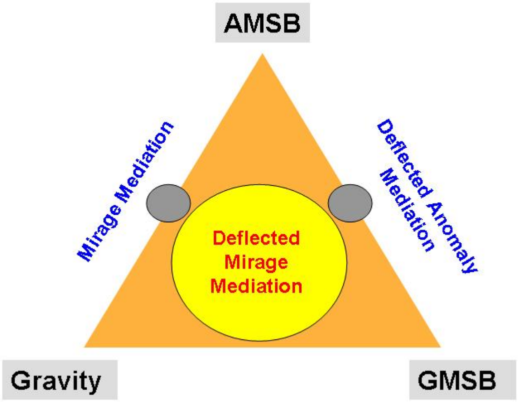
Combinations of all three mechanisms :

- **Deflected Mirage Mediation**

By moduli stabilization all three mechanisms, AMSB and GMSB and Gravity/Modulus, are operative yielding comparable soft SUSY breaking terms !

- Everett, Kim, Ouyang and Zurek 2008

The $\not{SUSY}$ mechanisms



The soft $\not{S}USY$ Lagrangian

The low energy Lagrangian breaks SUSY !

$$\mathcal{L} = \mathcal{L}^{SUSY} + \Delta\,\mathcal{L}^{\not{S}USY}$$

The part $\Delta\mathcal{L}^{\not{S}USY}$must not destabilize the hierarchy, **"soft breaking"** .
Corrections to scalar masses are logarithmic in the cut - off scale Λ .

$$\delta m^2_{scalar} \sim g^2\, M_S^2\, \log\Lambda^2$$

Allowed terms :

- Gaugino masses: $\frac{M_a}{2}\,\lambda^a\,\lambda^a + h.c.$
- Scalar masses: $m^2_{ij}\,\phi_i\,\phi^*_j + b_{ij}\,\phi_i\,\phi_j + h.c.$
- Trilinear scalar couplings : $A_{ijk}\,\phi_i\,\phi_j\,\phi_k + h.c.$

The soft $\not{S}USY$ Lagrangian

The low energy Lagrangian breaks SUSY !

$$\mathcal{L} = \mathcal{L}^{SUSY} + \Delta\, \mathcal{L}^{\not{S}USY}$$

The part $\Delta\mathcal{L}^{\not{S}USY}$must not destabilize the hierarchy, **"soft breaking"** .
Corrections to scalar masses are logarithmic in the cut - off scale Λ .

$$\delta m^2_{scalar} \sim g^2\, M^2_S\, \log \Lambda^2$$

Allowed terms :

- Gaugino masses: $\frac{M_a}{2}\, \lambda^a\, \lambda^a + h.c.$
- Scalar masses: $m^2_{ij}\, \phi_i\, \phi^*_j + b_{ij}\, \phi_i\, \phi_j + h.c.$
- Trilinear scalar couplings : $A_{ijk}\, \phi_i\, \phi_j\, \phi_k + h.c.$

The soft $\not{S}USY$ Lagrangian

- Must be allowed by gauge symmetries
- Be compatible with experimental constraints on FCNC, CP-violation, B, L conservation ...

- The origin and the magnitudes of the soft parameters depend on the particular mechanism which breaks SUSY.
- Phenomenology interesting if they fall in the TeV range, to be accessible at LHC, as demanded by the "gauge hierarchy".
- In the minimal SUSY (MSSM) their number is large ~ 105 but is reduced dramatically in particular schemes (mSUGRA ...)

The soft $\not{SUSY}$ Lagrangian

- Must be allowed by gauge symmetries
- Be compatible with experimental constraints on FCNC, CP-violation, B, L conservation ...

- The origin and the magnitudes of the soft parameters depend on the particular mechanism which breaks SUSY.
- Phenomenology interesting if they fall in the TeV range, to be accessible at LHC, as demanded by the "gauge hierarchy".
- In the minimal SUSY (MSSM) their number is large ~ 105 but is reduced dramatically in particular schemes (mSUGRA ...)

Gravity / Modulus mediated $S\!\!\!/USY$

Realized in Supergravity models. In N=1 SUGRA non-renormalizable interactions couple auxiliary F - terms, that break SUSY, to scalars and gaugino fields

$$\begin{aligned} \mathcal{L}_{mSUGRA} = & -\frac{f_a}{2\,M_P}\,F\,\lambda^a\lambda^a \\ & -\frac{K_{ij}}{2\,M_P^2}\,|F|^2\,\phi_i\,\phi_j^* - \frac{\beta_{ij}}{2!\,M_P}\,F\,\phi_i\,\phi_j \\ & -\frac{\lambda_{ijk}}{3!\,M_P}\,F\,\phi_i\,\phi_j\,\phi_k + h.c. \end{aligned}$$

- M_P is the Planck mass and f_a, K_{ij}, λ_{ijk}, β_{ij} depend on the particular SUGRA scheme !
- If F develops a vev $\langle F \rangle$ by some mechanism soft terms are generated

Gravity / Modulus mediated $\not{SUSY}$

- Gaugino masses: $M_a = f_a \dfrac{\langle F \rangle}{M_P}$
- Scalar masses: $m_{ij}^2 = K_{ij} \dfrac{|\langle F \rangle|^2}{M_P^2}$, $b_{ij} = \beta_{ij} \dfrac{|\langle F \rangle|}{M_P}$
- Trilinear scalar couplings : $A_{ijk} = \lambda_{ijk} \dfrac{\langle F \rangle}{M_P}$

$\langle F \rangle = M_S^2$ results to spontaneous symmetry breaking of local supersymmetry ! The Goldstino mode is absorbed by the gravitino which becomes massive (super-Higgs mechanism) :

$$m_{3/2} \simeq \frac{\langle F \rangle}{M_P}$$

■ The soft $S\!\!\!/USY$ parameters run with the energy scale. Their quoted values are valid at high scales $\simeq M_P$. At the EW scale are determined by running the appropriate RGEs

- All soft parameters

$$m_{soft} \sim m_{3/2} \simeq \frac{\langle F \rangle}{M_P}$$

  and are not flavor blind in general !

- For values

$$\sqrt{\langle F \rangle} \simeq 10^{11} - 10^{12}\ GeV$$

  m_{soft}, $m_{3/2}$ fall in the TeV range and SUSY is accessible to LHC !

- Watch if $m_{3/2} \simeq O(100)\ GeV - O(1)\ TeV$ since gravitino decays during and after Nucleosynthesis, if it is not the LSP, dissociating light element abundances (Gravitino problem)

mSUGRA option :

The particulat choice

$f_a = f$ (common), $K_{ij} = K\,\delta_{ij}$, $\lambda_{ijk} = \lambda\,(\text{Yukawa})_{ijk}$, $\beta_{ij} = \beta\,\mu_{ij}$

leads to common gaugino $M_{1/2}$, scalar m_0 and trilinear A_0 couplings :

$$M_{1/2} = f\,\frac{\langle F \rangle}{M_P}\,,\; m_0^2 = K\,\frac{|\langle F \rangle|^2}{M_P^2}\,,\; A_0 = \lambda\,\frac{\langle F \rangle}{M_P}\,,\; B_0 = \beta\,\frac{\langle F \rangle}{M_P}$$

The soft terms :

$$\mathcal{L}_{mSUGRA} = -\left(\frac{M_{1/2}}{2}\,\lambda^a\lambda^a + h.c.\right) - m_0^2\,|\phi_i|^2 - \left(\frac{A_0}{3!}\,Y_{ijk}\,\phi_i\,\phi_j\,\phi_k - \frac{B_0}{2!}\,\mu_{ij}\,\phi_i\,\phi_j + h.c.\right)$$

mSUGRA option :

The particulat choice

$f_a = f\ (\text{common}),\ K_{ij} = K\,\delta_{ij},\ \lambda_{ijk} = \lambda\,(\text{Yukawa})_{ijk}\ ,\ \beta_{ij} = \beta\,\mu_{ij}$

leads to common gaugino $M_{1/2}$, scalar m_0 and trilinear A_0 couplings :

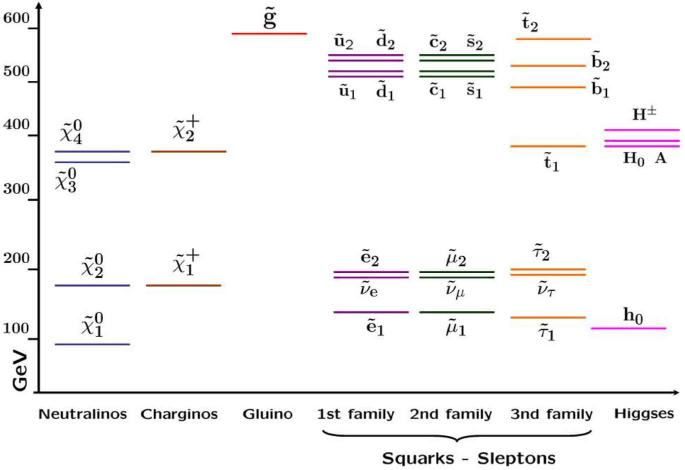
$$M_{1/2} = f\,\frac{\langle F \rangle}{M_P}\ ,\ m_0^2 = K\,\frac{|\langle F \rangle|^2}{M_P^2}\ ,\ A_0 = \lambda\,\frac{\langle F \rangle}{M_P}\ ,\ B_0 = \beta\,\frac{\langle F \rangle}{M_P}$$

The soft terms :

$$\begin{aligned}\mathcal{L}_{mSUGRA} = & -\left(\frac{M_{1/2}}{2}\,\lambda^a\lambda^a + h.c.\right) - m_0^2\,|\phi_i|^2 \\ & -\left(\frac{A_0}{3!}\,Y_{ijk}\,\phi_i\,\phi_j\,\phi_k - \frac{B_0}{2!}\,\mu_{ij}\,\phi_i\,\phi_j + h.c.\right)\end{aligned}$$

mSUGRA mass spectrum :

$$m_0 = 100\ ,\ M_{1/2} = 100\ ,\ A_0 = -100\ ,\ \tan\beta = 10\ ,\ \mu > 0$$



Particular classes of mSUGRA models can be more predictive

- **Polonyi :**

  Singlet hidden chiral multiplet Z and $\langle F_Z \rangle \neq 0$

$$m_0^2 = m_{3/2}^2 \, , \; A_0 \; = \; (\, 3 - \sqrt{3} \,) \; m_{3/2}$$

- **Dilaton - dominated :**

  The dilaton auxiliary field develops a vev $\langle F_{dilaton} \rangle \neq 0$

$$m_0^2 = m_{3/2}^2 \, , M_{1/2} \; = \; -A_0 \; = \; \sqrt{3} \; m_{3/2}$$

- **No - scale :**

  The *Kähler* metric allows for flat potential directions and the gravitino mass is determined by radiative corrections. In a subclass $\not{SUSY}$ is gaugino mass dominated

$$M_{1/2} \; \gg \; m_0 \; , \; A_0$$

Particular classes of mSUGRA models can be more predictive

- **Polonyi :**

  Singlet hidden chiral multiplet Z and $\langle F_Z \rangle \neq 0$

$$m_0^2 = m_{3/2}^2 \,,\ A_0 \ = \ (\, 3 - \sqrt{3}\,)\, m_{3/2}$$

- **Dilaton - dominated :**

  The dilaton auxiliary field develops a vev $\langle F_{dilaton} \rangle \neq 0$

$$m_0^2 = m_{3/2}^2 \,, M_{1/2} \ = \ -A_0 \ = \ \sqrt{3}\, m_{3/2}$$

- **No - scale :**

  The *Kähler* metric allows for flat potential directions and the gravitino mass is determined by radiative corrections. In a subclass $\not{S}USY$ is gaugino mass dominated

$$M_{1/2} \ \gg \ m_0 \ , \ A_0$$

Particular classes of mSUGRA models can be more predictive

- **Polonyi :**

  Singlet hidden chiral multiplet Z and $\langle F_Z \rangle \neq 0$

$$m_0^2 = m_{3/2}^2 \; , \; A_0 \; = \; (\, 3 - \sqrt{3} \,) \; m_{3/2}$$

- **Dilaton - dominated :**

  The dilaton auxiliary field develops a vev $\langle F_{dilaton} \rangle \neq 0$

$$m_0^2 = m_{3/2}^2 \; , M_{1/2} \; = \; -A_0 \; = \; \sqrt{3} \; m_{3/2}$$

- **No - scale :**

  The *Kähler* metric allows for flat potential directions and the gravitino mass is determined by radiative corrections. In a subclass $\not{S}USY$ is gaugino mass dominated

$$M_{1/2} \; \gg \; m_0 \; , \; A_0$$

Particular classes of mSUGRA models can be more predictive

- **Polonyi :**

  Singlet hidden chiral multiplet Z and $\langle F_Z \rangle \neq 0$

$$m_0^2 = m_{3/2}^2 \,,\; A_0 \;=\; (\,3 - \sqrt{3}\,)\; m_{3/2}$$

- **Dilaton - dominated :**

  The dilaton auxiliary field develops a vev $\langle F_{dilaton} \rangle \neq 0$

$$m_0^2 = m_{3/2}^2 \,, M_{1/2} \;=\; -A_0 \;=\; \sqrt{3}\; m_{3/2}$$

- **No - scale :**

  The *Kähler* metric allows for flat potential directions and the gravitino mass is determined by radiative corrections. In a subclass $\not{S}USY$ is gaugino mass dominated

$$M_{1/2} \;\gg\; m_0 \,,\; A_0$$

LSP is usually the lightest of the neutralinos. Gaugino masses run as the gauge couplings and the gluino is heavier than the chargino and neutralino states.

Gauge Mediated Susy Breaking - GMSB

- Transmission of $\not{S}USY$ from Hidden to Visible sector via $SU(3) \times SU(2) \times U(1)$ interactions $\Longrightarrow$ $\not{S}USY$ terms are automatically flavor blind !
- Messengers are heavy $\sim \boldsymbol{M_{mess}}$ chiral multiplets that feel $\not{S}USY$ through their coupling to $\langle \boldsymbol{F} \rangle$ inducing fermion - boson mass splittings within messenger's multiplet

Soft terms :

$$m_{soft} \simeq \frac{\alpha}{4\,\pi} \frac{\langle F \rangle}{M_{mess}}$$

- $M_S \equiv \sqrt{\langle F \rangle}$ can be as low as 10^4 GeV, if M_{mess} is comparable, and gravitino as low as ~ 0.1 eV .
- If $M_{mess} \ll M_P$ the gravitino is typically the LSP

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P} \ll \frac{\alpha}{4\,\pi} \frac{\langle F \rangle}{M_{mess}} \sim m_{soft}$$

Gauge Mediated Susy Breaking - GMSB

- Transmission of $\not{SUSY}$ from Hidden to Visible sector via $SU(3) \times SU(2) \times U(1)$ interactions $\Longrightarrow$ $\not{SUSY}$ terms are automatically flavor blind !
- Messengers are heavy $\sim M_{mess}$ chiral multiplets that feel $\not{SUSY}$ through their coupling to $\langle F \rangle$ inducing fermion - boson mass splittings within messenger's multiplet

Soft terms :

$$m_{soft} \simeq \frac{\alpha}{4\,\pi} \frac{\langle F \rangle}{M_{mess}}$$

- $M_S \equiv \sqrt{\langle F \rangle}$ can be as low as 10^4 GeV, if M_{mess} is comparable, and gravitino as low as ~ 0.1 eV .
- If $M_{mess} \ll M_P$ the gravitino is typically the LSP

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P} \ll \frac{\alpha}{4\,\pi} \frac{\langle F \rangle}{M_{mess}} \sim m_{soft}$$

Gauge Mediated Susy Breaking - GMSB

- Transmission of $\not{S}USY$ from Hidden to Visible sector via $SU(3) \times SU(2) \times U(1)$ interactions $\Longrightarrow$ $\not{S}USY$ terms are automatically flavor blind !
- Messengers are heavy $\sim \boldsymbol{M_{mess}}$ chiral multiplets that feel $\not{S}USY$ through their coupling to $\langle F \rangle$ inducing fermion - boson mass splittings within messenger's multiplet

Soft terms :

$$m_{soft} \simeq \frac{\alpha}{4\,\pi}\,\frac{\langle F \rangle}{M_{mess}}$$

■ $\boldsymbol{M_S} \equiv \sqrt{\langle F \rangle}$ can be as low as $\mathbf{10^4}$ $\boldsymbol{GeV}$, if $\boldsymbol{M_{mess}}$ is comparable, and gravitino as low as $\sim \mathbf{0.1}\ \boldsymbol{eV}$.

■ If $\boldsymbol{M_{mess}} \ll \boldsymbol{M_P}$ the gravitino is typically the LSP

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P} \ll \frac{\alpha}{4\,\pi}\,\frac{\langle F \rangle}{M_{mess}} \sim m_{soft}$$

Modelling GMSB

- N "quark", "lepton" like messenger multiplets $Q, \bar{Q}, L, \bar{L}$ with q.n.

  $Q \sim (3, 1, -1/6)$, $\bar{Q} \sim (\bar{3}, 1, 1/6)$, $L \sim (1, 2, 1)$, $\bar{L} \sim (1, 2, -1)$

  (Q.N. reminiscent of $SU(5)$ $\{5\}, \{\bar{5}\}$ - multiplets to guarantee gauge coupling unification. Other options are also available)

- Couple them to a singlet S chiral field whose scalar component s and its auxiliary field $\langle F_S \rangle$ get a vev.

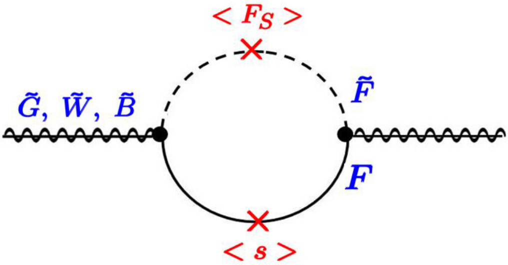
$$W \sim f_Q \, S \, Q \, \bar{Q} \, + \, f_L \, S \, L \, \bar{L}$$

  Upon $\langle F_S \rangle \neq 0$ the messenger fermions F and scalars $\tilde{F}$ are split !

- Thin splitting is transmitted through loops to the visible sector !

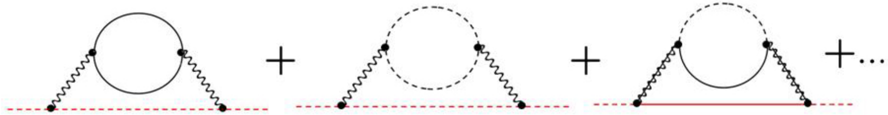
- **$F,\ \tilde{F}$ masses :** $m_F^2 \sim \langle s \rangle^2 \quad , \quad m_{\tilde{F}}^2 \sim \langle s \rangle^2 \left(1 \pm \frac{\Lambda}{\langle s \rangle}\right)$
- **Effective $SU\!\!\!/SY$ scale :** $\Lambda \sim \frac{\langle F_S \rangle}{\langle s \rangle}$

Gaugino mass corrections : $M_i = \frac{\alpha_i}{4\pi} N \Lambda$



1 - loop corrections to gluinos, winos, binos from messenger scalar - fermion exchange

Scalars get masses at 2-loops :



In the limit of small relative mass splitting, $\langle F_S \rangle \ll \langle s \rangle^2$

$$m_\phi^2 = 2\,\Lambda^2\,N\left[\left(\frac{\alpha_3}{4\,\pi}\right)^2 C_3^\phi + \left(\frac{\alpha_2}{4\,\pi}\right)^2 C_2^\phi + \left(\frac{\alpha_1}{4\,\pi}\right)^2 C_1^\phi\right] > 0$$

$C_3^\phi = \frac{4}{3}$ for squarks

$C_2^\phi = \frac{3}{4}$ for squark, slepton, Higgs doublets

$C_1^\phi = \frac{3}{5}\left(\frac{Y_\phi}{2}\right)^2$ for each ϕ of hypercharge Y_ϕ

Trilinear scalar couplings arise at 2-loops

Suppressed by additional $\frac{\alpha}{4\,\pi}$ relative to gaugino masses $\Longrightarrow$
Good approximation to take them zero !

- Soft gaugino and scalar masses comparable and dominant sources of $S\!\!\!/USY$

$$M_i\;,\;m_\phi\;\sim\;\frac{\alpha}{4\,\pi}\,\Lambda$$

- b.c. for soft masses hold at the messenger scale M_{mes} which along with Λ and the number of messenger multiplets, N , are the model parameters.

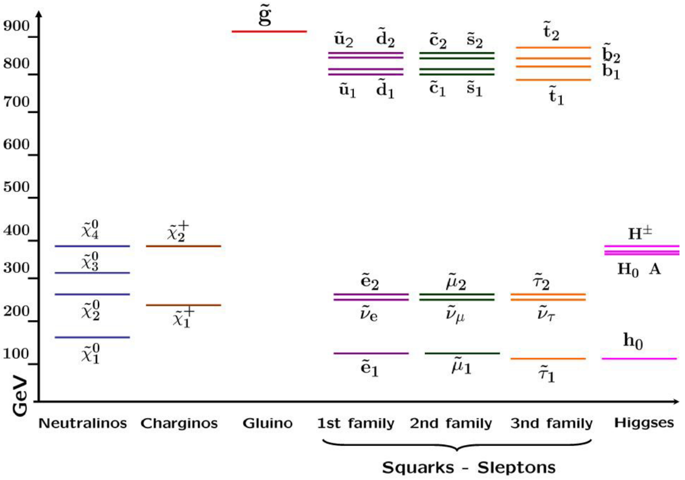
- Due to

$$\frac{M_i}{m_\phi}\sim\sqrt{N}$$

$N=1$ the NLSP is the bino-like neutralino
$N\geq 1$ the NLSP may be the $\tilde{\tau}$

GMSB mass spectrum :

$\Lambda = 40\ TeV\ ,\ M_{mess} = 80\ TeV\ ,\ N_{mess} = 3\ ,\ \tan\beta = 15\ ,\ \mu > 0$



Anomaly Mediated Susy Breaking - AMSB

- Scale invariant renormalizable couplings at the loop level depend on the renormalization scale which is an anomaly of the scaling symmetry $\Longrightarrow$ Soft terms are induced which may dominate $S\!\!\!/USY$!

$$\text{Gaugino masses} \quad : \; M_i = \frac{\beta_i}{g_i}\, m_{3/2}$$

$$\text{Scalar masses} \quad : \; m_\phi^2 = -\frac{1}{4}\left(\frac{\partial \gamma_\phi}{\partial g_i}\beta_i + \frac{\partial \gamma_y}{\partial y}\beta_y\right) m_{3/2}^2$$

$$\text{Trilinear couplings} \quad : \; A_{ijk} = \frac{1}{2}\left(\dot{\gamma}_i + \dot{\gamma}_j + \dot{\gamma}_k\right) m_{3/2}$$

$\beta_{i,y}$ beta functions of the gauge, Yukawa couplings, g_i, y

γ_i anomalous dimension of the field ϕ_i and $\dot{\gamma} \equiv d\gamma / d \ln \mu$

A_{ijk} defined as $\frac{1}{3!} A_{ijk}\, y_{ijk}\, \phi_i\, \phi_i\, \phi_i$,

- Soft contributions are scale invariant
- Solves SUSY flavor and CP - problem
- Soft terms depend on a single parameter $m_{3/2}$ and typically

$$m_{soft} \sim \frac{g^2}{16\,\pi^2}\, m_{3/2}$$

They are in the TeV range for $m_{3/2} \sim 100\ TeV$!

- Gaugino masses are non-universal as in the ratios

$$M_1 : M_2 : M_3 = 2.8 : 1 : 7.1$$

LSP $=$ neutral wino only slightly lighter, by a few $\mathcal{O}(100)\ MeV$, than the charged wino $\Longrightarrow$ Long - lived lightest chargino !

- Problematic since slepton masses tachyonic $m_{\tilde{L}}^2 < 0$ and charged breaking minima appear. Ad hoc addition of a m_0^2 to sfermion masses raises masses but RGE invariance of m_{soft} is lost. Other resolutions in Deflected Anomaly Mediation or other schemes ...

- Soft contributions are scale invariant
- Solves SUSY flavor and CP - problem
- Soft terms depend on a single parameter $m_{3/2}$ and typically

$$m_{soft} \sim \frac{g^2}{16\,\pi^2}\, m_{3/2}$$

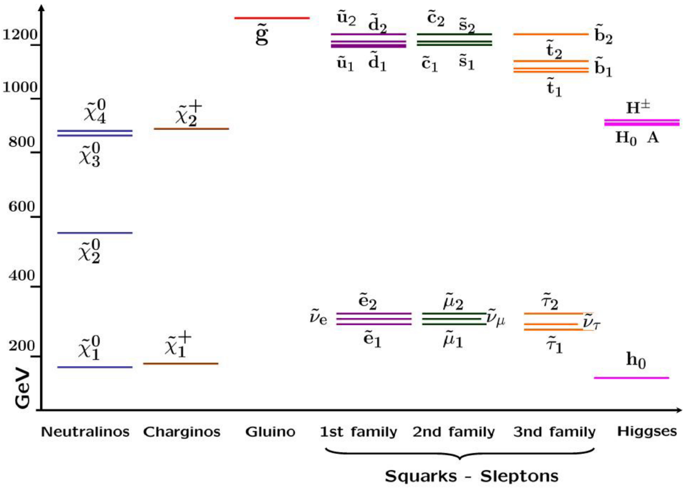
  They are in the TeV range for $m_{3/2} \sim 100\ TeV$!
- Gaugino masses are non-universal as in the ratios

$$M_1 : M_2 : M_3 = 2.8 : 1 : 7.1$$

  LSP = neutral wino only slightly lighter, by a few $\mathcal{O}(100)\ MeV$, than the charged wino $\Longrightarrow$ Long - lived lightest chargino !
- **Problematic since slepton masses tachyonic $m_{\tilde{L}}^2 < 0$ and charged breaking minima appear. Ad hoc addition of a m_0^2 to sfermion masses raises masses but RGE invariance of m_{soft} is lost. Other resolutions in Deflected Anomaly Mediation or other schemes ...**

AMSB mass spectrum :

$$m_0 = 40o\ GeV\ ,\ m_{3/2} = 60\ TeV\ ,\ \tan\beta = 10\ ,\ \mu > 0$$



- Supersymmetry breaking is one of the most important issues since is directly related to the experiment.
- There is on-going research and various theoretical proposals aim at a better understanding of SUSY breaking mechanisms.
- At the phenomenological level there is tremendous activity towards studying the various theoretical models.

Summary of this Lecture

- Motivation for Supergravity
- $N = 1$ SUGRA
- Local Supergavity breaking
- The role of the gravitino in Cosmology
- Mechanisms for Supersymmetry Breaking

Summary of this Lecture

- Motivation for Supergravity
- N = 1 SUGRA
- Local Supergavity breaking
- The role of the gravitino in Cosmology
- Mechanisms for Supersymmetry Breaking