Supersymmetry I

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Outline

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Open Questions

Although SM is in agreement with experimental observation fundamental questions still remain unanswered

Open Questions

- Too many parameters G_F, M_Z, α_{em}, α_s, θ_{QCD}, 12-fermion masses, ν - mixing parameters and masses, Higgs-boson mass.
- Why three generations ? Why quarks mix ?
- Substructute of Leptons and Quarks ?
- Higgs sector ?
- Strong and EW forces Unify ? Gravity how does it fit ?
- Why the EW scale is sixteen orders of magnitude smaller than the Planck scale ?
- ...

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Need move Beyond the SM physics !

Why Supersymmetry ?

- Good theoretical reasons to believe that SUSY will be the next big discovery !
- Some believe that SUSY is the low energy manifestation of a unified description valid at Planckian energies !
- Its mathematical beauty and its less divergent character, as a QFT, qualifies it as a powerful tool to build theoretical models.
- It is the only known symmetry that treats bosons and fermions on equal footing.
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Why Supersymmetry ?

Supersymmetry resolves the GHP and it is an indispensable ingredient of String Theories !

Historical note !

• $\sim$ **1970**, Supersymmetry (SUSY) was invoked to explain the masslessness of the neutrino. (Volkov , Akulov - Wess, Zumino) Goldstone modes of SB theories with **fermionic** generators are massless fermions (**Goldstinos**). These could be the neutrinos !

• $\sim$ **1981**, it was called back as resolution of the Gauge Hierarchy Problem in GUTs.

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Why Supersymmetry ?

With SM promoted to a SUSY model :

- SM gauge couplings unify at a scale $M_{GUT} \sim 10^{16}\ GeV$.
- The top quark mass drives the Higgs potential mass parameter to $\mu^2 < 0$ in a natural way and the hierarchy $M_W \sim 10^{-14}\, M_{GUT}$ is understood.
- Supersymmetric GUT models predict larger unification scales (better chance to reconcile proton lifetime with experimental data)
- The quartic Higgs self-couplings are not arbitrary, $\lambda \sim g^2$. Higgs masses are bounded $m_H < 135\ GeV$
- Rich phenomenology: Predicts more Higgses, and sparticles with masses in the TeV scale, likely to be discovered at LHC.
- Predicts WIMP candidates for DM.
- Gauged SUSY is a Supergravity Theory believed to be the low energy manifestation of String Theories !

How the Higgs stays light ?

The presence of a Higgs boson poses a severe problem if a theory at a highier scale, Λ, couples to the SM !

$$V(H) = -\mu^2 |H|^2 + \frac{\lambda}{2} |H|^4 \quad , \langle H \rangle = 175\, GeV$$

The quartic coupling is bounded by theory $\lambda < \mathcal{O}(1)$, (Unitarity, ...), and Higgs mass is bounded :

$$m_H^2 = 2\lambda \langle H \rangle^2 < (\text{few hundred GeV})^2$$

Higgs mass is subject to radiative corrections and its value changes !

$$m_H^2 = 2\lambda v^2 + \alpha \Lambda^2$$

- SM corrections pose no problem, $\Lambda \sim M_W$ and Higgs stays light.
- In the presence of a highier scale theory, $\Lambda \gg M_W$, Gravity is one example, $m_H \sim \Lambda$ and Higgs mass is driven to high values !

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To avoid Higgs mass from becoming large a fine - tuning of α is necessary

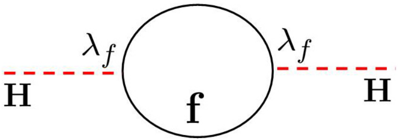
$$\textbf{Gravity} \implies \alpha < m_W^2/m_{Planck}^2 \sim 10^{-34}$$

$$\textbf{GUTs} \implies \alpha < m_W^2/m_{GUT}^2 \sim 10^{-26}$$

Couplings should be tuned to many decimal places, Unnatural !

Unless loop contributions conspire to cancelling each other !

For a Dirac fermion (4 d.o.f) coupled to Higgs



corrections are

$$\delta m_H^2 = \frac{\lambda_f^2}{16\pi^2}\left[-2\Lambda^2 + 6 m_f^2 \ln(\Lambda/m_f)\right]$$

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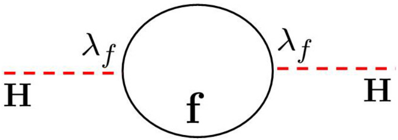

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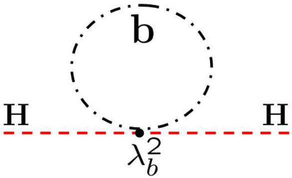

yields quadratic corrections with opposite sign

$$\delta m_H^2 = \frac{\lambda_b^2}{16\pi^2} \left[\Lambda^2 - 2 m_b^2 \ln(\Lambda/m_b) \right]$$

If $\lambda_f = \lambda_b$ and two complex scalars, to match the d.o.f of the Dirac fermion, the quadratic divergences of the two graphs cancel !
This occurs automatically is supersymmetric theories even if they are broken softly !

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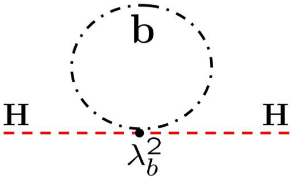

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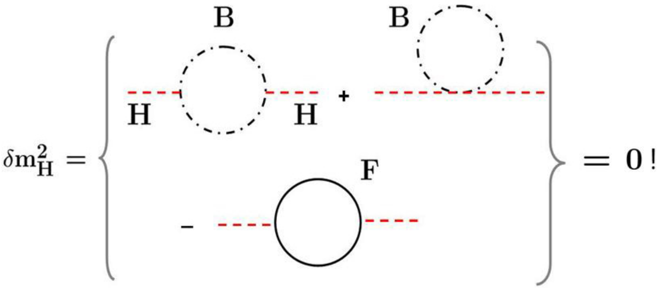
In Supersymmetry:

Particles go in pairs ! To each Fermion F there is a Boson B with same mass and same couplings. Only their spins differ by half a unit



Their contributions to Higgs mass exactly cancel !
When SUSY is broken "softly" couplings same but masses differ

$$m_B^2 - m_F^2 \simeq M_{SUSY}^2$$

Leading quadratic corrections, $\sim \Lambda^2$, cancel, next to leading $\sim \ln\Lambda$ survive

$$\delta m_H^2 = (m_B^2 - m_F^2) \ln\Lambda \sim M_{SUSY}^2 (\ln\Lambda)$$

corrections are not dangerous (large) due to their logarithmic nature !

To keep corrections of the order of the EW scale $\Longrightarrow$

$$M_{SUSY} \leq \mathcal{O}(1\ TeV)$$

Important for collider searches, supersymmetric particles have masses in the ***TeV*** range, supersymmetry is likely to be discovered at the LHC !

Supersymmetry controls the UV corrections more efficiently than an ordinary field theory !

Due to this it also ameliorates the cosmological constant problem!

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Why vacuum energy is so tiny ?

Vacuum energy of zero point fluctuations in gravity

$$\langle T_{\mu\nu} \rangle = m^4_{Planck}\, g_{\mu\nu}$$

Vacuum energy

$$E_{vac} \sim m^4_{Planck}$$

Observations point to a much lower value

$$E_{vac} \simeq (10^{-3}\, eV)^4 \sim 10^{-120}\, m^4_{Planck} \quad !$$

In Supersymmetry leading contributions cancel between fermions and bosons even if SUSY is broken

$$E^S_{vac} \simeq M^2_{SUSY}\, m^2_{Planck} = \left(\frac{M_{SUSY}}{m_{Planck}} \right)^2 m^4_{Planck} = 10^{-60}\, m^4_{Planck} \quad !$$

Much improvement but still far from $10^{-120}\, m^4_{Planck}$

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SUSY in QM

Bosonic oscillator

$$H_B = \frac{x^2 + p^2}{2} = \hbar\omega \left(a^\dagger a + \frac{1}{2} \right)$$

$$a \sim x + i\,p \quad \Longrightarrow \quad \left[a, a^\dagger \right] = 1$$

States $|n\rangle_B$ have energies $E_n = \hbar\omega\,(n + \frac{1}{2})$, $n = 0, 1, \ldots$

Fermionic oscillator

$$H_F = \hbar\omega \left(b^\dagger b - \frac{1}{2} \right)$$

$$\{ b, b^\dagger \} = 1 \quad , \quad b^2 = {b^\dagger}^2 = 0$$

Two states $|0\rangle_F$, $|1\rangle_F$ with energies $\mathcal{E}_0 = -\hbar\,\omega/2$, $\mathcal{E}_1 = +\hbar\,\omega/2$

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The combined Hamiltonian

$$H = H_B + H_F$$

States $|\,n\,,\,s\,\rangle = |n\rangle \otimes |s\rangle$ have energies $E_{n,s} = \hbar\omega\,(\,n + s\,)$

- States labelled by Quantum numbers : $\mathbf{s} = \mathbf{0}, \mathbf{1}$, $\mathbf{n} = \mathbf{0}, \mathbf{1}, \mathbf{2} \ldots$
- **s** declares the fermionic content : $\mathbf{s} = \mathbf{0}$ no-fermion, $\mathbf{s} = \mathbf{1}$ one fermion
- Except the vacuum $E_{0,0}$ the energy spectrum is degenerate !
- $|\,m\,,\,0\,\rangle$, $|\,m-1\,,\,1\,\rangle$ have same energy $E_{n,s} = \hbar\omega\; m$

Is there a symmetry behind ?

$Q = \sqrt{\hbar\omega}\; a^\dagger\, b$, $Q^\dagger = \sqrt{\hbar\omega}\; a\, b^\dagger$ and H close a graded Lie algebra !

$$\{Q\,,\,Q^\dagger\} = 2\,H$$
$$[\,Q\,,\,H\,] = [\,Q^\dagger\,,\,H\,] = 0$$
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$$Q^2 \;=\; {Q^\dagger}^2 \;=\; 0$$

For systems based on this algebra:

- $\boldsymbol{Q}$, $\boldsymbol{Q^\dagger}$ generate "Supersymmetric" (**SUSY**) transformations and they transform fermionic states to bosonic and v.v.
- The Hamiltonian $\boldsymbol{H}$ commutes with $\boldsymbol{Q}$, $\boldsymbol{Q^\dagger}$, respects **SUSY**. Degeneracy is a consequence of this symmetry !

The Hamiltonian has energies $E \geq 0$,

$$H = \frac{1}{2} (Q Q^\dagger + Q^\dagger Q)$$

If the symmetry breaks spontaneously, the vacuum state $|vac\rangle$ is not invariant

$$Q \, |vac\rangle \, , \, Q^\dagger \, |vac\rangle \neq 0$$

The vacuum energy is strictly positive

$$E_{vac} = || \, Q \, |vac\rangle \, ||^2 + || \, Q^\dagger \, |vac\rangle \, ||^2 > 0$$

and sets the scale of spontaneous breaking of Supersymmetry!

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Physical System ?

Electron moving on a plane under the influence of a constant magnetic field $\vec{B} \perp$ plane

$$H_0 = \frac{1}{2}({\pi_x}^2 + {\pi_y}^2) = \hbar\omega_B \left(a^\dagger a + \frac{1}{2}\right)$$

$$\omega_B = \frac{e\,B}{m_e\,c} \sim \text{Larmor frequency}$$

Electron carries spin - 1/2, couples to $\vec{B}$

$$H_s = -\vec{\mu}\cdot\vec{B} = g_s\,\frac{\hbar\omega_B}{2}\,\frac{\sigma_z}{2}$$

With the gyromagnetic ratio $g_s = 2$

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The total Hamiltonian $H_{tot} = H_0 + H_s$ is like the supersymmetric Hamiltonian studied before ! The energy spectrum

$$E_{n,\sigma} = \hbar\omega_B \left(n + \frac{1}{2} + \frac{\sigma}{2} \right) \quad , \qquad \begin{array}{l} \sigma = +1\,, \ \text{spin} \quad \uparrow \\ \sigma = -1\,, \ \text{spin} \quad \downarrow \end{array}$$

Two energy eigenstates for each Landau level $E_m = \hbar\omega_B \, m$, $m = 1, 2, \ldots$

$$|\, m, \downarrow\rangle \quad , \quad |\, m-1, \uparrow\rangle$$

One state $|\, 0, \downarrow\rangle$ to the vacuum energy, $E_0 = 0$, not degenerate !

Action of Q, $Q^\dagger$:

$$\begin{array}{ll} Q^\dagger \,|\, m, \downarrow\rangle = |\, m-1, \uparrow\rangle & \quad \text{flips spin up} \\ Q \ \ |\, m, \uparrow\rangle = |\, m+1, \downarrow\rangle & \quad \text{flips spin down} \end{array}$$

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In Particle Theory SUSY may play a fundamental role !
It is a Femion, Boson symmetry enlarging the Poincare symmetry.

The Algebra :

Poincare symmetry :

$$\begin{aligned} [\,P^m\,,\,P^n\,] &= 0 \\ [\,P^m\,,\,M^{rs}\,] &= i\,(\,\eta^{mr}\,P^s\,-(\,r \leftrightarrow s\,)\,) \\ [\,M^{mn}\,,\,M^{rs}\,] &= i\,(\,\eta^{ms}\,M^{nr}\,+\,\eta^{nr}\,M^{ms}\,-\,(\,r \leftrightarrow s\,)\,) \end{aligned}$$

Coleman - Mandula Theorem :

There is no fusion of internal with space-time symmetries, i.e. the maximal symmetry of the S - matrix is

$$\textbf{Poincare} \;\otimes\; \textbf{Internal symmetries}$$

unless

Haag-Lopuszanski-Sohnius :

The symmetry algebra is promoted to a **Graded Lie Algebra** (GLA) with Even E and Odd O elements closing the GLA

$$[\,E\,,\,E\,] = E \quad , \quad [\,E\,,\,O\,] = O \quad , \quad \{\,O\,,\,O\,\} = E$$

$\{\;,\;\} \equiv$ anticommutator

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unless
Haag-Lopuszanski-Sohnius :
The symmetry algebra is promoted to a **Graded Lie Algebra** (GLA) with Even $\boldsymbol{E}$ and Odd $\boldsymbol{O}$ elements closing the GLA

$$[\, E \,,\, E \,] = E \quad , \quad [\, E \,,\, O \,] = O \quad , \quad \{\, O \,,\, O \,\} = E$$

$\{\, ,\, \} \equiv$ anticommutator

- The minimal extension is the **N=1 SUSY** including two **Weyl**-type spinorial generators Q, $\overline{Q}$, in addition to P^m, M^{mn}

$$\begin{aligned} \{ Q_\alpha , \overline{Q}^{\dot\alpha} \} &= 2\,(\sigma^m)_\alpha^{\dot\alpha}\, P^m \\ [\, Q , P^m \,] &= \left[\overline{Q} , P^m \right] = 0 \\ [\, Q , M^{mn} \,] &= \sigma^{mn}\, Q \end{aligned}$$

Highier Supersymmeries have more spinorial charges, $N > 1$!

- Q, $\overline{Q}$ generate SUSY transformations and transform bosonic states to fermionic (and v.v.)

$$Q\,|B\rangle = |F\rangle\,,\; Q\,|F\rangle = |B\rangle \quad (\,idem\ \overline{Q}\,)$$

- The Hamiltonian is positive definite operator, and energies are $E \geq 0$.

$$H = \frac{1}{4}\,(\,Q_1^\dagger\, Q_1 + Q_1\, Q_1^\dagger + Q_2^\dagger\, Q_2 + Q_2\, Q_2^\dagger\,)$$

Exact SUSY : The vacuum state is invariant $Q\,|vac\rangle = \overline{Q}\,|vac\rangle = 0$ and the vacuum energy $E_{vac} = 0$!
SB SUSY : The vacuum state is non-invariant $Q\,|vac\rangle$, $\overline{Q}\,|vac\rangle \neq 0$ and the vacuum energy $E_{vac} > 0$!

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Spinology and Conventions

The fundamental spinor representations of the Poincare symmetry are the Weyl spinors

- A Left-handed Weyl spinor ψ has two components ψ_α

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

- The operation $\overline{\psi} \equiv i\,\sigma_2\,\psi^*$ defines a Right-handed antispinor with components $\overline{\psi}^{\dot{\alpha}}$

$$\overline{\psi} = \begin{pmatrix} {\psi_2}^* \\ -{\psi_1}^* \end{pmatrix}$$

- Invariant "mass" terms (indices raised lowered by $\epsilon^{\alpha\beta}$, $\epsilon_{\alpha\beta}$ ...)

$$\psi\chi \equiv \psi^\alpha\,\chi_\alpha \quad , \quad \overline{\psi}\,\overline{\chi} \equiv \overline{\psi}_{\dot{\alpha}}\,\overline{\chi}^{\dot{\alpha}}$$

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$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

L,R projection operators are diagonal

$$P_L = \frac{1+\gamma_5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, P_R = \frac{1-\gamma_5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

and the L and R -handed components of a Dirac fermion are Weyl spinors, ψ and $\overline{\phi}$

$$\psi_D = \begin{pmatrix} \psi \\ \overline{\phi} \end{pmatrix}$$

- The charge conjugation matrix $C = i\gamma^2\gamma^0$ and upon charge conjugation

$$\psi \Longrightarrow \psi_C = C\overline{\psi}^T \quad \text{equivalent to} \quad \psi = \begin{pmatrix} \psi \\ \overline{\phi} \end{pmatrix} \Longrightarrow \psi_C = \begin{pmatrix} \phi \\ \overline{\psi} \end{pmatrix}$$

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The building blocks :

The supersymmetry representations are called **supermultiplets**

Multiplet name	*Particle content*		*Spin content*	
Chiral	ϕ	ψ	0	1/2
Vector	A_μ	λ	1	1/2
Gravity	$g_{\mu\nu}$	ψ_μ	2	3/2

Every multiplet includes a boson and a fermion with spin differing by **1/2**. They are connected by SUSY transformations

$$\text{SUSY} \ : \quad \phi \Longleftrightarrow \psi \quad , \quad A_\mu \Longleftrightarrow \lambda \quad , \quad g_{\mu\nu} \Longleftrightarrow \psi_\mu$$

Besides the physical d.o.f. they include auxiliary fields to match the number of bosonic and fermionic components off-shell.
The most elegant description of supermultiplets is done through the notion of **superspace** !

Superspace & Superfields

- **Minkowski space-time :**
  - Coordinates are x^m
  - $G(a) = exp(\, i\, a_m\, P^m\,)$ generates translations by a_m
  - $G(a)\, G(x) = G(x')$ with $x'_m = x_m + a_m$
  - Fields transform as $G(a)\,\phi(x) G(a)^\dagger = \phi(x')$

- **Superspace :**
  - Coordinates are $z^m = (\, x^m, \theta, \bar{\theta}\,)$ with $\theta, \bar{\theta}$ Weyl spinors
  - $G(a, \xi, \bar{\xi}) = exp(\, i\, a_m\, P^m + i\, \xi\, Q + i\, \bar{\xi}\, \bar{Q}\,)$ generates translations by a_m, ξ, $\bar{\xi}$
  - $G(a, \xi, \bar{\xi})\, G(x, \theta, \bar{\theta}) = G(x', \theta', \bar{\theta}')$ with
    $x'_m = x_m + a_m + i\,(\xi\,\sigma_m\,\bar{\theta} - \theta\,\sigma_m\,\bar{\xi})\,,\ \theta' = \theta + \xi\,,\ \bar{\theta}' = \bar{\theta} + \bar{\xi}$
  - Superfields transform as $G(a, \xi, \bar{\xi})\,\Phi(x, \theta, \bar{\theta}) G(a, \xi, \bar{\xi})^\dagger = \Phi(x', \theta', \bar{\theta}')$

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- Superfields transform as $G(a, \xi, \overline{\xi})\, \Phi(x, \theta, \overline{\theta}) G(a, \xi, \overline{\xi})^\dagger = \Phi(x', \theta', \overline{\theta}')$

Expanding $\Phi(x,\theta,\bar\theta)$ in $\theta,\bar\theta$ no more than two powers of $\theta,\bar\theta$ can appear, the series terminates !

$$\begin{aligned}\Phi(x,\theta,\bar\theta) \;=\;& A(x) + \theta\Psi(x) + \bar\theta\,\bar\Sigma(x) + \\ & \theta\sigma^m\bar\theta\, V_m(x) + \theta\theta\, M(x) + \bar\theta\bar\theta\, N(x) + \\ & \theta\theta\bar\theta\,\bar\lambda(x) + \bar\theta\bar\theta\theta\,\xi(x) + \theta\theta\bar\theta\bar\theta\, D(x)\end{aligned}$$

- The fields $A, \Psi, \Sigma, V_m, M, N, \xi, \lambda, D$ are the "components" of $\Phi(x,\theta,\bar\theta)$ in increasing mass dimension, D has the highiest !
- They transform to one another by SUSY transformations.
- Since D carries the highiest dimension and the parameters of SUSY transformations have mass dimension $-1/2$

$$\delta_{SUSY}\, D \;=\; \partial_m K^m$$

The integral of the highiest dimensionality component of any superfield is supersymmetric invariant !

$$\int d^4x\, D \;=\; \text{SUSY invariant}$$

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Chiral & Antichiral multiplets

The general superfield involves many components. Multiplets with fewer components can be constructed !

Covariant derivatives :

$$D_\alpha = i\frac{\partial}{\partial\theta^\alpha} + (\sigma^m\bar{\theta})_\alpha \frac{\partial}{\partial x^m}\,,\ \bar{D}_{\dot{\alpha}} = -i\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - (\theta\sigma^m)_{\dot{\alpha}}\frac{\partial}{\partial x^m}$$

Commute with SUSY transformations :

$$[\delta_{SUSY}\,, D] = [\delta_{SUSY}\,, \bar{D}] = 0$$

Chiral superfield : $\bar{D}\Phi = 0$, Antichiral superfield : $D\Phi^\dagger = 0$

These properties are preserved by SUSY transformations !

- A chiral superfield contains a complex scalar A a Left - handed Weyl fermion ψ and an auxiliary complex field F. Its particle content is 2 spin-0 and 2 - spin 1/2 states .
- In terms of the variable $y^m \equiv x^m - i\,\theta\,\sigma^m\,\bar{\theta}$

$$\Phi = A(y) + \sqrt{2}\,\theta\,\psi(y) + \theta\,\theta\,F(y)$$

- The $\theta\theta$ component field F, or last component, carries the highiest dimensionality !

Under infinitesimal SUSY transformations by $\xi, \bar{\xi}$:

$$\begin{aligned}
\delta A &= \sqrt{2}\,\xi\,\psi \\
\delta\psi &= \sqrt{2}\,\xi\,F - i\,\sqrt{2}\,\sigma^m\,\bar{\xi}\,\partial_m A \\
\delta F &= -i\sqrt{2}\,\bar{\xi}\,\bar{\sigma}^m\,\partial_m\psi
\end{aligned}$$

The last component transform as a total derivative $\Longrightarrow$

$$\int d^4x\,F = \text{SUSY invariant}$$

- An **antichiral** superfield includes a complex scalar, a Right - handed Weyl fermion and an auxiliary complex field.
- If Φ is a **chiral** superfield with components (A, ψ, F) its Hermitian superfield $\Phi^\dagger$ is **antichiral** with components $(A^*, \overline{\psi}, F^*)$

Products of chiral superfields :

If $\Phi \sim (A, \psi, F)$ and $\Phi' \sim (A', \psi', F')$ are chiral superfields their product $\Phi'' = \Phi\,\Phi'$ is also a chiral field with components

$$\begin{aligned} A'' &= \; A\,A' \\ \psi'' &= \; A\,\psi' \;+\; A'\,\psi \\ F'' &= \; A'\,F + A\,F' - \psi\,\psi' \end{aligned}$$

The product $\Phi^\dagger\,\Phi$ is a real superfield which includes kinetic terms in its last $\theta^2\,\overline{\theta}^2$ component, producing SUSY invariant kinetic terms upon $\int d^4x$!

$$\Phi^\dagger\,\Phi|_{\theta^2\,\overline{\theta}^2} \;=$$
$$-\frac{1}{4}\,A\,\Box A^* - \frac{1}{4}\,A^*\,\Box A + \frac{1}{2}\,|\partial_m\,A|^2 + \frac{i}{2}\,(\,\psi\sigma^m\partial_m\overline{\psi} - h.c.\,) + FF^*$$

Vector multiplets

A Hermitian superfield V defines a vector multiplet

$$\text{Vector superfield :} \quad V = V^\dagger$$

including, among other components, a vector field A_μ a Weyl fermion λ and an auxiliary field D as its last $\theta^2 \bar{\theta}^2$ component.

- For any chiral field $\Phi \sim (A, \psi, F)$ the transformation

  $$V \Longrightarrow V + \Phi + \Phi^\dagger$$

  defines a gauge transformation with gauge parameter $\Lambda = -2\, Im\, A$
- Under the gauge transformation

  $$A_\mu \Longrightarrow A_\mu + \partial_\mu \Lambda \quad , \quad \lambda, D \Longrightarrow \text{themselves}$$

- The remaining components are not gauged invariant and can be gauged way ! This defines the **Wess - Zumino** gauge

- In the Wess - Zumino gauge

$$V = -(\theta\sigma^\mu\bar{\theta})A_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D$$

and $V^n = 0$ for $n \geq 3$

- The vector field A_μ and its partner, **gaugino** , λ are the physical d.o.f. describing 2 spin-1 and 2 spin-1/2 states.

Under infinitesimal SUSY transformations λ, D and the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ transform to each other :

$$\begin{aligned}
\delta F_{\mu\nu} &= -i(\xi\sigma_\nu\partial_\mu\bar{\lambda} + \bar{\xi}\bar{\sigma}_\nu\partial_\mu\lambda) - (\nu \leftrightarrow \mu) \\
\delta\lambda &= i\xi D + i\sigma^{\mu\nu}F_{\mu\nu} \\
\delta D &= \xi\sigma^\mu\partial_\mu\bar{\lambda} - \bar{\xi}\bar{\sigma}^\mu\partial_\mu\lambda
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Ungauged Lagrangians

The recipe to construct ungauged SUSY Lagrangians involving N chiral multiplets is easy

- Kinetic terms are included in the $\theta^2\,\overline{\theta}^2$, or D - terms, of $\Phi_i^\dagger\,\Phi_i$

$$(\Phi_i^\dagger\,\Phi_i)|_{\theta^2\,\overline{\theta}^2} \equiv \int d^2\theta\, d^2\overline{\theta}\,\Phi_i^\dagger\,\Phi_i$$

- Non - gauge interaction terms are included in the θ^2, F -terms, of a chiral field called **superpotential**, $W(\Phi_i)$, function of Φ_i 's

$$W(\Phi_i) = \frac{\lambda_{ijk}}{3!}\Phi_i\,\Phi_j\Phi_k + \frac{m_{ij}}{2!}\Phi_i\,\Phi_j$$

for renormalizable theories up to cubic terms are kept,

$$W(\Phi_i)|_{\theta^2} + h.c. \equiv \int d^2\theta\, W(\Phi_i) + h.c.$$

Auxiliary F - fields arise from the kinetic and superpotential terms

$$F_i^\dagger \, F_i \, + \, \left(F_i^\dagger \, \frac{\partial W(A_i)}{\partial A_i} \, + \, h.c. \right)$$

and are eliminated by their eqs. of motion

$$F_i^\dagger \, = \, - \, \frac{\partial W(A_i)}{\partial A_i}$$

- The SUSY Lagrangian is :

$$\mathcal{L} = |\partial_\mu A_i|^2 + i \, \bar{\psi}_i \, \bar{\sigma}^\mu \, \partial_\mu \psi_i - \left| \frac{\partial W}{\partial A_i} \right|^2 - \frac{1}{2} \left(\frac{\partial^2 W}{\partial A_i \partial A_j} \, \psi_i \psi_j + h.c. \right)$$

- A positive definite scalar potential arises :

$$V \, = \, F_i^\dagger \, F_i \, = \, \left| \frac{\partial W}{\partial A_i} \right|^2 \, \geq 0$$

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- A positive definite scalar potential arises :

$$V \,=\, F_i^\dagger\, F_i \,=\, \left|\frac{\partial W}{\partial A_i}\right|^2 \,\geq 0$$

Bosons and Fermions come in pairs with same mass as a **manifestation of supersymmetry**

$$m_B = m_F$$

as long as the vacuum energy vanishes, $V_0 = 0$. If $V_0 \neq 0$, due to some $\langle F_i \rangle \neq 0$, the mass degeneracy is lifted !

- Exact SUSY : $V_0 = 0$ and $m_B = m_F$
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Gauged SUSY Lagrangians

Generalize gauge transformations for chiral multiplets

$$\Phi \longrightarrow e^{-ig\Lambda}\,\Phi$$

$\Lambda \equiv \Lambda^a\, T^a$, $\Lambda^a =$ chiral superfields, $T^a =$ group generators.

- $\Phi^\dagger\,\Phi$ is not gauge invariant, Introduce gauge multiplet $V \equiv V^a\, T^a$ with $V^a = (\, A^a_\mu,\, \lambda^a,\, D^a\,)$ to make

  $$\Phi^\dagger\, e^{2gV}\,\Phi$$

  be gauge invariant. The gauge multiplet should transform as

  $$e^{2gV} \longrightarrow e^{-ig\Lambda^\dagger}\, e^{2gV}\, e^{ig\Lambda}$$

- In the Abelian case this reads

  $$V \longrightarrow V + \frac{i}{2}(\Lambda - \Lambda^\dagger)$$

- The gauge interactions of the chiral fields can be easily read in the Wess-zumino gauge since $V^n = 0\,,\ n \geq 3$.

$$\Phi^\dagger\, e^{2g\,V}\,\Phi \;=\; \Phi^\dagger\,\Phi + 2\,g\,\Phi^\dagger V\,\Phi + g^2\,\Phi^\dagger V^2\,\Phi$$

- The SUSY Yang-Mills Lagrangian

$$\mathcal{L}_{gauge} \;=\; -\frac{1}{4}\,G^{(a)\,2}_{\mu\nu} + \frac{i}{2}\,\overline{\lambda}^{(a)}\,\overline{\sigma}^\mu\,D_\mu\,\lambda^{(a)} + \frac{1}{2}\,D^{(a)2}$$

with $D_\mu\,\lambda^{(a)} = \partial_\mu\,\lambda^{(a)} + g\,f^{abc}\,A^b_\mu\,\lambda^{(c)}$ covariant derivative and $G^{(a)}_{\mu\nu}$ the non-Abelian gauge field strength is both gauge and SUSY invariant !

- A supersymmetric gauge Lagrangian with chiral multiplets Φ_i put in some representation Φ, reducible in general, is

$$\mathcal{L} \;=\; \mathcal{L}_{gauge} + \int d^2\theta\,d^2\overline{\theta}\,\Phi^\dagger\,e^{2g\,V}\,\Phi + \left(\int d^2\theta\,W(\Phi) + h.c.\right)$$

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- Auxiliary D - fields arise from the YM kinetic and gauge interaction terms and are eliminated by their eqs. of motion.

$$\frac{1}{2} D^{(a)2} + D^{(a)} \sum_i g\, A^* T^{(a)} A$$

- D and F type auxiliary fields are

$$D^{(a)} = -\sum_i g\, A^* T^{(a)} A \quad , \quad F_i^\dagger = -\frac{\partial W(A)}{\partial A_i}$$

The complete SUSY Lagrangian

$$\begin{aligned} \mathcal{L} &= \\ & -\frac{1}{4} G_{\mu\nu}^{(a)\,2} + |D_\mu A|^2 + \left(\frac{i}{2} \overline{\lambda}^{(a)} \bar{\sigma}^\mu D_\mu \lambda^{(a)} + \frac{i}{2} \bar{\Psi} \bar{\sigma}^\mu D_\mu \Psi + h.c. \right) \\ & - \left(\frac{1}{2} W_{ij}\, \psi_i \psi_j + i\sqrt{2} g\, A^* T^{(a)} \Psi \lambda^{(a)} + h.c. \right) - |F_i|^2 - \frac{1}{2} D^{(a)\,2} \end{aligned}$$

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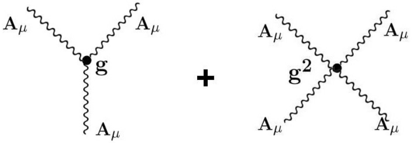
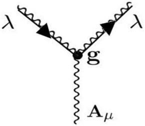
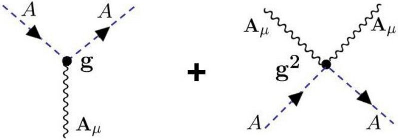
$\vec{G}_{\mu\nu}{}^{2}$ **gauge kinetic** $+$



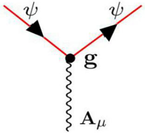
$\overline{\lambda}^{(a)}\,\overline{\sigma}^{\mu}\,D_\mu\,\lambda^{(a)}$ **gaugino kinetic** $+$



$|D_\mu\,A|^2$ **scalar kinetic** $+$



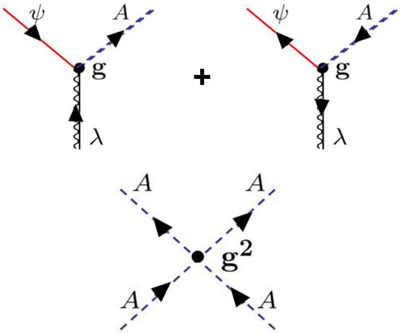
$\bar{\Psi}\,\bar{\sigma}^{\mu}\,D_\mu\Psi$ **fermion kinetic** $+$



Unconventional gauge interactions :

$g\, A^{*}\, T^{(a)}\, \psi\, \lambda^{(a)} + h.c.$

$D^{(a)\,2}$



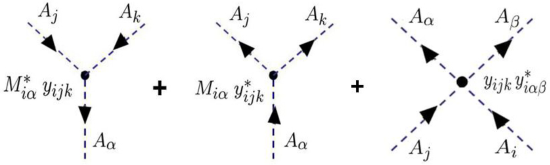
Non - gauge (superpotential) interactions :

With a superpotential $W(\Phi) = \frac{M_{ik}}{2}\Phi_i \Phi_j + \frac{y_{ijk}}{3!}\Phi_i \Phi_j \Phi_k$

- $|F_i|^2 = \left|\frac{\partial W}{\partial A_i}\right|^2$

Scalar mass terms : $A_i^* M_{ki}^* M_{kj} A_j$

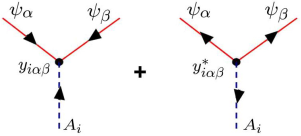
Scalar interactions :



- $W_{ij}\,\psi_i\,\psi_j + h.c.$

Fermion mass terms : $M_{ij}\,\psi_i\,\psi_j + h.c.$

Yukawa interactions :



- **Vertices are related by SUSY transformations!**

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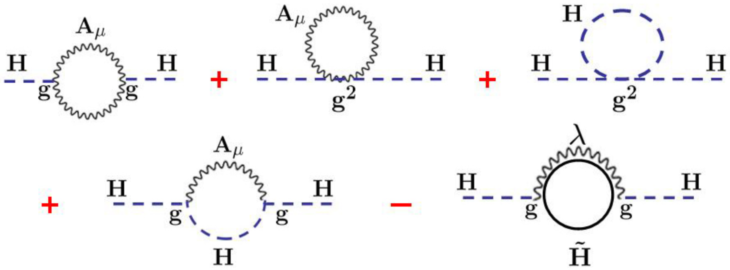
- **Essential for cancellations among graphs !**



Gauge-boson and Gaugino corrections to the Higgs mass

$\delta m_H^2 \; = \; 0$ if SUSY is exact !

$\delta m_H^2 \; \sim \; m_{soft}^2 \; \ln\left(\Lambda/m_{soft}\right)$ if SUSY is broken spontaneously or "softly"

The Wess - Zumino model

Describes one chiral multiplet $\Phi = (A, \psi, F)$ with superpotential,

$$W(\Phi) = \frac{m}{2}\Phi + \frac{\lambda}{3!}\Phi^3$$

Defining the Majorana fermion $\boldsymbol{\Psi}$, two d.o.f.!

$$\Psi = \begin{pmatrix} \psi \\ \bar{\psi} \end{pmatrix}$$

the SUSY Lagrangian is :

$$\mathcal{L}_{SUSY} =$$

$$|D_\mu A|^2 + \frac{i}{2}\bar{\Psi}\bar{\gamma}^\mu\partial_\mu\Psi \qquad \text{Kinetic terms}$$

$$-\frac{1}{2}\left[(m + \lambda A)\Psi_L\Psi_R + h.c.\right] \qquad \text{Fermion mass and Yukawa terms}$$

$$-\left|mA + \frac{\lambda}{2}A^2\right|^2 \qquad \text{Potential terms}$$

The potential is positive definite

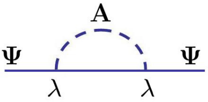
$$V = \left| mA + \frac{\lambda}{2} A^2 \right|^2 \geq 0$$

- There are two vacua, at $\langle A \rangle = 0$ or $\langle A \rangle = -2m/\lambda$, symmetric about the point $\langle A \rangle = -m/\lambda$.
- The vacuum energy vanishes, $V_{min} = 0$ and SUSY is unbroken !

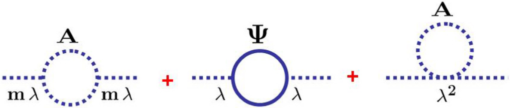
The model describes :

- A complex scalar boson A of mass m , two d.o.f.
- A Majorana fermion Ψ of mass m , two d.o.f !

Renormalization effects :



$$= Z_F \not{p} + finite$$



$$= Z_B \, p^2 + finite$$

Neither mass nor Yukawa coupling get renormalized. Only wave function renormalizations with $Z_B = Z_F$

No - Renormalization Theorem :

Superpotential parameters are not renormalized. The only infinities are those associated with the wave function renormalization of the chiral and vector multiplets and renormalization of the gauge couplings !

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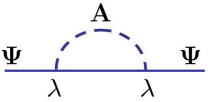

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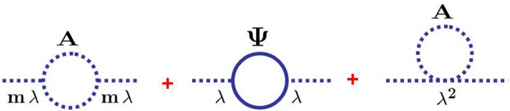

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Spontaneous Breaking of Global SUSY

- SSB of SUSY occurs if $Q_\alpha \, |vacuum> \neq 0$ or $\overline{Q}_{\dot{\alpha}} \, |vacuum> \neq 0$

$\Longrightarrow$

$$E_{vacuum} \; > \; 0$$

Lift of the vacuum energy signals SSB of global Supersymmetry !

- Tree level vacuum energy :

$$E_{vacuum} \; \equiv \; V_{min} \; = |\langle F_i \rangle|^2 + \frac{\langle D^a \rangle^2}{2}$$

and spontaneous breaking occurs for

- $\langle F_i \rangle \; \neq 0$, **F - type** breaking, (or O' Raifertaight breaking)
  Need special form of superpotential.
- $\langle D^a \rangle \; \neq 0$, **D - type** breaking, (or Fayet - Iliopoulos breaking)
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D - type breaking

Need a $U(1)$ gauge symmetry and a $U(1)$ vector multiplet V
Add to SUSY Lagrangian a term

$$\delta\mathcal{L} = 2\xi V = \xi D$$

This is $U(1)$ and SUSY invariant !

- The presence of the ξ - term alters the eqs. of motion for the auxiliary field D resulting to $\langle D \rangle \neq 0$.
- ξ sets the order parameter of global SUSY breaking and lifts the vacuum energy by $E_{vacuum} \sim \xi^2$.

Example: Two chiral multiplets Φ_1, Φ_2 with U(1) charges +1, -1 and a superpotential $W(\Phi) = m\,\Phi_1\,\Phi_2$

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Auxiliary fields :

$$D = -g\,(\,|\,A_1\,|^2 \;-\; |\,A_2\,|^2\,) \;-\; \xi$$
$$F_1 = -m\,A_2 \quad , \quad F_2 = -m\,A_2$$

Potential :

$$V = |\,F_1\,|^2 + |\,F_2\,|^2 + \frac{D^2}{2}$$
$$= (\,m^2 + g\,\xi\,)\,|\,A_1\,|^2 + (\,m^2 - g\,\xi\,)\,|\,A_2\,|^2$$
$$+ \frac{g^2}{2}(\,|\,A_1\,|^2 \;-\; |\,A_2\,|^2\,)^2 + \frac{\xi^2}{2} \geq 0$$

Case $|\,g\,\xi\,| < m^2$:

- Minimum of potential at $\langle\,A_{1,2}\,\rangle = 0 \implies U(1)$ unbroken
- $\langle\,D\,\rangle = -\xi, \quad \langle\,F_{1,2}\,\rangle = 0 \implies$ SUSY is broken by the D- term and $V_{min} = \xi^2/2 > 0$

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Potential :

$$\begin{aligned} V &= |F_1|^2 + |F_2|^2 + \frac{D^2}{2} \\ &= (\,m^2 + g\,\xi\,)\,|A_1|^2 + (\,m^2 - g\,\xi\,)\,|A_2|^2 \\ &+ \frac{g^2}{2}(\,|A_1|^2 - |A_2|^2\,)^2 + \frac{\xi^2}{2} \geq 0 \end{aligned}$$

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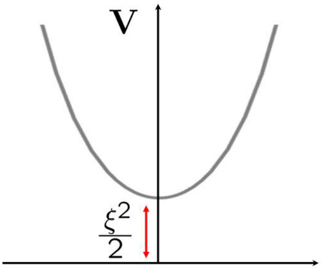
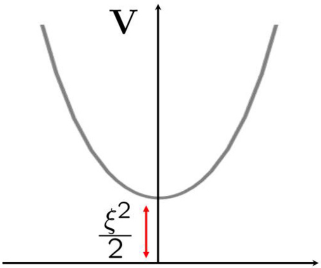


	Masses ^{2}	# of states	States
Spin - 0	$m^2 + g\,\xi$	2_B	A_1
	$m^2 - g\,\xi$	2_B	A_2
Spin - 1/2	m^2	4_F	Dirac : $\Psi = \begin{pmatrix} \psi_1 \\ \bar{\psi}_2 \end{pmatrix}$
	0	2_F	Goldstino : λ
Spin - 1	0	2_B	Gauge boson : A_μ



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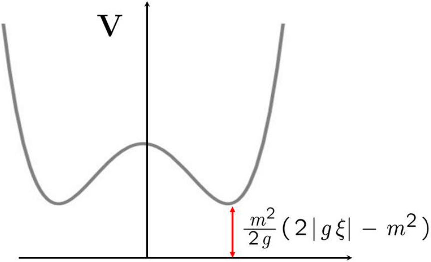
Case $|g\,\xi| > m^2$:

Take $\xi > 0$, if negative the role of A_1, A_2 is interchanged

- Minimum of potential at $\langle A_1 \rangle = 0\,,\, \langle A_2 \rangle \;=\; \dfrac{(g\xi - m^2)^{1/2}}{g}$
  $\Longrightarrow$ the gauge symmetry $U(1)$ is broken !
- $\langle D \rangle \neq 0,\ \langle F_1 \rangle \neq 0,\ \langle F_2 \rangle = 0 \;\Longrightarrow$ SUSY is broken, by the D- term, and also F_1
- The vacuum energy is $V_{min} \;=\; \dfrac{m^2}{2\,g}(2|g\xi| - m^2) \;>\; 0$
- Since both $\langle D \rangle$, $\langle F_1 \rangle$ non-zero the Goldstino $\equiv \chi$ is mixture of ψ_1 and the gaugino λ



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	0	1_B	Goldstone : $Im(A_2)$
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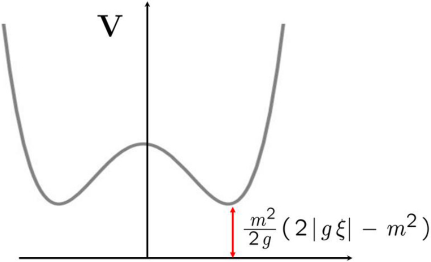

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F -type breaking

The form of the superpotential forces at least one F- term $\langle F_i \rangle \neq$ for .

Example :
Three chiral multiplets Φ_A, Φ_B, Φ_X coupled with a superpotential

$$W = g\,\Phi_X\,(\Phi_A^2 - \mu^2) + m\,\Phi_A\,\Phi_B$$

The equations for the auxiliary fields :

$$F_X^\dagger = -g\,(A^2 - \mu^2) \quad , \quad F_A^\dagger = -m\,A \quad , \quad F_B^\dagger = -(2g\,AX + m\,B)$$

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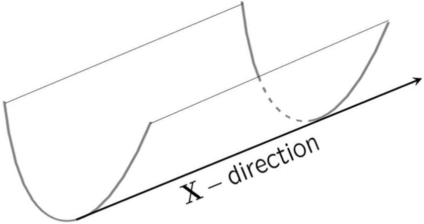
The potential has a flat direction along X :

$$V = g^2\,|A^2 - \mu^2|^2 + |2g\,AX + m\,B|^2 + m^2\,|A|^2$$



In the range $m^2 \, (\, 4 \, \mu^2 \, g^2 \, - \, m^2 \,) \, > 4 \, \mu^4 \, g^4$ the minimum of the potential is at

$$\langle A \rangle \, = \, \langle B \rangle \, = \, 0 \quad , \quad \langle X \rangle \, = \, \textbf{undetermined}$$

and the F - terms

$$\langle F_X \rangle \, = \, g \, \mu^2 \quad , \quad \langle F_{A,B} \rangle \, = \, 0$$

- Supersymmetry is broken by $\langle F_X \rangle \, \neq \, 0$.
- The vacuum energy is $V_{min} \, = g^2 \, \mu^4$ and the order parameter of SUSY breaking is $f \equiv g \, \mu^2$.
- The Goldstino is the partner of the scalar X .

Comments :

- In only F - type breaking of global SUSY some sfermions become lighter than their corresponding fermions, due to the relation

$$Str\,\mathcal{M}^2 \equiv \sum_J (-1)^{2J}(2J+1)\,m_J^2 = 0$$

which is preserved after SUSY breaking. In D - type breaking this problem is evaded since the r.h.s. receives $\langle D \rangle$ contributions !

- For D - type breaking an extra U(1) is required, a new neutral vector boson appears and new neutral current interactions are present !

- The presence of a U(1) vector multiplet may have disastrous consequences since quadratic loop corrections may emerge

$$\Lambda^2 \int d^2\theta\, d^2\bar{\theta}\;\; V = \Lambda^2\, D$$

This may be circumvented if U(1) is subgroup of a simple group. then the corrections are proportional to ***Trace Y*** $= 0$!.

Further

- The Goldstino mode is not observed in nature ! In local versions of Supersymmetry this is absorbed by a massless gravitino to make it massive. The Goldstino disappears and the gravitino provides for a Dark Matter candidate (in addition ...)
- Supergravity and String Theories provide additional mechanisms for SUSY breaking !

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- SUSY Algebra
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- SUSY Lagrangians
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What comes next ?

- **Build models encompassing the SM !**
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