Grand unification in the heterotic brane world

Patrick K. S. Vaudrevange
LMU München

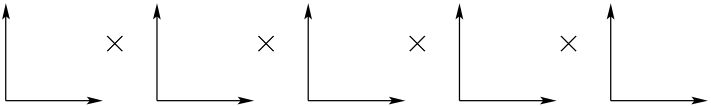
September 10, 2009

Based on:

M. Blaszczyk, S. Groot Nibbelink, M. Ratz, F. Ruehle, M. Trapletti and P. V.: arXiv:09xx.xxxx

Outline

Motivation



- Heterotic string theory: $E_8 \times E_8$ gauge group in 10D
- Aim: connection to observable world; MSSM
- Compactify six spatial dimensions on a compact space (e.g. 6-torus)
- 6D Orbifolds: compact space; flat like torus, except for some singularities (called fixed points)
- In this talk focus on $\mathbb{Z}_2 \times \mathbb{Z}_2$ and $\mathbb{Z}_6$-II orbifolds

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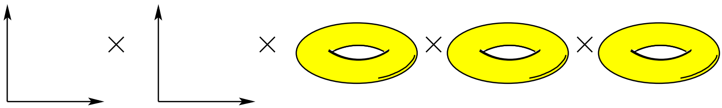

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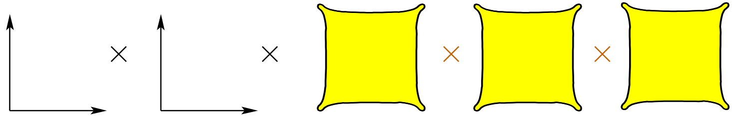

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Orbifold MSSMs

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$\mathbb{Z}_6$-II Mini-Landscape

$\mathcal{O}(100)$ $\mathbb{Z}_6$-II orbifold models with

- $SU(3) \times SU(2) \times U(1)_Y$ times hidden sector
- 3 generations of quarks and leptons + vector-like exotics
- exotics decouple
- (potentially) realistic flavor structure, e.g. heavy top

  O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. V., A. Wingerter 2006, 2007
- see talk by S. Ramos-Sanchez

Relation to other Constructions

- Can these models be obtained from a CY construction?

  ⇒No, at least not easily!

- $\mathbb{Z}_6$-II Mini-Landscape at special (symmetry enhanced) point in moduli space:
  - Wilson line breaks GUT to SM (locally) at fixed points
  - In full blow-up, SM gauge group (e.g. hypercharge) broken at these fixed points
  - (fixed points with only SM charged states ⇒ blow-up mode breaks SM)

  S. Groot Nibbelink, J. Held, F. Ruehle, M. Trapletti, P. V 2009

- Important: full blow-up of Mini-Landscape models not necessary

Full Blow-up possible?

Can MSSM orbifold models have a corresponding CY description in principle?

or

Can MSSM orbifold models be blown-up completely?

Non-local GUT Breaking

Non-local GUT Breaking

Non-local GUT Breaking

- One possibility: GUT broken to SM non-locally: freely acting orbifold
- In this talk: $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with freely acting twists
  R. Donagi and K. Wendland 2008
- Gauge coupling unification and M_{GUT} vs. M_{string}
  A. Hebecker and M. Trapletti 2004

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

(1-1) $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold by Donagi, Wendland:

- $T^6 = T^2 \times T^2 \times T^2$ spanned by orthogonal lattice e_i, $i = 1, \ldots, 6$
- $\mathbb{Z}_2 \times \mathbb{Z}_2$ generated by

$$v_1 = \left(0, \frac{1}{2}, -\frac{1}{2}\right)$$
$$v_2 = \left(-\frac{1}{2}, 0, \frac{1}{2}\right)$$

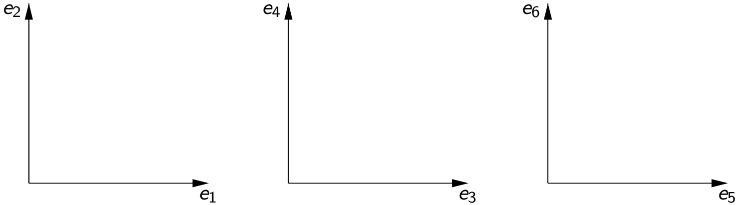
- freely acting twist: $\tau = \left(\frac{1}{2}e_2, \frac{1}{2}e_4, \frac{1}{2}e_6\right)$

R. Donagi and K. Wendland 2008

See also talks by Faraggi and Rizos

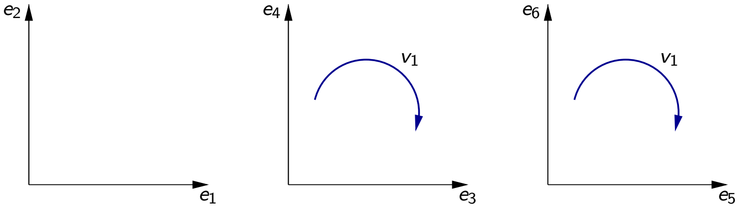
$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with freely acting twist

T^6 torus



$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

twist v_1 acting on T^6 torus



$$v_1 = \left(0, \frac{1}{2}, -\frac{1}{2}\right)$$

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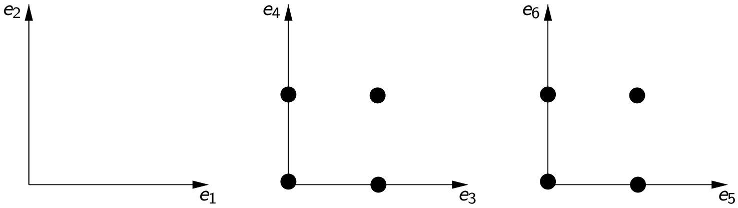
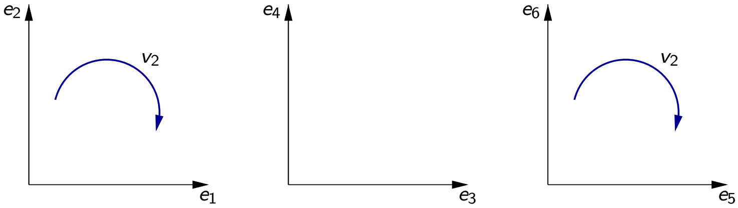


$$v_1 = \left(0, \frac{1}{2}, -\frac{1}{2}\right)$$

$\Rightarrow$ 16 fixed points

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

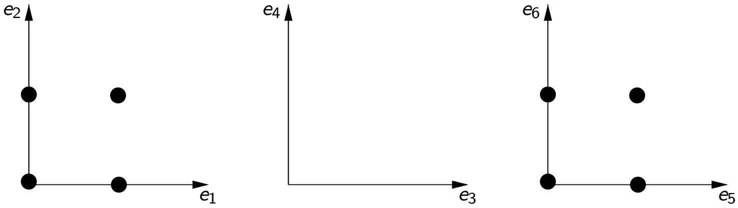
twist v_2 acting on T^6 torus



$$v_2 = \left(-\frac{1}{2}, 0, \frac{1}{2}\right)$$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

twist v_2 acting on T^6 torus



$$v_2 = \left(-\frac{1}{2}, 0, \frac{1}{2}\right)$$

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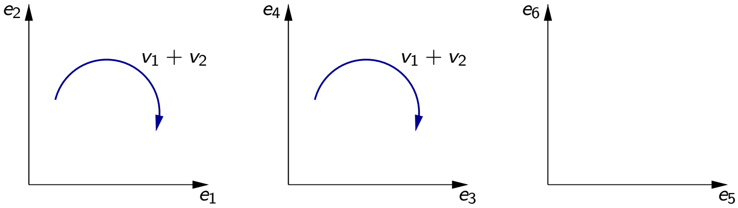
$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

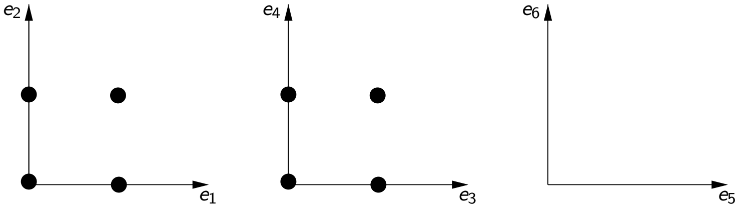
twist $v_1 + v_2$ acting on T^6 torus



$$v_1 + v_2 = \left(-\frac{1}{2}, \frac{1}{2}, 0\right)$$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

twist $v_1 + v_2$ acting on T^6 torus

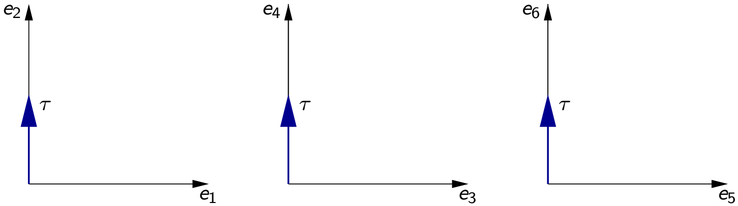


$$v_1 + v_2 = \left(-\frac{1}{2}, \frac{1}{2}, 0\right)$$

$\Rightarrow$ 16 fixed points

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

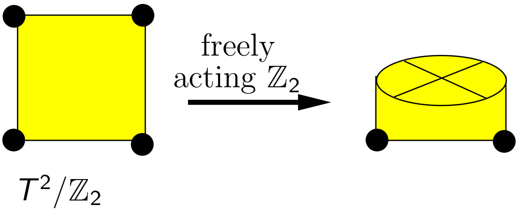
freely acting twist τ acting on T^6 torus



$\Rightarrow$ half the number of fixed points: $(16+16+16)/2 = 24$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

action of freely acting twist in 2d:



$\Rightarrow$ half the number of fixed points: $4/2 = 2$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

- setup:

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with 6 generations of SU(5)

$\downarrow$ freely acting $\mathbb{Z}_2$

3 generations of $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$

- where freely acting Wilson line induces GUT breaking
- Potentially: one SM singlet per fixed point $\Rightarrow$ full blow-up

Example

MSSM from $\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

3 generations of quarks and leptons
plus vectorlike exotics

Input data

- Shifts

$$V_1 = \left(\frac{1}{2}, \frac{1}{2}, 2, 0, 0, 0, 1, -1, 0, 1, 1, 0, 1, 0, 0, -1\right)$$
$$V_2 = \left(\frac{5}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, \frac{1}{2}, 0^5, 4\right)$$

- Wilson lines

$$A_1 = 0$$
$$A_2 = \left(-1, -1, 0, -1, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$$
$$A_3 = \left(1, -1, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{3}{4}, \frac{1}{4}, -\frac{3}{4}, \frac{1}{4}\right)$$
$$A_5 = \left(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, -\frac{1}{2}, -\frac{1}{2}\right)$$
$$A_6 = A_4 = A_2$$

- Freely acting Wilson line

$$A_\tau = \frac{1}{2} A_2$$

(1) without freely acting Wilson line

- 4d gauge group: $SU(5) \times U(1)^4 \times [SU(4)^2 \times U(1)^2]$
- massless spectrum

15	$(5, 1, 1)$	9	$(\overline{5}, 1, 1)$
6	$(\overline{10}, 1, 1)$	52	$(1, 1, 1)$
6	$(1, 4, 1)$	6	$(1, \overline{4}, 1)$
8	$(1, 1, 4)$	8	$(1, 1, \overline{4})$
2	$(1, 1, 6)$		

- (remark: generically, SU(5) vector-like exotics decouple linear in VEVs!)

(2) with freely acting Wilson line

- 4d gauge group:
  $SU(3) \times SU(2) \times U(1)_Y \times U(1)^4 \times [SU(3) \times SU(2)^2 \times U(1)^4]$
- massless spectrum

3	$(\overline{3}, 2, 1, 1, 1)_{1/6}$	q
8	$(3, 1, 1, 1, 1)_{1/3}$	$\overline{d}$, $\overline{\delta}$
7	$(1, 2, 1, 1, 1)_{-1/2}$	ℓ, h_d
3	$(1, 1, 1, 1, 1)_1$	$\overline{e}$
5	$(1, 1, \overline{3}, 1, 1)_0$	$\overline{x}$
6	$(1, 1, 1, 1, 2)_0$	y

3	$(3, 1, 1, 1, 1)_{-2/3}$	$\overline{u}$
5	$(\overline{3}, 1, 1, 1, 1)_{-1/3}$	δ
4	$(1, 2, 1, 1, 1)_{1/2}$	h_u
33	$(1, 1, 1, 1, 1)_0$	s
5	$(1, 1, 3, 1, 1)_0$	x
6	$(1, 1, 1, 2, 1)_0$	z

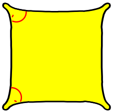
- (remark: generically, vector-like “exotics” still decouple linear in VEVs! Draw back: also the Higgs!)

String couplings

The superpotential

$$\mathcal{W} \supset m\bar{\delta}\,\delta + Y_u\, q\, \overline{u}\, h_u + \overline{u}\,\overline{d}\,\overline{d} + \ldots$$

String couplings

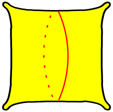


String states carry some "charges" that must be conserved:

- Space group selection rule (geometric)
- R-charge conservation
- Gauge Invariance

$\Rightarrow$ allowed terms in superpotential $\mathcal{W}$

String couplings



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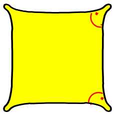

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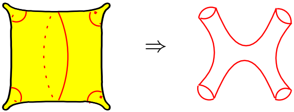

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- Space group selection rule (geometric)
- R-charge conservation
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Effective couplings

- couplings like

$$\begin{aligned}\mathcal{W} &= \bar{\delta}_2\, \delta_1\, s_1 \\ &+ q_1\, \bar{u}_2\, h_u\, s_4\, s_5\, s_{26}\, s_{30} \\ &+ \bar{u}_1\, \bar{d}_1\, \bar{d}_2\, s_2\, s_3\, s_{11}\, s_{30} + \ldots\end{aligned}$$

- induce effective operators if SM singlets s_i develop (large) VEVs

$$\begin{aligned}\mathcal{W} &= \bar{\delta}_2\, \delta_1\, \langle s_1 \rangle \\ &+ q_1\, \bar{u}_2\, h_u\, \langle s_4 \rangle \langle s_5 \rangle \langle s_{26} \rangle \langle s_{30} \rangle \\ &+ \bar{u}_1\, \bar{d}_1\, \bar{d}_2\, \langle s_2 \rangle \langle s_3 \rangle \langle s_{11} \rangle \langle s_{30} \rangle\end{aligned}$$

- like Froggatt-Nielsen:
(hierarchical) Yukawa couplings + Proton decay operator

$\Rightarrow$ vacuum selection very difficult

Vacuum selection criterions

- SUSY preserving vacua!
  - F-Terms (global SUSY)

$$F_i \sim \frac{\partial \mathcal{W}}{\partial \phi_i} = 0$$

  - D-Terms

$$\sum_i q_i |\langle \phi_i \rangle|^2 + \text{FI} = 0$$

- decoupling of exotics
- (hierarchical) Yukawa couplings
- strongly suppressed Proton decay

$\Rightarrow$ Symmetries help!

- $\text{U}(1)_{B-L}$ generator (from $\text{U}(1)^9$)

$$t_{B-L} = \left(0,0,0,1,\frac{1}{3},\frac{1}{3},\frac{1}{3},0,\frac{7}{2},-\frac{5}{2},\frac{3}{2},\frac{3}{2},\frac{3}{2},\frac{3}{2},-\frac{1}{2},-\frac{1}{2}\right)$$

- SM charged spectrum (distinguish between ℓ & h_d and $\overline{d}$ & $\overline{\delta}$)

3	$(\overline{3},2)_{1/6,1/3}$	q
3	$(3,1)_{-2/3,-1/3}$	$\overline{u}$
3	$(1,1)_{1,1}$	$\overline{e}$
4	$(1,2)_{1/2,0}$	h_u
5	$(3,1)_{1/3,2/3}$	$\overline{\delta}$

3	$(1,2)_{-1/2,-1}$	ℓ
3	$(3,1)_{1/3,-1/3}$	$\overline{d}$
4	$(1,2)_{-1/2,0}$	h_d
5	$(\overline{3},1)_{-1/3,-2/3}$	δ

- $\text{U}(1)_{B-L}$ forbids dangerous dim. 4 operators:

$$\overline{u}\,\overline{d}\,\overline{d},\quad q\,\overline{d}\,\ell,,\quad \ell\,\ell\,\overline{e}\quad\text{and}\quad \ell h_u\quad(\text{total } B-L=-1)$$

and allows for wanted couplings (Yukawas), e.g.

$$q\,\overline{u}\,h_u,\quad q\,\overline{d}\,h_d,\quad \ell\,\overline{e}\,h_d$$

- SM $\times U(1)_{B-L}$ singlets obtain VEVs:

$$\{s_1, s_2, s_3, s_4, s_5, s_9, s_{10}, s_{15}, s_{16}, s_{18}, s_{19}, s_{20}, s_{22}, s_{23}, s_{25}, s_{26}, s_{28}, s_{30}, s_{31}, s_{33}, y_1, y_2, y_3, y_4, y_5, y_6, z_1, z_2, z_3, z_4, z_5, z_6\}$$

- symmetry breaking: hidden $SU(2)^2 \times U(1)^6$ broken
- exotics decouple
- $U(1)_{B-L}$ broken to matter parity by VEVs of fields with even $B - L$ charge:

$$\{s_7, s_8, s_{17}, s_{21}, s_{27}, s_{32}, x_3, x_4, x_5, \overline{x}_2, \overline{x}_4, \overline{x}_5\}$$

- symmetry breaking: only SM $\times$ matter parity unbroken

$\Rightarrow$ Unwanted couplings remain forbidden!

Unbroken symmetries help for $F = 0$

- Consider set of fields $\{\phi_i\}$ and a symmetry group G (e.g. $G = \mathrm{U}(1)$)
- Split the set:

$$\{\phi_i\} = \{s_i\} \cup \{r_i\}$$

where s_i uncharged and r_i charged w.r.t G

- Choose VEV: $\langle r_i \rangle = 0$ and $\langle s_i \rangle \neq 0 \Rightarrow G$ unbroken
- Then, the F-terms $F(\phi_i) \sim \frac{\partial \mathcal{W}}{\partial \phi_i}$ split:

$$\begin{aligned} \langle F(r_i) \rangle &= 0 \quad \text{no term linear in } r_i \\ \langle F(s_i) \rangle &\neq 0 \quad \text{(model dependent)} \end{aligned}$$

i.e. number of VEVs $\langle s_i \rangle$ equals number of potential non-trivial F-terms $\langle F(s_i) \rangle$

$\Rightarrow$ Generically, $F = 0$ has solutions!

$D = 0$

- Gauge invariant monomial involving all s_i ensures $D = 0$ configuration

Buccella, Derendinger, Ferrara, Savoy

- Exotics decouple linear in VEVs: $\delta_i \, M^\delta_{ij} \, \overline{\delta}_j$

$$M^\delta = \begin{pmatrix} s^3 & \langle s_1 \rangle & s^3 & s^3 & s^3 \\ \langle s_2 \rangle & s^3 & s^5 & \langle s_{16} \rangle & \langle s_{20} \rangle \\ s^5 & s^3 & s^5 & \langle s_{26} \rangle & \langle s_{31} \rangle \\ \langle s_{28} \rangle & s^3 & \langle s_{19} \rangle & \langle s_{10} \rangle & s^3 \\ \langle s_{33} \rangle & s^3 & \langle s_{23} \rangle & s^3 & \langle s_{10} \rangle \end{pmatrix}$$

- But (generically) so do the Higgs: $(h_d)_i \, M^h_{ij} \, (h_u)_j$

$$M^h = \begin{pmatrix} s^3 & \langle s_3 \rangle & s^3 & s^3 \\ \langle s_{15} \rangle & s^5 & \langle s_{19} \rangle & \langle s_{23} \rangle \\ s^3 & \langle s_{26} \rangle & \langle s_{10} \rangle & s^3 \\ s^3 & \langle s_{31} \rangle & s^3 & \langle s_{10} \rangle \end{pmatrix}$$

- $q_i \, M^u_{ij} \, \overline{u}_j$

$$M^u = \begin{pmatrix} \langle h_u \rangle s^4 & \langle h_u \rangle s^4 & 0 \\ \langle h_u \rangle s^4 & \langle h_u \rangle s^4 & 0 \\ 0 & 0 & \langle (h_u)_1 \rangle \end{pmatrix}$$

- $q_i \, M^d_{ij} \, \overline{d}_j$

$$M^d = M^e = \begin{pmatrix} 0 & 0 & \langle (h_d)_3 \rangle \\ 0 & 0 & \langle (h_d)_4 \rangle \\ \langle (h_d)_3 \rangle & \langle (h_d)_4 \rangle & 0 \end{pmatrix}$$

- SU(5) relation survives freely acting Wilson line
- D_4 family symmetry with geometrical origin: third generation: singlet; first/second: doublet

... and there are many more vacua!

Conclusion

Conclusion

Summary

- $\mathbb{Z}_6$-II Mini-Landscape at special (symmetry enhanced) point, but full blow-up not necessary
- $\mathbb{Z}_2 \times \mathbb{Z}_2$ with freely acting twist $\Rightarrow$ non-local GUT breaking
- Example: promising model (potentially also in full blow-up)