Electroweak Theory

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- The electroweak Standard Model
- Precision tests
- Higgs bosons
- Outlook: SUSY extension of the SM

Matter fields in SM

SU(2) × U(1) gauge symmetry

weak charges: weak isospin I_W

weak hypercharge Y_W

left-handed fermions are SU(2) doublets ($I_\mathrm{W} = 1/2$)

$$\begin{pmatrix} \nu_\mathrm{e} \\ \mathrm{e} \end{pmatrix}_\mathrm{L}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_\mathrm{L}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_\mathrm{L}, \begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix}_\mathrm{L}, \begin{pmatrix} \mathrm{c} \\ \mathrm{s} \end{pmatrix}_\mathrm{L}, \begin{pmatrix} \mathrm{t} \\ \mathrm{b} \end{pmatrix}_\mathrm{L}$$

right-handed fermions are SU(2) singlets ($I_\mathrm{W} = 0$)

$$\mathrm{e_R},\ \mu_\mathrm{R},\ \tau_\mathrm{R},\ \mathrm{u_R},\ \mathrm{d_R},\ \mathrm{c_R},\ \mathrm{s_R},\ \mathrm{t_R},\ \mathrm{b_R}$$

minimal model: no right-handed neutrino ν_R by convention

ν_R can easily be added

electric charge is fixed by Gell-Mann–Nishijima relation: $Q = I^3_\mathrm{W} + \frac{Y_\mathrm{W}}{2}$

different representations for left-handed and right-handed fermions ⇔ violation of P and C invariance

Gauge fields

Isotriplet W_μ^a $(a = 1, 2, 3)$ and isosinglet B_μ

$$W_\mu^\pm = (W_\mu^1 \mp i W_\mu^2)/\sqrt{2}$$

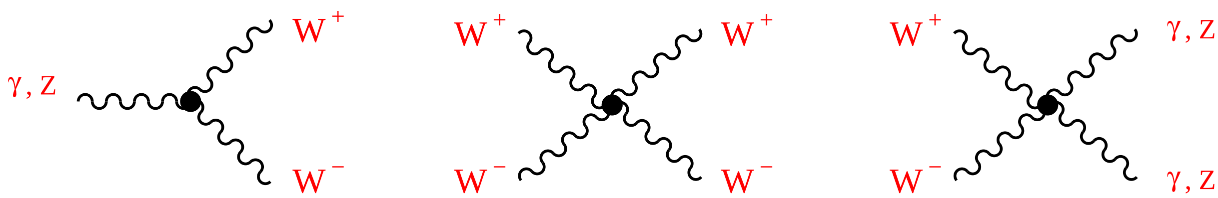
$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

field strength tensors

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\,\epsilon_{abc}\,W_\mu^b W_\nu^c$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian

$$\mathcal{L}_G = -\frac{1}{4} W_{\mu\nu}^a W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$



Interaction with fermions

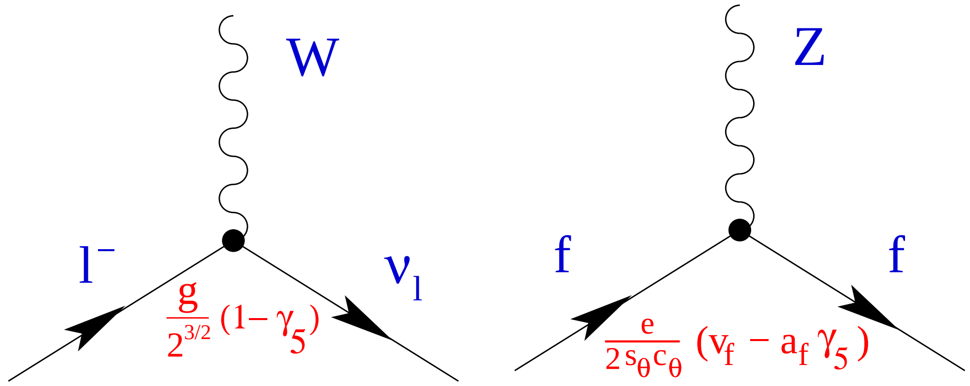
through covariant derivative

$$D_\mu = \partial_\mu \; - \; i\, g\, I_a W_\mu^a \, + \, i\, g' \, \frac{Y}{2} \, B_\mu$$

for left and right-handed fermion fields

$$\bar{\psi}^L D_\mu \gamma^\mu \psi^L + \bar{\psi}^R D_\mu \gamma^\mu \psi^R =$$

$$\bar{\psi}^L \partial_\mu \gamma^\mu \psi^L + \bar{\psi}^R \partial_\mu \gamma^\mu \psi^R + \text{interaction terms}$$



$$g = \frac{e}{s_\theta}, \quad s_\theta = \sin\theta_W, \quad c_\theta = \cos\theta_W$$

$$\begin{aligned} a_f &= I_3^f \\ v_f &= I_3^f - 2\, Q_f\, s_\theta^2 \end{aligned}$$

Problem:

gauge fields Z, W^+, W^- are massive

explicit mass terms $\Leftrightarrow$ gauge invariance broken

$\Rightarrow$ **Higgs mechanism**

scalar field postulated, gauge-invariant mass terms from coupling to Higgs field

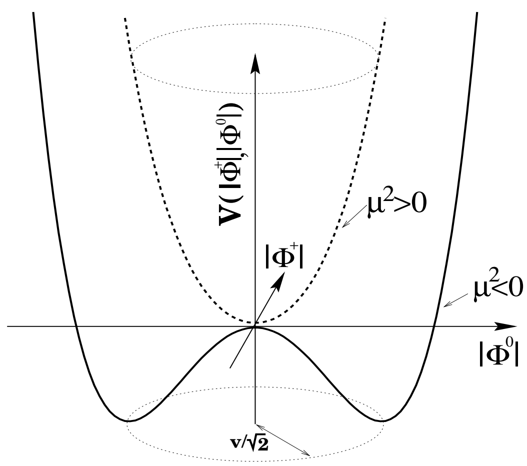
Higgs sector of the Standard Model:

scalar SU(2) doublet: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Higgs potential:

$$V(\phi) = \mu^2 \left|\Phi^\dagger \Phi\right| + \lambda \left|\Phi^\dagger \Phi\right|^2, \quad \lambda > 0$$

$\mu^2 < 0$: spontaneous symmetry breaking



minimum of the potential at $\langle\Phi\rangle = \frac{1}{\sqrt{2}}\sqrt{\frac{-\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$

Gauge-invariant interaction with gauge fields:

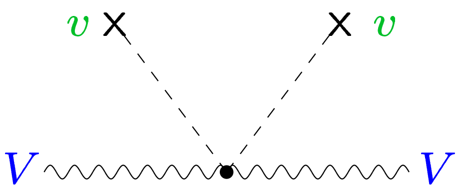
$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)$$

$\Rightarrow$ mass terms

Unitary gauge:

$$\Phi = \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$VV\Phi\Phi$ coupling:



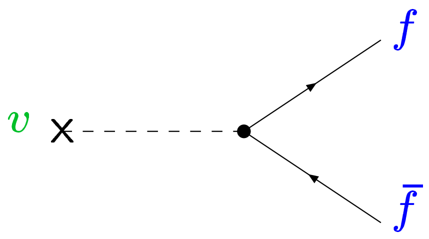
$\Rightarrow$ VV mass terms

$$\frac{1}{2} g_2^2 v^2 \equiv M_{\text{W}}^2, \quad \frac{1}{2}(g_1^2 + g_2^2) v^2 \equiv M_{\text{Z}}^2$$

3 components of Higgs doublet $\longrightarrow$ longitudinal components of $W^\pm$, Z

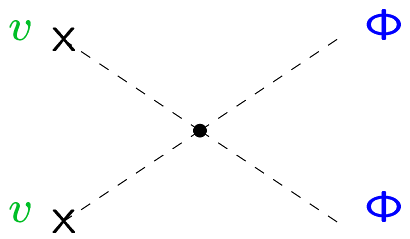
H: elementary scalar field, Higgs boson

Fermion mass terms: Yukawa couplings



$m_f = v\, g_f$ free parameters

Mass of the Higgs boson: self-interaction

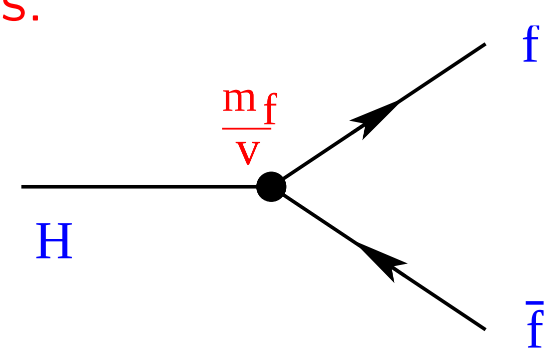


$M_\mathrm{H} = v\sqrt{\lambda}$ free parameter

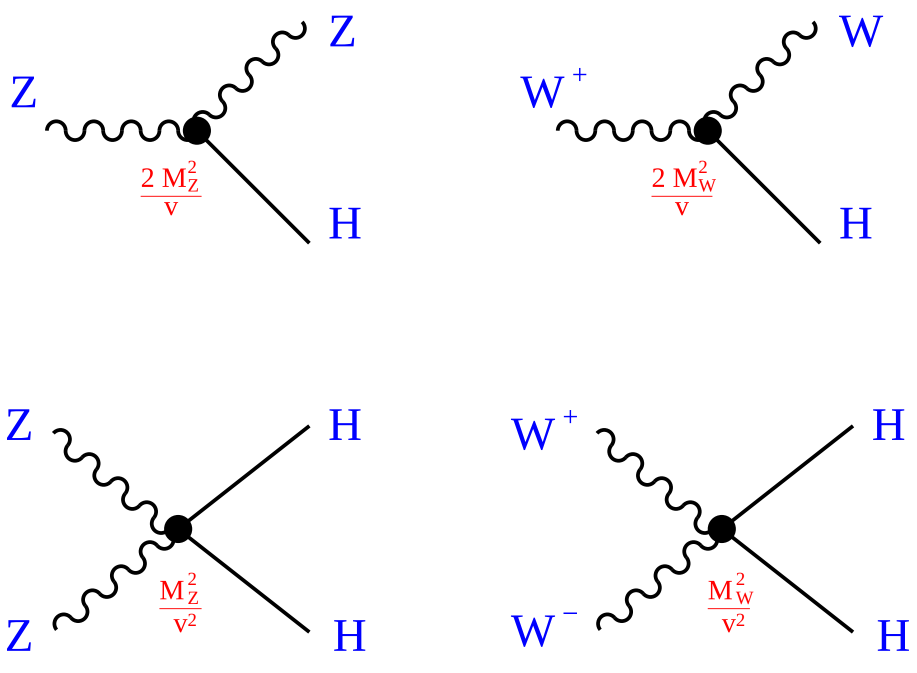
$\Rightarrow$ Higgs couplings proportional to masses of the particles

Interactions of the Higgs boson

with fermions:



with gauge bosons: $\left(\frac{2M_W}{v} = g\right)$



Quark mixing and $\mathcal{CP}$-violation

The weak eigenstates (d', s', b') of quarks differ from the mass eigenstates d, s, b:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$

V_{CKM}: unitary transformation, Cabibbo-Kobayashi-Maskawa (CKM) matrix

GIM Mechanism:

unitarity of CKM-matrix $\Rightarrow$ no flavor changing neutral current transitions at tree level

$b \to s\gamma, \ldots$ are loop-induced in SM $\Rightarrow$ high sensitivity to new physics effects

$V^\dagger_{\text{CKM}} V_{\text{CKM}} = 1$

$\Rightarrow$ parameterized by 3 angles $\underbrace{+\ 1 \text{ phase}}$

gives rise to $\mathcal{CP}$-violation in SM

$\Rightarrow$ weak interaction violates $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{CP}$

Physical parameters of the Standard Model

- gauge sector (2 parameters)
  elementary charge e
  weak mixing angle θ_W, $\cos\theta_\mathrm{W} = \frac{M_\mathrm{W}}{M_\mathrm{Z}}$

- Higgs sector (2 parameters)

  Higgs-boson mass M_H
  W-boson mass M_W

- fermion sector (9+4 parameters)
  fermion masses $m_\mathrm{e}, m_\mu, m_\tau$
  $m_\mathrm{u}\, m_\mathrm{d}, m_\mathrm{c}, m_\mathrm{s}, m_\mathrm{t}, m_\mathrm{b}$
  quark-mixing matrix (V_{ij}) $\theta_{12}, \theta_{23}, \theta_{13}, \delta_\mathrm{CP}$

  if right-handed neutrino is included in addition
  neutrino masses $m_{\nu_\mathrm{e}}, m_{\nu_\mu}, m_{\nu_\tau}$
  lepton-mixing matrix $\theta^l_{12}, \theta^l_{23}, \theta^l_{13}, \delta^l_\mathrm{CP}$
  $\Rightarrow$ 12+8 parameters

$\Rightarrow$ most parameters originate from fermion sector

Gauge-invariant Lagrangian:

$$\mathcal{L}_{\mathrm{EW}}(\underbrace{g_2, g_1, v}_{M_\mathrm{W},\, M_\mathrm{Z},\, \alpha}, \quad \underbrace{\lambda}_{M_\mathrm{H}}, \quad \underbrace{g_f}_{m_f}) + \mathcal{L}_{\mathrm{QCD}}(\alpha_\mathrm{s})$$

gauge invariance $\Rightarrow$ **theory is renormalizable**

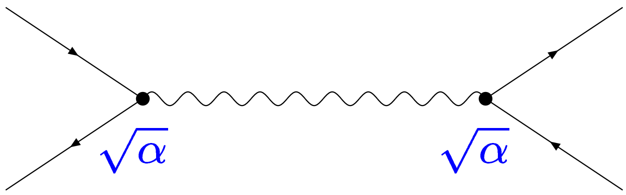
[G. 't Hooft '71] *Nobel price '99*

[G. 't Hooft, M. Veltman '72]

$\Rightarrow$ quantum field theory: quantum effects calculable

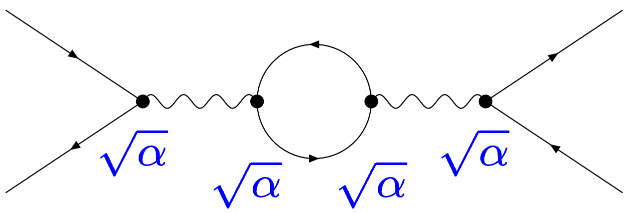
expansion in coupling constant:

lowest order, classical limit



$\alpha \approx \frac{1}{137}$

quantum corrections: loop diagrams

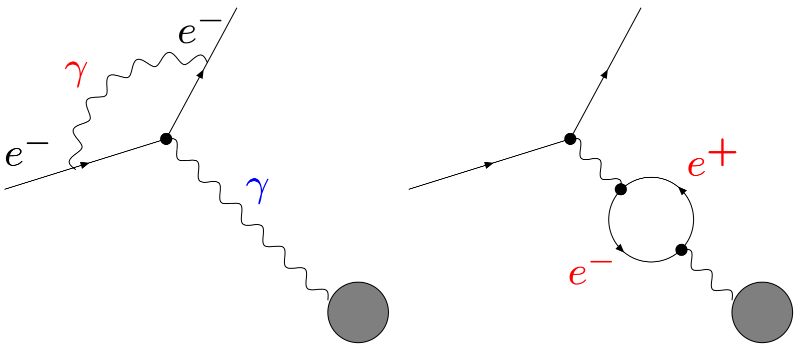


via loop corrections: all particles of the model enter

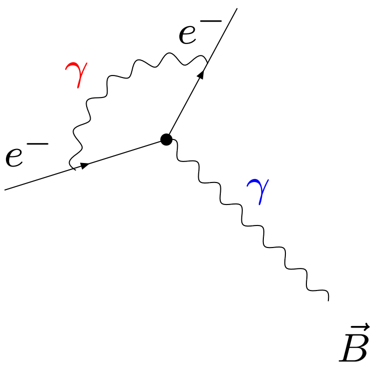
Electroweak precision tests

a theoretical concept becomes precision physics

Lamb shift:



$g-2$:



$$a = \tfrac{1}{2}(g-2)$$

$$a_{\text{exp}} = 1\,159\,652\,188(\pm 4)\times 10^{-12}$$

$$a_{\text{theo}} = 1\,159\,652\,157(\pm 28)\times 10^{-12}$$

- LEP1/SLC: $e^+e^- \to Z \to f\bar{f}$
  LEP1: $\sim 4 \times 10^6$ events/experiment
  4 experiments (1989 – 1995)

- LEP2: $e^+e^- \to W^+W^-$
  $\mathcal{O}(10^4)$ W pairs (1996 – 2000)

- Tevatron: $q\bar{q}' \to W \to l\nu, q\bar{q}'$
  (p$\bar{\text{p}}$) $q\bar{q}' \to t\bar{t},\ t \to W^+b \to \ldots$

- low-energy experiments (μ decay, νN scattering, νe scattering, atomic parity violation, ...)

exp. results

M_Z [GeV]	$= 91.1875 \pm 0.0021$	0.002%
Γ_Z[GeV]	$= 2.4952 \pm 0.0023$	0.09%
$\sin^2\theta_{\text{eff}}^{\text{lept}}$	$= 0.23148 \pm 0.00017$	0.07%
M_W [GeV]	$= 80.410 \pm 0.032$	0.04%
m_t [GeV]	$= 172.7 \pm 2.9$	1.7%
G_F [GeV^{-2}]	$= 1.16637(1)10^{-5}$	0.001%

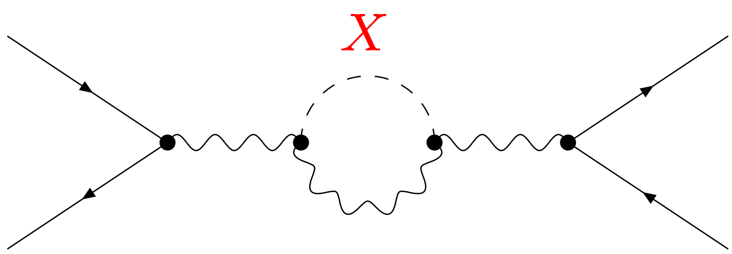
Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
$\Delta\rho, \Delta r, M_W, \sin^2\theta_{\text{eff}}$	$\leftrightarrow$	SM, MSSM , . . .

$\Downarrow$

Test of theory at quantum level:

Sensitivity to loop corrections



$\Downarrow$

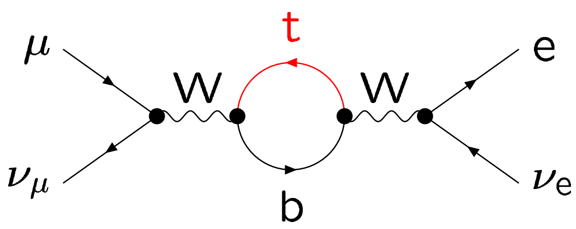
sensitivity to internal particles (X)

Loop contributions

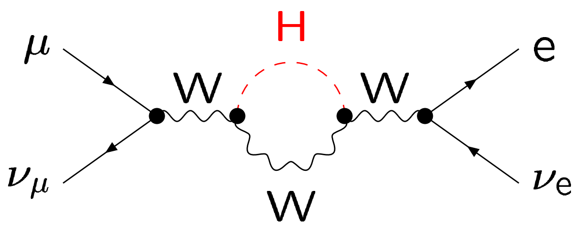
quantum corrections, of $\mathcal{O}(1\%)$

contain all details of the theory

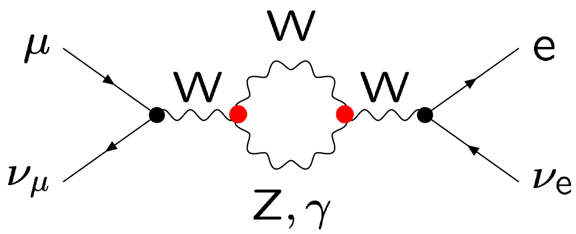
- top quark



- Higgs boson

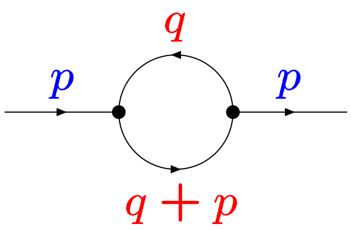


- gauge-boson self-couplings



$\Rightarrow$ allow for indirect experimental tests of not directly accessible quantities

Example of loop integral:



$$\sim \int d^4q \frac{1}{(q^2 - m_1^2)\left[(q+p)^2 - m_2^2\right]}$$

$$q \to \infty: \quad \sim \int^{\infty} \frac{q^3 dq}{q^4} = \int^{\infty} \frac{dq}{q} \quad \to \infty$$

$\Rightarrow$ integral diverges for large q

$\Rightarrow$ theory in this form not physically meaningful

$\Rightarrow$ further concept needed: renormalization

Renormalizable theories: infinities can consistently be absorbed into parameters of theory

Two step procedure:

Regularization:

theory modified such that expressions become mathematically meaningful

$\Rightarrow$ "regulator" introduced, removed at the end

e.g. cut-off in loop integral

$$\int_0^\infty d^4q \;\rightarrow \int_0^\Lambda d^4q; \quad \Lambda \rightarrow \infty \text{ at the end}$$

technically more convenient: dimensional regularization

$$\int d^4q \;\rightarrow \int d^Dq, \quad D = 4 - \varepsilon; \quad D \rightarrow 4 \text{ at the end}$$

Renormalization:

original "bare" parameters replaced by renormalized parameters + counterterms

reparameterization:

$$\underbrace{g_0}_{\text{bare parameter}} = \underbrace{g}_{\text{renormalized parameter}} + \underbrace{\delta g}_{\text{counterterm}}$$

Renormalizable theory:
divergencies compensated by counterterms

Renormalization:

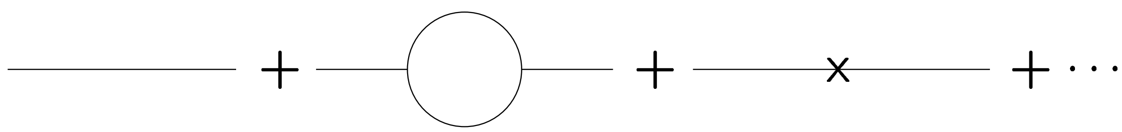
- absorption of divergencies
- determination of physical meaning of parameters order by order in perturbation theory

Example:

mass renormalization, $m_0^2 = m^2 + \delta m^2$

Physical mass: pole of propagator

inverse propagator up to 1-loop order:



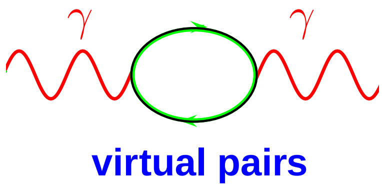
$p^2 - m^2$ $\qquad \Sigma(p^2)$ $\qquad -\,\delta m^2$

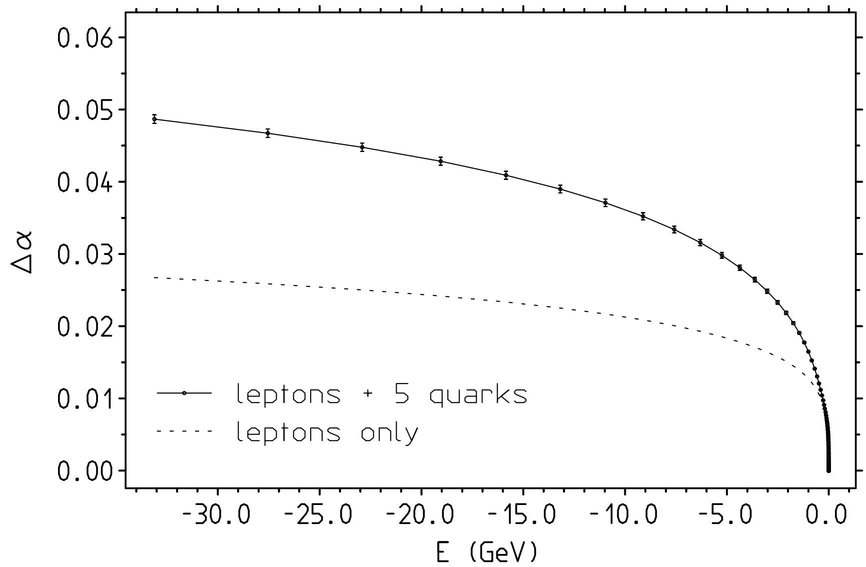
on-shell renormalization: $\delta m^2 = \mathrm{Re}\,\Sigma(m^2)$

charge renormalization: $e + \delta e$

δe for $q^2 = 0$ (real photons) involves

photon vacuum polarization





$$\Pi^\gamma(M_Z^2) - \Pi^\gamma(0) \equiv \Delta\alpha$$

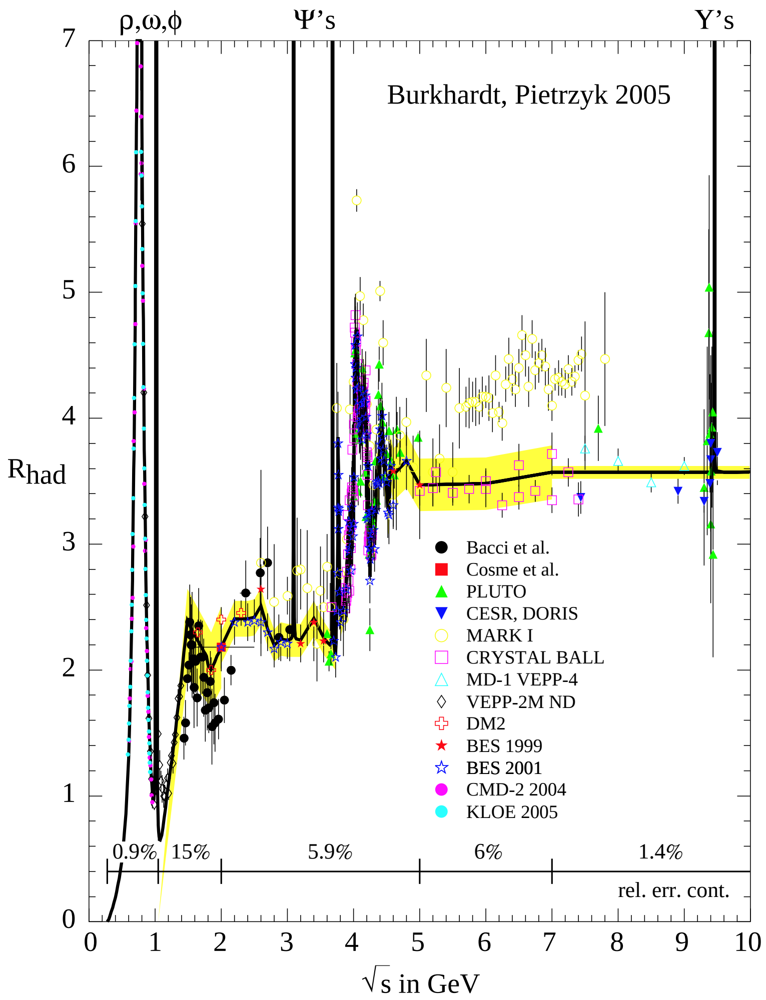
$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha}$$ effective charge

$$\Delta\alpha = \Delta\alpha_{\mathrm{lept}} + \Delta\alpha_{\mathrm{had}},$$

$$\Delta\alpha_{\mathrm{lept}} = 0.031498 \quad (3-\mathrm{loop})$$

$$\Delta\alpha_{\mathrm{had}} = 0.02758 \pm 0.00035$$

$$\Delta\alpha_{\mathrm{had}} = -\frac{\alpha}{3\pi} M_Z^2 \,\mathrm{Re} \int_{4m_\pi^2}^{\infty} ds' \, \frac{R_{\mathrm{had}}(s')}{s'(s'-M_Z^2-i\epsilon})$$

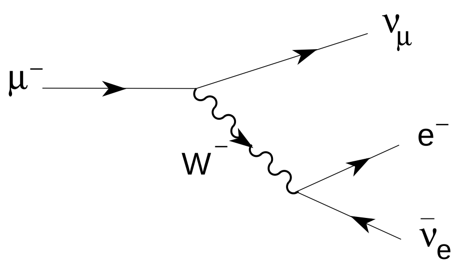


$M_W - M_Z$ correlation

Definition of Fermi constant G_F via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right) \left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right) (1 + \Delta q)$$

Δq: QED corrections in Fermi Model,
included in definition



SM prediction:

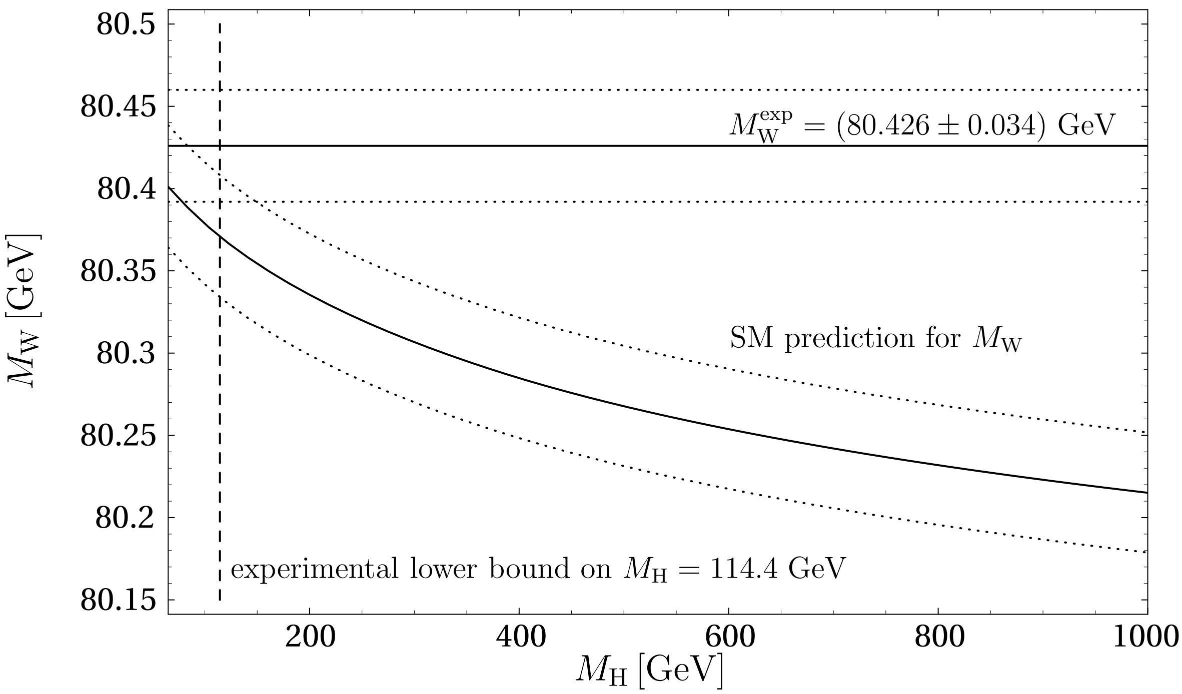
$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2\left(1 - M_W^2/M_Z^2\right)} (1 + \Delta r)$$

Δr: quantum correction, $\Delta r = \Delta r(m_t, M_H, \dots)$

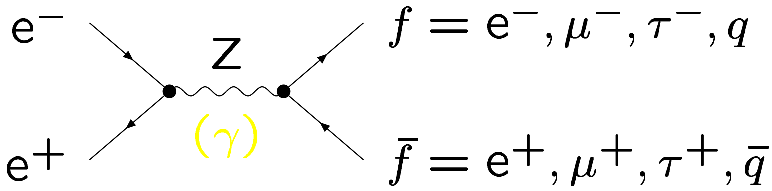
complete at 1-loop and 2-loop order

$$\rightarrow \quad M_W = M_W(\alpha, G_F, M_Z, m_t, M_H)$$

[Awramik, Czakon, Freitas, Weiglein]



Z-boson resonance

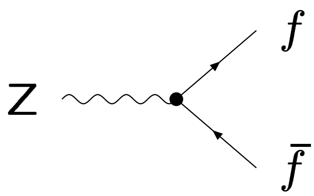


LEP1: $\sim 16 \cdot 10^6$ events (1989–1995)

resonance cross-section
(approximate Breit-Wigner) $\qquad s = E^2_{\mathrm{CMS}}$

$$\sigma_f(s) = 12\pi \frac{s}{M_Z^2} \frac{\Gamma(Z \to e^+e^-)\,\Gamma(Z \to f\bar{f})}{(s - M_Z^2)^2 + s^2(\Gamma_Z)^2/M_Z^2}$$

Z-boson width

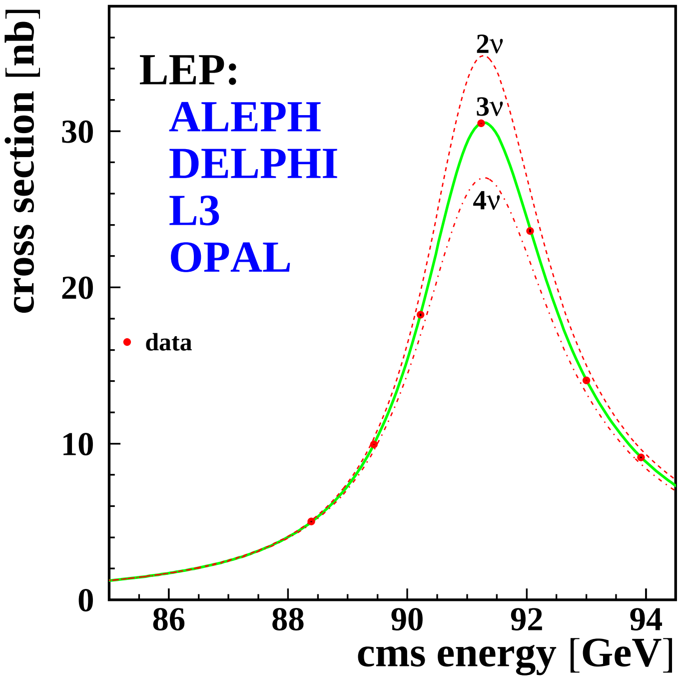


$$\Gamma_Z = \underbrace{\Gamma(e^-e^+) + \Gamma(\mu^-\mu^+) + \Gamma(\tau^-\tau^+)}_{\text{leptonic}}$$

$$+ \underbrace{\sum_q \Gamma(q\bar{q})}_{\text{hadronic}} + \underbrace{N_\nu \Gamma(\nu\bar{\nu})}_{\text{invisible}}$$

- Line shape $\Rightarrow M_Z, \Gamma_Z$
- peak cross section
  $\Rightarrow \Gamma(Z \to l^+l^-), \Gamma(Z \to \text{hadrons})$

Z resonance



Z-boson observables can be expressed in terms of

- effective Z boson couplings:

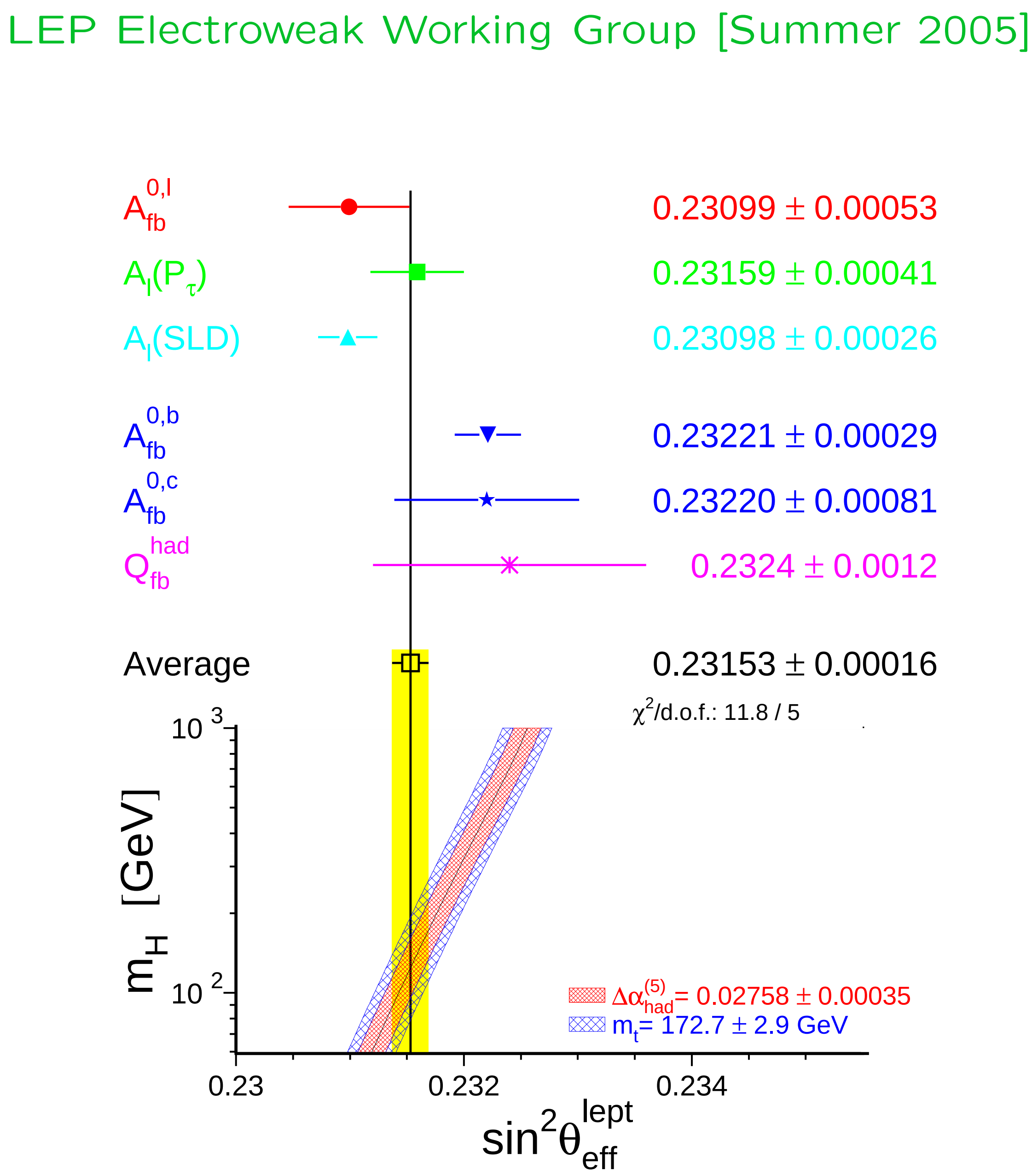
$$g_V^f \to g_V^f + \Delta g_V^f, \qquad g_A^f \to g_A^f + \Delta g_A^f$$

with higher order contributions in $\Delta g_{V,A}^f$

- effective ew mixing angle (for $f = e$):

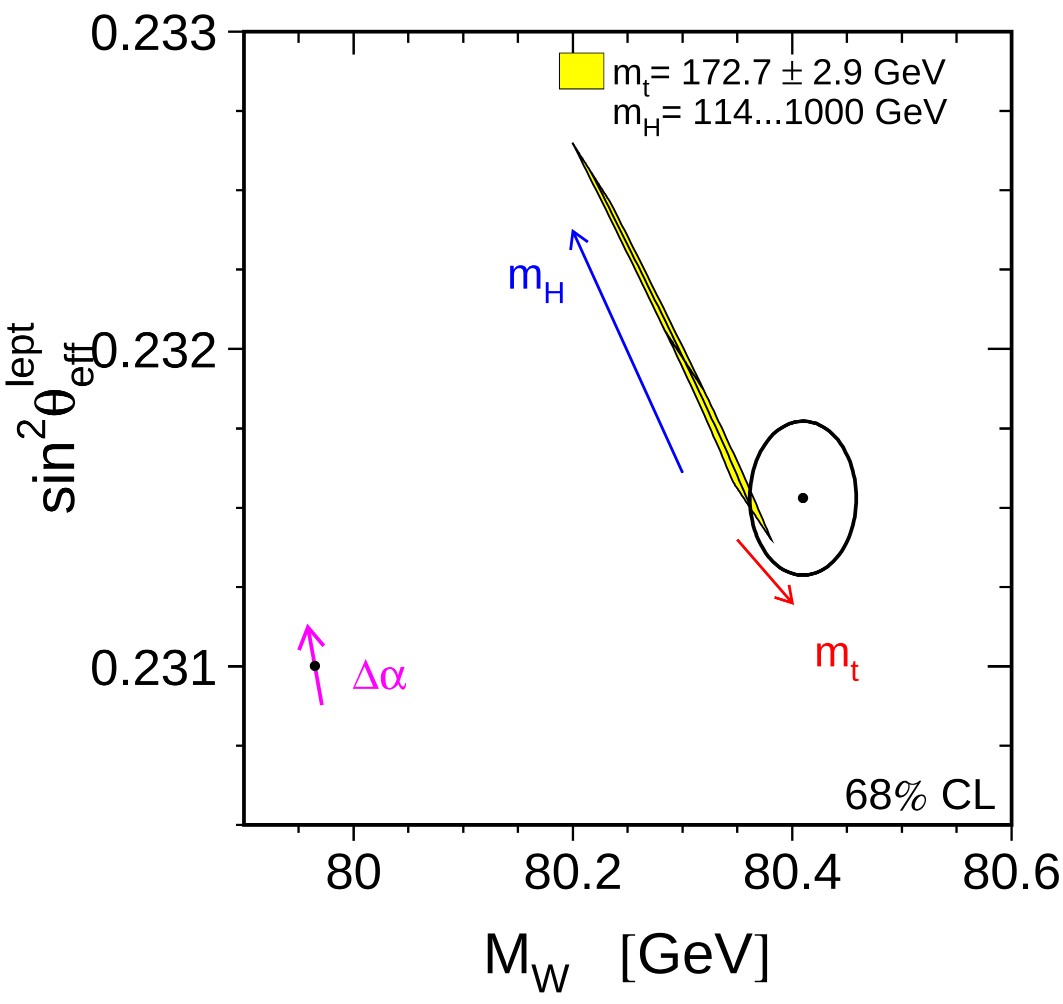
$$\sin^2\theta_{\text{eff}} = \frac{1}{4}\left(1 - \text{Re}\,\frac{g_V^e}{g_A^e}\right)$$

complete at 1-loop order, 2-loop fermionic contributions



LEP Electroweak Working Group [Summer 2005]
$A_{fb}^{0,l}$ 0.23099 ± 0.00053
$A_l(P_\tau)$ 0.23159 ± 0.00041
A_l(SLD) 0.23098 ± 0.00026
$A_{fb}^{0,b}$ 0.23221 ± 0.00029
$A_{fb}^{0,c}$ 0.23220 ± 0.00081
Q_{fb}^{had} 0.2324 ± 0.0012
Average 0.23153 ± 0.00016
χ^2/d.o.f.: 11.8 / 5
10^3
10^2
m_H [GeV]
$\Delta\alpha_{had}^{(5)}$ = 0.02758 ± 0.00035
m_t = 172.7 ± 2.9 GeV
0.23
0.232
0.234
$\sin^2\theta_{eff}^{lept}$

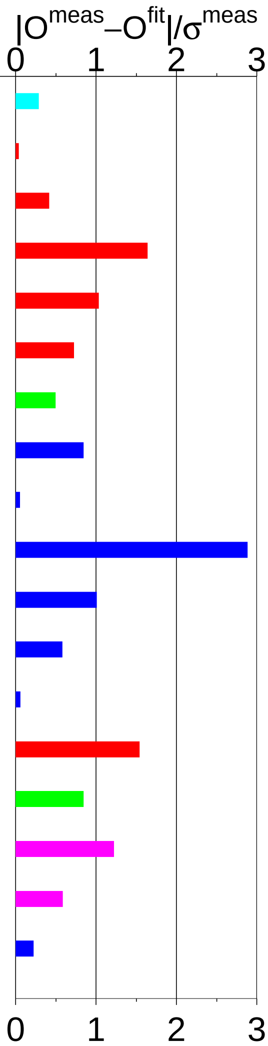
LEP Electroweak Working Group



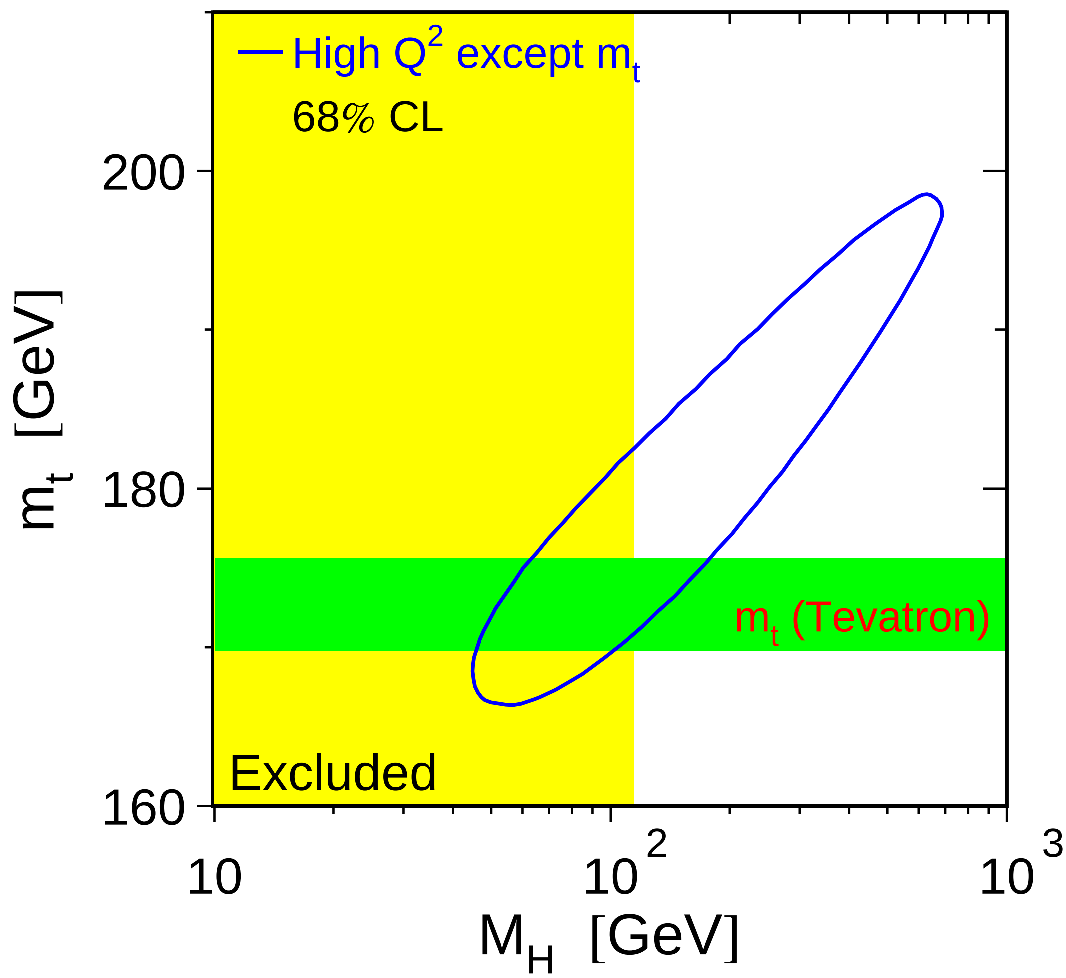
Global fit [M. Grünewald, EPS Lisbon 2005]

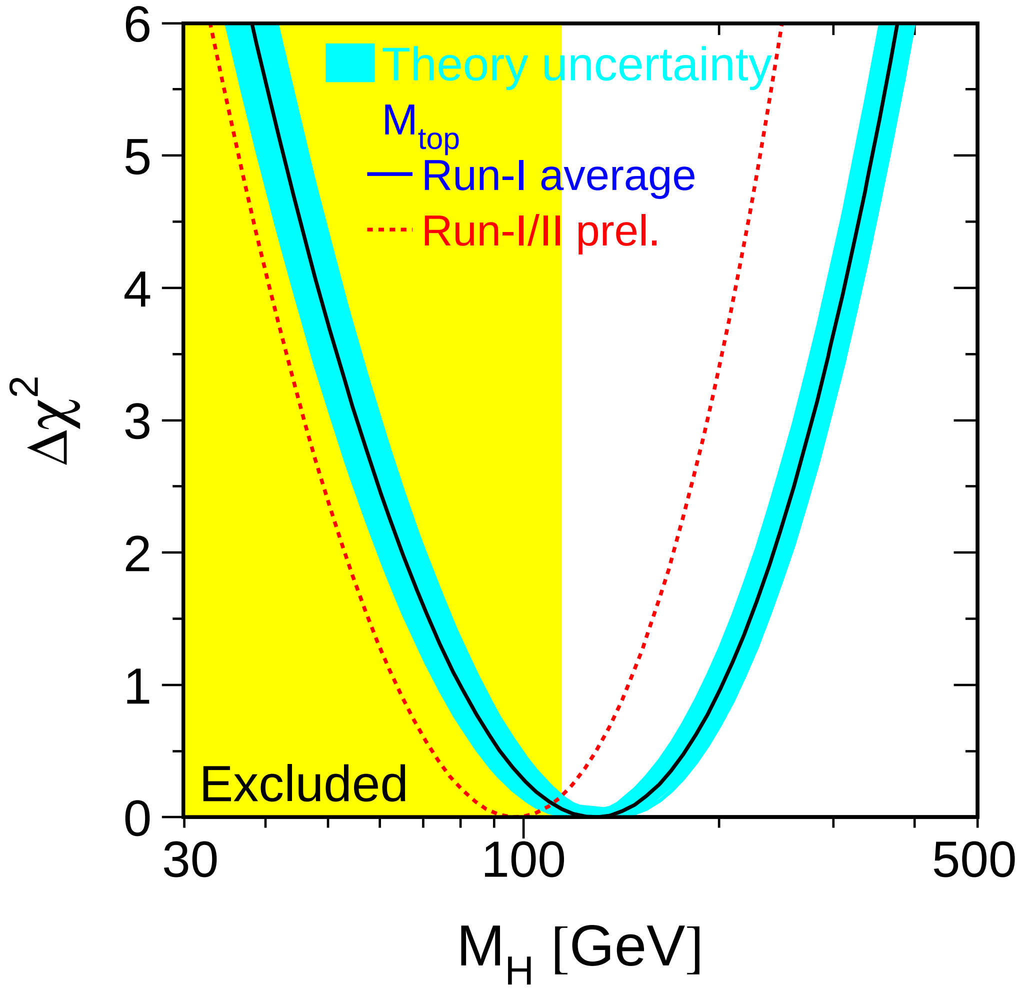
Preliminary

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4962
σ^0_{had} [nb]	41.540 ± 0.037	41.479
R_l	20.767 ± 0.025	20.741
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21573
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.383
Γ_W [GeV]	2.133 ± 0.069	2.092
m_t [GeV]	174.3 ± 3.4	175.1



Bounds on m_t and M_H



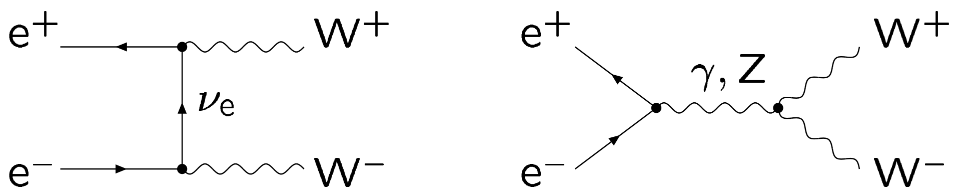


$$M_{\mathsf{H}} < 186 \text{ GeV} \quad (95\%\text{C.L.})$$

renormalized probability for $M_{\mathsf{H}} > 114$ GeV to 100%:

$$M_{\mathsf{H}} < 219 \text{ GeV} \quad (95\%\text{C.L.})$$

W-pair production



LEP2: $\mathcal{O}(10^4)$ events (1996-2000)

study of W-pair production allows

- precise measurement of M_W

  $\Delta M_W \approx 40\,\text{MeV}$, $\Delta M_W/M_W \approx 0.05\%$

- measurement of triple-gauge-boson couplings

  total cross-section

  $\Delta\sigma_{WW}\,/\,\sigma_{WW} \;\sim\; 1\%$

  triple-gauge-boson couplings

  $\sim 3\%$

anomalous gauge couplings

generalization of gauge boson self couplings

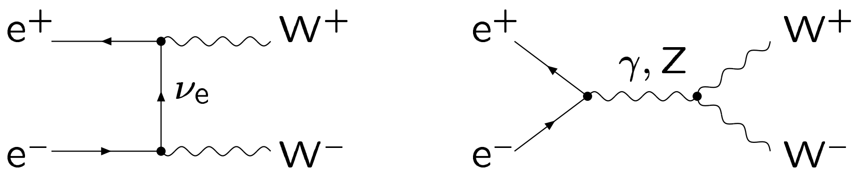
($F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\quad Z_{\mu\nu}$, $W^\pm_{\mu\nu}$ analogously)

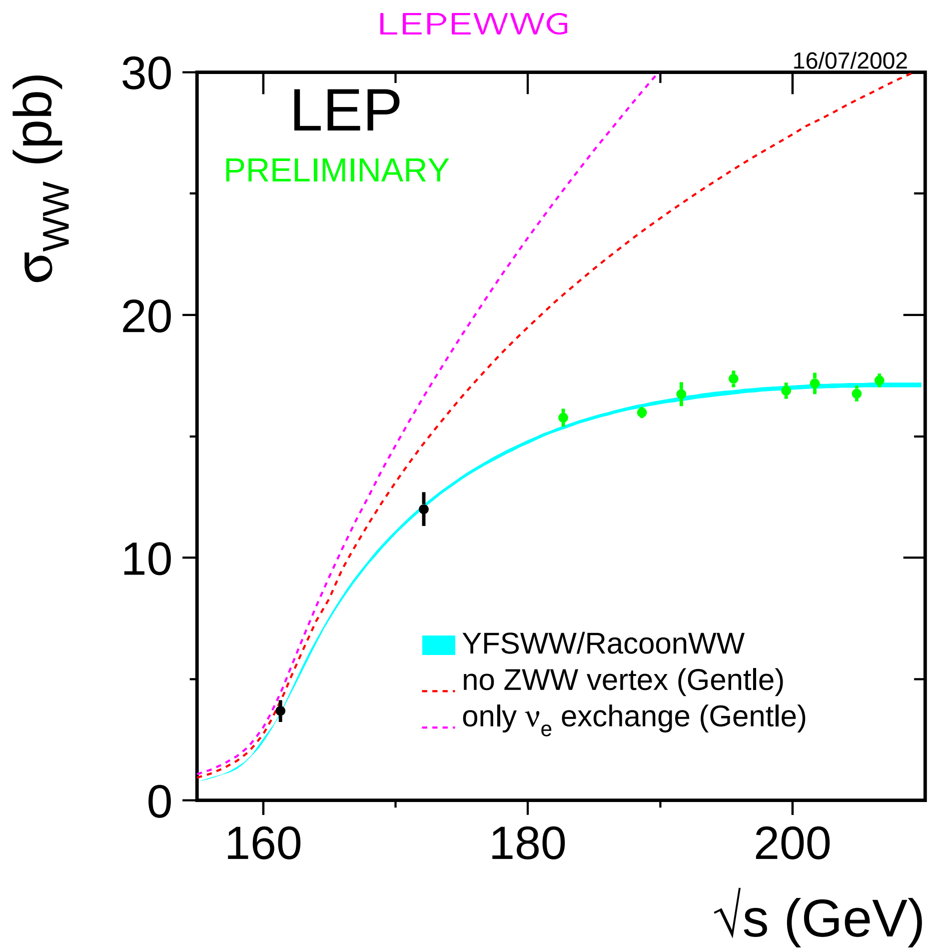
$$\begin{aligned}
\mathcal{L}_{WW\gamma/Z} \; = \; & e \Big[(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)\, W^{-\mu} A^\nu \\
& + \kappa_\gamma \, W^+_\mu W^-_\nu \, F^{\mu\nu} \\
& + \frac{\lambda_\gamma}{M_W^2} W^+_{\rho\mu} W^{-\mu}_\nu \, F^{\rho\nu} \; + \text{h. c.} \Big] \\
& + e \cot\theta_W \Big[(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)\, W^{-\mu} Z^\nu \\
& + \kappa_Z \, W^+_\mu W^-_\nu \, Z^{\mu\nu} \\
& + \frac{\lambda_Z}{M_W^2} W^+_{\rho\mu} W^{-\mu}_\nu \, Z^{\rho\nu} + \text{h. c.} \Big]
\end{aligned}$$

Standard Model:

$$\kappa_\gamma = \kappa_Z = 1, \quad \lambda_\gamma = \lambda_Z = 0$$

W-pair-production cross-section





contributions of non-abelian couplings
relevance of gauge invariance

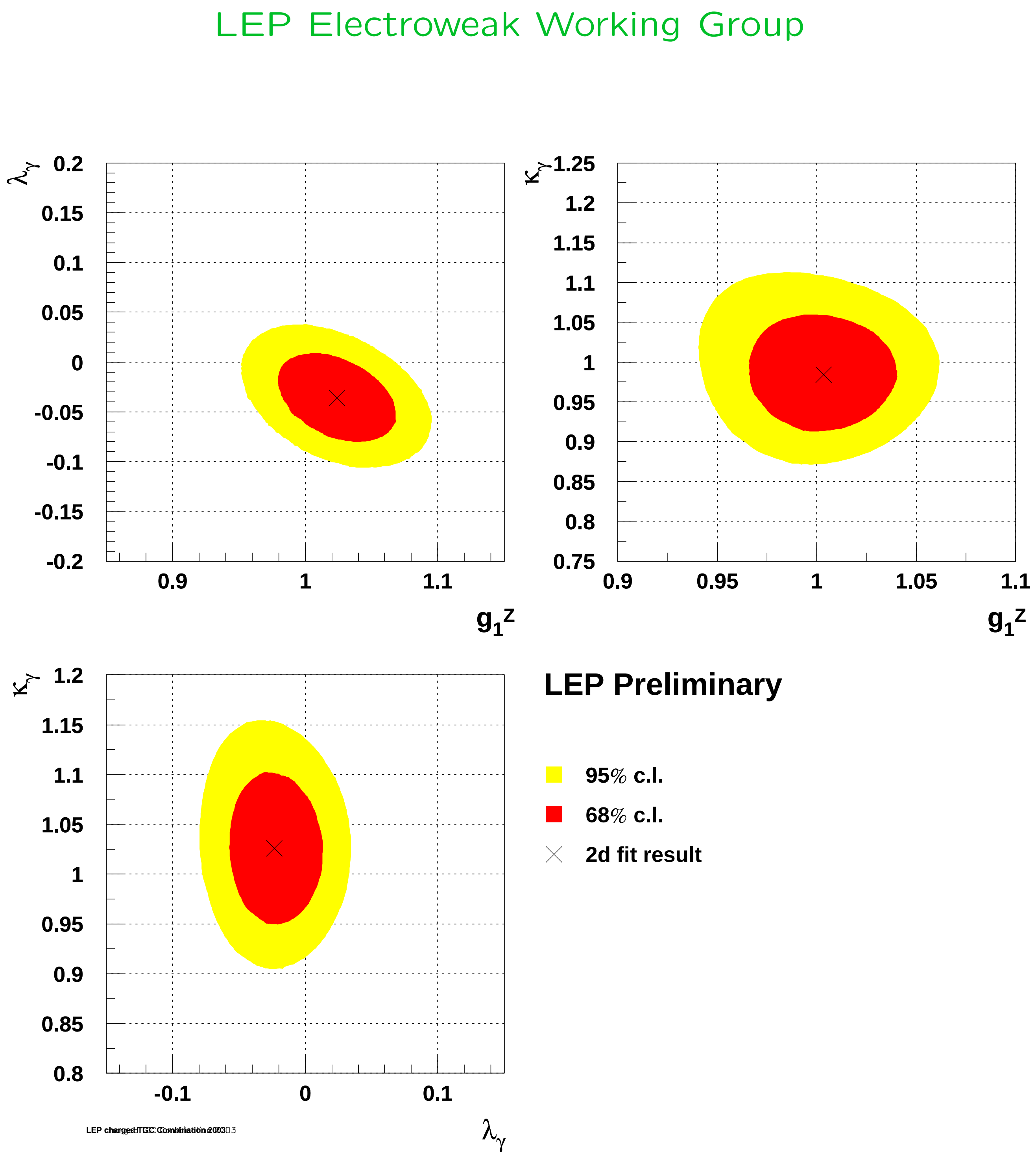


Figure 11.2: The 68% and 95% confidence level contours for the three two-parameter fits to the charged TGCs g_1^Z-λ_γ, g_1^Z-κ_γ and λ_γ-κ_γ. The fitted coupling value is indicated with a cross; the Standard Model value for each fit is in the centre of the grid. The contours include the contribution from systematic uncertainties.

Anomalous g-factor of the muon

- Dirac theory: $g = 2$

- QED, 1-loop order: $g = 2 + \frac{\alpha}{\pi}$

- Standard Model prediction

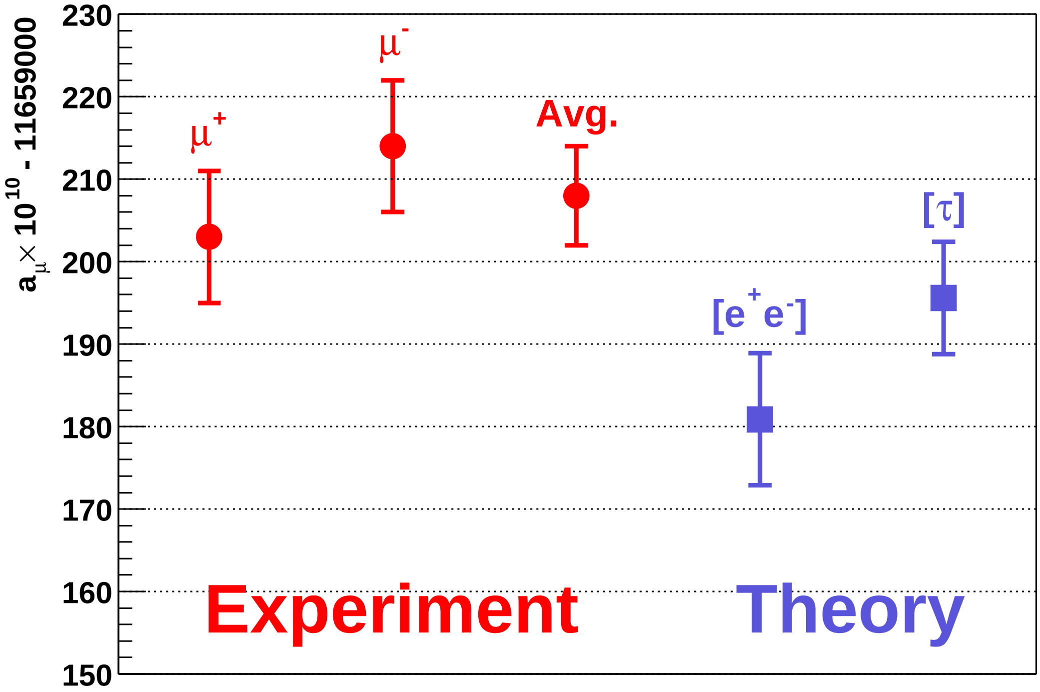
  QED part: 4-loop (5-loop estimate)
  Electroweak part: 2-loop

- **Experiment 2004:** Brookhaven E821

$$a_\mu = \frac{g-2}{2} = 11659208(6) \cdot 10^{-10}$$

above the SM prediction

Theory versus experiment



e^+e^- data based prediction:
2.7 σ below exp. value

τ data based prediction:
0.7 σ below exp. value

uncertainty mainly from hadronic vacuum polarization

Summary of precision tests

- Electroweak precision physics

  $\Rightarrow$ Sensitivity to quantum effects of the theory

  $\Rightarrow$ test consistency of the model

  constraints on unknown parameters

- Precision tests of the SM

  $\Rightarrow$ light Higgs preferred, $M_H \lesssim 200$ GeV

  preference for light Higgs is not an artefact of observables deviating by $\approx 3\sigma$ from SM prediction

- Prospects for next generation of colliders:

  improved accuracy of precision observables
  M_W, $\sin^2\theta_{eff}$, m_h, ... and
  input parameters m_t, ...

  $\Rightarrow$ Highly sensitive test of electroweak theory

(expected) experimental precision

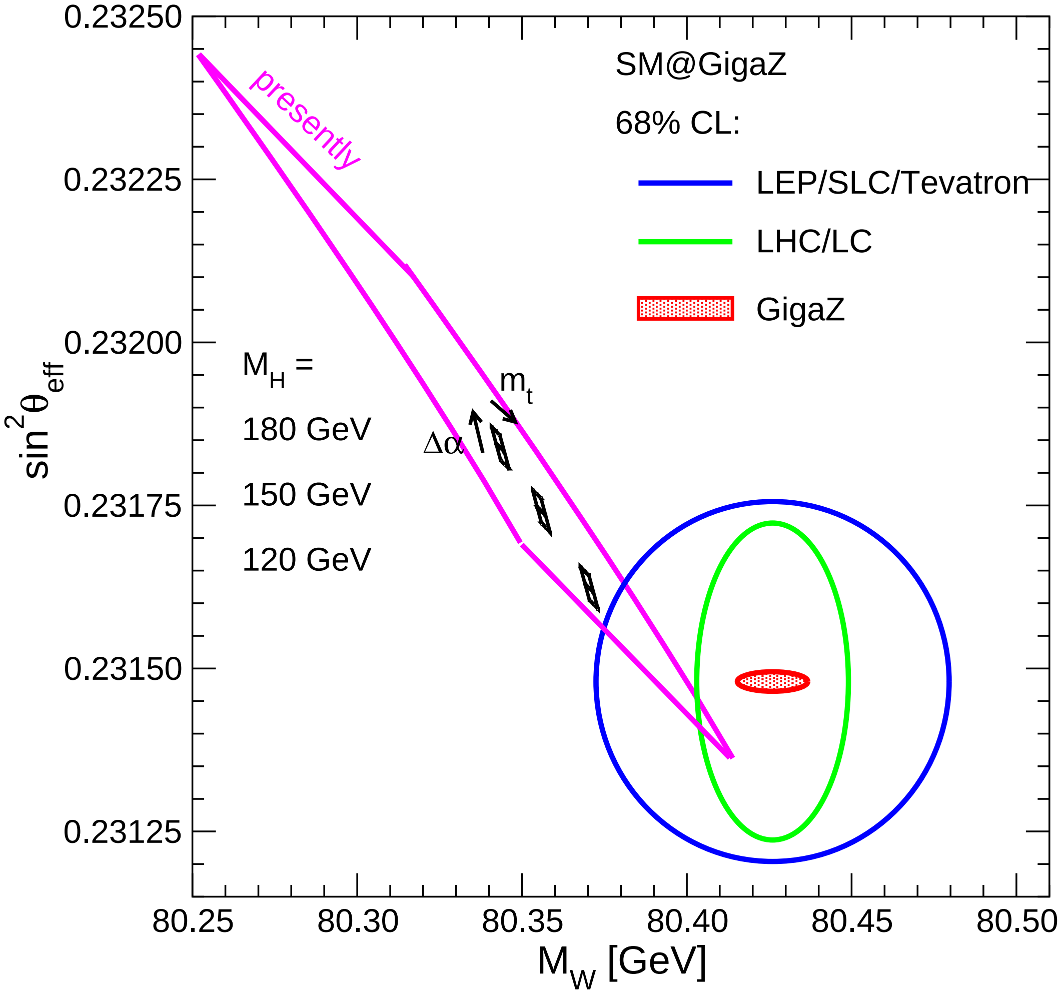
error for	LEP/Tev	Tev/LHC	LC	GigaZ
M_W [MeV]	33	15	15	7
$\sin^2\theta_{\mathrm{eff}}$	0.00017	0.00021		0.000013
m_{top} [GeV]	4.3	2	0.2	0.13
M_{Higgs} [GeV]	–	0.1	0.05	0.05

together with

$\delta M_Z = 2.1$ MeV (LEP)

$\delta G_F/G_F = 1 \cdot 10^{-5}$ (μ lifetime)

[Erler, Heinemeyer, Hollik, Weiglein, Zerwas]



Higgs bosons

Higgs boson is the only missing ingredient of the SM

$\Rightarrow$ Higgs search (& Higgs physics) is one of the main goals of collider physics

$\leq$ 2000: LEP:
e^+e^- collider, $E_{\text{CM}} \lesssim 206$ GeV

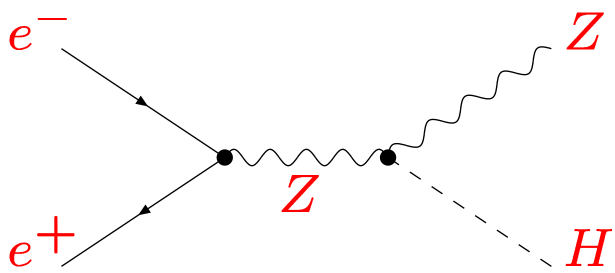
$\geq$ 2001: Tevatron, Run II:
$p\bar{p}$ collider, $E_{\text{CM}} \approx 2$ TeV

$\gtrsim$ 2007: LHC:
pp collider, $E_{\text{CM}} \approx 14$ TeV

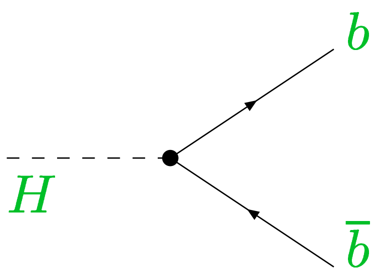
$\gtrsim$ 2012: ? LC:
e^+e^- collider, $E_{\text{CM}} \approx 500$–$1000$ GeV

Search for the Standard Model Higgs at LEP

Dominant production process: $e^+e^- \to ZH$



Dominant decay process: $H \to b\bar{b}$



exclusion limit (95% C.L.):

$$M_{\mathrm{H}} > 114.4\ \mathrm{GeV}$$

Theoretical bounds on Higgs boson mass from

- perturbativity
  $\longrightarrow$ upper bound

- unitarity
  $\longrightarrow$ upper bound

- triviality (Landau pole)
  $\longrightarrow$ upper bound

- vacuum stability
  $\longrightarrow$ lower bound

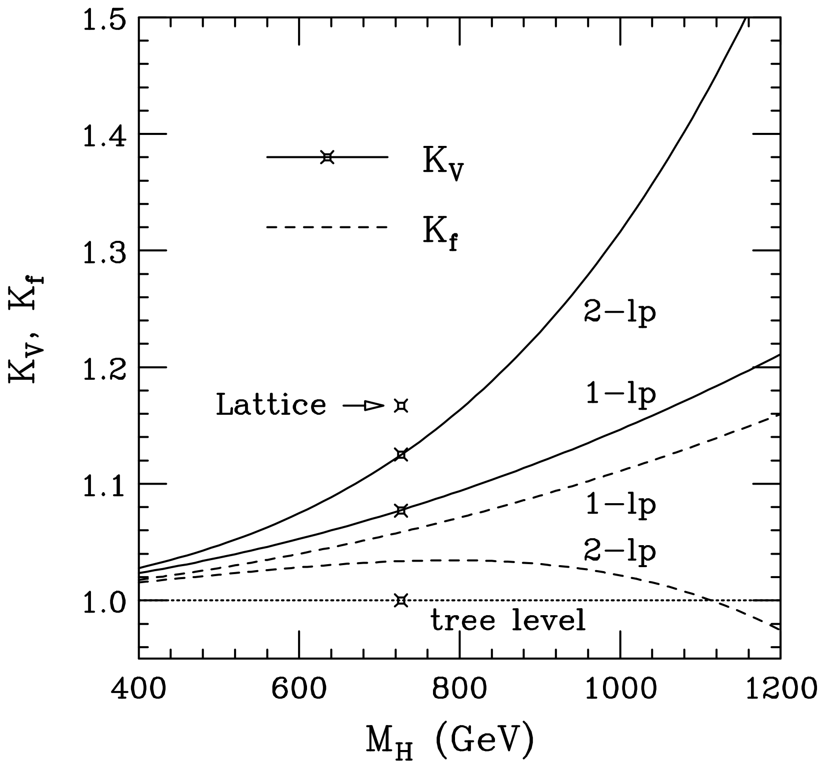
perturbativity

Higgs decay widths into fermions:

$$\Gamma(H \to f\bar{f}) = \Gamma_{\text{tree}} \cdot K_f$$
$$K_f = 1 + (1-\text{loop}) + (2-\text{loop}) + \cdots$$

Higgs decay widths into vector bosons:

$$\Gamma(H \to V\bar{V}) = \Gamma_{\text{tree}} \cdot K_V$$
$$K_V = 1 + (1-\text{loop}) + (2-\text{loop}) + \cdots$$



[Ghinculov; Frinck, Kniehl, Riesselmann]

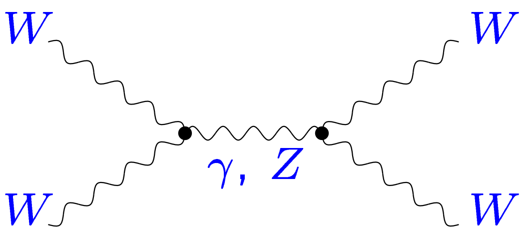
(1-loop) = (2-loop) for $M_H = 930$ GeV

unitarity

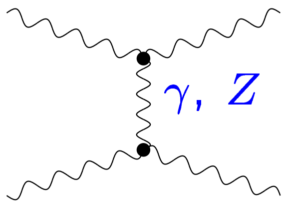
scattering of longitudinally polarized W bosons:

$$W_L W_L \to W_L W_L$$

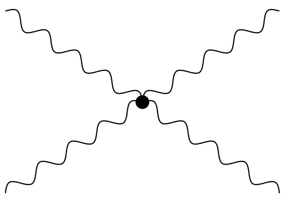
$\mathcal{M}_V =$



$+$



$+$

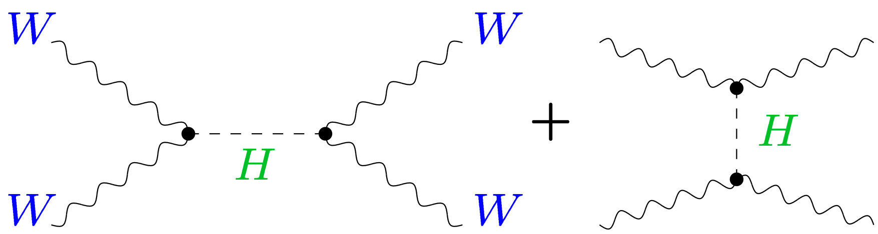


$$= -g^2 \frac{E^2}{M_{\mathrm{W}}^2} + \mathcal{O}(1) \qquad \text{for } E \to \infty$$

$\Rightarrow$ violation of probability conservation

Extra contribution from scalar particle:

$\mathcal{M}_S =$



$$= g_{WWH}^2 \frac{E^2}{M_{\mathrm{W}}^4} + \mathcal{O}(1) \qquad \text{for } E \to \infty$$

$$\mathcal{M} \quad = \mathcal{M}_V + \mathcal{M}_S$$

$\Rightarrow$ terms with bad high-energy behavior cancel for

$$g_{WWH} = g\, M_{\mathrm{W}}$$

$$\text{for } s >> M_W^2, \qquad \text{with } t = -\tfrac{s}{2}\,(1 - \cos\theta),$$

$$\mathcal{M} \approx \frac{M_H^2}{v^2}\left(2 + \frac{M_H^2}{s - M_H^2} + \frac{M_H^2}{t - M_H^2}\right)$$

partial wave expansion:

$$\mathcal{M}(s,t) \;=\; 8\pi \sum_{l=0}^{\infty} (2l+1)\, P_l(\cos\theta)\, a_l$$

unitarity condition:

$$|a_l| < 1$$

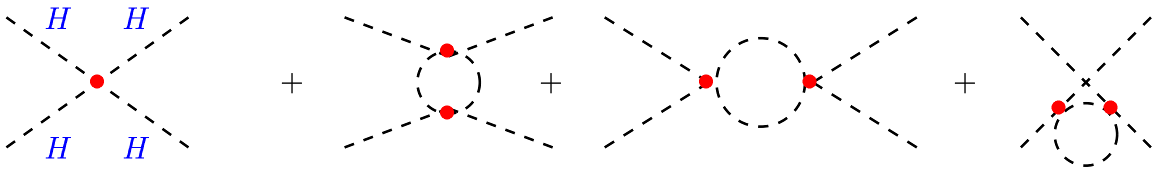
project on $l = 0$ partial wave:

$$\begin{aligned} a_0 &= \frac{1}{16\pi} \int_{-1}^{1} d\cos\theta\, \mathcal{M}(s,t) \\ &= \frac{M_H^2}{8\pi v^2} \left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log\left(1 + \frac{s}{M_H^2}\right) \right] \\ &\approx \frac{M_H^2}{4\pi v^2} \quad \text{for} \quad s >> M_H^2 \end{aligned}$$

$$\boxed{a_0 < 1 \quad \rightarrow \quad M_H < 872\,\text{GeV}}$$

triviality (Landau pole)

Higgs self coupling is scale dependent, $\lambda(Q)$



variation with scale Q described by RGE

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2}\lambda^2, \qquad t = \log\frac{Q^2}{v^2}$$

solution:

$$\lambda(Q) = \frac{\lambda(v)}{1 - \frac{3}{4\pi^2}\lambda(v)\log\frac{Q^2}{v^2}} \quad \text{with} \quad \lambda(v) = \frac{M_H^2}{2v^2}$$

diverges at scale $Q = \Lambda_C$ (Landau pole)

$$\Lambda_C = v \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right)$$

maximum Higgs mass by condition $\Lambda_C > M_H$

$$\rightarrow \quad M_H < 800\,\text{GeV}$$

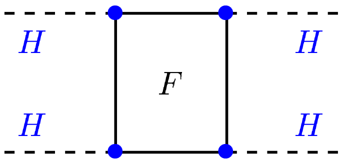
vacuum stability

top-quark Yukawa coupling

$$g_t = \frac{\sqrt{2} m_t}{v}$$

contributes to the running Higgs self coupling $\lambda(Q)$ through

top loop $\quad \sim g_t^4$



variation with scale Q described by RGE

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2} \left(\lambda^2 - \frac{m_t^4}{v^4} \right)$$

approximate solution:

$$\lambda(Q) = \lambda(v) - \frac{3 m_t^4}{2\pi^2 v^4} \log \frac{Q}{v}$$

$$\lambda(Q) < 0 \text{ for } Q > \Lambda_C \rightarrow \text{vacuum not stable}$$

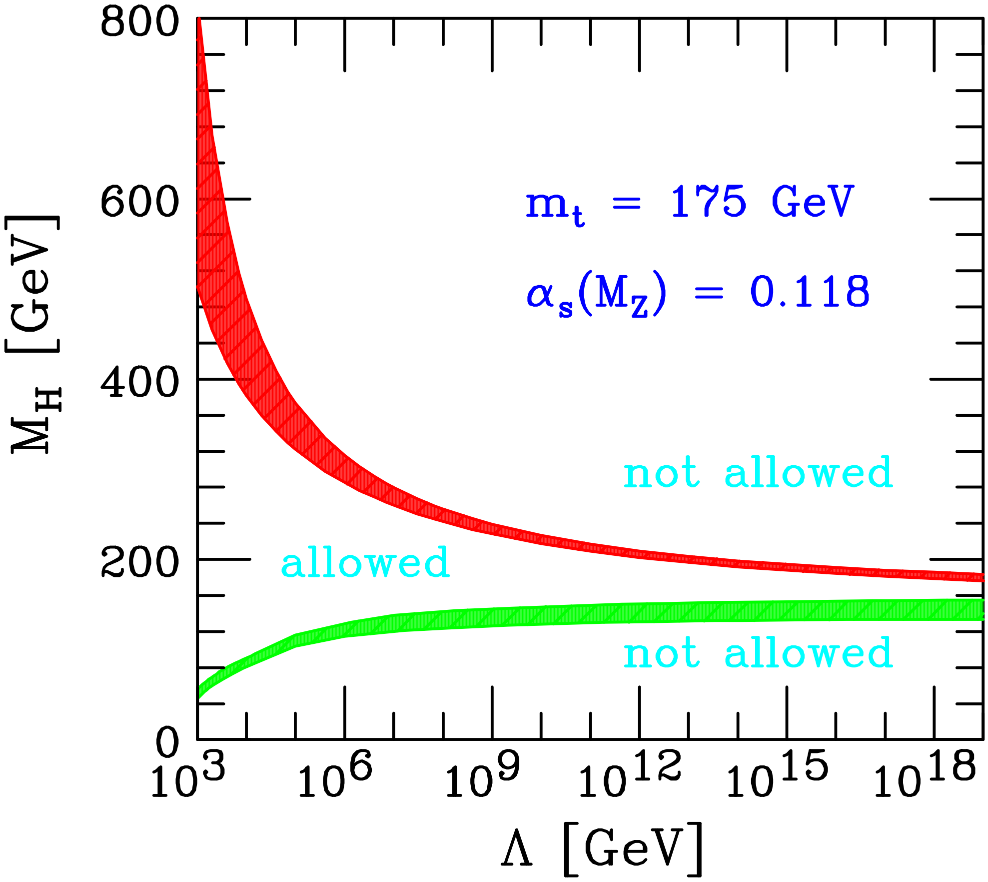
high value of Λ_C needs M_H large enough

$$\Lambda_C \sim 10^{16} : \quad M_H > 130 \text{ GeV}$$

$$\Lambda_C \sim 10^{3} : \quad M_H > 70 \text{ GeV}$$

combined effects, RGE in two-loop order:

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left(12\lambda^2 - 3\,g_t^2 + 6\,\lambda\,g_t^2 + \cdots\right)$$



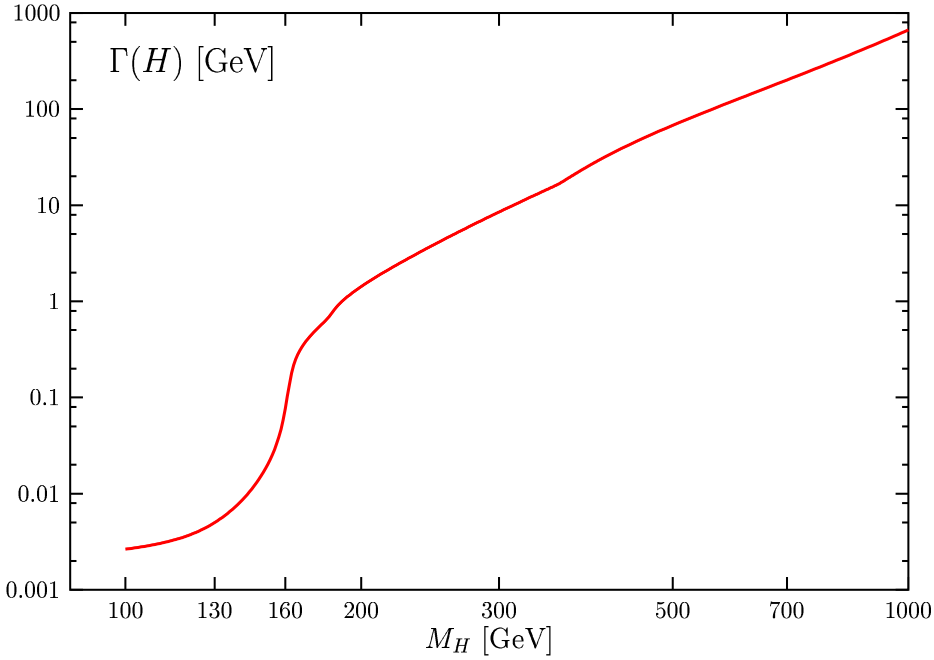
[Hambye, Riesselmann]

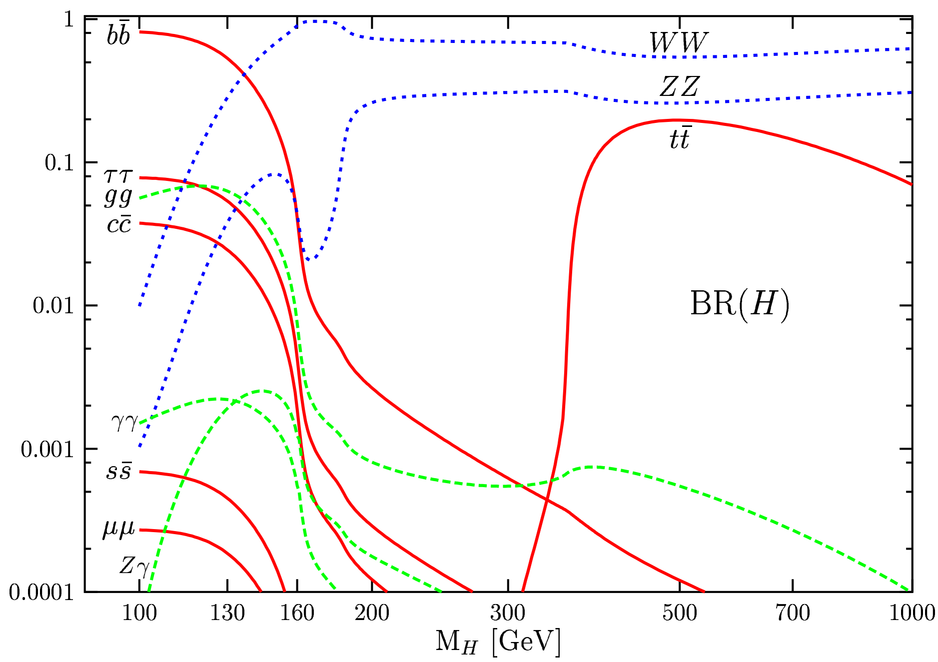
Higgs boson(s)

- questions to be answered:
  - numbers of Higgs particles
  - masses and quantum numbers (spin, partity, charges, CP, ...)
  - couplings to fermions / gauge bosons
  - self couplings $\rightarrow$ Higgs potential

- needs precise determination of mass(es) and coupling constants
  - production cross sections
  - decay rates/ branching ratios
  - inclusion of higher-order effects

total width and branching ratios





Production mechanisms

- gluon-gluon fusion:

$$gg \to H$$

NNLO QCD [Harlander, Kilgore]
NL EW [Degrassi, Maltoni]

- WW (ZZ) fusion:

$$qq \to Hq'q'$$

NLO QCD [Figy, Oleari, Zeppenfeld]

- Higgs-strahlung processes:

$$qq' \to WH$$

$$q\bar{q} \to ZH$$

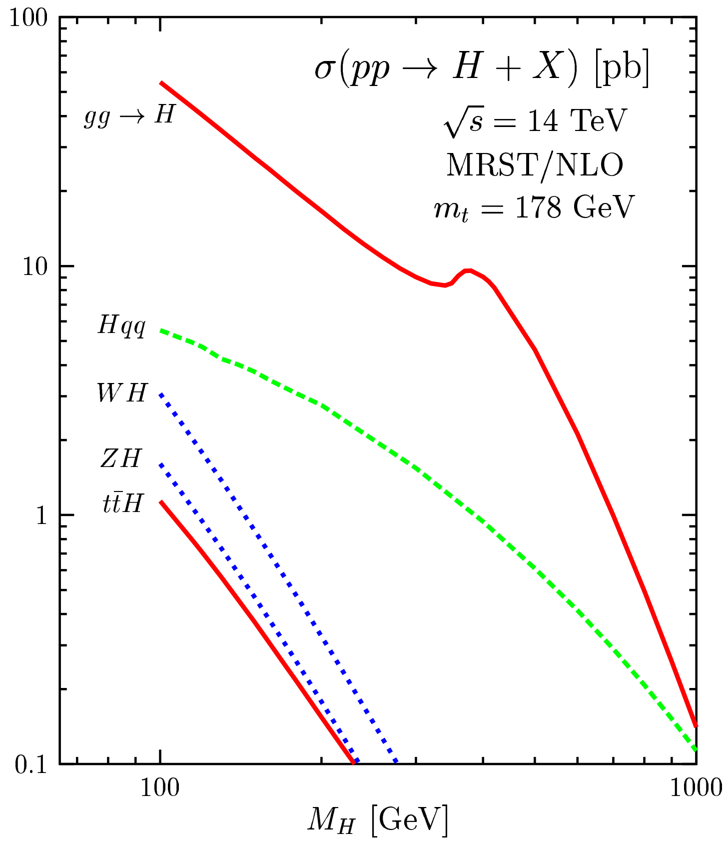
NNLO QCD + NLO EW [Brein et al.]

- radiation from heavy quarks:

$$gg,\ q\bar{q} \to t\bar{t}H\ (b\bar{b}H$$

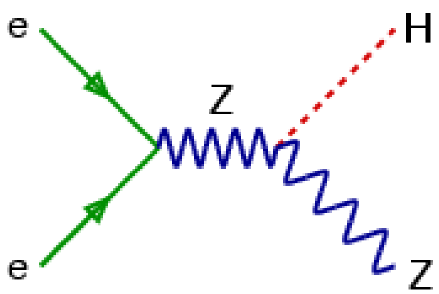
NLO QCD [Beenakker et al., Dawson et al.]
NLO EW [Denner et al.]

Higgs production at the LHC

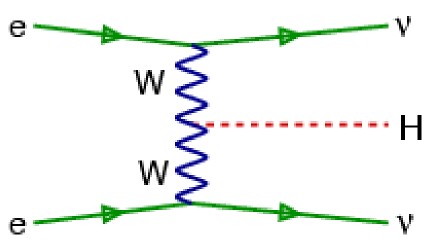


The Profile of the Higgs Boson

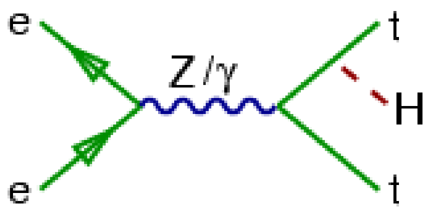
Production Processes



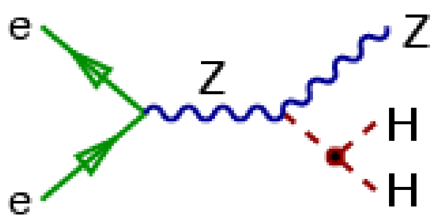
	500 fb^{-1}	500 fb^{-1}	1000 fb^{-1}
	350 GeV	500 GeV	800 GeV
m_H = 120	74000	35000	27000
m_H = 160	52000	29000	24000
m_H = 250	5500	16500	19000



	500 fb^{-1}	500 fb^{-1}	1000 fb^{-1}
	350 GeV	500 GeV	800 GeV
m_H = 120	15500	37500	158000
m_H = 160	7500	25000	126000
m_H = 250	6500	8000	71000

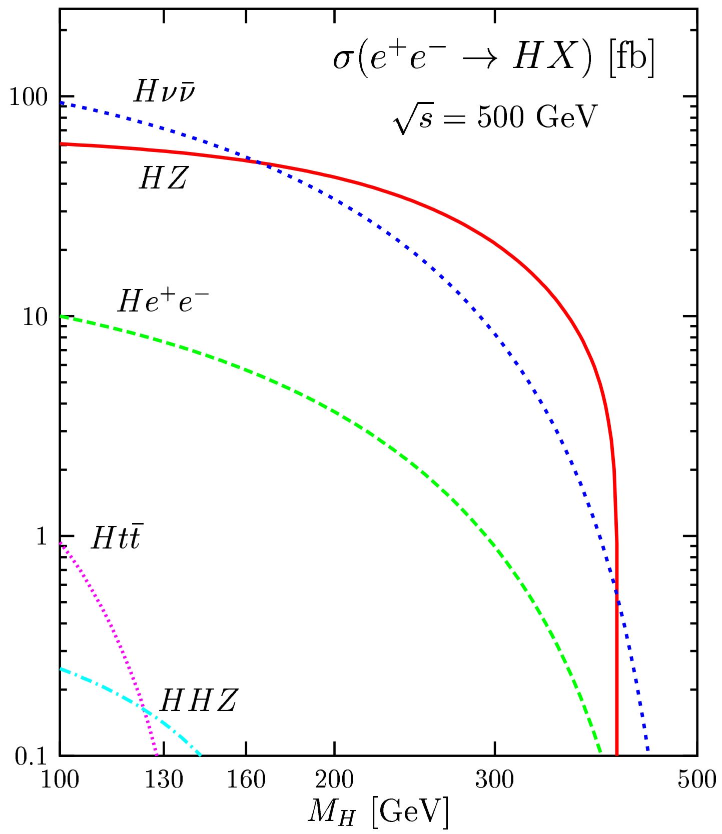


	500 fb^{-1}	500 fb^{-1}	1000 fb^{-1}
	350 GeV	500 GeV	800 GeV
m_H = 120	–	90	2600
m_H = 160	–	–	1500
m_H = 250	–	–	390



	500 fb^{-1}	500 fb^{-1}	1000 fb^{-1}
	350 GeV	500 GeV	800 GeV
m_H = 120	–	80	160
m_H = 160	–	20	120
m_H = 250	–	–	30

Higgs production at a Linear Collider



Standard Model established as a quantum field theory

- in agreement with (almost) all experiments (accuracy $\gtrsim 0.1\%$)
- quantum corrections are established
- indirect and direct determination of m_{t} agree
- constraints on the Higgs-boson mass
  $\Rightarrow$ light Higgs boson
- triple-gauge boson self-interactions established at per-cent level

not yet directly tested

- existence of Higgs boson
- Higgs-boson self-interaction $\Rightarrow$ Higgs potential
- Yukawa interaction

$\Rightarrow$ future experiments

Open questions of the Standard Model

- Large number of free parameters, in particular in fermion sector
- origin of gauge group $SU(3) \times SU(2) \times U(1)$ with three different gauge couplings
- origin of charge quantization
- origin and number of fermion generations
- origin of mass pattern
- origin of baryon asymmetry in universe
- inclusion of gravity ($\Rightarrow$ string theories)

Minimal Supersymmetric Standard Model (MSSM)

SM		Spin	SUSY		Spin
leptons	$\ell,\ \nu_\ell$	$\frac{1}{2}$	sleptons	$\tilde{\ell},\ \tilde{\nu}_\ell$	0
quarks	q	$\frac{1}{2}$	squarks	$\tilde{q}$	0
gluons	g	1	gluinos	$\tilde{g}$	$\frac{1}{2}$
EW bosons	γ, Z, W	1	charginos	$\tilde{\chi}^{\pm}_{1,2}$	$\frac{1}{2}$
Higgs	$h, H, A, H^{\pm}$	0	neutralinos	$\tilde{\chi}^{0}_{1,2,3,4}$	$\frac{1}{2}$

lightest SUSY particle stable LSP $= \tilde{\chi}^0_1$

- masses of SUSY partners > 100 GeV (experimentally)
- lightest Higgs boson < 135 GeV (theoretically)

SUSY Higgs sector

SM Higgs:

- $\lambda\Phi^4$ term ad hoc
- Higgs boson mass: free parameter
- no a-priori reason for a light Higgs boson
- SM (perturbatively) unstable at some high energy

SUSY Standard Model avoids these questions

minimal model: MSSM

$$H_2 = \begin{pmatrix} H_2^+ \\ v_2 + H_2^0 \end{pmatrix}, \qquad H_1 = \begin{pmatrix} v_1 + H_1^0 \\ H_1^- \end{pmatrix}$$

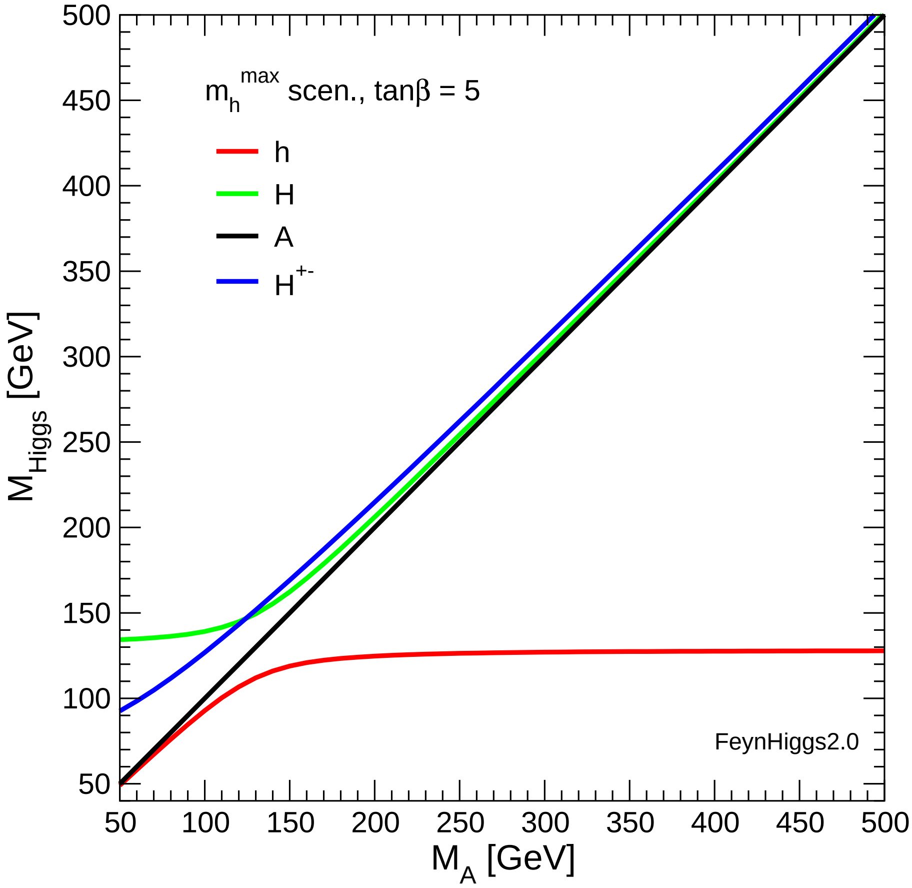
couples to u couples to d

- SUSY gauge interaction $\rightarrow$ H^4 terms
- self coupling remains weak

physical Higgs bosons: $h^0,\ H^0,\ A^0,\ H^\pm$

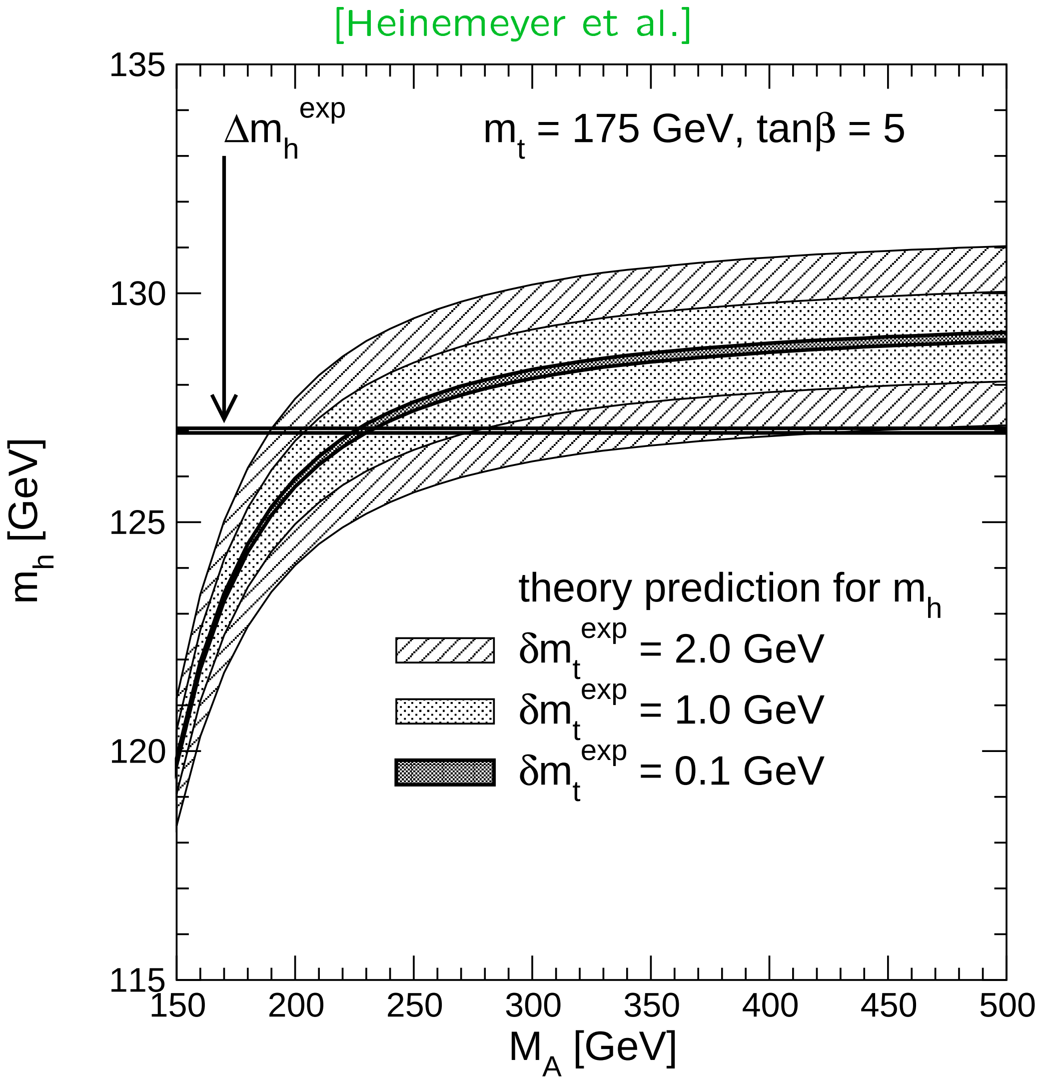
2 vacuum expectation values: $\frac{v_2}{v_1} = \tan\beta$

Spectrum of Higgs bosons in the MSSM (example)



large M_A: h^0 like SM Higgs boson

m_h^0 strongly influenced by quantum effects



[Heinemeyer et al.]
Δm_h^exp
m_t = 175 GeV, tanβ = 5
theory prediction for m_h
δm_t^exp = 2.0 GeV
δm_t^exp = 1.0 GeV
δm_t^exp = 0.1 GeV
m_h [GeV]
M_A [GeV]
115
120
125
130
135
150
200
250
300
350
400
450
500

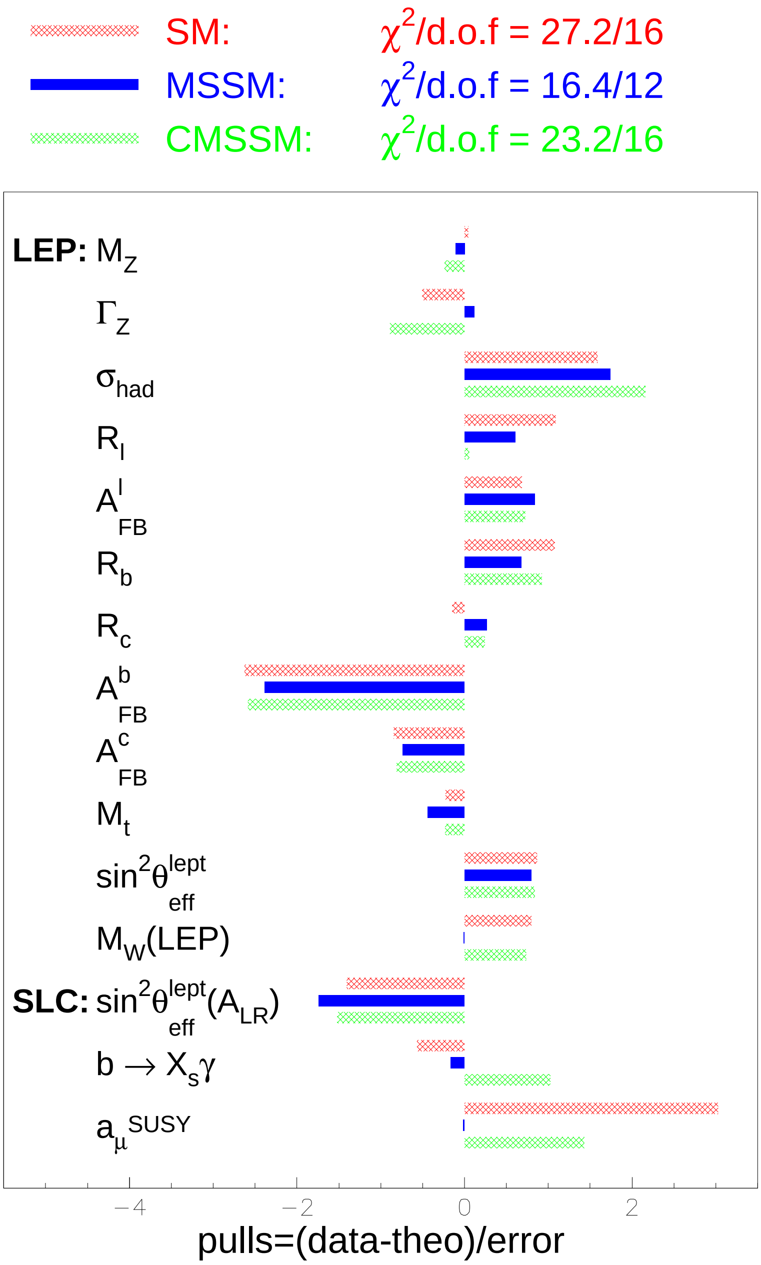
Possible scenarios

- a single light Higgs boson
  - SM Higgs boson?
  - SUSY light Higgs boson?
    with H, A, $H^{\pm}$ heavy (decoupling scenario)
    $h \sim H_{\rm SM}$
- a light Higgs boson + more $(H, A, H^{\pm})$
  - SUSY Higgs?
  - non-SUSY 2-Higgs-Doublet model?
- a single heavy Higgs boson ($\gg 200$ GeV)
  - SUSY ruled out
  - SM + (?) strong interaction?
- no Higgs boson
  - strongly interacting weak interaction
    new strong force $\sim$ TeV scale

Global fits in the MSSM

[de Boer, Dabelstein, WH, Mösle, Schwickerath]

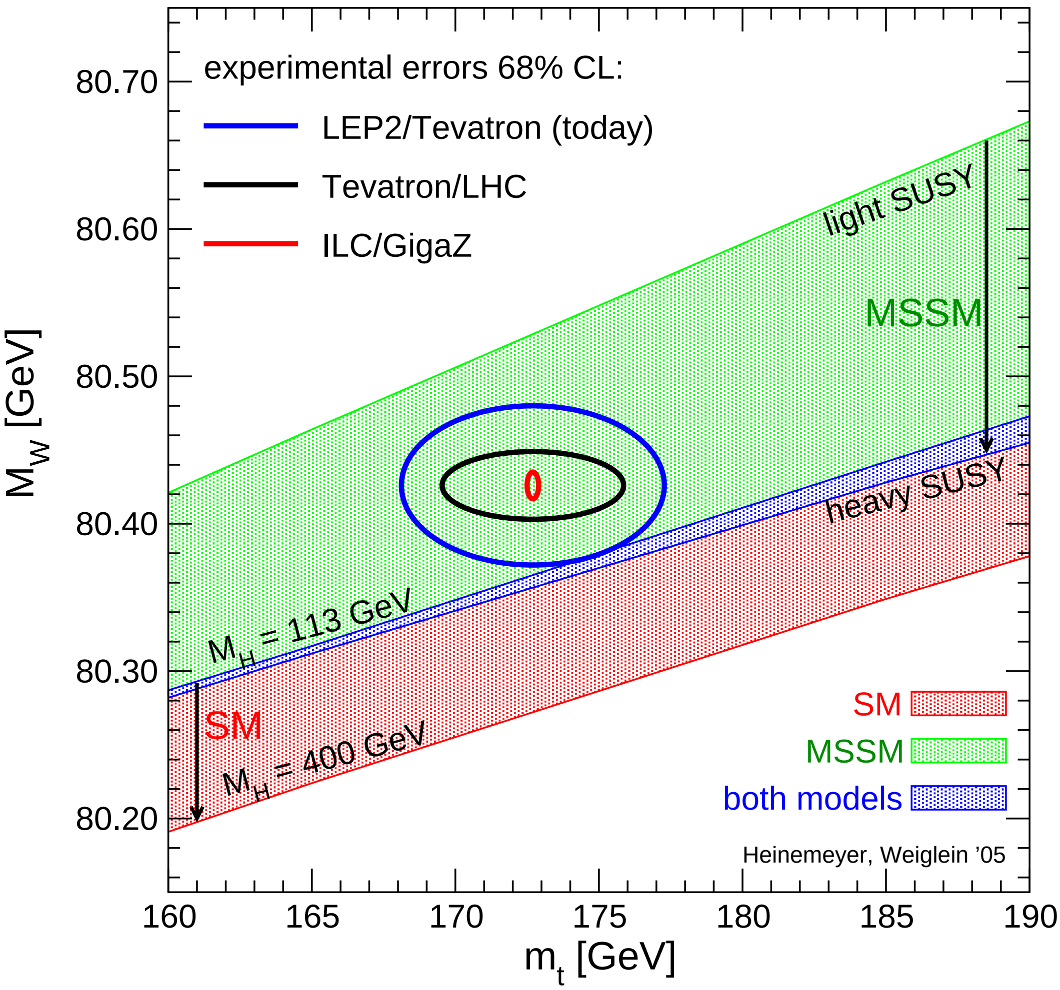
[de Boer, Sander]



special: M_W and $a_\mu = (g-2)/2$ for muon

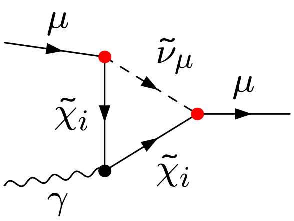
[Chankowski, Dabelstein, WH, Mösle, Pokorski, Rosiek]

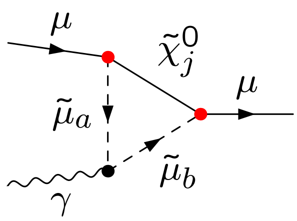
[update: Heinemeyer, Weiglein]



$g-2$

Feynman diagrams for MSSM 1L corrections:





– Diagrams with chargino/sneutrino exchange

– Diagrams with neutralino/smuon exchange

Enhancement factor as compared to SM:

$$\mu - \tilde{\chi}^\pm_i - \tilde{\nu}_\mu \;:\; \sim m_\mu \tan\beta$$
$$\mu - \tilde{\chi}^0_j - \tilde{\mu}_a \;:\; \sim m_\mu \tan\beta$$

SM, EW 1L: $\frac{\alpha}{\pi}\frac{m_\mu^2}{M_W^2}$

MSSM, 1L: $\frac{\alpha}{\pi}\frac{m_\mu^2}{M_{\text{SUSY}}^2} \times \tan\beta$

Beyond the Standard Model

further substructure	elementary fundamental fields
effects from new strong interaction	interactions remain weak
	Grand Unified Theories
new strong dynamics at high enery scale	new symmetry supersymmetry