# Scalarized black holes in Gauss-Bonnet gravity

**Stoytcho Yazadjiev**

***In collaboration with* D. Doneva**

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**The talk is based on following paper:**

**D. Doneva and S.Y., PRL (2018); arXiv: 1711.01187[gr-qc].**

**Other papers studying similar problems:**

H. Silva, J. Sakstein, L. Gualtieri, T. Sotiriou, E. Berti, PRL (2018): arXiv:1711.02080 [gr-qc].

G. Antoniou, A. Bakopoulos, P. Kanti, PRL (2018); arXiv:1711.03390 [hep-th].

G. Antoniou, A. Bakopoulos, P. Kanti, arXiv: 1711.07431 [hep-th];

## Plan of the talk

- Extended scalar-tensor-Gauss-Bonnet theories (ESTGBT) of gravity
- Observational constraints on a particular class of ESTGBT
- Instability of the Schwarzschild solution within the framework of a class of ESTGBT
- New scalarized black hole solutions in ESTGBT
- Possible observational signatures
- Conclusion

# Extended scalar-tensor theories of gravity- motivation

- There are both phenomenological and theoretical reasons for the modification of the original Einstein's theory .

- General Relativity is well-tested in the weak-field regime, whereas the strong-field regime still remains essentially unexplored and unconstrained.

- The attempts to construct a unified theory of all the interactions, naturally lead to scalar-tensor type generalizations of general relativity with an additional dynamical scalar field and with Lagrangians containing various kinds of curvature corrections to the usual Einstein-Hilbert Lagrangian coupled to the scalar field.

- The most natural modifications of this class are the extended scalar-tensor (ESTT) theories where the usual Einstein-Hilbert action is supplemented with all possible algebraic curvature invariants of second order with a dynamical scalar field non-minimally coupled to these invariants.

- The equations of the ESTT in their most general form are of order higher than two. This in general can lead to the Ostrogradski instability and to the appearance of ghosts.

**Particular sector of the ESTT - Extended scalar-tensor-Gauss-Bonnet theories**

- The scalar field is coupled exactly to the Gauss-Bonnet invariant.

- The field equations are of second order as in general relativity and the theory is free from ghosts.

**The general action of ESTGB theories**

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - 2\nabla_\mu\varphi\nabla^\mu\varphi - V(\varphi) + \lambda^2 f(\varphi)\mathcal{R}^2_{GB}\right]$$

$$\mathcal{R}^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

# Field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Gamma_{\mu\nu} = 2\nabla_\mu\varphi\nabla_\nu\varphi - g_{\mu\nu}\nabla_\alpha\varphi\nabla^\alpha\varphi - \frac{1}{2}g_{\mu\nu}V(\varphi).$$

$$\nabla_\alpha\nabla^\alpha\varphi = \frac{1}{4}\frac{dV(\varphi)}{d\varphi} - \frac{\lambda^2}{4}\frac{df(\varphi)}{d\varphi}\mathcal{R}^2_{GB}$$

where

$$\Gamma_{\mu\nu} = -R(\nabla_\mu\Psi_\nu + \nabla_\nu\Psi_\mu) - 4\nabla^\alpha\Psi_\alpha\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + 4R_{\mu\alpha}\nabla^\alpha\Psi_\nu + 4R_{\nu\alpha}\nabla^\alpha\Psi_\mu$$
$$-4g_{\mu\nu}R^{\alpha\beta}\nabla_\alpha\Psi_\beta + 4R^{\beta}_{\ \mu\alpha\nu}\nabla^\alpha\Psi_\beta$$

with

$$\Psi_\mu = \lambda^2\frac{df(\varphi)}{d\varphi}\nabla_\mu\varphi.$$

# Special class of ESTGBT

**The coupling function satisfies**

$$\frac{df}{d\varphi}(0) = 0 \quad \text{and} \quad \frac{d^2 f}{d\varphi^2}(0) > 0$$

- **The class of ESTGB theories defined above is a class of viable theories.**
- **This class of ESTGB theories, as we will see, differ from General relativity in strong curvature regime only.**

# Observational constraints

- **The current weak field constraints on the theories considered by us are very weak. (**Sotiriou and Barausse, PRD (2007). **In fact the class of ESTGBT is indisguishable from general relativity in the weak field regime.**

- **The electromagnetic counterpart GRB 170817A to the gravitational wave signal GW170817 from the merger of two neutron stars can impose stringent constraints on many scalar-tensor and vector-tensor theories (**Baker et al. PRL (2017) **).**

- **In our case this approach can impose no constraints on our class of ESTGBT. The reasons are the following.**

- **Since the Gauss-Bonnet invariant drops off very rapidly outside the source zone, the tensor and scalar degrees of freedom in the wave zone decouple, as the tensor part satisfies the same equation as in GR while the scalar degree satisfies the ordinary wave equation, i.e. the tensor and scalar waves propagate with the speed of light. Therefore, the dispersion relation is the standard one for both the tensor and scalar part of the gravitational waves and the event GRB 170817A can not impose constraints on our theories.**

- **Concerning the cosmological background, it is easy to see that it can not modify the dispersion relation because the scalar field does not need to play any role in the late time cosmology – the Gauss-Bonnet curvature invariant drops off very rapidly in cosmic time and the scalar field is not excited in the late time cosmology.**

- **The possible constraints on our theories from the binary pulsars and the event GW170817 itself, need more sophisticated investigations. The first step in this direction is the development of the neutron star models in the theories under consideration and this step has been undertaken in (**Doneva and Yazadjiev, JCAP (2018)**). In advance we can say that our results show that the coupling parameter and the coupling function can be chosen in such a way so that the scalar gravitational radiation from binary systems of scalarized neutron stars to be strongly suppressed and the theories to be in agreement with the binary pulsars observations and the event GW170817.**

# Static and spherically symmetric field equations

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\frac{2}{r}\left[1 + \frac{2}{r}(1 - 3e^{-2\Lambda})\Psi_r\right]\frac{d\Lambda}{dr} + \frac{(e^{2\Lambda} - 1)}{r^2} - \frac{4}{r^2}(1 - e^{-2\Lambda})\frac{d\Psi_r}{dr} - \left(\frac{d\varphi}{dr}\right)^2 = 0,$$

$$\frac{2}{r}\left[1 + \frac{2}{r}(1 - 3e^{-2\Lambda})\Psi_r\right]\frac{d\Phi}{dr} - \frac{(e^{2\Lambda} - 1)}{r^2} - \left(\frac{d\varphi}{dr}\right)^2 = 0,$$

$$\frac{d^2\Phi}{dr^2} + \left(\frac{d\Phi}{dr} + \frac{1}{r}\right)\left(\frac{d\Phi}{dr} - \frac{d\Lambda}{dr}\right) + \frac{4e^{-2\Lambda}}{r}\left[3\frac{d\Phi}{dr}\frac{d\Lambda}{dr} - \frac{d^2\Phi}{dr^2} - \left(\frac{d\Phi}{dr}\right)^2\right]\Psi_r$$
$$- \frac{4e^{-2\Lambda}}{r}\frac{d\Phi}{dr}\frac{d\Psi_r}{dr} + \left(\frac{d\varphi}{dr}\right)^2 = 0,$$

$$\frac{d^2\varphi}{dr^2} + \left(\frac{d\Phi}{dr} - \frac{d\Lambda}{dr} + \frac{2}{r}\right)\frac{d\varphi}{dr}$$
$$- \frac{2\lambda^2}{r^2}\frac{df(\varphi)}{d\phi}\left\{(1 - e^{-2\Lambda})\left[\frac{d^2\Phi}{dr^2} + \frac{d\Phi}{dr}\left(\frac{d\Phi}{dr} - \frac{d\Lambda}{dr}\right)\right] + 2e^{-2\Lambda}\frac{d\Phi}{dr}\frac{d\Lambda}{dr}\right\} = 0,$$

**Special class of ESTGBT - the coupling function satisfies** $\frac{df}{d\varphi}(0) = 0$ and $\frac{d^2 f}{d\varphi^2}(0) > 0$

- The natural and the important question is whether the class of ESTGBT defined above admits static and spherically symmetric black hole solutions.

- From the dimensionally reduced field equations it is clear that the usual Schwarzschlild black hole solution is also a black hole solution to the ESTGBT under consideration with a trivial scalar field $\varphi = 0$.

- We shall however show that the Schwarzschild solution within the certain range of the mass is unstable in the framework of the special class of ESTGBT under consideration.

- For this purpose we consider the perturbations of the Schwarzschild solution with mass **M** within the framework of the described class of ESTGBT.

# Instability of Schwarzschild black hole within the ESTGBT

- In the considered class of ESTGBT the equations governing the perturbations of the metric are decoupled from the equation governing the perturbation of the scalar field. The equations for metric perturbations are in fact the same as those in the pure Einstein gravity and therefore we shall focus only on the scalar field perturbations.

- The equation governing the scalar perturbations is

$$\Box_{(0)}\delta\varphi + \frac{1}{4}\lambda^2\mathcal{R}^2_{GB(0)}\delta\varphi = 0$$

- The equation can be cast in Schrodinger form $\delta\varphi = \frac{u(r)}{r}e^{-i\omega t}Y_{lm}(\theta,\phi),$

$$\frac{d^2u}{dr_*^2} + [\omega^2 - U(r)]u = 0$$

with an effective potential

$$U(r) = (1 - \frac{2M}{r})\left[\frac{2M}{r^3} + \frac{l(l+1)}{r^2} - \lambda^2\frac{12M^2}{r^6}\right]$$

# Instability of Schwarzschild black hole within the ESTGBT

- A sufficient condition for the existence of a unstable mode is

$$\int_{-\infty}^{+\infty} U(r_*)dr_* = \int_{2M}^{\infty} \frac{U(r)}{1-\frac{2M}{r}}dr < 0$$

- We can conclude that the Schwarzshild black holes with mass satisfying $\boldsymbol{M^2 < 0.3\,\lambda^2}$ are unstable within the framework of the ESTGBT under consideration. Stated differently, the Schwarzshild black holes become unstable when the curvature of the horizon exceeds a certain critical value – in terms of the Kretschmann scalar of the horizon $K_H$, the instability occurs when $K_H > 8.3/\lambda^4$ .

- In other words, the scalar field can be excited only in the strong curvature regime when the spacetime curvature exceeds ceratin critical value, namely $K_H > 8.3/\lambda^4$.

- This result naturally leads us to the conjecture that, in our class of ESTGBT and in the interval where the Schwarzschild is unstable, there exist black hole solutions with nontrivial scalar field. We numerically proved that such black hole solutions really exist.

- In order to obtain the black hole solutions with a nontrivial scalar field we solve numerically the system of reduced field equations with the following boundary conditions:

$$\Phi|_{r\rightarrow\infty} \rightarrow 0, \;\; \Lambda|_{r\rightarrow\infty} \rightarrow 0, \;\; \varphi|_{r\rightarrow\infty} \rightarrow 0$$

$$e^{2\Phi}|_{r\rightarrow r_H} \rightarrow 0, \;\; e^{-2\Lambda}|_{r\rightarrow r_H} \rightarrow 0.$$

- The regularity of the scalar field and its first and second derivatives on the black hole horizon gives one more condition, namely

$$\left(\frac{d\varphi}{dr}\right)_H + \frac{2\lambda^2}{r_H}\frac{df}{d\varphi}(\varphi_H)\left(\frac{d\varphi}{dr}\right)_H^2 + \frac{2\lambda^2}{r_H^3}\frac{df}{d\varphi}(\varphi_H) = 0.$$

- Hence black hole solution exist only when:

$$r_H^4 > 24\lambda^4 \left(\frac{df}{d\varphi}(\varphi_H)\right)^2$$

## Numerics

Finding the solutions with nontrivial scalar field is numerically difficult and it is of great help to know the exact points of bifurcation. In the previous slides we discussed that for $\boldsymbol{M^2 < 0.3\,\lambda^2}$ the Schwarzschild black holes are unstable but this is only a sufficient condition for instability and the true point of the first bifurcations is actually at a little bit larger masses. In order to find the points of bifurcation we can use the fact that they are the same as the points where new unstable modes appear. That is why instead of solving the reduced field equations, we can determine the bifurcations points using the perturbation equation for the scalar field, that is numerically easier. Since we are interested in unstable modes, $\omega^2$ should be negative which leads to the fact that the boundary conditions are zero both at the black hole horizon and infinity . Therefore we have a self-adjoint Sturm-Liouville problem.

# Instability of Schwarzschild black hole within the ESTGBT

- We employed a shooting procedure to find the eigenvalues and the eigenfunction of the Schrodinger equation and determined the regions of the parameter space where the Schwarzschild solution is stable, where it is unstable and only one unstable mode is present, where two unstable modes are present and so on. This means that we have determined the points of bifurcation of the Schwarzschild solution which significantly simplifies the search for black holes with nontrivial scalar field.

- Solving the fully nonlinear problem we found that, in addition to the trivial (Schwarzschild) solution, there exist several nontrivial branches of black hole solutions.

- The different nontrivial branches of solutions are characterized by different number of zeros of the scalar field. For the first branch (the red dashed line in Fig. 1) there are no zeros of $\varphi$ as one can see in the left panel of Fig. 2, the next one (green line) has one zero while the third one (blue line) has two zeros as one can see in Fig. 3.

- All the nontrivial branches start from a bifurcation point at the trivial branch and they span either to $r_H = 0$ (the first nontrivial branch) or they are terminated at some nonzero $r_H$ (all the other nontrivial branches). The reason for termination of the branch at nonzero $r_H$ is that beyond this point the black hole existence condition is violated.

- For smaller values of $r_H$ there are more bifurcation points but our investigations show the corresponding nontrivial branches would be even shorter and that is why we have not plotted them.

- It is expected that only the first nontrivial branch characterized by a scalar field without zeros will be stable while the rest of the branches correspond to unstable solutions. A good indication for the stability/instability is the entropy of the black hole branches.

**Results for coupling function** $f(\varphi) = \frac{1}{12}\left[1 - \exp(-6\varphi^2)\right]$

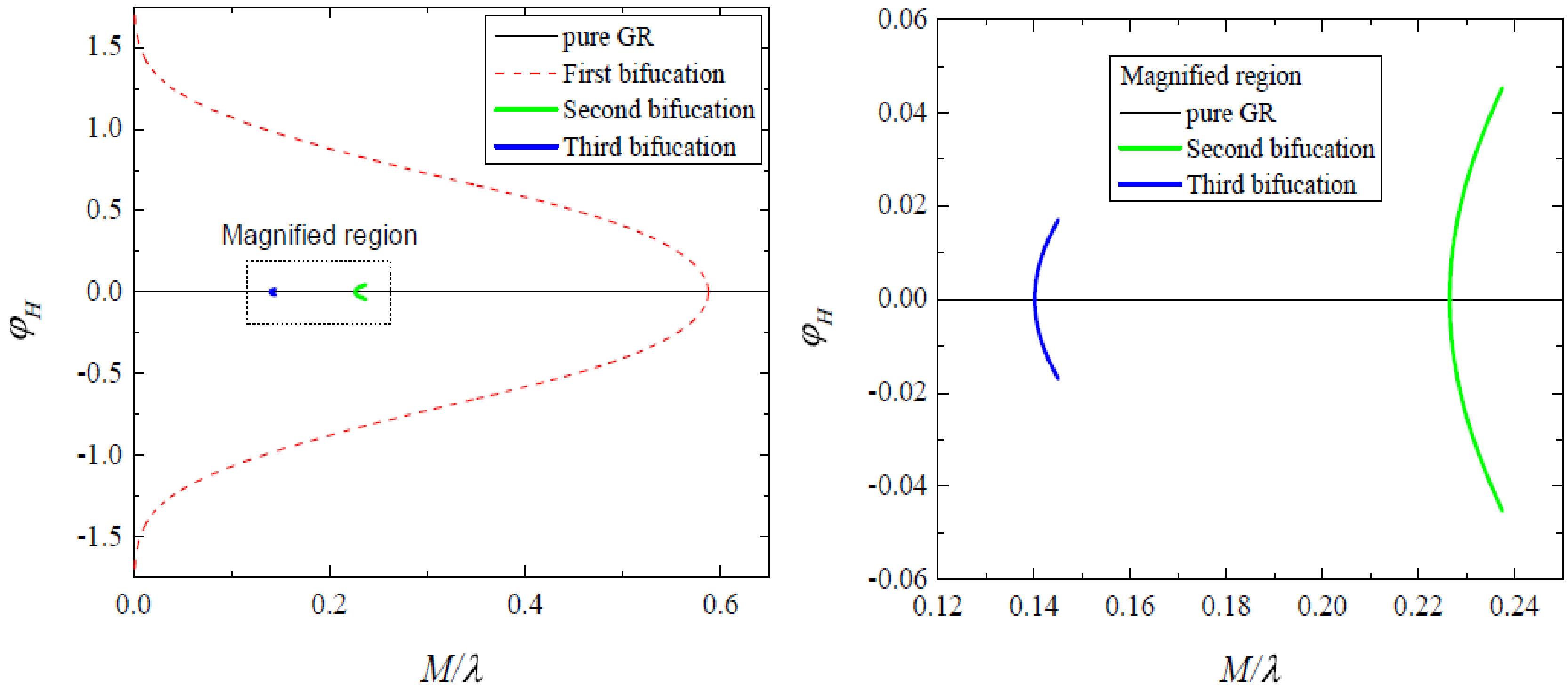


FIG. 1: The scalar field at the horizon as a function of the black hole mass. The right figure is a magnification of the left one.

# New black holes with curvature induced scalarization in ESTGBT

**Results for coupling function** $f(\varphi) = \frac{1}{12}\left[1 - \exp(-6\varphi^2)\right]$

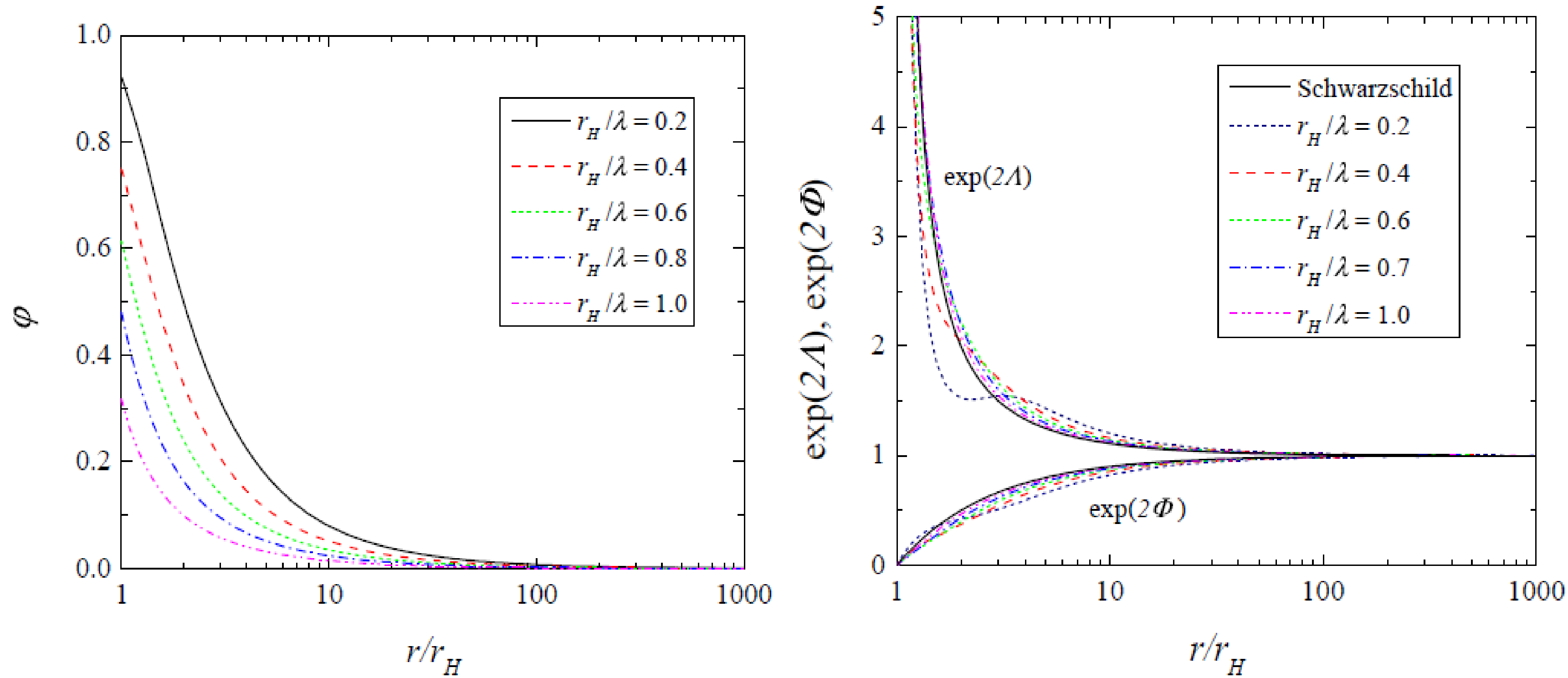


FIG. 2: The scalar field and the $g_{tt}$ and $g_{rr}$ components of the metric as a function of the normalized radial coordinate $r/r_H$ for several black hole solutions from the first nontrivial branch with different values of $r_H$.

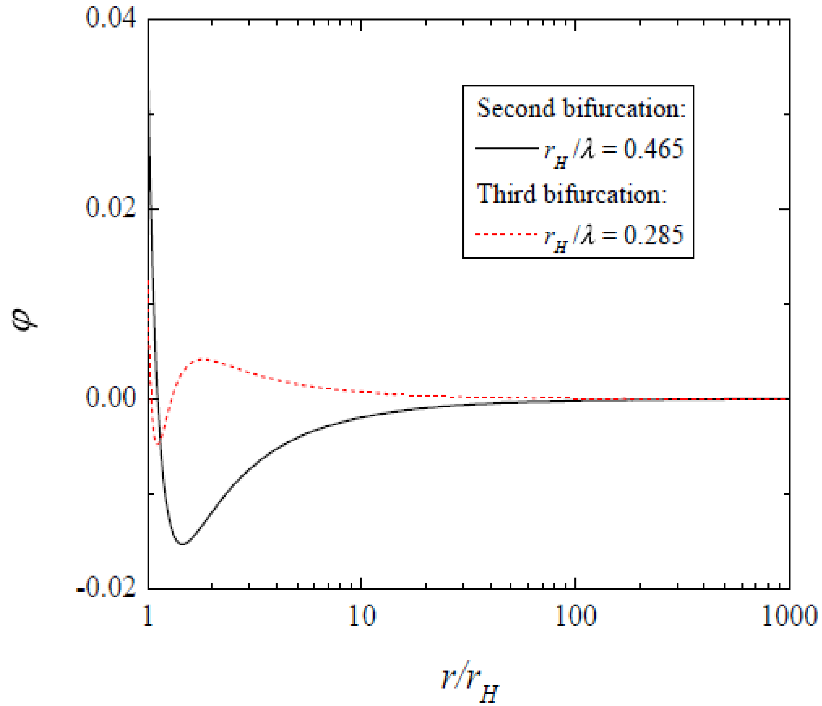


FIG. 3: The scalar field as a function of the normalized radial coordinate $r/r_H$ for two representative solution from the second and the third branch of nontrivial solutions. The components of the metric $g_{tt}$ and $g_{rr}$ are not shown since they are almost indistinguishable from the Schwarzschild case.

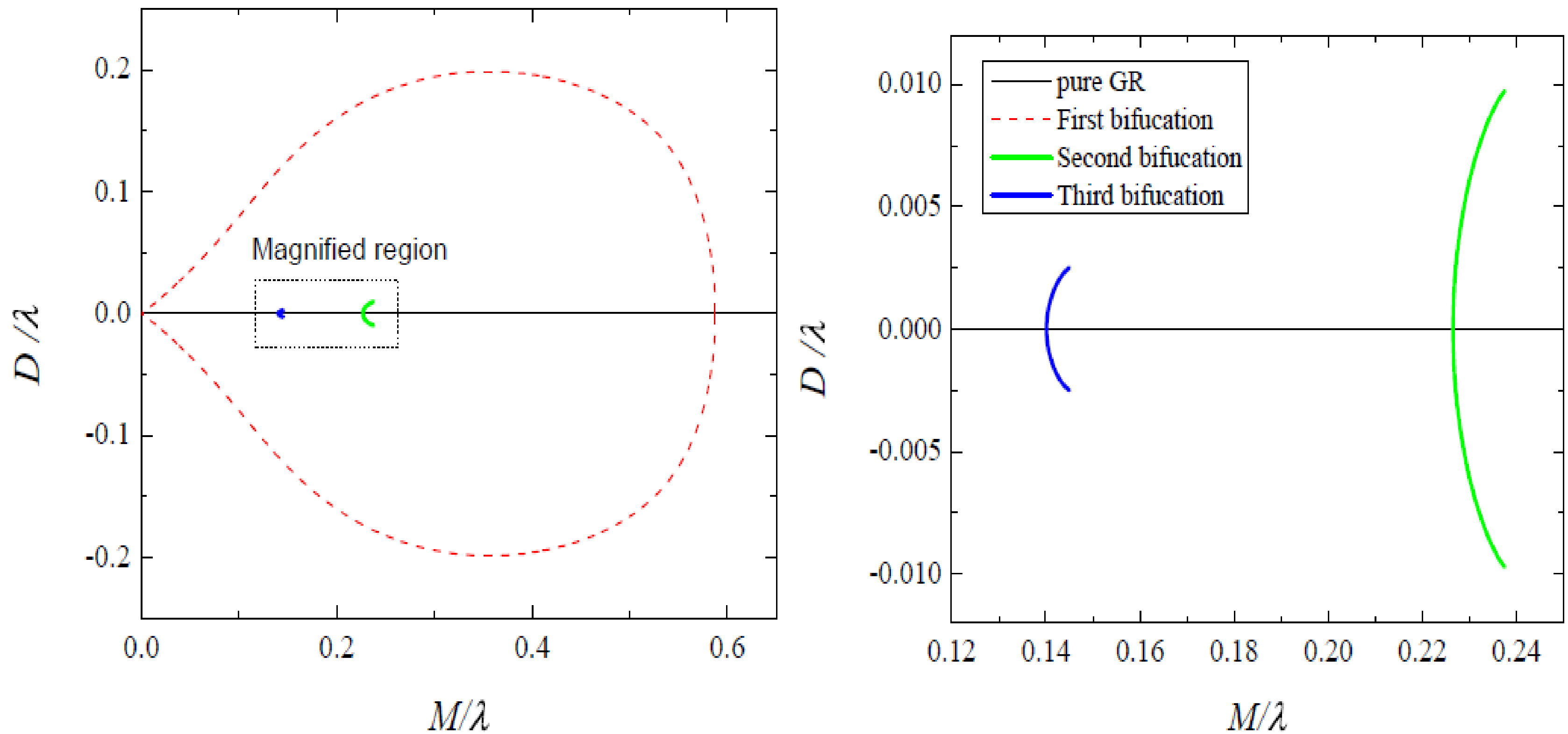


FIG. 4: The dilaton charge of the black hole as a function of its mass. The right figure is a magnification of the left one.

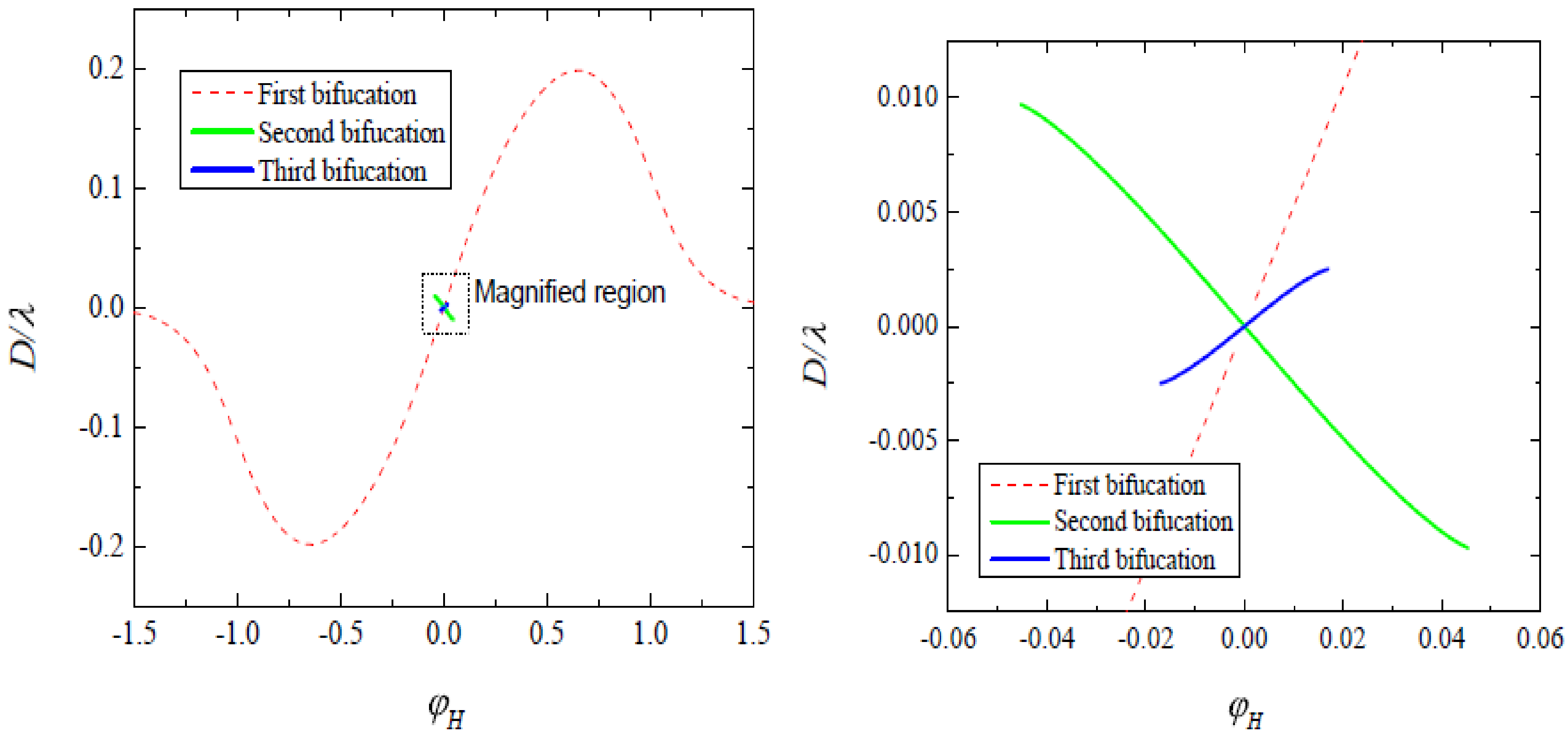


FIG. 5: The dilaton charge of the black hole as a function of the scalar field at the horizon. The right figure is a magnification of the left one.

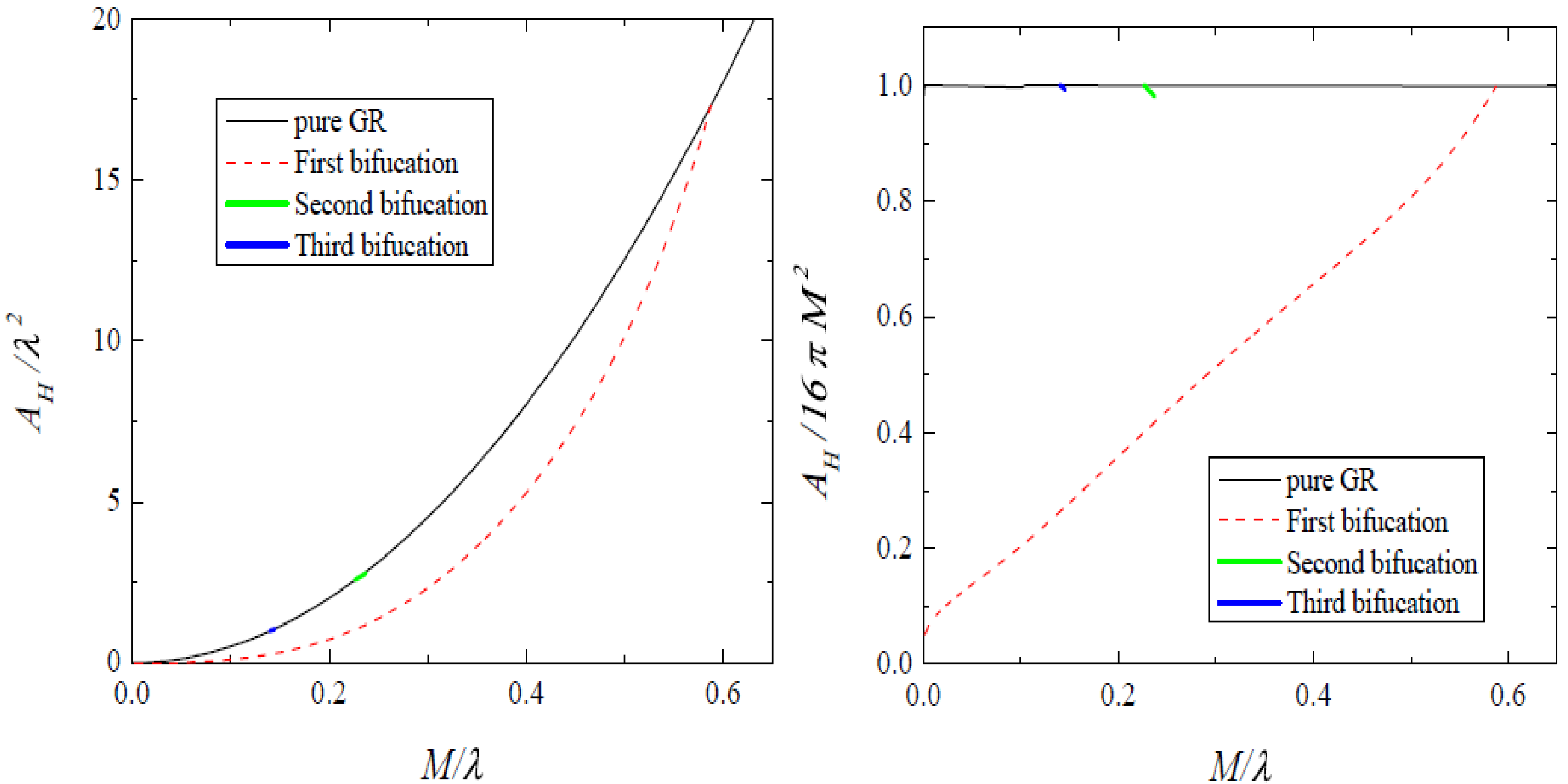


FIG. 6: The area of the black hole horizon $A_H$ as a function of the mass. In the right figure the black hole area is normalized to the corresponding value in the Schwarzschild limit, i.e. $A_H^{Schwarschild} = 16\pi M^2$.

In order to have an indicator for the stability of the black hole branches one can study the entropy. The black hole entropy in the presence of the Gauss-Bonnet invariant in the action is not just one fourth of the horizon area and is more complicated. The entropy is given by Wald's formula

$$S_H = 2\pi \int_H \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\alpha\beta}} \epsilon_{\mu\nu} \epsilon_{\alpha\beta}$$

where £ is the Lagrangian density and $\epsilon_{\alpha\beta}$ is the volume 2-form of the 2-dimensional cross section H of the horizon.

In our case we find

$$S_H = \frac{1}{4} A_H + 4\pi f(\varphi_H)$$

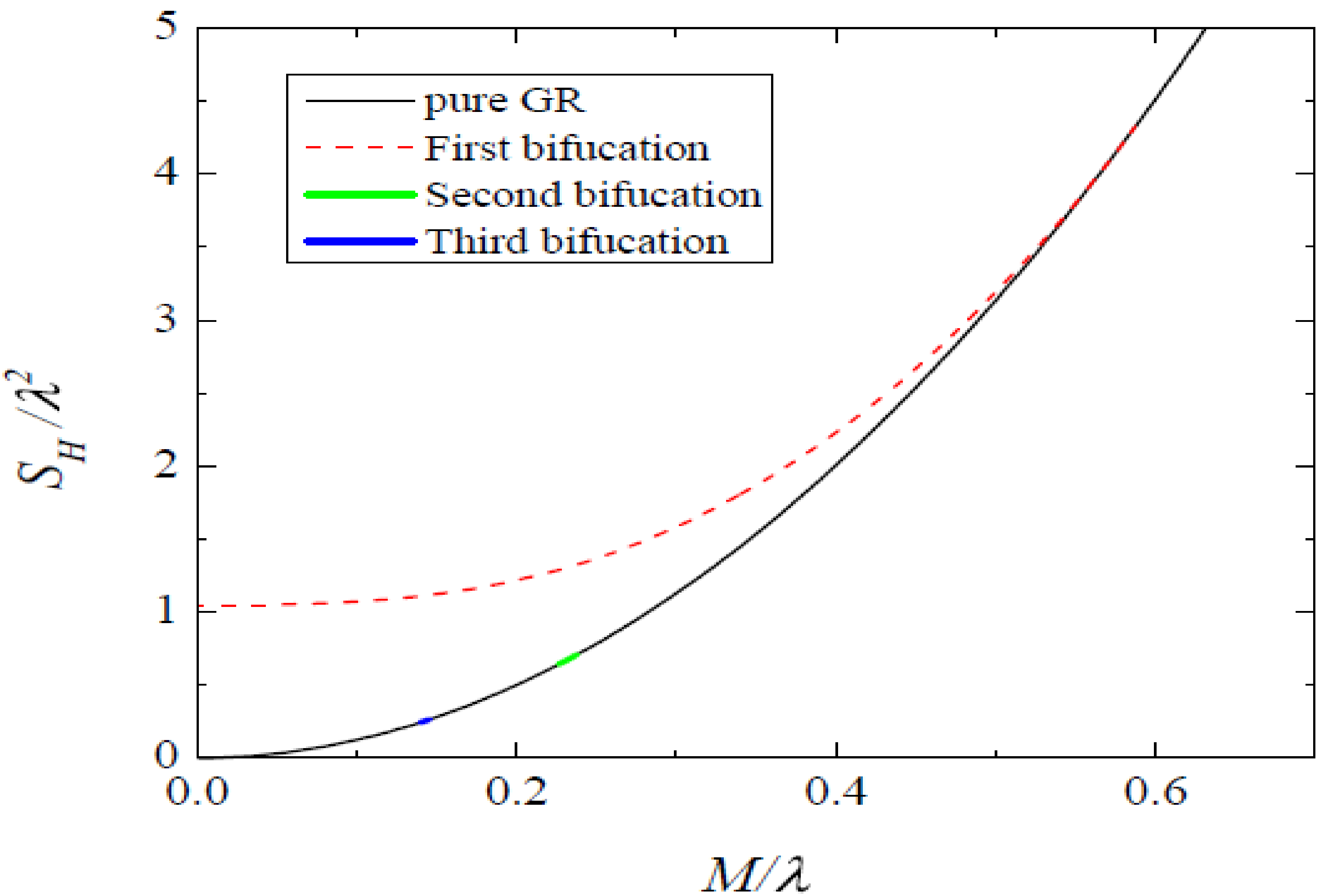


FIG. 7: The entropy of the black hole as a function of its mass. .

## Possible observational signatures

**1.** Let us comment on the importance of our results in view of the recent detection of gravitational wave emission by binary black hole mergers. One of the most prominent effects would come from the fact that if the black holes are scalarized there will be an additional channel of energy loss during the inspiral phase via the emission of dipole scalar field radiation. As a results the inspiral will be faster in comparison with the pure general relativistic case similar to the mergers of scalarized binary neutron stars. The describe effect is expected for small black holes with $\beta < 30$.

**2.** Our preliminary investigations show that the quasinormal frequencies of the scalarized black holes can differ from those in General relativity especially in the case for small black holes with $\beta < 10$

**3.** The tidal Love numbers (both polar and axial) can be differ significantly from those of the Schwarzschild black hole (i.e. from zero).

# Conclusion

- We have shown that for such theories an effect similar to the spontaneous scalarization of neutron stars exists – the Schwarzschild solution becomes unstable below certain mass and new branches of black hole solutions with nontrivial scalar field appear that bifurcate from the Schwarzschild one at certain masses. The first branch of nontrivial solutions is characterized by a scalar field that has no zeroes while the scalar field has one zero for the second branch, two zeros for the third branch and so on. The general expectation, though, is that only the first branch of solutions would be stable and the rest would be unstable. The main difference with the spontaneous scalarization of neutron stars is that the scalar field is not sourced the by matter, but instead by the extreme curvature of the spacetime around black holes.

- We have explicitly constructed such solutions with nonzero scalar field and it was shown that the first branch of nontrivial solutions is thermodynamically more stable compared to the Schwarzschild one. The results presented here are for a particular coupling function that can produce non-negligible deviations from pure general relativity. We have tested, though, several other functions satisfying the above given conditions and the results are qualitatively very similar.

# THANK YOU!