(Part of)Ph.D. Thesis

Metric-Affine Gravity and Cosmology

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Outline

- Brief Intro to the Geometry of Metric-Affine Gravity
- Metric-Affine f(R) Theories of Gravity
- Cosmological Solutions/ Torsion $\Longleftrightarrow$ Non-metricity duality
- $1+(n-1)$ Spacetime Split with Torsion and Non-metricity
- Raychaudhuri Equation with Torsion and Non-metricity
- Solutions
- Conclusions

Metric, Palatini, and Metric-Affine Gravity

Metric Gravity

- $\Gamma^{\alpha}{}_{\mu\nu} \rightarrow$ *torsionless* , metric compatibility $\nabla_{\sigma} g_{\mu\nu} = 0$
- $S = S_{Gravity} + S_{Matter} = \int d^n x \sqrt{-g}\,[\mathcal{L}_G(g_{\mu\nu}) + \mathcal{L}_M(g_{\mu\nu})]$

Palatini Gravity

- $\Gamma^{\alpha}{}_{[\mu\nu]} \neq 0$, $\nabla_{\sigma} g_{\mu\nu} \neq 0$, $\Gamma^{\alpha}{}_{\mu\nu}, g_{\mu\nu}$ are left independent
- $S = \int d^n x \sqrt{-g}\left[\mathcal{L}_G(g_{\mu\nu}, \Gamma^{\alpha}{}_{\mu\nu}) + \mathcal{L}_M(g_{\mu\nu})\right]$

Metric-Affine Gravity (generalization of Palatini)

- $S = \int d^n x \sqrt{-g}\left[\mathcal{L}_G(g_{\mu\nu}, \Gamma^{\alpha}{}_{\mu\nu}) + \mathcal{L}_M(g_{\mu\nu}, \Gamma^{\alpha}{}_{\mu\nu})\right]$

Geometrical Properties

Non Riemannian geometry

- Non metricity tensor : $Q_{\alpha\mu\nu} \equiv -\nabla_\alpha g_{\mu\nu}$
  $\rightarrow$ Dot products and lengths of vectors not preserved!
  $\frac{d}{d\lambda}(ab)|_{along\ \mathcal{C}} \neq 0$
- Cartan torsion tensor : $S_{\mu\nu}{}^{\lambda} \equiv \Gamma^{\lambda}{}_{[\mu\nu]}$
  $\rightarrow$ Infinitesimal Parallelograms do not exist!
- Limited symmetries of Riemann Tensor. For instance

$$R_{(\mu\nu)\alpha\beta} = \nabla_{[\alpha} Q_{\beta]\mu\nu} - S_{\alpha\beta}{}^{\lambda} Q_{\lambda\mu\nu} \neq 0$$

New tensors are introduced in this context and as a result more scalar combinations can be formed

Torsion/Non-metricity related vectors

Torsion/Non-metricity related vectors

$$S_{\mu} = S_{\mu\lambda}{}^{\lambda} \, , \qquad \tilde{S}^{\mu} = \epsilon^{\mu\nu\rho\sigma} S_{\nu\rho\sigma}$$

$$Q_{\mu} = g^{\alpha\beta} Q_{\mu\alpha\beta} \, , \qquad \tilde{Q}_{\mu} = g^{\rho\alpha} Q_{\rho\alpha\mu}$$

Simplest forms of Torsion/Non-metricity

$$S_{\mu\nu}{}^{\lambda} = \frac{2}{n-1} S_{[\mu}\delta^{\lambda}_{\nu]} \, , \qquad Q_{\alpha\mu\nu} = \frac{1}{n} Q_{\alpha} g_{\mu\nu}$$

- Another interesting form of non-metricity is one for which there exist fixed length vectors! Then $Q_{(\alpha\mu\nu)} = 0$ and one possible form is: $Q_{\alpha\mu\nu} = A_{\alpha} g_{\mu\nu} - g_{\alpha(\mu} A_{\nu)}$

Connection decomposition

Affine connection

$$\Gamma^{\lambda}{}_{\mu\nu} = \tilde{\Gamma}^{\lambda}{}_{\mu\nu} + \frac{1}{2} g^{\alpha\lambda}(Q_{\mu\nu\alpha} + Q_{\nu\alpha\mu} - Q_{\alpha\mu\nu}) - g^{\alpha\lambda}(S_{\alpha\mu\nu} + S_{\alpha\nu\mu} - S_{\mu\nu\alpha})$$

where $\tilde{\Gamma}^{\lambda}{}_{\mu\nu} := \frac{1}{2} g^{\alpha\lambda}(\partial_{\mu} g_{\nu\alpha} + \partial_{\nu} g_{\alpha\mu} - \partial_{\alpha} g_{\mu\nu})$ is the Levi-Civita part of the connection. We often write

- $\Gamma^{\lambda}{}_{\mu\nu} = \tilde{\Gamma}^{\lambda}{}_{\mu\nu} + N^{\lambda}{}_{\mu\nu}$

where $N^{\lambda}{}_{\mu\nu}$ is called the distortion. Then, each quantity can be decomposed into its Riemannian and non-Riemannian counterparts. Example:

$$R = \tilde{R} + \tilde{\nabla}_{\mu}(A^{\mu} - B^{\mu}) + B_{\mu} A^{\mu} - N_{\alpha\mu\nu} N^{\mu\nu\alpha}$$

where $A^{\mu} \equiv g^{\nu\beta} N^{\mu}{}_{\nu\beta}$ and $B^{\mu} \equiv N^{\alpha\mu}{}_{\alpha}$.

Geodesics Vs Autoparalles

Geodesic Curves

$$\frac{d^2x^\mu}{d\lambda^2}+\tilde{\Gamma}^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}=0$$

$\rightarrow$ Solution=Curve of shortest length (joining two points locally)

Autoparallel Curves

$$\frac{d^2x^\mu}{d\lambda^2}+\Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}=0$$

$\rightarrow$ Solution=Straightest Curve

Note 1: The two coincide for Standard GR .
Note 2: Test particles that 'experience' torsion and non-metricity follow Autoparallels! Those that do not $\Rightarrow$ Geodesics!

Hypermomentum

From a physical perspective

- Not only $T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_M[g_{\mu\nu}, \Gamma^{\alpha}{}_{\mu\nu}]}{\delta g^{\mu\nu}}$,

  but also $\Delta_{\alpha}{}^{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_M[g_{\mu\nu}, \Gamma^{\alpha}{}_{\mu\nu}]}{\delta \Gamma^{\alpha}{}_{\mu\nu}}$
- $\Delta_{\alpha}{}^{\mu\nu}$ is the hypermomentum tensor

Example

- Spinless particles (scalars) : $\nabla_{\mu}\phi \to \partial_{\mu}\phi$ (no Γ - dependence) $\Rightarrow \Delta_{\alpha}{}^{\mu\nu} = 0$
- Spin $\iff$ torsion , $\Gamma^{\alpha}{}_{[\mu\nu]} \neq 0 \Rightarrow \Delta_{\alpha}{}^{\mu\nu} \neq 0$
- Note: Spin is not the only source of torsion!

Einstein-Hilbert Action

$S_{EH}[g_{\mu\nu}, \Gamma^{\lambda}{}_{\alpha\beta}] = \int d^n x \sqrt{-g} R = \int d^n x \sqrt{-g} g^{\mu\nu} R_{(\mu\nu)}$

Independent variations w.r.t. the metric and the connection give

$$R_{(\mu\nu)} - \frac{g_{\mu\nu}}{2} R = 0$$

$$-\frac{\nabla_\lambda(\sqrt{-g} g^{\mu\nu})}{\sqrt{-g}} + \frac{\nabla_\sigma(\sqrt{-g} g^{\mu\sigma})\delta^{\nu}_{\lambda}}{\sqrt{-g}} + 2(S_\lambda g^{\mu\nu} - S^{\mu}\delta^{\nu}_{\lambda} + g^{\mu\sigma} S_{\sigma\lambda}{}^{\nu}) = 0$$

The left hand side is denoted with $P_{\lambda}{}^{\mu\nu}$-Palatini tensor

End Result

After some manipulation of the field eqns$\rightarrow$ GR+ An unspecified Vectorial dof: $\Gamma^{\lambda}{}_{\mu\nu} = \tilde{\Gamma}^{\lambda}{}_{\mu\nu} - \frac{2}{(n-1)} S_{\nu} \delta^{\lambda}_{\mu}$

This can be gauged away by means of a projective transformation!

Projective transformations of the Connection

$$\Gamma^{\lambda}{}_{\mu\nu} \longrightarrow \Gamma^{\lambda}{}_{\mu\nu} + \delta^{\lambda}_{\mu}\xi_{\nu}$$

- The Ricci scalar is left invariant under projective transformations $R \to R$
- This implies $P_{\mu}{}^{\mu\nu} = 0(\text{identically}) \Longrightarrow$ Unspecified Vectorial dof
- Any f(R) action has this attribute and this causes problems when one tries to add matter!(More about it later on)

To conclude

$\tilde{R}_{\mu\nu} = \frac{\tilde{R}}{2} g_{\mu\nu}$, $Q_{\alpha\mu\nu} = \frac{Q_{\alpha}}{n} g_{\mu\nu}$, $S_{\mu\nu}{}^{\lambda} = \frac{2}{n-1} S_{[\mu}\delta^{\lambda}_{\nu]}$
with $Q^{\mu} = -\frac{4n}{(n-1)} S^{\mu} \Rightarrow$ Can be gauged away and finally
$Q_{\alpha\mu\nu} = 0 = S_{\alpha\mu\nu}$

Vacuum f(R) Theories

$S = \frac{1}{2\kappa}\int d^n x\sqrt{-g}f(R)$

Field Equations

$$f^{'}(R)R_{(\mu\nu)} - \frac{f(R)}{2}g_{\mu\nu} = 0$$

$$-\nabla_\lambda(\sqrt{-g}f^{'}g^{\mu\nu}) + \nabla_\alpha(\sqrt{-g}f^{'}g^{\mu\alpha}\delta^\nu_\lambda) +$$
$$2\sqrt{-g}f^{'}(S_\lambda g^{\mu\nu} - S^\mu\delta^\nu_\lambda - S_\lambda{}^{\mu\nu}) = 0$$

$\rightarrow$ Taking the trace of the first

$$f^{'}(R)R - \frac{n}{2}f(R) = 0$$

This is an algebraic equation in R and it will have number of solutions (except $f(R) = \alpha R^{n/2}$)[Ferraris]

End result

$R = c_i = const. \Rightarrow$

$$\tilde{R}_{\mu\nu} = \frac{f(c_i)}{2f'(c_i)} g_{\mu\nu} = \Lambda(c_i) g_{\mu\nu}$$

(Many) Einstein's Gravity with Cosmological constant!

- As before, after gauging away the unspecified vectorial dof

$$Q_{\alpha\mu\nu} = 0 = S_{\alpha\mu\nu}$$

Conclusion

Vacuum Metric Affine f(R)$\Rightarrow$ A class (number of solutions c_i) of Einstein Gravities with Cosmological Constant

Adding Matter

$S = S_G + S_M = \frac{1}{2\kappa} \int d^n x \sqrt{-g} f(R) + \int d^n x \sqrt{-g} \mathcal{L}_M$

Field Equations:

$$f'(R) R_{(\mu\nu)} - \frac{f(R)}{2} g_{\mu\nu} = \kappa T_{\mu\nu}$$
$$P_\lambda{}^{\mu\nu}(h) = \kappa \Delta_\lambda{}^{\mu\nu}$$

where $P_\lambda{}^{\mu\nu}(h)$ is computed wrt the metric $h^{\mu\nu} = f'(R) g^{\mu\nu}$. Now, since $P_\mu{}^{\mu\nu} = 0$ it follows that $\Delta_\mu{}^{\mu\nu} = 0 \Rightarrow$ Unreasonable constraint on matter fields! Inconsistent Theory!

Resolving the inconsistency

To resolve the inconsistency and obtain a self-consistent theory one needs to brake the projective invariance of the $f(R)$ action and fix a vectorial dof! Two Proposals 1)Helh et al $\Rightarrow$ Works only for $f(R) = R$. 2) Sotiriou and Liberati $\Rightarrow$ Works for any $f(R)$

Special Case $f(R) = \alpha R^2$ in $4 - dim$

Consider the theory

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\alpha R^2$$

where our connection is non-metric but torsionless! The field equations are in this case

- $R_{(\mu\nu)} - \frac{R}{4}g_{\mu\nu} = 0$
- $\nabla_\alpha\left(R\sqrt{-g}g^{\mu\nu}\right) - \nabla_\beta\left(R\sqrt{-g}g^{\beta(\mu}\right)\delta_\alpha^{\nu)} = 0$

Solution of the system

$$\frac{\partial_\mu R}{R} = \frac{1}{4}Q_\mu\ ,\quad Q_{\lambda\mu\nu} = \frac{1}{4}g_{\mu\nu}Q_\lambda\ ,\quad R = \tilde{R} - \frac{3}{4}\tilde{\nabla}_\mu Q^\mu - \frac{3}{32}Q_\mu Q^\mu$$

Cosmological Solutions

For a flat FLRW Universe we have to solve the set of equations

$$(\dot{H}+H^2)+\frac{\dot{Q}}{8}+\frac{1}{8}HQ=\frac{C}{12}e^{\frac{1}{4}\int Qdt}\ , \quad \Big(Q=Q_0(t)\Big)$$
$$6(\dot{H}+2H^2)+\frac{3}{4}\dot{Q}+\frac{9}{4}HQ+\frac{3}{32}Q^2=Ce^{\frac{1}{4}\int Qdt}$$

This can be solved exactly, to yield

- $H(t)=H_0e^{\frac{1}{8}\int Qdt}-\frac{Q}{8}$
- $a(t)=a_0e^{\int\left[H_0e^{\frac{1}{8}\int Qdt}-\frac{Q}{8}\right]dt}$

For $Q=const.$

$$a(t)=a_0e^{\frac{8H_0}{Q}e^{\frac{Q}{8}t}-\frac{Q}{8}t}$$

$\Rightarrow$ Non-metric Cosmological expansion!

Model with torsion

The same model $f(R) = \alpha R^2$ but with $S_{\mu\nu\alpha} \neq 0$, $Q_{\alpha\mu\nu} = 0$ was studied by Capozziello et al. Their solutions there we

- $H(t) = H_0 e^{\frac{1}{3}\int T dt} - \frac{T}{3}$, (T=2S)
- $a(t) = a_0 e^{\int \left[H_0 e^{\frac{T}{3}\int Q dt} - \frac{T}{3} \right] dt}$

Duality

Looking back at our solution we see that one solution maps to another by exchanging

$$T \leftrightarrow \frac{3}{8} Q$$

- This duality can be seen in at least two more places

Ricci Scalars

The duality is also apparent when looking at the Ricci scalar decomposition

- $R = \tilde{R} - 2\tilde{\nabla}_\mu T^\mu - \frac{2}{3} T_\mu T^\mu \quad , \qquad S_{\alpha\mu\nu} \neq 0 \;\; Q_{\alpha\mu\nu} = 0$
- $R = \tilde{R} - \frac{3}{4}\tilde{\nabla}_\mu Q^\mu - \frac{3}{32} Q_\mu Q^\mu \quad , \qquad S_{\alpha\mu\nu} = 0 \;\; Q_{\alpha\mu\nu} \neq 0$

Autoparallels (For general n)

- $\ddot{x}^\alpha + \tilde{\Gamma}^\alpha{}_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = \frac{2}{n-1} S^a \dot{x}^2 \quad , \qquad S_{\alpha\mu\nu} \neq 0 \;\; Q_{\alpha\mu\nu} = 0$
- $\ddot{x}^\alpha + \tilde{\Gamma}^\alpha{}_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = \frac{1}{2n} Q^a \dot{x}^2 \quad , \qquad S_{\alpha\mu\nu} = 0 \;\; Q_{\alpha\mu\nu} \neq 0$

Duality for general dim

$$S_\mu \leftrightarrow \frac{(n-1)}{4n} Q_\mu$$

Mixed Model

If we assume that both $S_{\alpha\mu\nu} \neq 0, Q_{\alpha\mu\nu} \neq 0$ the solution is now

- $H = H_0 e^{\frac{1}{2}\int w dt} - \frac{w}{2}$
- $a(t) = a_0 e^{\int \left[H_0 e^{\frac{1}{2}\int w dt} - \frac{w}{2} \right] dt}$, where $w_\mu := \frac{1}{n} Q_\mu + \frac{4}{n-1} S_\mu = \frac{\partial_\mu R}{R}$

Affine connection

The form of the affine connection is

- $\Gamma^\lambda{}_{\mu\nu} = \tilde{\Gamma}^\lambda{}_{\mu\nu} + \frac{1}{2}\left(\delta^\lambda_\nu w_\mu - g_{\mu\nu} w^\lambda\right)$
- Note: It may just so happen that $w_\mu = 0$ but with $S_\mu \neq 0$, $Q_\mu \neq 0$
- In this case torsion and non-metricity cancel each other and we are effectively left with a Riemannian space!

$1+(n-1)$ Spacetime split

Let $u^{\mu}=\frac{dx^{\mu}}{d\lambda}$ be a tangent vector ($4-$*velocity* for $n=4$) to a curve $\mathcal{C}$. Then

$$u_{\mu}u^{\mu}\neq -1 \quad but \quad u_{\mu}u^{\mu}=-l^{2}(x)$$

since any vector's length changes because of non-metricity. This suggests that acceleration and velocity are no longer perpendicular.

Projection Tensor

The naive generalization

- $h_{\mu\nu}=g_{\mu\nu}+u_{\mu}u_{\nu}$ does not work here since $h_{\mu\nu}u^{\mu}\neq 0$.
  Proper definition
- $h_{\mu\nu}=g_{\mu\nu}+\frac{u_{\mu}u_{\nu}}{l^{2}}$
  which now satisfies $h_{\mu\nu}u^{\mu}=0=h_{\mu\nu}u^{\nu}$ and $h_{\mu\nu}h^{\mu\nu}=n-1$ as can be easily checked

The Setup

Projections along *'time'* and spatial space

- $\dot{T}_{\alpha_1 \dots \alpha_n}{}^{\beta_1 \dots \beta_m} = u^\mu \nabla_\mu T_{\alpha_1 \dots \alpha_n}{}^{\beta_1 \dots \beta_m}$
- $D_\mu T_{\alpha_1 \dots \alpha_n}{}^{\beta_1 \dots \beta_m} = h_\mu^\lambda h_{\alpha_1}^{\gamma_1} \dots h_{\alpha_n}^{\gamma_n} h_{\delta_1}^{\beta_1} \dots h_{\delta_m}^{\beta_m} \nabla_\lambda T_{\gamma_1 \dots \gamma_n}{}^{\delta_1 \dots \delta_m}$

Two kinds of Accelerations

→Path Acceleration

- $A^\mu \equiv \dot{u}^\mu \equiv u^\lambda \nabla_\lambda u^\mu$ ($g_{\mu\nu}$ does not commute with ∇_α)

→Hyper Acceleration

- $a_\mu \equiv \dot{u}_\mu \equiv u^\lambda \nabla_\lambda u_\mu$

Note

The former vanishes for autoparallel motion while the latter does not! The two are related through $A^\mu = a^\mu + Q^{\lambda\mu\nu} u_\lambda u_\nu$

Expansion, Shear, Vorticity and the rest

- $\Theta \equiv g^{\mu\nu}\nabla_\mu u_\nu \ , \quad \Theta_D \equiv g^{\mu\nu}D_\mu u_\nu = \Theta + \frac{a\cdot u}{l^2}$
- $\omega_{\nu\mu} \equiv D_{[\mu}u_{\nu]}$
- $\sigma_{\nu\mu} \equiv D_{<\mu}u_{\nu>} \equiv D_{(\mu}u_{\nu)} - \frac{(h^{\alpha\beta}D_\alpha u_\beta)}{n-1}h_{\mu\nu}$
- $\nabla_\mu u_\nu = \omega_{\nu\mu} + \sigma_{\nu\mu} + \left(\Theta + \frac{(a\cdot u)}{l^2}\right)\frac{h_{\mu\nu}}{n-1} - \frac{\xi_\mu u_\nu + u_\mu a_\nu}{l^2} - \frac{u_\mu u_\nu (a\cdot u)}{l^4}$

where $\xi_\mu \equiv u^\alpha \nabla_\mu u_\alpha$ and $(a\cdot u) = a_\mu u_\nu g^{\mu\nu}$

Note

- $\Theta \neq \nabla_\mu u^\mu \ , \quad \nabla_\mu u^\mu = \Theta + u^\mu \tilde{Q}_\mu$
- $\Theta = \tilde{\Theta} + \Big(-\tilde{Q}_\mu + 1/2 Q_\mu + 2S_\mu\Big)u^\mu \ , \quad \tilde{\Theta} = \tilde{\nabla}_\mu u^\mu$

The Raychaudhuri Equation

Starting by Ricci's identity and using the above definitions, we find

$$\dot{\Theta}_D + \frac{\Theta_D^2}{n-1} = -R_{\mu\nu}u^\mu u^\nu - \sigma^2 + \omega^2 + g^{\mu\nu}\nabla_\mu a_\nu$$
$$+\frac{d}{d\lambda}\left(\frac{a\cdot u}{l^2}\right) + \frac{(a\cdot u)^2}{l^4} + 2\frac{(a\cdot\xi)}{l^2} + u_\alpha Q^{\alpha\beta\mu}\nabla_\beta u_\mu - u_\alpha Q^{\mu\nu\alpha}\nabla_\nu u_\mu$$
$$+u^\mu u^\beta\left(g^{\nu\alpha}\nabla_{[\alpha}Q_{\beta]\mu\nu} - S_{\alpha\beta}{}^\lambda Q_{\lambda\mu}{}^\alpha\right) + 2u_\alpha S^{\alpha\mu\nu}\nabla_\nu u_\mu$$

Comments

- The first line is formalistically the same with the standard one!
- All the extra terms depend on non-metricity except from the last one which depends on torsion!

Cosmological Solution for Vectorial Torsion (and $Q_{\alpha\mu\nu}=0$)

For a vectorial torsion of the form
$S_{\mu\nu}{}^{\lambda}=\frac{2}{n-1}S_{[\mu}\delta^{\lambda}_{\nu]}$, assuming a flat and empty FLRW universe and an autoparallel motion ($A^{\mu}=0$)we obtain

$$\dot{\Theta}+\frac{\Theta^2}{3}=\frac{2}{3}(u^{\mu}S_{\mu})\Theta$$

This can be solved for generic $S_{\mu}=\delta^{0}_{\mu}S_0(t)$ and the solution reads

- $a(t)=e^{-\frac{2}{3}\int S_0(t)dt}\left[C_1+C_0\int e^{\frac{2}{3}\int S_0(t)dt}dt\right]$

Cosmological Solution for Weyl non-metricity (and $S_{\alpha\mu\nu}=0$)

Upon the same assumptions but now Weyl non-metricity and vanishing torsion, finally we get

- $a(t)=e^{-\frac{1}{8}\int Q_0(t)dt}\left[C_1+C_0\int e^{\frac{1}{8}\int Q_0(t)dt}dt\right]$

The duality appears again!

Collecting the two solutions

- $a(t) = e^{-\frac{2}{3}\int S_0(t)dt}\left[C_1 + C_0\int e^{\frac{2}{3}\int S_0(t)dt}dt\right], (S \neq 0 \;\; Q = 0)$
- $a(t) = e^{-\frac{1}{8}\int Q_0(t)dt}\left[C_1 + C_0\int e^{\frac{1}{8}\int Q_0(t)dt}dt\right], (S = 0 \;\; Q \neq 0)$

We observe an astonishing result! The two solutions look similar, in fact one maps to another by interchanging $S_\mu \longleftrightarrow \frac{3}{16}Q_\mu$. This is the same duality we saw earlier!

Conclusion

As far as the evolution of scale factor is concerned, vectorial torsion is indistinguishable from Weyl non-metricity in FLRW universes!

Fixed Length Vector Non-Metricity

For flat FLRW with

- $Q_{\alpha\mu\nu} = A(t)\Big(u_\alpha g_{\mu\nu} - g_{\alpha(\mu}u_{\nu)}\Big) \Rightarrow$

$$\dot{\Theta} + \frac{\Theta^2}{3} = -\frac{1}{4}\left(5A\Theta + 3\dot{A} + \frac{3}{2}A^2\right)$$

Solution for $A(t) = A_0 = const.$

- $\Theta(t) = -\frac{3A_0}{8}\left(1 + \frac{8}{C_1 e^{A_0 t}+1}\right), \quad \dot{\Theta} + \frac{1}{3}\Theta^2 > 0$
- $a(t) = C_2(C_1 + e^{-A_0 t})e^{\frac{7}{8}A_0 t}$

Result

Accelerated expansion regardless of the sign of A_0!

Conclusions

- We studied a peculiar case of metric affine gravity in vacuum and found exact cosmological solutions
- As it turns out turning on non-metricity (and no torsion) is the same as turning on only torsion and the two solutions map to one another by a duality transformation
- When both torsion and non-metricity are present they may cancel each other out and result in a Riemannian geometry
- We presented (for the first time) the Raychaudhuri Equation in spaces with both torsion and non-metricity
- We found Cosmological solutions for solely torsion and solely non-metricity and the duality reappeared!
- Conclusion: As far as the expansion is concerned, Vectorial torsion looks indistinguishable from Weyl's non-metricity in Cosmology!

...Thank you!!!