Axionic black branes with conformal coupling

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Motivation

- Black holes are important objects whose relevance ranges from astrophysical grounds to quantum gravity effects.
- The topological censorship theorem as well as the no-hair conjecture complicate to find black hole solutions interacting with matter fields.[Friedman, Schleich and Witt, PRL 71],[Bekenstein, gr-qc/9808028]
- In $D = 4$, the inclussion of Λ allows to access different BH's topologies. [Birmingham, CQG 16]
- Higher dimensions: the Schwarzschild-Tangherlini black hole and the black p-brane. [Tangherlini, Nuovo Cim. 27]

- No-hair conjecture: black holes can not be described by any different quantity apart from its mass, electric charge and angular momentum. [Ruffini and Wheeler, Phys. Today 24]

- Non-minimal couplings: BMBB black hole

$$I = \int \sqrt{-g} d^4 x \left(\frac{R}{16\pi G} - \frac{1}{2}\nabla_\mu \phi \nabla^\mu \phi - \frac{1}{12} R\phi^2 - \frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} \right)$$

$$ds^2 = -\left(1 - \frac{M}{r}\right)^2 dt^2 + \left(1 - \frac{M}{r}\right)^{-2} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

$$A = -\frac{Q_E}{r} dt,$$

$$\phi = \frac{\phi_0}{r - M},$$

with $M^2 = Q_E^2 + \dfrac{4\pi\phi_0^2}{3}$.

[Bocharova, Bronnikov and Melnikov, Vestn. Mosk. Univ. Ser. III Fiz. Astron., no. 6, 706 (1970)],[Bekenstein, Annals Phys. 82, Annals Phys. 91, 75 (1975)]

- Non-minimal couplings: MTS black hole

$$I=\int\sqrt{-g}d^4x\left(\frac{R-2\Lambda}{16\pi G}-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi-\frac{1}{12}R\phi^2-\alpha\phi^4-\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}\right)$$

$$ds^2=-\left[-\frac{\Lambda r^2}{3}+\gamma\left(1+\frac{G\mu}{r}\right)^2\right]dt^2+\left[-\frac{\Lambda r^2}{3}+\gamma\left(1+\frac{G\mu}{r}\right)^2\right]^{-1}dr^2+r^2d\sigma^2,$$

$$A=-\frac{Q_E}{r}dt,$$

$$\phi=\sqrt{\frac{-\Lambda}{6\alpha}}\frac{G\mu}{r+G\mu},$$

with $Q_E^2=\gamma G\mu^2\left(1+\frac{2\pi\Lambda G}{9\alpha}\right)$ provided $0<\alpha\leq-\frac{2\pi\Lambda G}{9}$.
[Martínez, Troncoso and Zanelli, PRD 67]

- Only trivial solutions for $\gamma=0$.
- Interaction with p-forms allows planar solutions.[Bardoux, Caldarelli and Charmousis, JHEP 1205]

- Regular solutions can be found by charging the horizon with homogeneously distributed axionic charges along planar directions.

$$I = \int \sqrt{-g}d^4x \left(\frac{R-2\Lambda}{16\pi G} - \frac{1}{2}\nabla_\mu \phi \nabla^\mu \phi - \frac{1}{12}R\phi^2 - \alpha\phi^4 - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} \right)$$

$$- \int \sqrt{-g}d^4x \left[\frac{1}{2}\left(1 - \frac{4\pi G}{3}\phi^2\right)^{-1} \sum_{i=1}^{2} \frac{1}{3!} H^{(i)}_{abc} H^{(i)abc} \right]$$

$$ds^2 = -\left[-\frac{\Lambda r^2}{3} - p^2\left(1 + \frac{G\mu}{r}\right)^2 \right] dt^2 + \left[-\frac{\Lambda r^2}{3} - p^2\left(1 + \frac{G\mu}{r}\right)^2 \right]^{-1} dr^2 + r^2 \delta_{ij} dx^i dx^j,$$

$$A = -\frac{Q_E}{r}dt, \quad \mathcal{H}^{(i)} = -\frac{p}{\sqrt{8\pi G}}\left(1 - \frac{4\pi G}{3}\phi^2\right) dt \wedge dr \wedge dx^i$$

$$\phi = \sqrt{\frac{-\Lambda}{6\alpha}} \frac{G\mu}{r + G\mu},$$

with $Q_E^2 = -p^2 G\mu^2 \left(1 + \frac{2\pi\Lambda G}{9\alpha}\right)$ provided $0 < \alpha \leq -\frac{2\pi\Lambda G}{9}$.

[Bardoux, Caldarelli and Charmousis, JHEP 1209]

- **AdS/CFT**: Planar/toroidal black holes with scalar fields possess special relevance due, in particular, to their applications in the dual description of superconductor systems. [Maldacena, Int. J. Theor. Phys. 38],[Hartnoll, CQG 26],[Horowitz, Lect. Notes Phys. 828]

- Real materials $\Longrightarrow$ Momentum dissipation. [Gubser, Klebanov and Polyakov, Nucl. Phys. B 636], [Witten, Adv. Theor. Math. Phys. 2]

- Scalar lattice technique, massive gravity, Q-lattice model.[Horowitz, Santos and Tong, JHEP 1207],[Davison, PRD 88],[Kuang, Papantonopoulos, Wu and Zhou, PRD 97]

- An effective and simple way is allowing interaction with massless scalar fields linearly dependent on the base manifold coordinates (translational invariance breaking) [Andrade and Withers, JHEP 1405].

The aim of this work

The construction of AdS black brane solutions with a conformally coupled scalar field where the translational invariance at the boundary is broken by means of axion fields that depend linearly on the base manifold directions.

Outline

Section

- We consider the following action,

$$I[g,\phi,\psi_I]=\int d^4x\sqrt{-g}\left[\kappa(R-2\Lambda)-\frac{1}{2}(\partial\phi)^2-\frac{1}{12}\phi^2R-\frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right],$$

where $\kappa\equiv\frac{1}{16\pi G}$ and G is the four dimensional Newton's constant.

- The field equations are

$$\begin{aligned}\kappa(G_{\mu\nu}+\Lambda g_{\mu\nu}) &= \frac{1}{2}T^{\phi}_{\mu\nu}+\frac{1}{2}T^{\psi}_{\mu\nu},\\ \left(\Box-\frac{1}{6}R\right)\phi &= 0,\\ \Box\psi_I &= 0,\end{aligned}$$

where $\Box\equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$.

- The energy-momentum tensor is given by contributions from the scalar and axion field which are respectively

$$T^{\phi}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 + \frac{1}{6}(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + G_{\mu\nu})\phi^2,$$

$$T^{\psi}_{\mu\nu} = \sum_{I=1}^{2}\left(\partial_\mu\psi_I\partial_\nu\psi_I - \frac{1}{2}g_{\mu\nu}(\partial\psi_I)^2\right).$$

- We look for static and planar four dimensional metrics, whose expression are given by

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(dx^2 + dy^2),$$

where $0 \leq r < \infty$, $0 \leq x \leq \beta_x$ and $0 \leq y \leq \beta_y$.

- Imposing the axion fields to depend only on the boundary directions the Klein-Gordon equation for each axion field is trivially integrated yielding

$$\psi_I = \zeta_{Ii} x^i + \alpha_I ,$$

where $x^1 \equiv x$, $x^2 \equiv y$ and ζ_{Ii}, α_I are integration constants.

- Translational symmetry is broken having $\alpha_I = 0$. In this way, we may write the solution as $\psi_I = \lambda x_I$.[Caldarelli, Christodoulou, Papadimitriou and Skenderis, JHEP 1704]

Section

- The field equations admit an exact solution where the metric, scalar and axion fields are given by

$$\begin{aligned} ds^2 &= -\frac{(r-3\lambda l)(r+\lambda l)^3}{r^2 l^2}dt^2 + \frac{r^2 l^2}{(r-3\lambda l)(r+\lambda l)^3}dr^2 + r^2(dx^2+dy^2), \\ \phi &= 2\sqrt{3}\frac{\lambda l}{r+\lambda l}, \\ \psi_I &= 2\sqrt{3}\lambda x_I, \end{aligned}$$

where we have redefined the axion parameter $\lambda \to 2\sqrt{3}\lambda$, set $\kappa = 1$ for simplicity and defined the AdS radius $l^{-2} := -\frac{\Lambda}{3}$.

- Asymptotically AdS behavior $g_{tt} \sim -\frac{r^2}{l^2} + \mathcal{O}(r^0)$, $g_{rr} \sim \frac{l^2}{r^2} + \mathcal{O}(r^0)$

- Curvature singularity

$$R = -\frac{12}{l^2} + \frac{12\lambda^2}{r^2}.$$

- Event horizon: $r_+ = 3\lambda l$ $(\lambda > 0)$, $r_+ = -\lambda l$ $(\lambda < 0)$.

- Economic way to obtain a toroidal black hole of with regular conformally coupled scalar field.

- Arbitrary cosmological constant.

- An electrically and magnetically charged black hole is obtained by adding the Maxwell term,

$$-\frac{1}{4}\int d^4x\sqrt{-g}F^{\mu\nu}F_{\mu\nu},$$

to the action.

- Gauge potential of the form

$$\phi=\frac{\sqrt{Q_E^2+Q_M^2+12\lambda^4 l^2}}{\lambda(r+\lambda l)}, \qquad A=-\frac{Q_E}{r}dt+\frac{Q_M}{2}(xdy-ydx).$$

At large r, scalar field is approximated by

$$\phi=\frac{\phi_1}{r}+\frac{\phi_2}{r^2}+\mathcal{O}(r^{-3}),$$

where

$$\phi_1\equiv\lambda^{-1}\sqrt{Q_E^2+Q_M^2+12l^2\lambda^4}, \qquad \phi_2\equiv -l\sqrt{Q_E^2+Q_M^2+12l^2\lambda^4}.$$

- Boundary conditions that respect AdS symmetry of the scalar field asymptotic behavior
  - $\{\phi_1 = 0, \phi_2 \neq 0\}$
  - $\{\phi_1 \neq 0, \phi_2 = 0\}$
  - $\{\phi_1^2 = \alpha\phi_2\}$

$$\lambda^4 = \frac{Q_E^2 + Q_M^2}{(\alpha^2 - 12)l^2}, \qquad \alpha^2 > 12.$$

[Henneaux, Martínez, Troncoso and Zanelli, Annals Phys. 322], [Hertog and Maeda, JHEP 0407]

- Dominant energy condition (DEC) is satisfied

$$T^{ab} = \text{diag}\left(\frac{12r^2 - 6l^2\lambda^4}{r^4}, -\frac{12r^2 - 6l^2\lambda^4}{r^4}, -\frac{6l^2\lambda^4}{r^4}, -\frac{6l^2\lambda^4}{r^4}\right).$$

- We can identify the energy density ρ and the principal pressures p_a $(a = 1, 2, 3)$, as

$$\rho = -p_1 = \frac{12r^2 - 6l^2\lambda^4}{r^4}, \qquad p_2 = p_3 = -\frac{6l^2\lambda^4}{r^4}.$$

verifying directly that $\rho \geq 0$ and $-\rho \leq p_a \leq \rho$ for $r \geq \lambda l$.

- DEC is satisfied at least outside the event horizon.

- The solution can be endowed with angular momentum.

Section

- The thermodynamical analysis can be done by following the Euclidean approach: The partition function for a thermodynamical ensemble is identified with the Euclidean path integral in the saddle point approximation around the classical Euclidean solution.[Gibbons and Hawking, PRD 15]

- Euclidean continuation of the black hole metric

$$ds_E^2 = N^2(r)F(r)d\tau^2 + \frac{dr^2}{F(r)} + r^2(dx^2 + dy^2),$$

and the scalar, axionic and gauge field are

$$\phi = \phi(r), \qquad \psi_I = \psi_I(x^i), \qquad A = A_\tau(r)d\tau + A_x(y)dx + A_y(x)dy,$$

where $x^1 = x$ and $x^2 = y$.

- $0 \leq \tau \leq \beta$, $r_+ \leq r < \infty$, $0 \leq x \leq \beta_x$, $0 \leq y \leq \beta_y$ where $\beta = T^{-1}$ and $I_E = \beta\mathcal{G}$.

- The Euclidean action can take a Hamiltonian form,

$$I_E = \beta \int_{r_+}^{\infty} dr \int_0^{\beta_x} dx \int_0^{\beta_y} dy (N\mathcal{H} - A_\tau \mathcal{G}) + B_E$$

with B_E a surface term.

- The reduced constraints are given by

$$\begin{aligned}\mathcal{H} =& \frac{r^2}{8\pi G}\left[\left(1 - \frac{4\pi G}{3}\phi^2\right)\left(\frac{\partial_r F}{r} + \frac{F}{r^2}\right) + \Lambda\right] + \frac{r^2}{6}\left[F(\partial_r\phi)^2 - \left(\partial_r F + \frac{4F}{r}\right)\phi\partial_r\phi - 2\right.\\ &+ \frac{1}{2}\sum_{i=1}^{2}(\partial_{x^i}\psi_I)^2 + \frac{1}{2r^2}(\partial_x A_y - \partial_y A_x)^2 + \frac{1}{2r^2}(\pi^r)^2,\\ \mathcal{G} =& \partial_r \pi^r,\end{aligned}$$

where π^r stands for the electromagnetic field momentum defined by,

$$\pi^r = -\frac{r^2 A'_\tau}{N}.$$

- Canonical variables $\{F, A_x, A_y, \phi, \pi^r, \psi_I\}$.

- $\delta I_E = 0 \implies B_E$

- Variations of the reduced action with respect to $\{N, F, A_\tau, A_x, A_y, \phi, \pi^r\}$ are consistent with the original Einstein equations.

- The variational principle on the boundary term gives

$$\begin{aligned}\delta B_E =& \beta\sigma \left[A_\tau \delta\pi^r - \frac{rN}{8\pi G}\left(1 - \frac{4\pi G}{3}\phi^2 - \frac{4\pi G}{3} r\phi\phi'\right)\delta F - \frac{r^2 N}{6}\left(4F\phi' + F'\phi\right)\delta\phi + \frac{r^2 N}{3} F\phi\delta\phi'\right]_{r_+}^{\infty} \\ & - \int_{r_+}^{\infty} dr \frac{N}{r^2}\left\{\left[\int_0^{\beta_x} dx(\partial_y A_x - \partial_x A_y)\delta A_x\right]_{y=0}^{y=\beta_y} - \left[\int_0^{\beta_y} dy(\partial_y A_x - \partial_x A_y)\delta A_y\right]_{x=0}^{x=\beta_x}\right\} \\ & - \int_{r_+}^{\infty} dr \left\{\left[\int_0^{\beta_y} dy \partial_x\psi_1\delta\psi_1\right]_{x=0}^{x=\beta_x} + \left[\int_0^{\beta_x} dx \partial_y\psi_2\delta\psi_2\right]_{y=0}^{y=\beta_y}\right\},\end{aligned}$$

where we have used the fact that the volume of the base manifold is $\sigma = \beta_x\beta_y$.

- Regularity condition at the horizon $\beta F'(r_+) = 4\pi$, gives temperature $T = \dfrac{16}{9\pi}\dfrac{\lambda}{l}$.

- The variation of the fields on the event horizon is given by

$$\delta F|_{r_+} = -\frac{4\pi}{\beta}\delta r_+, \quad \delta\phi|_{r_+} = \delta\phi(r_+) - \phi'|_{r_+}\delta r_+,$$

$$\delta\psi_I|_{r_+} = 2\sqrt{3}x^i\delta\lambda, \quad \delta\pi^r|_{r_+} = \delta Q_E, \quad \delta A_x|_{r_+} = \frac{\delta Q_M}{2}y, \quad \delta A_y|_{r_+} = -\frac{\delta Q_M}{2}x.$$

- Effective Newton constant at the horizon is defined as $\tilde{G}_+ = \dfrac{G}{\left(1 - \frac{4\pi G}{3}\phi(r_+)^2\right)}$

- The variation of the boundary term at the horizon is

$$\delta B_E(r_+) = \delta\left(\frac{A_+}{4\tilde{G}_+}\right) + \beta\Phi_e\delta(\sigma Q_E) + \beta\Phi_M\delta(\sigma Q_M) + \beta\Phi_{\psi_1}\delta(-2\sqrt{3}\sigma\lambda) + \beta\Phi_{\psi_2}\delta(-2\sqrt{3}\sigma\lambda),$$

  where $A_+ = \sigma r_+^2$ is the horizon area.

- Chemical potentials $\Phi_E, \Phi_M, \Phi_{\psi_I}$,

$$\Phi_E \equiv \frac{Q_E}{r_+}, \qquad \Phi_M \equiv \frac{Q_M}{r_+}, \qquad \Phi_{\psi_1} \equiv 2\sqrt{3}\lambda r_+, \qquad \Phi_{\psi_2} \equiv 2\sqrt{3}\lambda r_+,$$

- Grand canonical ensemble: β and $\Phi_E, \Phi_M, \Phi_{\psi_I}$ are fixed.

- Using these boundary conditions we get,

$$B_E(r_+) = \frac{A_+}{4\tilde{G}_+} + \beta\Phi_e(r_+)(\sigma Q_E) + \beta\Phi_M(\sigma Q_M) + \beta\Phi_{\psi_1}(-2\sqrt{3}\sigma\lambda) + \beta\Phi_{\psi_2}(-2\sqrt{3}\sigma\lambda).$$

- The variation of fields at infinity are

$$\delta F|_\infty = -12\lambda\delta\lambda - \frac{24l\lambda^2\delta\lambda}{r} - \frac{12l^2\lambda^3\delta\lambda}{r^2}, \quad \delta\phi|_\infty = \frac{\delta\phi_1}{r} + \frac{\delta\phi_2}{r^2} + \mathcal{O}(r^{-3}),$$

$$\delta\psi_I|_\infty = 2\sqrt{3}x^i\delta\lambda, \quad \delta\pi^r|_\infty = \delta Q_E, \quad \delta A_x|_\infty = \frac{\delta Q_M}{2}y, \quad \delta A_y|_\infty = -\frac{\delta Q_M}{2}x.$$

- Then the variation of the boundary term at inftinity is,

$$\delta B_E(\infty) = \beta\sigma 48l\lambda^2\delta\lambda + \frac{\beta\sigma}{3l^2}(2\phi_2\delta\phi_1 - \phi_1\delta\phi_2).$$

- Integrability condition $\delta^2 B_E(\infty) = 0$ implies $\phi_2 = \phi_2(\phi_1)$.

- The boundary term at infinity generically takes the form

$$B_E(\infty) = \beta\sigma 16l\lambda^3 + \frac{\beta\sigma}{3l^2}\int\left(2\phi_2 - \phi_1\frac{d\phi_2}{d\phi_1}\right)d\phi_1.$$

- Then the value of the reduced action on-shell is

$$I_E = B_E(\infty) - B_E(r_+)$$
$$= \beta\sigma 16 l\lambda^3 + \frac{\beta\sigma}{3l^2}\int\left(2\phi_2 - \phi_1\frac{d\phi_2}{d\phi_1}\right)d\phi_1 - \frac{A_+}{4\tilde{G}_+}$$
$$- \beta\Phi_e(r_+)(\sigma Q_E) - \beta\Phi_M(\sigma Q_M) - \beta\Phi_{\psi_1}(-2\sqrt{3}\sigma\lambda) - \beta\Phi_{\psi_2}(-2\sqrt{3}\sigma\lambda),$$

up to an arbitrary additive constant without variation.

- Gibbs free energy: $I_E = \beta\mathcal{G} = \beta\mathcal{M} - \mathcal{S} - \beta\Phi_E\mathcal{Q}_E - \beta\Phi_M\mathcal{Q}_M - \beta\Phi_{\psi_1}\mathcal{Q}_1 - \beta\Phi_{\psi_2}\mathcal{Q}_2$
- Conserved charges,

$$\mathcal{M} = \left(\frac{\partial}{\partial\beta} - \beta^{-1}\Phi_E\frac{\partial}{\partial\Phi_E} - \beta^{-1}\Phi_M\frac{\partial}{\partial\Phi_M} - \beta^{-1}\Phi_{\psi_I}\frac{\partial}{\partial\Phi_{\psi_I}}\right)I_E$$
$$= 16\sigma l\lambda^3 + \frac{\sigma}{3l^2}\int\left(2\phi_2 - \phi_1\frac{d\phi_2}{d\phi_1}\right)d\phi_1,$$

$$\mathcal{S} = \left(\beta\frac{\partial}{\partial\beta} - 1\right)I_E = \frac{A_+}{4\tilde{G}_+}, \qquad \mathcal{Q}_i = -\frac{1}{\beta}\frac{\partial I_E}{\partial\Phi_{\psi_I}} = -2\sqrt{3}\lambda\sigma,$$

$$\mathcal{Q}_E = -\frac{1}{\beta}\frac{\partial I_E}{\partial\Phi_E} = \sigma Q_E, \qquad \mathcal{Q}_M = -\frac{1}{\beta}\frac{\partial I_E}{\partial\Phi_M} = \sigma Q_M.$$

- First law of black hole thermodynamics

$$d\mathcal{M} = TdS + \Phi_E d\mathcal{Q}_E + \Phi_M d\mathcal{Q}_M + \Phi_{\psi_1} d\mathcal{Q}_1 + \Phi_{\psi_2} d\mathcal{Q}_2,$$

- Scalar field respect the asymptotic AdS invariance requires,
$\phi_1^2 = \alpha\phi_2 \implies \lambda^4 = \dfrac{Q_E^2 + Q_M^2}{(\alpha^2 - 12)l^2}.$

- The positivity of the entropy requires $\tilde{G}_+ > 0$ providing a lower bound for the axion parameter,

$$\lambda^4 > \frac{Q_E^2 + Q_M^2}{180 l^2}.$$

- Then $2\sqrt{3} < |\alpha| < 8\sqrt{3}$.

- Asymptotic AdS symmetry implies $\mathcal{M} = 16\sigma l \lambda^3$.

- The local thermal stability of the black hole can be analyzed by computing the specific heat at fixed angular velocity and chemical potentials,

$$C = \left(\frac{\partial M}{\partial T}\right)_{\Phi_{\psi_1}, \Phi_{\psi_2}} = \left[\left(\frac{\partial M}{\partial r_+}\right)\left(\frac{\partial T}{\partial r_+}\right)^{-1}\right]_{\Phi_{\psi_1}, \Phi_{\psi_2}},$$

which gives

$$C = 3\pi\sigma r_+^2 .$$

The specific heat is always positive, and in consequence, the black brane always attains equilibrium with a heat bath.

Section

- We study the DC conductivities σ_{DC} and the Hall angle θ_H of the holographic theory dual to the charged hairy black hole.

- $\sigma_{DC} = \sigma_{ccs} + \sigma_{diss}$, where σ_{ccs} is the 'charge-conjugation symmetric' part while σ_{diss} is related to the charge Q_E of the black hole and it is divergent in a translationally invariant theory.[Charmousis, Gouteraux, Kim, Kiritsis and Meyer, JHEP 1011]

- Considering relevant perturbations

$$\delta A_x = -E_x t + a_x, \quad \delta A_y = -E_y t + a_y,$$
$$\delta g_{tx} = r^2 h_{tx}, \quad \delta g_{rx} = r^2 h_{rx}, \quad \delta g_{ty} = r^2 h_{ty}, \quad \delta g_{ry} = r^2 h_{ry},$$
$$\delta\psi_1 = \Psi_1, \quad \delta\psi_2 = \Psi_2,$$

where E_x and E_y are constant, while $a_x, a_y, h_{tx}, h_{ty}, h_{rx}, h_{ry}, \Psi_1, \Psi_2$ are all functions of the radial coordinate r.
(More details of this technic in [A. Donos and J. P. Gauntlett, JHEP 1406])

- Perturbed Maxwell equations

$$F'a_x' + Fa_x'' + Q_E h_{\rm tx}' + Q_m\left(F'h_{\rm ry} + Fh_{\rm ry}'\right) = 0,$$
$$F'a_y' + Fa_y'' + Q_E h_{\rm ty}' - Q_m\left(F'h_{\rm rx} + Fh_{\rm rx}'\right) = 0,$$

where the prime denotes the derivative to r.

- Two radially conserved currents can be defined,

$$J_x = -r^2 F^{rx} = -Fa_x' - Q_E h_{tx} - Q_M F h_{ry},$$
$$J_y = -r^2 F^{ry} = -Fa_y' - Q_E h_{ty} + Q_M F h_{rx},$$

which satisfy $\frac{dJ_x(r)}{dr} = \frac{dJ_y(r)}{dr} = 0$ due to the Maxwell equations.

- Regularity conditions at the horizon: In Eddington-Finkelstein coordinates (v, r) with $v = t - \int \frac{dr}{F}$,

$$a_x = -E_x \int \frac{dr}{F}, \;\; a_y = -E_y \int \frac{dr}{F}$$

while the perturbed metric reads

$$h_{rx} = \frac{h_{tx}}{F}, \;\; h_{ry} = \frac{h_{ty}}{F}.$$

[Donos and Gauntlett, JHEP 1411, 081]

- $F(r_+) \sim 4\pi T(r - r_+)$ and we set $\Psi_{1,2}$ to be constant near the horizon.
- Plugging regularity conditions into ${E^r}_x$ and ${E^r}_y \implies h_{tx}(r_+)$ and $h_{ty}(r_+)$.
- Thus, the conductivities can be obtained

$$\begin{aligned}
\sigma_{xx} &= \frac{\partial J_x(r_+)}{\partial E_x} = \frac{3\left(12\lambda^4 + Q_M^2 + Q_E^2\right)\left(36\lambda^4 + 3Q_M^2 + Q_E^2\right)}{2Q_E^2\left(36\lambda^4 + 5Q_M^2\right) + 9\left(12\lambda^4 + Q_M^2\right)^2 + Q_E^4}, \\
\sigma_{yy} &= \frac{\partial J_y(r_+)}{\partial E_y} = \sigma_{xx}, \\
\sigma_{xy} &= \frac{\partial J_x(r_+)}{\partial E_y} = \frac{6Q_E Q_M\left(12\lambda^4 + Q_M^2 + Q_E^2\right)}{2Q_E^2\left(36\lambda^4 + 5Q_M^2\right) + 9\left(12\lambda^4 + Q_M^2\right)^2 + Q_E^4}, \\
\sigma_{yx} &= \frac{\partial J_y(r_+)}{\partial E_x} = -\frac{6Q_E Q_M\left(12\lambda^4 + Q_M^2 + Q_E^2\right)}{2Q_E^2\left(36\lambda^4 + 5Q_M^2\right) + 9\left(12\lambda^4 + Q_M^2\right)^2 + Q_E^4} = -\sigma_{xy},
\end{aligned}$$

where we have considered the event horizon $r_+ = 3\lambda$ and the temperature $T = 16\lambda/9\pi$ by setting $l = 1$.

- The DC conductivity and the Hall angle are

$$\sigma_{DC} \quad := \quad \sigma_{xx}(Q_M = 0) = \frac{3\left(12\lambda^4 + Q_E^2\right)}{36\lambda^4 + Q_E^2},$$

$$\theta_H \quad := \quad \frac{\sigma_{xy}}{\sigma_{xx}} = \frac{2Q_E Q_M}{36\lambda^4 + 3Q_M^2 + Q_E^2},$$

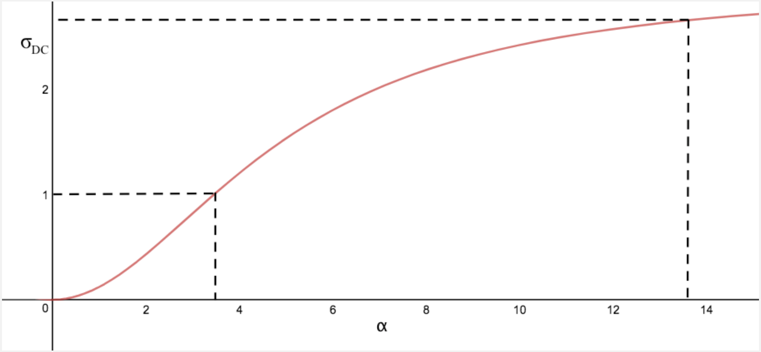
and we see that σ_{DC} and θ_H are finite.

- We rewrite the DC conductivity as

$$\sigma_{DC} = 1 + \frac{2Q_E^2}{36\lambda^4 + Q_E^2} = 1 + \frac{2(\alpha^2 - 12)}{24 + \alpha^2} = \frac{3\alpha^2}{24 + \alpha^2},$$

where we have used the asymptotic AdS invariance criterium in the second equality.

- DC conductivity gets restricted $1 < \sigma_{DC} < \frac{8}{3}$



- Some interesting remarks,
  - $Q_E \to 0 \implies \sigma_{DC} \to 1$ [A. Donos and J. P. Gauntlett, JHEP 1406]
  - Unexpectedly, the conformally coupled scalar field $\phi(r)$ modifies the backreaction of the black brane solution such that σ_{DC} is temperature independent.
  - Hall angle decays as $\theta_H \sim 1/T^2$ at high temperature limit, resembling the same behavior as in cuprates.[Blake and Donos, PRL 114]

Ending remarks

- We have constructed a black brane solution in a conformally coupled scalar theory. The black brane interacts with two axion fields homogeneously distributed along the horizon that depend linearly on the transverse manifold coordinates. No self-interaction for the scalar field has been needed.

- In the neutral black brane all parameters appearing in the solution are free from fine tuning.

- We constructed the dyonic black brane solution whose asymptotic AdS behavior relates the axionic and electromagnetic charges.
  - The scalar field is regular everywhere.
  - The black brane always attains equilibrium with a heat bath.
  - The DC conductivity $\sigma_{DC} > 1$ is temperature independent, in contrast with other charged black branes.[Bardoux, Caldarelli and Charmousis, JHEP 1205]

$Ευχαριστώ$!

Section

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