Production of PBH and observational prospects

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Based on a work in collaboration with
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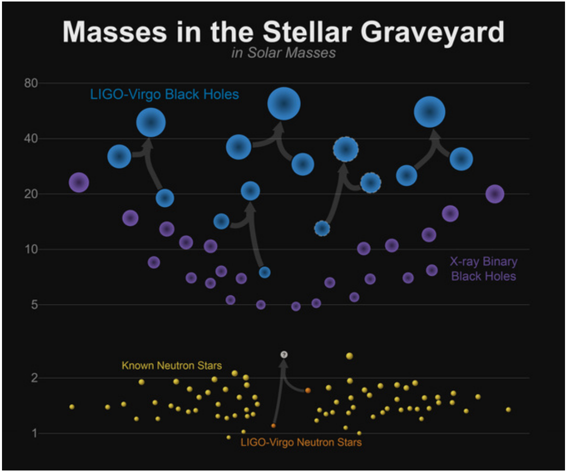
Motivation

More than the 80% of the matter in the universe,
***the dark matter**,*
remains elusive

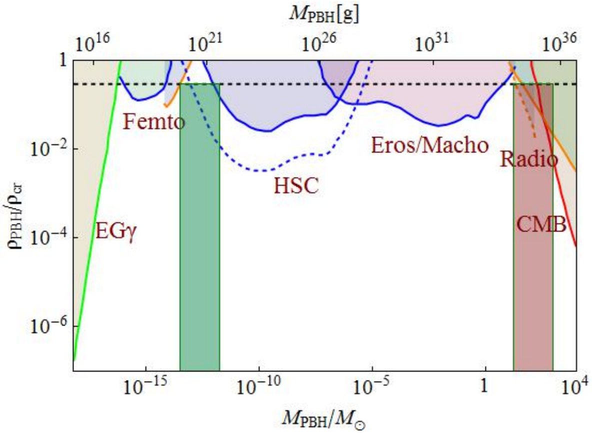
The dark sector of the universe (which we know that exists)



Motivation and Outline

- The recent detection of gravitation waves created by the merging of two massive ($\sim 30M_{\odot}$) black holes has revived the idea that the dark matter in the universe, or some fraction of it, is composed of Primordial Black Holes (PBHs)
- Such PBH are distinguished from astrophysical black holes since they are not produced by stellar collapse. Instead, they are formed due to gravitational collapse of density perturbations which are of order unity upon horizon entry
- The mass of such PBH is not limited by the usual bound of the $3M_{\odot}$ for astrophysical black holes but can attain much larger masses.
- Also PBH can be as light as $10^{-18}M_{\odot}$, but not lighter in order the evaporation rate in the late universe to be suppressed.

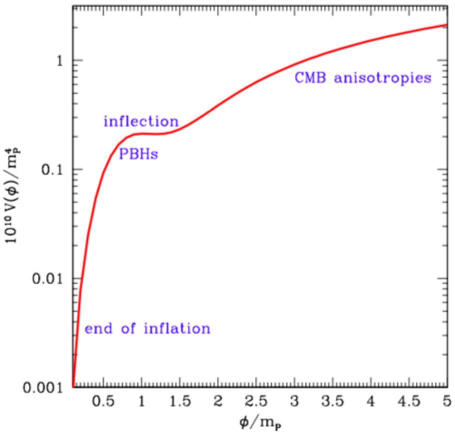
The allowed windows



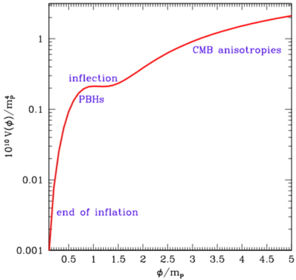
PBH Production mechanism

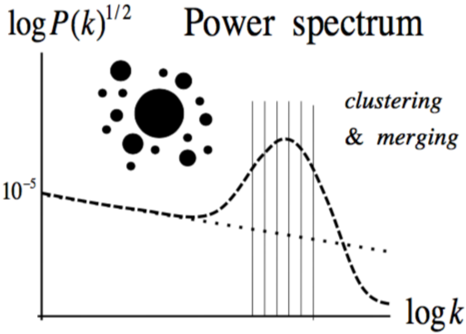
- If PBH are possible to be created in the early universe it is natural to contemplate upon the production mechanism and the possibility to comprise the elusive dark matter
- Recent studies indicate that the PBH production can be successfully attributed to the inflationary phase.
- PBH are found to be formed with a relic abundance possibly large enough to account for a significant fraction of the bulk dark matter in the universe.
- The power spectrum has to be CMB normalized at large scales, $\mathcal{P}_\mathcal{R} \sim 10^{-9}$ and increase about seven orders of magnitude in small scales, $\mathcal{P}_\mathcal{R} \sim 10^{-2}$.

A sketch of the required inflationary potential



The peak in the Power Spectrum





The inflationary set up

Superconformal attractors

We consider here superconformal model of $\mathcal{N} = 1$ supergravity coupled to chiral multiplets in the context of supersymmetric α attractor models[1] Along the trajectory $S = Im\Phi = 0$, the effective Lagrangian for the field $\phi = Re\Phi$ turns out to be

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{\alpha}{\left(1 - \frac{\phi^2}{3}\right)^2}\left(\partial_\mu\phi\right)^2 - f^2\left(\phi/\sqrt{3}\right).$$

Defining

$$\phi = \sqrt{3}\tanh\frac{\varphi}{\sqrt{6\alpha}},$$

one finds

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - f^2\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right).$$

[1] Cecotti, Kalosh, Linde, Roest

The inflationary set up

Superconformal attractors

In this class of models inflation occurs for large fields $\varphi \gg 0$ (or $\phi \approx \sqrt{3}$). In this case, we can approximate the potential as

$$V = f_1^2 - 4 f_1 f_1' e^{-\sqrt{\frac{2}{3}}\varphi} + \mathcal{O}(e^{-2\sqrt{\frac{2}{3}}\varphi}),$$

where

$$f_1 = f(\tanh \varphi/\sqrt{3})|_{\varphi\to\infty}, \quad f_1' = \partial_\varphi f(\varphi)|_{\varphi\to\infty}. \tag{1}$$

It is then straightforward to verify that for the potential the spectral index n_s and the scalar-to-tensor ratio r are given to leading order in the number of e-folds N as

$$1 - n_s = \frac{2}{N}, \quad r = \frac{12}{N^2}. \tag{2}$$

In the universality class of superconformal models, inflation gives the correct CMB anisotropies and normalization if $f(1) \approx 10^{-5} M_{\rm Pl}^2$.

Conditions for inflection point

The potential of the scalar lagrangian is $V(\varphi) = f^2(\tanh \varphi/\sqrt{6})$. The conditions that the potential should satisfy are:

1. A global minimum at $\varphi = \varphi_0$ where the potential vanish
$$V(\varphi_0) = 0.$$
This is the point where the inflaton will settle down without giving rise to a cosmological constant.
2. An inflection point $\varphi = \varphi_{\text{infl}}$,
$$V'(\varphi_{\text{infl}}) \approx 0, \quad V''(\varphi_{\text{infl}}) = 0,$$
that is the point where the inflaton slows down and generates large amplification in power spectrum. Both points have to lie into a specific interval before the potential becomes asymptotically flat.

CMB and inflection point

- The main feature of the cosmological attractor mechanism is that for a arbitrary potential function chosen, it is compatible with the CMB constraints due to the flat potential for $\varphi \gg 1$. The flattening of the potential for the boost $\phi \to \mathtt{tanh}\, \varphi/\sqrt{6}$ happens close to the boundary $\varphi = 6$ of the moduli space where the SO(1,1) approximate symmetry of superpotential is not broken yet. In the ϕ space this means when ϕ approaches unity.
- This very strong feature of attractors may make someone think that it is impossible to construct a second small plateau into the φ interval $\mathcal{O}(6)$ around zero, that will serve for PBHs production.
- A family of functions which share the IP characteristics is possible to make the attractors a candidate to generate primordial black holes.

Specific inflationary models

Model I: Polynomial Superpotential

- Generaly, an n-th degree polynomial has $n - 2$ inflection points. Hence, the simplest polynomial with one inflection point is the cubic

$$f(\phi) = a_0 + a_1\phi + a_2\phi^2 + a_3\phi^3,$$

- the inflationary trajectory is determined by the canonically normalized inflaton potential

$$V = |f(\varphi)|^2 = \left\{a_0 + a_1 \tanh\left(\frac{\varphi}{\sqrt{6}}\right) + a_2 \tanh^2\left(\frac{\varphi}{\sqrt{6}}\right) + a_3 \tanh^3\left(\frac{\varphi}{\sqrt{6}}\right)\right\}^2$$

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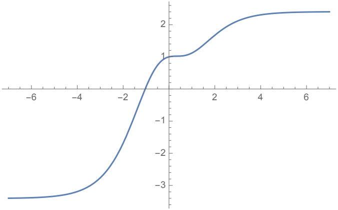
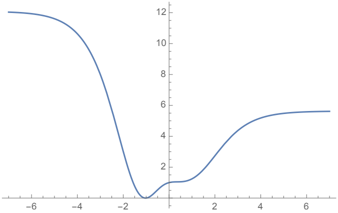




Specific inflationary models

Model II: Modulated Chaotic Inflation Potentials

- Beside the obvious polynomial functions which possess this behavior, sinusoidal functions appear periodical inflection points and roots.
- We study a class of models with potentials inspired from the natural modulated potentials[2] and the axion monodromy models[3].

$$f(\phi) = \lambda\left(\phi + \sqrt{3}A\sin\frac{\phi}{f_\phi}\right)$$

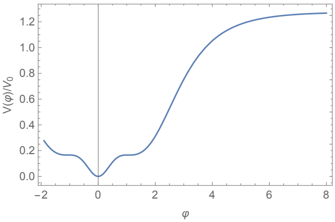
- The potential for the canonical normalized field φ turns out to be

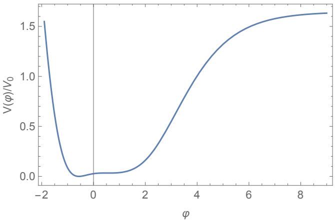
$$V(\varphi) = V_0\left[\tanh(\varphi/\sqrt{6}) + A\sin\left(\sqrt{3}\tanh(\varphi/\sqrt{6})/f_\phi\right)\right]^2, \quad V_0 = 3\lambda^2.$$

[2] Kallosh & Linde

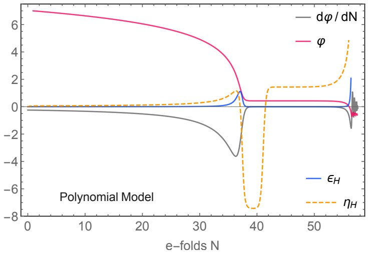
[3] Silverstein etal, Easther etal, Flauger etal, Kobayashi etal

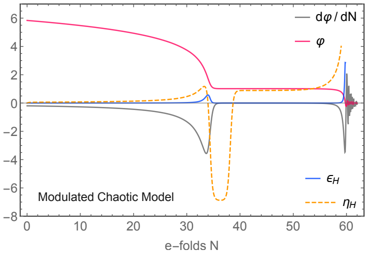
Inflationary potentials with inflection point from superconformal attractors



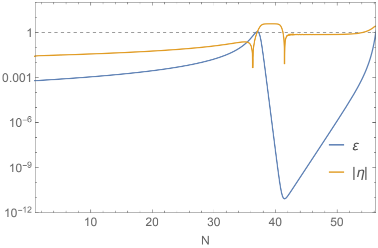


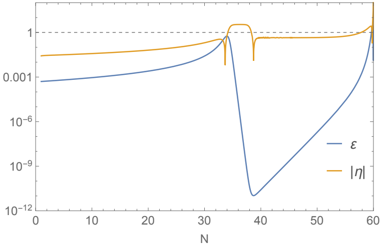
The evolution of the inflaton field





The Hubble flow parameters





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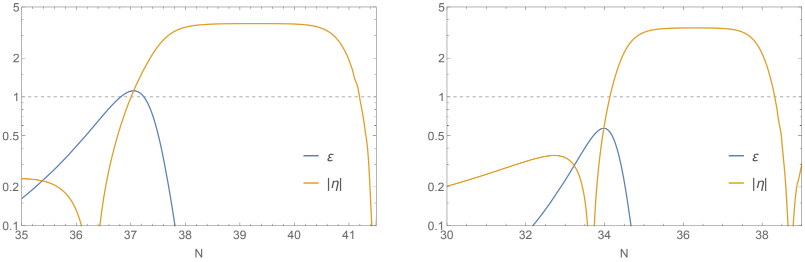

The curvature power spectrum

- Expanding the inflaton-gravity action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

  to second order in $\mathcal{R}$ one obtains

$$S_{(2)} = \frac{1}{2} \int d^4x \sqrt{-g} a^3 \frac{\dot{\phi}^2}{H^2} \left[\dot{\mathcal{R}}^2 - \frac{(\partial_i \mathcal{R})^2}{a^2} \right] .$$

- After the variable redefinition $v = z\mathcal{R}$ where $z^2 = a^2 \dot{\phi}^2 / H^2 = 2a^2\epsilon$ and switching to conformal time τ, $d\tau = dt/a$, the action is recast into

$$S_{(2)} = \frac{1}{2} \int d\tau d^3x \left[(v')^2 - (\partial_i v)^2 + \frac{z''}{z} v^2 \right] . \qquad (2)$$

The curvature power spectrum

- The evolution of the Fourier modes v_k of $v(x)$ are described by the Mukhanov-Sasaki equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

- The z''/z can be analyzed in terms of the Hubble flow slow-roll parameters

$$\frac{z''}{z} = (aH)^2 \left[2 - \epsilon + \frac{3}{2}\eta - \frac{1}{2}\epsilon\eta + \frac{1}{4}\eta^2 + \frac{1}{2}\eta\kappa\right]$$

- The power spectrum of $\mathcal{R}$ is obtained once the solution v_k of the Mukhanov-Sasaki equation is known

$$P_{\mathcal{R}} = \frac{k^3}{2\pi^2} \frac{|v_k|^2}{z^2}\bigg|_{k=aH}$$

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- In de Sitter space it is simplified, since it is $z''/z = 2/\tau^2$, and one can solve it explicitly. In such a case the power spectrum for $\mathcal{R}$ in scales larger than the Hubble radius is found to be

$$P_{\mathcal{R}} = \frac{H^2}{8\pi^2\epsilon}\bigg|_{k=aH} \tag{3}$$

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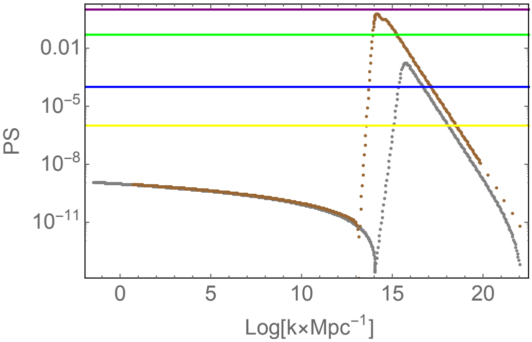
- The analytic result is a very good approximation as long as the Hubble flow slow-roll parameters are much less than one during the inflationary phase. If this is not the case the numeric solution of the exact Mukhanov-Sasaki equation has to be pursued

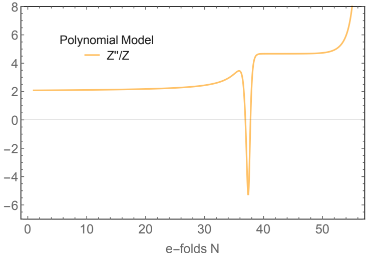
The numerical estimation of the curvature power spectrum

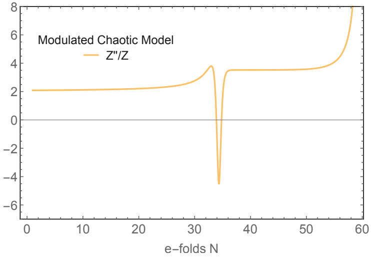
- We calculate the evolution of the coupled inflaton-metric system in the background level.
- We solve numerically the Mukhanov-Sasaki equation and find the evolution of the real and the imaginary part of the solution v_k. A numerical iteration is carried out for more than 2500 modes k that range from $k_* = 0.05\ \mathrm{Mpc}^{-1}$ to $k = k_*(H_{\mathrm{end}}/H_*)\, a^N$. We apply the Bunch-Davies initial conditions for each mode five e-folds before it exits the Hubble horizon.
- We calculate the power spectrum of each $\mathcal{R}_k$ once the scale exits the Hubble horizon and its value freezes out. In total more than 2500 points produce the $\mathcal{P}_{\mathcal{R}}(k)$ that allows the estimation of the variance $\sigma^2(M)$ of the density perturbations and in turn the fraction of the mass that collapses to form PBHs.

The numerical estimation of the curvature power spectrum

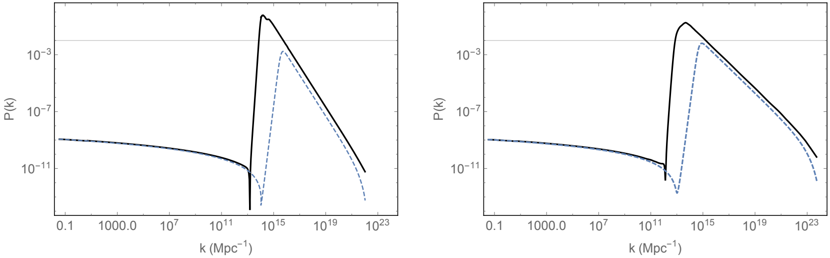


What sources the PS enhancement?

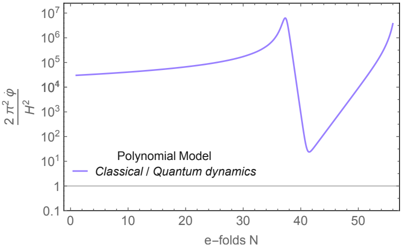


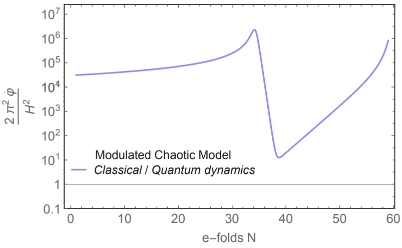


The power spectrum



The diffusion issue





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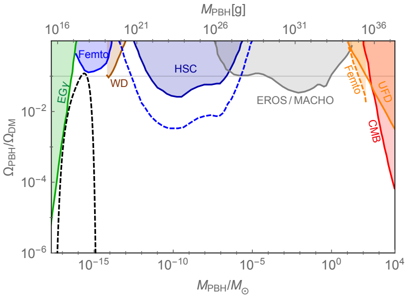

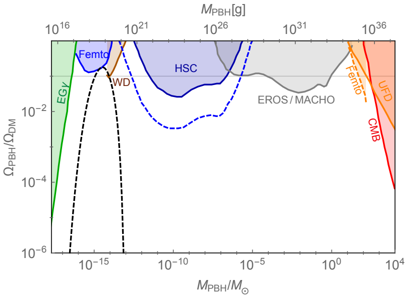

Results -Signatures

	n_s	P_R^{peak}	δ_c	$M_{PBH}^{peak}/M_\odot$	Ω_{PBH}/Ω_{DM}
Model I	0.943	0.56	0.48	2.5×10^{-16}	0.22
Model II	0.943	0.18	0.30	3.3×10^{-15}	0.42

Summary-Outlook

- We studied the PBH production from inflationary superconformal attractors, where the inflationary fluctuations provide the seeds for the CMB anisotropies an the PBH formation
- The increase in the power spectrum is achieved by the rapid change of the Hubble flow parameters ϵ_H, η_H and κ_H about the inflection point.
- The amplification of the power spectrum requires a notable tuning of the potential parameters.
- The superconformal attractors customized to generate PBH of mass $10^{-16} - 10^{-15}\, M_\odot$ predict a relatively small n_s value, and larger r and α_s values compared to the coventional inflationary attractor models. T
- Henceforth, these scenarios are possible to be tested by the next generation CMB probes that aim at pinning down the scalar tilt value with per mile accuracy. Large values for the n_s and microlensing results will rule out these sort of models.

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THANK YOU!