Hairy black holes in scalar tensor theories

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Christos Charmousis

Gravitational waves in modified gravity theories

1. Introduction : Motivating modified gravity
2. Scalar tensor: From BD to Horndenski...
3. Black holes and no hair
4. Constructing black hole solutions: Examples
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GR is a unique theory

- Theoretical consistency: In 4 dimensions, consider $\mathcal{L} = \mathcal{L}(\mathcal{M}, g, \nabla g, \nabla\nabla g)$. Then Lovelock's theorem in $D = 4$ states that GR with cosmological constant is the unique metric theory emerging from,

$$S_{(4)} = \int_{\mathcal{M}} d^4x \sqrt{-g^{(4)}} \, [R - 2\Lambda]$$

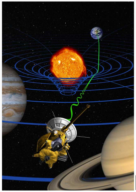
giving,

  - Equations of motion of $2^{\rm nd}$-order (Ostrogradski no-go theorem 1850!)
  - given by a symmetric two-tensor, $G_{\mu\nu} + \Lambda g_{\mu\nu}$
  - and admitting Bianchi identities.

Under these hypotheses GR is the unique massless-tensorial 4 dimensional theory of gravity!

Observational data

- Experimental consistency:
  -Excellent agreement with solar system tests and strong gravity tests on binary pulsars
  -Observational breakthrough GW170817: Non local, $40Mpc$ and strong gravity test from binary neutron stars. $c_T = 1 \pm 10^{-15}$

Time delay of light  Planetary tajectories 

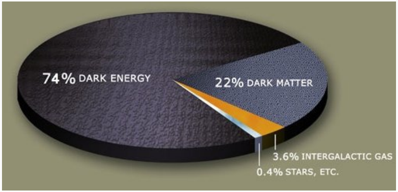
Neutron star binary

Q: What is the matter content of the Universe today?

Assuming homogeinity-isotropy and GR

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

cosmological and astrophysical observations dictate the matter content of the Universe today:



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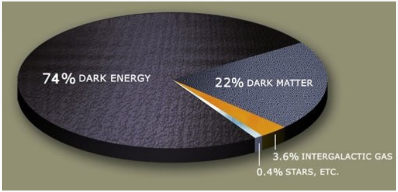

A: -Only a 4% of matter has been discovered in the laboratory. We hope to see more at LHC. But even then...

If we assume only ordinary sources of matter (DM included) there is disagreement between local, astrophysical and cosmological data.

Universe is accelerating → Enter the cosmological constant

Simplest way out: Assume a tiny cosmological constant $\rho_\Lambda = \frac{\Lambda_{obs}}{8\pi G} = (10^{-3} eV)^4$, ie modify Einstein's equation by,

$$G_{\mu\nu} + \Lambda_{obs} g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Cosmological constant introduces $\sqrt{\Lambda}$ and generates a cosmological horizon
- $\sqrt{\Lambda}$ is as tiny as the inverse size of the Universe today, $r_0 = H_0^{-1}$
- Note that $\frac{\text{Solar system scales}}{\text{Cosmological Scales}} \sim \frac{10 \text{ A.U.}}{H_0^{-1}} = 10^{-14}$
- Typical mass scale for neutrinos...
- Theoretically the cosmological constant should be huge.
- What if GR is modified at astrophysical or cosmological scales.

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Modified Gravity : General issues to deal with

- Since GR is unique we need to introduce new and genuine gravitational degrees of freedom!
- They generically must not lead to higher derivative equations of motion. Additional degrees of freedom can lead to ghosts (Ostrogradski theorem 1850 [Woodard 2006]). Since [Gleyzes et al] we know that higher derivative EOM do not always lead to ghosts. What is essential is the number of propagating dof.
- Matter does not directly couple to novel gravity degrees of freedom. Matter sees only the metric and evolves in metric geodesics. As such EEP is preserved and space-time can be put locally in an inertial frame.
- Novel degrees of freedom need to be screened from local gravity experiments. Need a well defined GR local limit.
- Exact solutions essential in modified gravity in order to understand strong gravity regimes and novel characteristics. Need to deal with no hair paradigm, absence of Birkhoff theorem etc.
- A modified gravity theory should tell us something about the cosmological constant problem and in particular how to screen an a priori enormous cosmological constant. Self tuning and self acceleration.

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Scalar-tensor theories

- are the simplest modification of gravity with one additional degree of freedom
- Admit a uniqueness theorem due to Horndeski 1973 and extended to DHOST theories [Langlois et.al] [Crisostomi et.al.]
- contain or are limits of other modified gravity theories.
- Include terms that can screen classically a big cosmological constant or give self accelerating solutions. Need a non trivial scalar field.
- Have non trivial hairy black hole solutions even around non trivial self accelerating vacua
- New: Theories are strongly constrained from gravity waves.

Galileons/Horndeski [Horndeski 1973]

What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x\sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$\begin{aligned}
L_2 &= G_2(\phi, X),\\
L_3 &= G_3(\phi, X)\Box\phi,\\
L_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],\\
L_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}$$

the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- In fact same action as covariant Galileons [Deffayet, Esposito-Farese, Vikman].

Galileons/Horndeski [Horndeski 1973]

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- Examples: $G_4 = 1 \longrightarrow R$.
  $G_4 = X \longrightarrow G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$.
  $G_3 = X \longrightarrow$ "DGP" term, $(\nabla\phi)^2\Box\phi$
  $G_5 = lnX \longrightarrow$ gives GB term, $\hat{G} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2$
  Action is unique modulo integration by parts.

Galileons/Horndeski [Horndeski 1973]

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- Horndeski theory admits self accelerating vacua with a non trivial scalar field in de Sitter spacetime. A subset of Horndeski theory self tunes the cosmological constant.

Galileons/Horndeski [Horndeski 1973]

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the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- Horndeski theory includes Shift symmetric theories where G_i's depend only on X and $\phi \to \phi + c$.
  Associated with the symmetry there is a Noether current, J^μ which is conserved $\nabla_\mu J^\mu = 0$.
  Presence of this symmetry permits a very general no hair argument

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Black holes have no hair [recent review Herdeiro and Radu 2015]

During gravitational collapse...
Black holes eat or expel surrounding matter
their stationary phase is characterized by a limited number of charges
and no details
black holes are bald...

No hair arguments/theorems dictate under some reasonable hypotheses that adding degrees of freedom lead to singular solutions...

For example in vanilla scalar-tensor theories black hole solutions are GR black holes with constant scalar.

Warning : beyond GR Birkhoff's theorem is not valid.
Spherical symmetry thus does not guarantee staticity.
Scalar tensor black holes radiate monopole gravity waves.
There is no reason for metric and scalar not to radiate for spherical symmetry

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No hair [Hui, Nicolis] [Sotiriou, Zhou] [Babichev, CC, Lehébel]

Static no hair theorem

Consider shift symmetric Horndeski theory with G_2, G_3, G_4, G_5 arbitrary functions of X. We have a Noether current J^μ which is conserved, $\nabla_\mu J^\mu = 0$.

We now suppose that:

1. spacetime and scalar are spherically symmetric and static,
$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dK^2, \ \phi = \phi(r)$$
2. spacetime is asymptotically flat, $\phi' \to 0$ as $r \to \infty$ and the norm of the current J^2 is finite on the horizon,
3. there is a canonical kinetic term X in the action,
4. and the G_i functions are such that their X-derivatives contain only positive or zero powers of X.

Under these hypotheses, ϕ is constant and thus the only black hole solution is locally isometric to Schwarzschild.

Most interesting part of no go theorem: Breaking any of these hypotheses leads to black hole solutions!

Theorem can be extended for star solutions. [Lehébel et al.]

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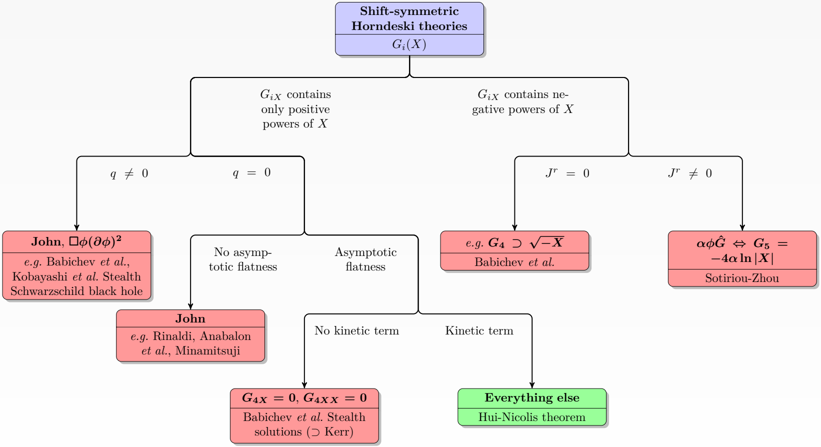
Hair versus no hair [figure: Lehébel]



Introducing time dependence, $q \neq 0$

Spherical symmetry certainly does not impose staticity (not like GR).

- Furthermore, for self accelerating or self tuning solutions one has a time dependence for the scalar in FRW coordinates
- In spherical symmetry this leads to a time and radially depending scalar already for flat spacetime.
- So let us allow time dependence for the scalar while keeping for a static and spherically symmetric spacetime.

But is this consistent with respect to the field equations:

$$\mathcal{E}_\phi = 0, \qquad \mathcal{E}_{\mu\nu} = 0$$

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The question of time dependence, $qt + \psi(r)$

Consistency theorem [Babichev, CC, Hassaine]

Consider :
-an arbitrary shift symmetric Horndeski theory $\phi \to c + \phi$
-a scalar-metric ansatz $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dK^2$, $\phi = qt + \psi(r)$ with $q \neq 0$.
The unique solution to the scalar field equation $\mathcal{E}_\phi = 0$ and the "matter flow" metric equation $\mathcal{E}_{tr} = 0$ is given by $J^r = 0$.

- We are killing two birds with one stone.
- The current now reads, $J^\mu J_\mu = -h(J^t)^2 + (J^r)^2/f$ and is regular. Time dependence renders no hair theorem irrelevant.
- If $J^r = 0$ allows $\phi' \neq 0$ solutions then we may construct hairy solutions.
- This is where the higher order nature of Horndeski theory is essential!!

General solution

Consider, $L = R - \eta(\partial\phi)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\Lambda$ For static and spherically symmetric spacetime.

The general solution of theory L for static and spherically symmetric metric and $\phi = \phi(t, r)$ is given as a solution to the following third order algebraic equation with respect to $\sqrt{k(r)}$:

$$(q\beta)^2 \left(1 + \frac{r^2}{2\beta}\right)^2 - \left(2 + (1 - 2\beta\Lambda)\frac{r^2}{2\beta}\right) k(r) + C_0 k^{3/2}(r) = 0$$

All metric and scalar functions given with respect to k and $\phi = qt + \psi(r)$.

For general shift symmetric G_2, G_4 the result can be extended, [Kobayashi, Tanahashi]

Let us now give some specific examples for the different cases...

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2 Scalar tensor: From BD to Horndenski...

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Scalar with constant velocity $q \neq 0$

Consider the action,

$$S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda - \eta (\partial\phi)^2 + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right] \ldots,$$

- Scalar field equation and conservation of current,

$$\nabla_\mu J^\mu = 0, \quad J^\mu = (\eta g^{\mu\nu} - \beta G^{\mu\nu}) \partial_\nu \phi.$$

- Take $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$, and $\phi = \phi(t, r)$ then
- $\phi = \psi + qt$ while $\mathcal{E}_{tr} = -\frac{q^2 J^r}{f} \longrightarrow J^r = 0$ solves both equations...
- $\beta G^{rr} - \eta g^{rr} = 0$ ie. $f = \frac{(\beta + \eta r^2) h}{\beta (rh)'}$ or $\phi' = 0$

For a higher order theory $J^r = 0$ does not necessarily imply $\phi = const.$

$J^r = 0$ means that we kill primary hair since, $\nabla_\mu J^\mu = 0 \rightarrow \sqrt{-g}(\beta G^{rr} - \eta g^{rr}) \partial_r \phi = c$

- We now solve for the remaining field eqs...

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Solving the remaining EoM

- From (rr)-component get ψ'

$$\psi' = \pm \frac{\sqrt{r}}{h(\beta + \eta r^2)} \left(q^2 \beta(\beta + \eta r^2) h' - \frac{\zeta\eta + \beta\Lambda}{2} (h^2 r^2)' \right)^{1/2}.$$

- and finally (tt)-component gives $h(r)$ via,

$$h(r) = -\frac{\mu}{r} + \frac{1}{r} \int \frac{k(r)}{\beta + \eta r^2} dr,$$

with

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Any solution to the algebraic eq for $k = k(r)$ gives full solution to the system!

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Lets take $\eta = \Lambda = 0$

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Asymptotically flat limit : $\Lambda = 0$, $\eta = 0$

- Consider $S = \int d^4x\sqrt{-g}\,[\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]$
- Algebraic equation to solve: $q^2\beta^3 - 2\zeta\beta k + C_0 k^{3/2} = 0 \to k = constant$!
- $f(r) = h(r) = 1 - \mu/r$
- $\phi_\pm = qt \pm q\mu\left[2\sqrt{\frac{r}{\mu}} + \log\frac{\sqrt{r}-\sqrt{\mu}}{\sqrt{r}+\sqrt{\mu}}\right] + \phi_0 \ldots$
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  Regular chart for horizon, EF coordinates
- $\phi_+ = q\left[v - r + 2\sqrt{\mu r} - 2\mu\log\left(\sqrt{\frac{r}{\mu}} + 1\right)\right] + \text{const}$
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Self tuning de Sitter black hole

- $$S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda - \eta (\partial\phi)^2 + \beta G^{\mu\nu} \partial_\mu\phi \partial_\nu\phi\right]$$

... $q^2\beta(\beta + \eta r^2)^2 - \left(2\zeta\beta + (\zeta\eta - \beta\Lambda)\, r^2\right) k + C_0 k^{3/2} = 0$

- $f = h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta} r^2$ de Sitter Schwarzschild!
- $\psi' = \pm\frac{q}{h}\sqrt{1-h}$ and $\phi(t,r) = q\,t + \psi(r)$
- The effective cosmological constant is not the vacuum cosmological constant. In fact,
- Self tuning relation : $q^2\eta = \Lambda - \Lambda_{eff} > 0$
- Hence for any $\Lambda > \Lambda_{eff}$ fixes q, integration constant.
- where $\Lambda_{eff} = -\frac{\eta}{\beta}$ is fixed by effective theory.
- Solution hides vacuum cosmological constant leaving a smaller effective cosmological constant [Gubitosi, Linder]

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Galileons/Horndeski [Horndeski 1973]

What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x \sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$\begin{aligned}
L_2 &= G_2(\phi, X), \\
L_3 &= G_3(\phi, X)\Box\phi, \\
L_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
L_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}$$

the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

-

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$$L_2 = G_2(\phi, X),$$
$$L_3 = G_3(\phi, X)\Box\phi,$$
$$L_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$
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- Examples: $G_4 = 1 \longrightarrow R$.
  $G_4 = X \longrightarrow G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$.
  $G_3 = X \longrightarrow$ "DGP" term, $(\nabla\phi)^2\Box\phi$
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  Action is unique modulo integration by parts.

Going beyond Horndeski [Gleyzes et.al], [Zumalacarregui et.al],[Deffayet et.al], [Langlois et.al], [Crisostomi et.al]

What is the most general scalar-tensor theory with three propagating degrees of freedom?

It is beyond Horndeski but not quite DHOST yet...

$$S_H = \int d^4x\sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)\ ,$$

where

$$L_2 = G_2(\phi, X), \qquad L_3 = G_3(\phi, X)\Box\phi,$$

$$L_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\,\epsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'},$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

$$+ F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}$$

where $XG_{5,X}F_4 = 3F_5\left[G_4 - 2XG_{4,X} - (X/2)G_{5,\phi}\right]$. Beyond Horndeski acquires one extra function. BH has similar SA and ST solutions.

Conformal and disformal relations [Bellido, Zumalacarregui]

How are theories mapped under conformal and disformal transformations?

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X) g_{\mu\nu} + D(\phi, X) \nabla_\mu \phi \nabla_\nu \phi$$

- Horndeski theory has G_2, G_3, G_4, G_5 free functions.
- For $C(\phi)$ and $D(\phi)$ we remain within Horndeski.
- However if we take a disformal $D(X)$ we jump to
- Beyond Horndeski (one more free function)
- Take a conformal $C(X)$ and jump to
- DHOST Type I (one more free function) [Langlois, Noui], [Crisostomi, Koyama]

In other words DHOST type I are all related to some Horndeski theory. Remaining DHOST theories are pathological [Langlois, Noui, Vernizzi]

Most general acceptable scalar tensor theories are related to Horndeski theory via a disformal and conformal transformation.

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GW170817 constraints on scalar tensor theories [Creminelli, Vernizzi], [Ezquiaga, Zumalacarregui]

- The combined observation of a gravity wave signal from a binary neutron star and its GRB counterpart constraints $c_T = 1$ to a 10^{-15} accuracy.
- For dark energy the scalar field (ST or SA) is non trivial at such distance scales (40Mpc) and generically mixes with the tensor metric perturbations modifying the light cone for gravity waves.
- For Horndeski the surviving theory has free $G_2(\phi, X)$, $G_3(\phi, X)$, $G_4(\phi)$ and $G_5 = 0$.
- For beyond Horndeski we have $G_5 = 0, F_5 = 0, 2G_{4,X} + XF_4 = 0$ and theory,

$$L_{c_T=1} = G_2(\phi, X) + G_3(\phi, X)\Box\phi + B_4(\phi, X)\,{}^{(4)}R - \frac{4}{X}B_{4,X}(\phi, X)(\phi^\mu\phi^\nu\phi_{\mu\nu}\Box\phi - \phi^\mu\phi_{\mu\nu}\phi_\lambda\phi^{\lambda\nu}) \,,$$

- For DHOST we just make a conformal transformation of the above, $G_2(\phi, X) G_3(\phi, X)$, $B_4(\phi, X)$, $C(\phi, X)$

Galileons/Horndeski [Horndeski 1973]

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-

Galileons/Horndeski [Horndeski 1973]

$$
\begin{aligned}
L_2 &= G_2(\phi, X), \\
L_3 &= G_3(\phi, X)\Box\phi, \\
L_4 &= G_4(\phi)R \\
L_5 &= 0
\end{aligned}
$$

-
- $G_4 = 1 \longrightarrow R$.
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Physical and disformed frames

Most general scalar tensor theory with $c_T = 1$ minimally coupled to matter parametrized by G_2, G_3, B_4, C

$$L_{c_T=1} = G_2 + G_3\Box\phi + B_4 C\,^{(4)}R - \frac{4B_{4,X}C}{X}\phi^\mu\phi^\nu\phi_{\mu\nu}\Box\phi$$
$$+\left(\frac{4B_{4,X}C}{X} + \frac{6B_4 C_{,X}{}^2}{C} + 8C_{,X}B_{4,X}\right)\phi^\mu\phi_{\mu\nu}\phi_\lambda\phi^{\lambda\nu}$$
$$+\frac{8C_{,X}B_{4,X}}{X}(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2\,.$$

- Horndeski is related via a transformation
$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\nabla_\mu\phi\nabla_\nu\phi$$
to the $L_{c_T=1}$ for given C and D.
- One can start with a $c_T \neq 1$ Horndeski theory and map it to a DHOST $c_T = 1$ theory for a specific function D.
- The former is what we could have called the Einstein → Horndeski frame respective to the latter, the Jordan frame...
- except that the metric is disformed in the procedure...

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The physical frame and the disformed solution [Babichev, CC, GEFarèse, Lehébel]

- The theory

$$S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda - \eta \left(\partial\phi\right)^2 + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right],$$

  is excluded or it is not in the physical frame.
- Solution: $f = h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta} r^2$, $\phi = qt \pm \frac{q}{h}\sqrt{1-h}$ with $\Lambda_{\text{eff}} = -\zeta\eta/\beta$.
- The physical frame is :

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\beta}{\zeta + \frac{\beta}{2}\varphi_\lambda^2} \varphi_\mu \varphi_\nu .$$

- Indeed the $\tilde{g}_{\mu\nu}$ frame is a beyond Horndeski theory with $c_T = 1$ for a cosmological background.
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- we have exactly $c_{grav} = 1$ for a highly curved background!

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Conclusions

- Modifying GR is a difficult task but with countable possibilities. Even more so after the GW experiments. :)))
- Scalar tensor theories are parametrized by 4 free functions which we hope will be further constrained.
- Numerous spherically symmetric solutions known. Black holes, neutron stars. One has to adjust them to acceptable theories. One has to study GW-compatible theories independently.
- Self tuning vacua and black holes can be found with $c_{grav} = 1$.
- Exact solutions are important to understand the MG theory and find novel effects. Are there solutions free of singularities? What of rotating hairy black holes and Neutron stars?

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