## The Weak Equivalence Principle in the strong gravity regime

Andrew Coates

March 25, 2018

Eberhard Karls Univerity of Tübingen Talk based on work carried out at the University of Nottingham

## Outline

## Outline

Topics:

• Equivalence Principles

- Equivalence Principles
- Brans-Dicke and the solar system

- Equivalence Principles
- Brans-Dicke and the solar system
- Spontaneous scalarization

- Equivalence Principles
- Brans-Dicke and the solar system
- Spontaneous scalarization
- Solar system WEP tests

- Equivalence Principles
- Brans-Dicke and the solar system
- Spontaneous scalarization
- Solar system WEP tests
- The gravitational Higgs mechanism: a toy model

- Equivalence Principles
- Brans-Dicke and the solar system
- Spontaneous scalarization
- Solar system WEP tests
- The gravitational Higgs mechanism: a toy model
- Possible implications

- Equivalence Principles
- Brans-Dicke and the solar system
- Spontaneous scalarization
- Solar system WEP tests
- The gravitational Higgs mechanism: a toy model
- Possible implications
- Future work

Definitions based on<sup>1</sup>

- Weak Equivalence Principle
  - Test particles have universal free-fall

- Weak Equivalence Principle
  - Test particles have universal free-fall
- Einstein Equivalence Principle
  - ► WEP + Local Lorentz Invariance

- Weak Equivalence Principle
  - Test particles have universal free-fall
- Einstein Equivalence Principle
  - ► WEP + Local Lorentz Invariance
- Strong Equivalence Principle
  - EEP but test particles  $\rightarrow$  self-gravitating systems<sup>\*</sup>

- Weak Equivalence Principle
  - Test particles have universal free-fall
- Einstein Equivalence Principle
  - ► WEP + Local Lorentz Invariance
- Strong Equivalence Principle
  - EEP but test particles  $\rightarrow$  self-gravitating systems<sup>\*</sup>
  - can also be formulated in terms of existence of "isolated systems"

<sup>&</sup>lt;sup>1</sup>Will 2014.

Jordan frame:

$$\mathcal{S} = rac{1}{16\pi}\int\mathrm{d}^4x\sqrt{- ilde{g}}\left[\Phi ilde{R} - rac{\omega_0}{\Phi}\partial_\mu\Phi\partial^\mu\Phi
ight] + \mathcal{S}_m[ ilde{\mathbf{g}},\psi_m].$$

 $<sup>^2 {\</sup>rm See}$  Jordan 1959; Brans et al. 1961 and more generally e.g. Fujii et al. 2007; Faraoni 2004

Jordan frame:

$$S = rac{1}{16\pi}\int \mathrm{d}^4x \sqrt{- ilde{g}}\left[\Phi ilde{R} - rac{\omega_0}{\Phi}\partial_\mu \Phi\partial^\mu \Phi
ight] + S_m[ ilde{\mathbf{g}},\psi_m].$$

Einstein frame:

$$S = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ R - 2\partial_\mu \phi \partial^\mu \phi \right] + S_m [A^2(\phi) \mathbf{g}, \psi_m]$$

 $<sup>^2 {\</sup>rm See}$  Jordan 1959; Brans et al. 1961 and more generally e.g. Fujii et al. 2007; Faraoni 2004

Jordan frame:

$$S = rac{1}{16\pi}\int \mathrm{d}^4x \sqrt{- ilde{g}}\left[\Phi ilde{R} - rac{\omega_0}{\Phi}\partial_\mu \Phi\partial^\mu \Phi
ight] + S_m[ ilde{\mathbf{g}},\psi_m].$$

Einstein frame:

$$S = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ R - 2\partial_\mu \phi \partial^\mu \phi \right] + S_m [A^2(\phi) \mathbf{g}, \psi_m]$$

Where:

$$ilde{g}=A^2(\phi)\mathbf{g},\quad A^2(\phi)=rac{1}{G\Phi(\phi)},\quad \mathrm{d}\phi=rac{\sqrt{3+2\omega_0}}{\Phi}\mathrm{d}\Phi.$$

 $<sup>^2 {\</sup>rm See}$  Jordan 1959; Brans et al. 1961 and more generally e.g. Fujii et al. 2007; Faraoni 2004

Two parameters at first PPN order:

- $\gamma = (1 + \omega_0) / (2 + \omega_0)$
- $\phi_{\infty}$  *c.f.* isolated systems definition of SEP

Only  $\gamma$  from pure solar system. Current constraints effectively render BD uninteresting  $^{3}.$ 

<sup>&</sup>lt;sup>3</sup>Bertotti et al. 2003.

For our purposes need Einstein frame:

$$S = \frac{1}{16\pi G} \int \mathrm{d}^4 x \sqrt{-g} \left[ R - 2\partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + S_m [A^2(\phi) \mathbf{g}, \psi_m].$$

<sup>&</sup>lt;sup>4</sup>Again see Fujii et al. 2007; Faraoni 2004

For our purposes need Einstein frame:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ R - 2\partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + S_m [A^2(\phi) \mathbf{g}, \psi_m].$$

Addition of potential and  $A^2(\phi)$  now much less restricted.

<sup>&</sup>lt;sup>4</sup>Again see Fujii et al. 2007; Faraoni 2004

For our purposes need Einstein frame:

$$S = rac{1}{16\pi G}\int \mathrm{d}^4x \sqrt{-g}\left[R - 2\partial_\mu \phi \partial^\mu \phi - V(\phi)\right] + S_m[A^2(\phi)\mathbf{g},\psi_m].$$

Addition of potential and  $A^2(\phi)$  now much less restricted.

Useful to define:

$$\alpha(\phi) = \frac{\mathrm{d}}{\mathrm{d}\phi} \log \phi$$

<sup>&</sup>lt;sup>4</sup>Again see Fujii et al. 2007; Faraoni 2004

Simple, and relevant, example of a "screening mechanism" <sup>5</sup>:

<sup>&</sup>lt;sup>5</sup>Damour et al. 1993; Damour et al. 1996.

Simple, and relevant, example of a "screening mechanism" 5:

Linear scalar field equation around GR solution with constant scalar  $\phi_0$ :

$$\Box \varphi = \left( m^2 - 4\pi GT\beta \right) \varphi$$

where

$$\phi = \phi_0 + \varphi + \cdots, \quad m^2 = \frac{1}{4} V''(\phi_0), \quad \beta = \alpha'(\phi_0).$$

<sup>&</sup>lt;sup>5</sup>Damour et al. 1993; Damour et al. 1996.

Simple, and relevant, example of a "screening mechanism" 5:

Linear scalar field equation around GR solution with constant scalar  $\phi_0$ :

$$\Box \varphi = \left( m^2 - 4\pi GT\beta \right) \varphi$$

where

$$\phi = \phi_0 + \varphi + \cdots, \quad m^2 = \frac{1}{4} V''(\phi_0), \quad \beta = \alpha'(\phi_0).$$

Heuristically:

$$\omega^2 = k^2 - \mu^2$$

<sup>&</sup>lt;sup>5</sup>Damour et al. 1993; Damour et al. 1996.

Simple, and relevant, example of a "screening mechanism"<sup>5</sup>:

Linear scalar field equation around GR solution with constant scalar  $\phi_0$ :

$$\Box \varphi = \left( m^2 - 4\pi GT\beta \right) \varphi$$

where

$$\phi = \phi_0 + \varphi + \cdots, \quad m^2 = \frac{1}{4} V''(\phi_0), \quad \beta = \alpha'(\phi_0).$$

Heuristically:

$$\omega^2 = k^2 - \mu^2$$

Upshot: screened SEP violations which show up in strong gravity (just one example).

<sup>&</sup>lt;sup>5</sup>Damour et al. 1993; Damour et al. 1996.

 $<sup>^{6}\</sup>mbox{This}$  is based on Coates et al. 2017; Franchini et al. 2018 and my thesis (which is not yet available)

1. Overlooked possibilities (c.f. spontaneous scalarization)

 $<sup>^{6}\</sup>mbox{This}$  is based on Coates et al. 2017; Franchini et al. 2018 and my thesis (which is not yet available)

- 1. Overlooked possibilities (c.f. spontaneous scalarization)
- 2. Theoretical considerations

 $<sup>^{6}\</sup>mbox{This}$  is based on Coates et al. 2017; Franchini et al. 2018 and my thesis (which is not yet available)

- 1. Overlooked possibilities (c.f. spontaneous scalarization)
- 2. Theoretical considerations
- 3. Additional ways to search for extra degrees of freedom

 $<sup>^{6}\</sup>mbox{This}$  is based on Coates et al. 2017; Franchini et al. 2018 and my thesis (which is not yet available)

Properties:

Properties:

• A screening mechanism

Properties:

- A screening mechanism
- Tractablilty

Properties:

- A screening mechanism
- Tractablilty
- Some WEP violation

## The gravitational Higgs mechanism: the action

Einstein frame (because there is no true Jordan frame):

Einstein frame (because there is no true Jordan frame):

$$\begin{split} &\frac{1}{16\pi G}\int \mathrm{d}^4 x \sqrt{-g}\left[R-2g^{\mu\nu}\overline{\mathcal{D}_{\mu}\phi}\mathcal{D}_{\nu}\phi-V(|\phi|)\right] \\ &-\frac{1}{4}\int \mathrm{d}^4 x \sqrt{-g}\left[F_{\mu\nu}F^{\mu\nu}\right]+S_m[\mathcal{A}^2(|\phi|)\mathbf{g},\psi_m], \end{split}$$

where:

$$\mathcal{D}_{\mu}\phi=\partial_{\mu}\phi-iqA_{\mu}\phi,$$

 $A_{\mu}$  is a U(1) gauge field and  $\phi$  is U(1) charged.
Einstein frame (because there is no true Jordan frame):

$$\frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ R - 2g^{\mu\nu} \overline{\mathcal{D}_{\mu}\phi} \mathcal{D}_{\nu}\phi - V(|\phi|) \right] \\ -\frac{1}{4} \int d^4 x \sqrt{-g} \left[ F_{\mu\nu} F^{\mu\nu} \right] + S_m [\mathcal{A}^2(|\phi|)\mathbf{g}, \psi_m],$$

where:

$$\mathcal{D}_{\mu}\phi=\partial_{\mu}\phi-iqA_{\mu}\phi,$$

 $A_{\mu}$  is a U(1) gauge field and  $\phi$  is U(1) charged.

In SI the U(1) field has an effective mass:

$$m_{\gamma}^2(|\phi|) = rac{\mu_0 q^2 c^2}{4\pi G} \overline{\phi} \phi.$$

Einstein frame (because there is no true Jordan frame):

$$\frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ R - 2g^{\mu\nu} \overline{\mathcal{D}_{\mu}\phi} \mathcal{D}_{\nu}\phi - V(|\phi|) \right]$$
$$-\frac{1}{4} \int d^4 x \sqrt{-g} \left[ F_{\mu\nu} F^{\mu\nu} \right] + S_m [\mathcal{A}^2(|\phi|)\mathbf{g}, \psi_m],$$

where:

$$\mathcal{D}_{\mu}\phi=\partial_{\mu}\phi-iqA_{\mu}\phi,$$

 $A_{\mu}$  is a U(1) gauge field and  $\phi$  is U(1) charged.

In SI the U(1) field has an effective mass:

$$m_{\gamma}^2(|\phi|) = rac{\mu_0 q^2 c^2}{4\pi G} \overline{\phi} \phi.$$

If the spontaneous scalarization mechanism is undamaged, this is the U(1) Higgs mechanism<sup>\*</sup>.

$$abla^{\mu}F_{\mu
u}=m_{\gamma}^{2}(|\phi|)A_{
u}.$$

$$abla^{\mu}F_{\mu
u}=m_{\gamma}^{2}(|\phi|)A_{
u}.$$

This relies on that the 4-current contribution of the scalar field is

$$J_{\mu} \propto \overline{\phi} \partial_{\mu} \phi - \phi \partial_{\mu} \overline{\phi}.$$

$$abla^{\mu}F_{\mu
u}=m_{\gamma}^{2}(|\phi|)A_{
u}.$$

This relies on that the 4-current contribution of the scalar field is

$$J_{\mu} \propto \overline{\phi} \partial_{\mu} \phi - \phi \partial_{\mu} \overline{\phi}.$$

This can then be transformed away ( $\phi = |\phi| \exp i\theta \rightarrow |\phi| \exp i(\theta + q\lambda)$  and so  $\lambda = -\theta/q$  does this for us).

$$abla^{\mu}F_{\mu
u}=m_{\gamma}^{2}(|\phi|)A_{
u}.$$

This relies on that the 4-current contribution of the scalar field is

$$J_{\mu} \propto \overline{\phi} \partial_{\mu} \phi - \phi \partial_{\mu} \overline{\phi}.$$

This can then be transformed away ( $\phi = |\phi| \exp i\theta \rightarrow |\phi| \exp i(\theta + q\lambda)$  and so  $\lambda = -\theta/q$  does this for us).

In spherical symmetry and staticity the r (areal radius) component of the gauge fields equation reads:

$$m_{\gamma}^2 A_r = 0$$

(in the above guage) and so  $A_{\mu} = (A_t, 0, 0, 0)$ .

## The gravitational Higgs mechanism: spherical symmetry

Following the Bekenstein proof<sup>7</sup> of the no-hair theorem for the Einstein-Proca system we can also prove that in this situation the U(1) field must be trivial.

<sup>&</sup>lt;sup>7</sup>Bekenstein 1972.

## The gravitational Higgs mechanism: spherical symmetry

Following the Bekenstein proof<sup>7</sup> of the no-hair theorem for the Einstein-Proca system we can also prove that in this situation the U(1) field must be trivial.

Sketch: Contract the Proca-like equation with  $A^{\nu}$  and integrate over some spacetime volume,  $\mathcal{V}$ ,

$$\int_{\mathcal{V}} \mathrm{d}^4 x \sqrt{-g} \left[ A^{\nu} \nabla^{\mu} F_{\mu\nu} - m_{\gamma} A^{\nu} A_{\nu} \right] = 0.$$

<sup>&</sup>lt;sup>7</sup>Bekenstein 1972.

## The gravitational Higgs mechanism: spherical symmetry

Following the Bekenstein proof<sup>7</sup> of the no-hair theorem for the Einstein-Proca system we can also prove that in this situation the U(1) field must be trivial.

Sketch: Contract the Proca-like equation with  $A^{\nu}$  and integrate over some spacetime volume,  $\mathcal{V}$ ,

$$\int_{\mathcal{V}} \mathrm{d}^4 x \sqrt{-g} \left[ A^{\nu} \nabla^{\mu} F_{\mu\nu} - m_{\gamma} A^{\nu} A_{\nu} \right] = 0.$$

Integrate the first term by parts:

$$\int_{\mathcal{V}} \mathrm{d}^4 x \sqrt{-g} \left[ \frac{1}{2} F^{\mu\nu} F_{\mu\nu} + m_{\gamma} A^{\nu} A_{\nu} \right] = \int_{\partial \mathcal{V}} \mathrm{d}^3 \sigma n^{\nu} A^{\nu} \nabla^{\mu} F_{\mu\nu}.$$

<sup>&</sup>lt;sup>7</sup>Bekenstein 1972.

Following the Bekenstein proof<sup>7</sup> of the no-hair theorem for the Einstein-Proca system we can also prove that in this situation the U(1) field must be trivial.

Sketch: Contract the Proca-like equation with  $A^{\nu}$  and integrate over some spacetime volume,  $\mathcal{V}$ ,

$$\int_{\mathcal{V}} \mathrm{d}^4 x \sqrt{-g} \left[ A^{\nu} \nabla^{\mu} F_{\mu\nu} - m_{\gamma} A^{\nu} A_{\nu} \right] = 0.$$

Integrate the first term by parts:

$$\int_{\mathcal{V}} \mathrm{d}^4 x \sqrt{-g} \left[ \frac{1}{2} F^{\mu\nu} F_{\mu\nu} + m_{\gamma} A^{\nu} A_{\nu} \right] = \int_{\partial \mathcal{V}} \mathrm{d}^3 \sigma n^{\nu} A^{\nu} \nabla^{\mu} F_{\mu\nu}.$$

We're interested in stars so we can take a single boundary, a constant r surface, and take  $r \to \infty$ . Asymptotic flatness then kills the boundary integral.

<sup>&</sup>lt;sup>7</sup>Bekenstein 1972.

Following the Bekenstein proof<sup>7</sup> of the no-hair theorem for the Einstein-Proca system we can also prove that in this situation the U(1) field must be trivial.

Sketch: Contract the Proca-like equation with  $A^{\nu}$  and integrate over some spacetime volume, V,

$$\int_{\mathcal{V}} \mathrm{d}^4 x \sqrt{-g} \left[ A^{\nu} \nabla^{\mu} F_{\mu\nu} - m_{\gamma} A^{\nu} A_{\nu} \right] = 0.$$

Integrate the first term by parts:

$$\int_{\mathcal{V}} \mathrm{d}^4 x \sqrt{-g} \left[ \frac{1}{2} F^{\mu\nu} F_{\mu\nu} + m_{\gamma} A^{\nu} A_{\nu} \right] = \int_{\partial \mathcal{V}} \mathrm{d}^3 \sigma n^{\nu} A^{\nu} \nabla^{\mu} F_{\mu\nu}.$$

We're interested in stars so we can take a single boundary, a constant r surface, and take  $r \to \infty$ . Asymptotic flatness then kills the boundary integral.

Finally look at the integrand of the left-hand-side, it is sign definite and one gets  $A_t = 0$ .

<sup>&</sup>lt;sup>7</sup>Bekenstein 1972.

 $<sup>^{8}</sup>$  Note that, if we interpret the U(1) field as the photon, we would have to use different equations of state

It turns out that this mechanism is extremely effective. Using the expression for the effective mass from earlier:

$$m_{\gamma} pprox \left(rac{|q|}{e}
ight) |\phi| \left(0.1 M_{
m Pl}
ight).$$

 $<sup>^{8}</sup>$  Note that, if we interpret the U(1) field as the photon, we would have to use different equations of state

It turns out that this mechanism is extremely effective. Using the expression for the effective mass from earlier:

$$m_{\gamma} pprox \left(rac{|q|}{e}
ight) |\phi| \left(0.1 M_{
m Pl}
ight). \qquad (M_{
m Pl} pprox 20 \mu g)$$

 $<sup>^{8}</sup>$  Note that, if we interpret the U(1) field as the photon, we would have to use different equations of state

It turns out that this mechanism is extremely effective. Using the expression for the effective mass from earlier:

$$m_{\gamma} pprox \left(rac{|q|}{e}
ight) |\phi| \left(0.1 M_{
m Pl}
ight). \qquad (M_{
m Pl} pprox 20 \mu g)$$

For comparison, weak field tests of the photon mass give an upper bound  $\sim 10^{-42} M_{\rm Pl}$ 

 $<sup>^{8}</sup>$  Note that, if we interpret the U(1) field as the photon, we would have to use different equations of state

It turns out that this mechanism is extremely effective. Using the expression for the effective mass from earlier:

$$m_{\gamma} pprox \left(rac{|q|}{e}
ight) |\phi| \left(0.1 M_{
m Pl}
ight). \qquad (M_{
m Pl} pprox 20 \mu g)$$

For comparison, weak field tests of the photon mass give an upper bound  $\sim 10^{-42} M_{\rm Pl}$ , *i.e.* even for small amounts of scalarization there can be large changes in the matter sector.

 $<sup>^{8}</sup>$  Note that, if we interpret the U(1) field as the photon, we would have to use different equations of state

• Smoking guns.

 Smoking guns. Based on nathematical analogy between field dependent mass and parametric oscillators (ω<sup>2</sup>(t)).

- Smoking guns. Based on nathematical analogy between field dependent mass and parametric oscillators (ω<sup>2</sup>(t)).
  - Collapse

- Smoking guns. Based on nathematical analogy between field dependent mass and parametric oscillators (ω<sup>2</sup>(t)).
  - Collapse
  - Cosmology

- Smoking guns. Based on nathematical analogy between field dependent mass and parametric oscillators (ω<sup>2</sup>(t)).
  - Collapse
  - Cosmology
- Beyond the U(1) (toy) model

- Smoking guns. Based on nathematical analogy between field dependent mass and parametric oscillators (ω<sup>2</sup>(t)).
  - Collapse
  - Cosmology
- Beyond the U(1) (toy) model
  - More directly relevant examples

- Smoking guns. Based on nathematical analogy between field dependent mass and parametric oscillators (ω<sup>2</sup>(t)).
  - Collapse
  - Cosmology
- Beyond the U(1) (toy) model
  - More directly relevant examples
  - $\phi$  dependent equations of state

Compare the flatspace Klein-Gordon equation with a field dependent mass:

$$-\partial_t^2 \psi = \left(k^2 + m^2(|\phi|)\right)\psi,$$

Compare the flatspace Klein-Gordon equation with a field dependent mass:

$$-\partial_t^2 \psi = \left(k^2 + m^2(|\phi|)\right)\psi,$$

to the parametric oscillator:

$$-\frac{\mathrm{d}^2}{\mathrm{d}t^2}x = \omega^2(t)x.$$

Compare the flatspace Klein-Gordon equation with a field dependent mass:

$$-\partial_t^2 \psi = \left(k^2 + m^2(|\phi|)\right)\psi,$$

to the parametric oscillator:

$$-\frac{\mathrm{d}^2}{\mathrm{d}t^2}x = \omega^2(t)x.$$

So in dynamical situations can expect some excitation of  $\psi$  (c.f. reheating).

For:

• Testing robustness of screening mechanisms to WEP violations

For:

- Testing robustness of screening mechanisms to WEP violations
- Making astrophysically relevant predictions

For:

- Testing robustness of screening mechanisms to WEP violations
- Making astrophysically relevant predictions
- Maintaining some tractability

1. Overlooked possibilities (c.f. spontaneous scalarization)

- 1. Overlooked possibilities (c.f. spontaneous scalarization)
  - Demonstrated the possibility of large WEP violations in strong gravity

- 1. Overlooked possibilities (c.f. spontaneous scalarization)
  - Demonstrated the possibility of large WEP violations in strong gravity
  - Future work on  $\phi$  dependent equations of state

- 1. Overlooked possibilities (c.f. spontaneous scalarization)
  - Demonstrated the possibility of large WEP violations in strong gravity
  - $\blacktriangleright$  Future work on  $\phi$  dependent equations of state
- 2. Theoretical considerations
- 1. Overlooked possibilities (c.f. spontaneous scalarization)
  - Demonstrated the possibility of large WEP violations in strong gravity
  - Future work on  $\phi$  dependent equations of state
- 2. Theoretical considerations
  - Given the strength of the mechanism: possible strong requirements for QG

- 1. Overlooked possibilities (c.f. spontaneous scalarization)
  - Demonstrated the possibility of large WEP violations in strong gravity
  - Future work on  $\phi$  dependent equations of state
- 2. Theoretical considerations
  - Given the strength of the mechanism: possible strong requirements for QG
- 3. Additional ways to search for extra degrees of freedom

- 1. Overlooked possibilities (c.f. spontaneous scalarization)
  - Demonstrated the possibility of large WEP violations in strong gravity
  - Future work on  $\phi$  dependent equations of state
- 2. Theoretical considerations
  - Given the strength of the mechanism: possible strong requirements for QG
- 3. Additional ways to search for extra degrees of freedom
  - Even very mild scalarization can lead to large effects

- 1. Overlooked possibilities (c.f. spontaneous scalarization)
  - Demonstrated the possibility of large WEP violations in strong gravity
  - Future work on  $\phi$  dependent equations of state
- 2. Theoretical considerations
  - Given the strength of the mechanism: possible strong requirements for QG
- 3. Additional ways to search for extra degrees of freedom
  - ► Even very mild scalarization can lead to large effects
  - Possible smoking gun phenomena

Questions?