

# Homage to Giannis Bakas: The Formative Years

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National Technical University  
Athens

28th March 2018



**Xmas Theoretical Physics Workshop @ University of Athens 20-21/12/2016**

<https://sites.google.com/site/xmasathens2016/programv2>

**Crete String meeting 9-16/07/2017 (Kolymbari)**

<http://hep.physics.uoc.gr/mideast9/Bakas-memories-Vladikas.pdf>

## John (later Giannis...) Bakas: the formative years

- Arrived at Imperial College Theory Group in 1982-83 as a postgrad
  - The group was basking in the sun of fame:
    - Abdus Salam (Nobel in 1979) then in Trieste, visited the group about twice a year
    - Tom Kibble was head; Iain Halliday, Chris Isham, Hugh Jones, David Olive, Ray Rivers, are senior staff; Michael Duff was the new arrival, and Kelly Stelle was about to arrive....
  - Students in the period 1981-1985 who stayed in Physics (apologies to many I forget):
    - Frank Gomez (Brazil), Mark Hindmarsh (Sussex), Desmond Johnson (Herriot Watt), Tony Kakas (Cyprus), Martin Lavelle (Plymouth), David McMullan (Plymouth), Kostas Panagiotakopoulos (Salonica), Regina Ricotta (Brazil), Neil Turok (Perimeter)...
  - Postdocs: Ian Jack, Peter Orland, Chris Pope, Graham Shore...
  - The group had many activities:
    - perturbative QCD; lattice field theory (scalar fields and triviality)
    - group theoretic aspects of field theory
    - unification (SUSY, SUGRA, later strings, cosmology...) and **QUANTUM GRAVITY**
- (NB: strings arrived in 1984-85 through a M.Green seminar at King's)

## John (later Giannis...) Bakas: the formative years

- John knew from the start he wanted to work with Chris Isham on quantum gravity
- Isham accepted him as his PhD student after a very successful personal interview
- His first paper was with an older Isham student, Tony Kakas (1985):
  - I. Bakas and A.C.Kakas, “Quantization and Deformations: I. General Construction”
- ... followed by other papers with Isham’s students McMullan, Kakas and working on his own.
- He built a reputation of being a young, very serious and reliable mathematical physicist, always well focused on his objectives
- The group had a lot of illustrious guests who interacted with students. John had always been singled out by them for his focused dedication (Iliopoulos, Kuchar, Jackiw,...)

# John (later Giannis...) Bakas: the formative years

Frankie Gomez & Regina Ricotta



The co-authors of the Bakas-Kakas paper

## John (later Giannis...) Bakas: the formative years



Life in 3 Old Oak Rd.,  
Acton (West London)

relaxing...

calling Greece...

## John (later Giannis...) Bakas: the formative years



getting to grips with QED, QFT, symmetries, anomalies, and most importantly gravity beyond the classical level

it was clear from the start that the two flatmates, though living in complete harmony, had very different approaches to life and Physics, as shown in the next photo...

## John (later Giannis...) Bakas: the formative years



## John (later Giannis...) Bakas: the formative years



... a show of Greek temperament  
in a London back garden

but John's life had also many happy breaks, partying with the  
other students (not only of Imperial College)

## John (later Giannis...) Bakas: the formative years



theory group party at the  
theoretical physics library

Chris Pope

Mirjam McMullan

Annie  
Andrikopoulou

Des Johnson & Frankie Gomez

John Bakas

Nigel Gent

Susan Mokhtari



## John (later Giannis...) Bakas: the formative years



life in Putney (southwest London) :dinning with food that just passed its sell-by date from the delicatessen-shop owned by Tony Kakas' family!

Bakas gourmet proposal: Scottish salmon on toast, topped with feta cheese !!!!!

# John (later Giannis...) Bakas: the formative years

The Overlapping Divergence [i.e. we have logarithmic & quadratic diverg.]

This completes our discussion of the analytic continuation. We now wish to use the technique to examine another basic diagram in  $\lambda\phi^4/4!$  theory. The diagram in Fig. 6 is the lowest order, non-trivial diagram.



FIG. 6

contributing to the propagator. The symmetry factor for this diagram is  $\frac{1}{6}$  and the integral to be examined is

$$I = \frac{i\lambda^2}{6(2\pi)^8} \int \frac{dk dl}{[k^2 - m^2][l^2 - m^2][(p+l-k)^2 - m^2]} \quad (2.52)$$

Inspection of  $I$  shows that it is divergent. There are various regions of  $l$  and  $k$  in which the integral diverges, e.g.  $l$  small,  $k$  large,  $k$  small and  $l$  large, these give rise to logarithmic divergences. But if we take both  $k$  and  $l$  large the integral diverges quadratically. Furthermore this is what is called an overlapping divergence because one cannot ascribe the divergence to any subdiagram of Fig. 6. Also there does not exist a change of variables from  $l, k$  to  $l', k'$  for which the divergence only appears when one does only one of the loop integrations  $l'$  and  $k'$ . Overlapping divergences are much more difficult to deal with than the simpler kind (i.e. just associated with one loop of a diagram). They have the property that if one uses the Feynman parameters of eq. 2.5 and the formula of eq. 2.10 to evaluate the integral the divergence moves in part from the loop-momenta integrations to the integrations over the Feynman parameters. This usually makes the dimensional method very difficult to use and caution must be exercised in the face of multiloop calculations. Now as implied above not only does the overall diagram of Fig. 6 diverge but so do the various subdiagrams of which it is made up. We shall need therefore counter terms to eliminate the divergences in the subdiagrams as well as a counter term to eliminate the divergence due to the whole diagram. The subdiagrams are shown in Fig. 7(a)-(c). The subdiagrams of Fig. 7 can easily be checked to have logarithmic divergences. This reasoning leads one to believe that four counter terms are needed to eliminate these four

logarithmic divergences  
quadratic divergences  
Casual point  
Difficulties

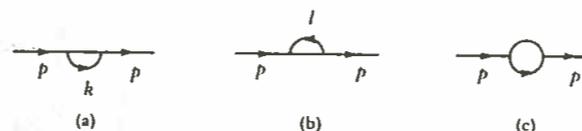


FIG. 7

sources of divergence. This is indeed the case, and is the proper way to tackle the overlapping divergence. The origins of the four subtractions that must be made from  $I$  can be isolated very simply in the dimensional method. They are as follows

- (a) a subtraction due to a divergence exposed by integrating by parts with respect to  $k$ ;
- (b) a subtraction due to a divergence exposed by integrating by parts with respect to  $l$ ;
- (c) a subtraction due to a divergence exposed by changing variables from  $l$  to  $k-l$  and integrating by parts with respect to  $k$ ;
- (d) a subtraction due to a divergence exposed by using the identity

$$1 = \frac{1}{2D} \sum_{i=1}^D \left( \frac{\partial k_i}{\partial k_i} + \frac{\partial l_i}{\partial l_i} \right) \quad (2.53)$$

in the integrand. Taking the operation described in (d) first and using the method that we have just developed we find

$$I = \frac{1}{2D-6} I_c,$$

$$I_c = + \frac{i\lambda^2}{6(2\pi)^8} \int dk dl \left[ \frac{2m^2}{[k^2 - m^2]^2 [l^2 - m^2] [(p+l-k)^2 - m^2]} \right. \\ \left. + \frac{2m^2}{[k^2 - m^2] [l^2 - m^2]^2 [(p+l-k)^2 - m^2]} \right. \\ \left. + \frac{-2p^2 + 2m^2 + 2p \cdot (k-l)}{[k^2 - m^2] [l^2 - m^2] [(p+l-k)^2 - m^2]^2} \right]. \quad (2.54)$$

When  $D=4$ ,  $2D-6=2$  verifying the presence of the quadratic divergence already mentioned. For the other three diagrams we introduce the notation of putting a cross in the subdiagram to denote the counter term. The counter terms of (a), (b) and (c) can then be

# John (later Giannis...) Bakas: the formative years

(N.B)

The idea for the overlapping divergences is the following one:

In diagrams like:  in order to deal with U.V. divergences we must put at least one of  $k$  or  $l$  tending to infinity.

∴

$\frac{l}{k}$	small	large
small	///	logarithmic
large	logarithmic	quadratic

↓  
we drop this region as  $(k \text{ small})$   
 $(l \text{ small})$   
∴ No U.V. behaviour.

∴ Result: In diagrams like this we can have both logarithmic or quadratic divergences. These are the so called overlapping divergences.

# John (later Giannis...) Bakas: the formative years

JOHN BAKAS (DIP)

ESSAY IN ALGEBRAIC TOPOLOGY

TITLE:

"TOPOLOGICAL SOLITONS IN FIELD THEORIES"

## 1. INTRODUCTION

It has been observed that non-linear field theories admit the existence of stable finite energy solutions, which are of topological origin, called topological solitons. In this essay we study their nature and classification by using methods of algebraic topology, which seems to be a very powerful tool.

For topological solitons to exist, there must be an internal degree of freedom for the fields, which gives rise to an internal field space.

A most striking stability feature of these solutions is an associated topological charge, a consequence solely of the continuity of the fields. This charge or kink number takes on a discrete set of values, which remain invariant under any continuous deformations, particularly in the course of time evolution. Thus obeying homotopic conservation laws, these kinks arise not from any symmetry of the Lagrangian, but rather from the global topology of the field manifold, a structure specified by appropriate boundary conditions. So topological charge is absolutely conserved; the field topology acts like an infinite potential barrier.

Soliton solutions can not be reached by standard perturbation theory, because they are associated with the global properties of the field manifold; these properties are unexpressed by perturbation methods, as whenever we use perturbation theory and expand about one point of the field manifold, we thereby effectively replace it by the tangent space to the

Coffee stained pages  
from his notes on a  
student seminar and/or  
DIC thesis

## John (later Giannis...) Bakas: the formative years

- Once in the States, he quickly matured to the scientist we knew and appreciated
- Utah
- Texas
- Maryland
- CERN
- Greece (Patras, Athens)



## John (later Giannis...) Bakas: the formative years

- ~1992: I met him in Crete (a soldier on leave with girl friend!)
- ~1996: Returning to Greece (first Patras, then NTU Athens) was what his heart desired
- By 2016 he had reached full scientific maturity, as an esteemed researcher and senior Greek academic in a top Institute of his homeland, enjoying international prestige
- He was also rewarded by his wife's and daughter's love, affection, and admiration
- He had a lot to give to family and Physics. It is truly sad that the Gods decided otherwise. He will always be remembered fondly by all who knew him.



# $B_K$ in the SM and Beyond

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Athens

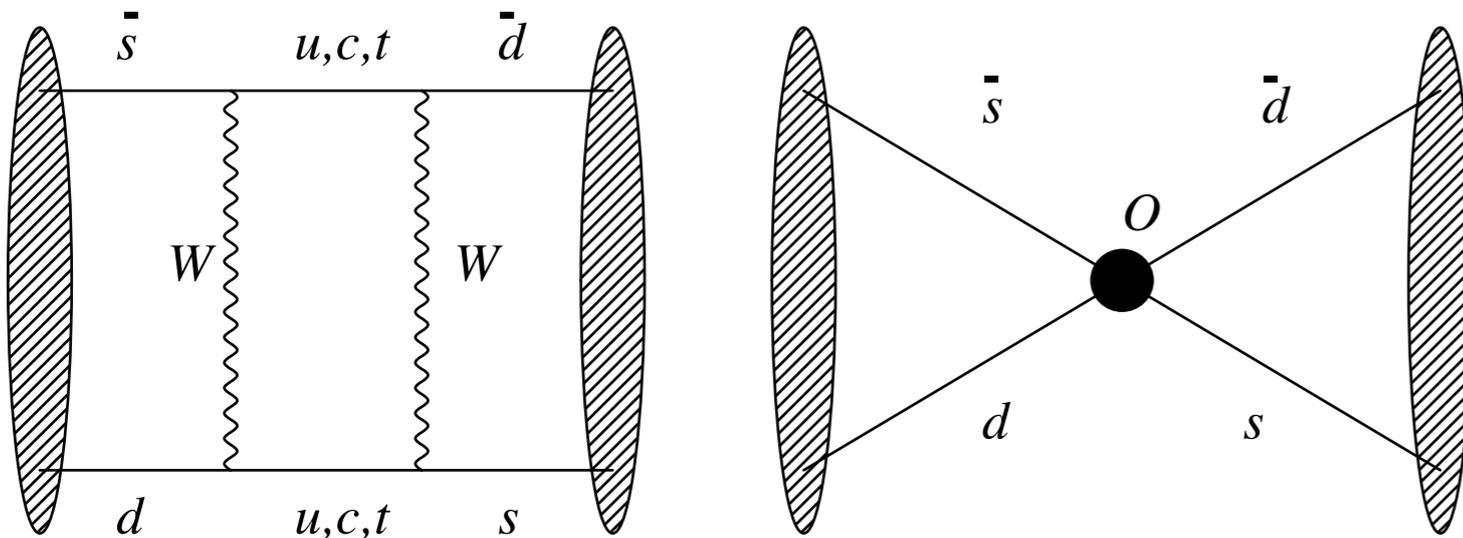
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# $B_K$ in the SM

- indirect CP-violation

$$\epsilon_K = \frac{\mathcal{A}[K_L \rightarrow (\pi\pi)_{I=0}]}{\mathcal{A}[K_S \rightarrow (\pi\pi)_{I=0}]} = [2.282(17) \times 10^{-3}] \exp(i\pi/4)$$

can also be expressed in terms of neutral K-oscillations: dominant EW process is FCNC (2-W exchange)

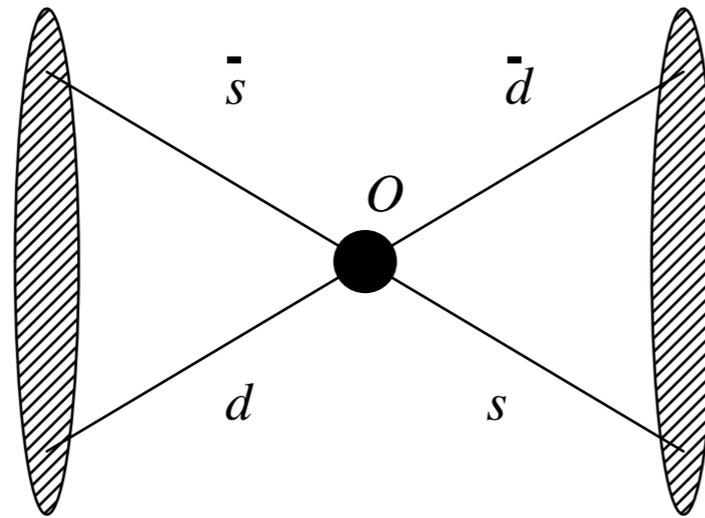
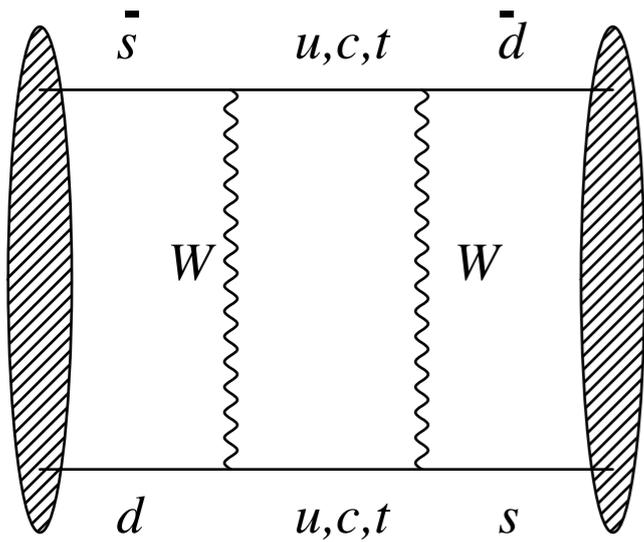


QCD effects consist in gluon and internal quark-loop exchanges (not shown here)

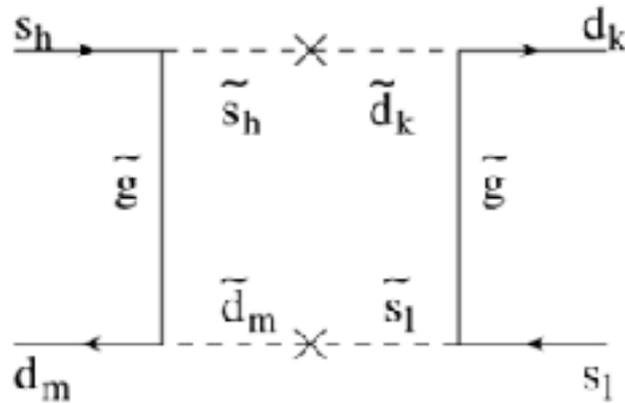
- $\Delta S = 2$  oscillations are governed by the transition amplitude of an effective Hamiltonian, obtained by successively integrating out  $W$ 's and  $t$ - (b-) and  $c$ -quarks
- We are left with an OPE with a single, dim-6, 4-fermion,  $L \otimes L$  operator, in a 3-quark approximation of QCD ( $N_f = 3$ )

$$Q^{\Delta S=2} = [\bar{s}\gamma_\mu(1 - \gamma_5)d] [\bar{s}\gamma_\mu(1 - \gamma_5)d] \equiv O_{VV+AA} - O_{VA+AV}$$

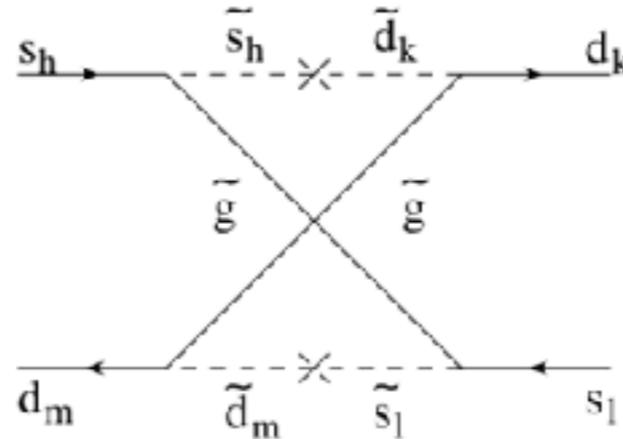
# B<sub>K</sub> BSM



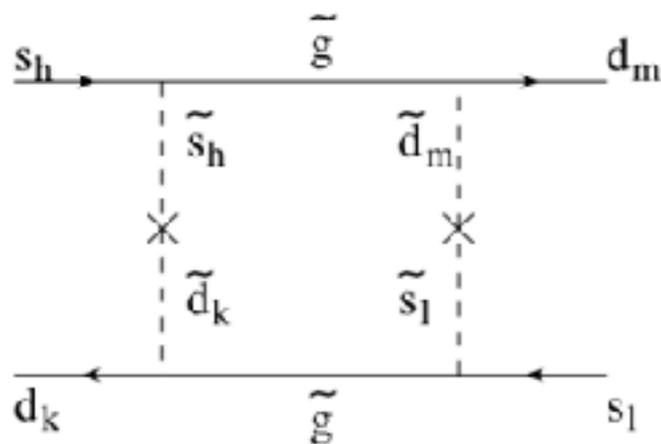
- More particles on the box diagrams
- More operators in the OPE



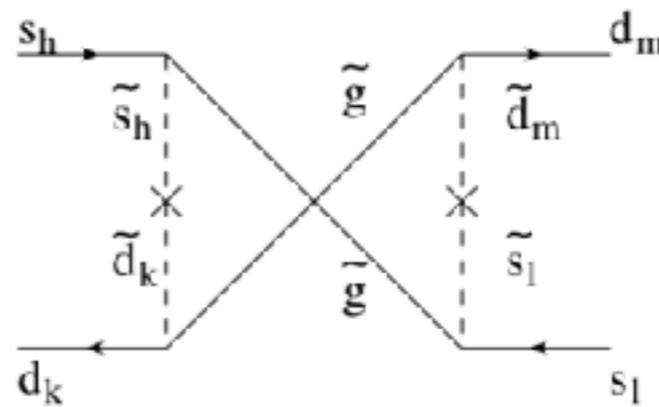
a)



c)



b)



d)

$$O_1 = [\bar{s}^\alpha \gamma_\mu (1 - \gamma_5) d^\alpha] [\bar{s}^\beta \gamma_\mu (1 - \gamma_5) d^\beta]$$

$$O_2 = [\bar{s}^\alpha (1 - \gamma_5) d^\alpha] [\bar{s}^\beta (1 - \gamma_5) d^\beta]$$

$$O_3 = [\bar{s}^\alpha (1 - \gamma_5) d^\beta] [\bar{s}^\beta (1 - \gamma_5) d^\alpha]$$

$$O_4 = [\bar{s}^\alpha (1 - \gamma_5) d^\alpha] [\bar{s}^\beta (1 + \gamma_5) d^\beta]$$

$$O_5 = [\bar{s}^\alpha (1 - \gamma_5) d^\beta] [\bar{s}^\beta (1 + \gamma_5) d^\alpha]$$

NB: MIXING of (O<sub>2</sub>, O<sub>3</sub>) & (O<sub>4</sub>, O<sub>5</sub>)

# B<sub>K</sub> in the SM

- $\Delta S = 2$  transitions are governed by the transition amplitude of the effective Hamiltonian:

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = \frac{G_F^2 M_W^2}{16\pi^2} \left[ \lambda_c^2 S_0(x_c) \eta_1 + \lambda_t^2 S_0(x_t) \eta_2 + 2\lambda_c \lambda_t S_0(x_c, x_t) \eta_3 \right]$$

$$\times \left( \frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[ \frac{\beta_1 \gamma_0 - \beta_0 \gamma_1}{2\beta_0^2} \right] \right\} \langle \bar{K}^0 | Q_R^{\Delta S=2}(\mu) | K^0 \rangle + \text{h.c.}$$

$$x_{c,t} \equiv m_{c,t}^2 / M_W^2 \qquad \lambda_f = V_{fs}^* V_{fd} \qquad f = c, t$$

Inami-Lim functions:  $S_0(x_{c,t})$        $S_0(x_c, x_t)$

# $B_K$ in the SM

- $\Delta S = 2$  transitions are governed by the transition amplitude of the effective Hamiltonian:

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$$\times \left( \frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[ \frac{\beta_1 \gamma_0 - \beta_0 \gamma_1}{2\beta_0^2} \right] \right\} \langle \bar{K}^0 | Q_R^{\Delta S=2}(\mu) | K^0 \rangle + \text{h.c.}$$

- Wilson coefficient
- Known to NLO in PT; afflicted by PT errors
- The higher the renormalisation scale  $\mu$ , the more reliable PT (typically chose 2-4 GeV)

# $B_K$ in the SM

- $\Delta S = 2$  transitions are governed by the transition amplitude of the effective Hamiltonian:

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$$\times \left( \frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[ \frac{\beta_1 \gamma_0 - \beta_0 \gamma_1}{2\beta_0^2} \right] \right\} \langle \bar{K}^0 | Q_R^{\Delta S=2}(\mu) | K^0 \rangle + \text{h.c.}$$

- Matrix element is computed on the lattice through its  $B_K$ -parameter; afflicted by usual errors
- Habitually this is renormalised either:
  - in 1-loop PT (lattice regularisation  $\rightarrow$  MS renormalisation)
  - NPLy (lattice regularisation  $\rightarrow$  MOM-subtraction renormalisation at 2-4 GeV  $\rightarrow$  MS renormalisation with finite matching)
- Often normalised by a constant  $\rightarrow \rightarrow \rightarrow$  “B-parameter”

$$B_K(\mu) = \frac{\langle \bar{K}^0 | Q_R^{\Delta S=2}(\mu) | K^0 \rangle}{\frac{8}{3} f_K^2 m_K^2}$$

# $B_K$ in the SM

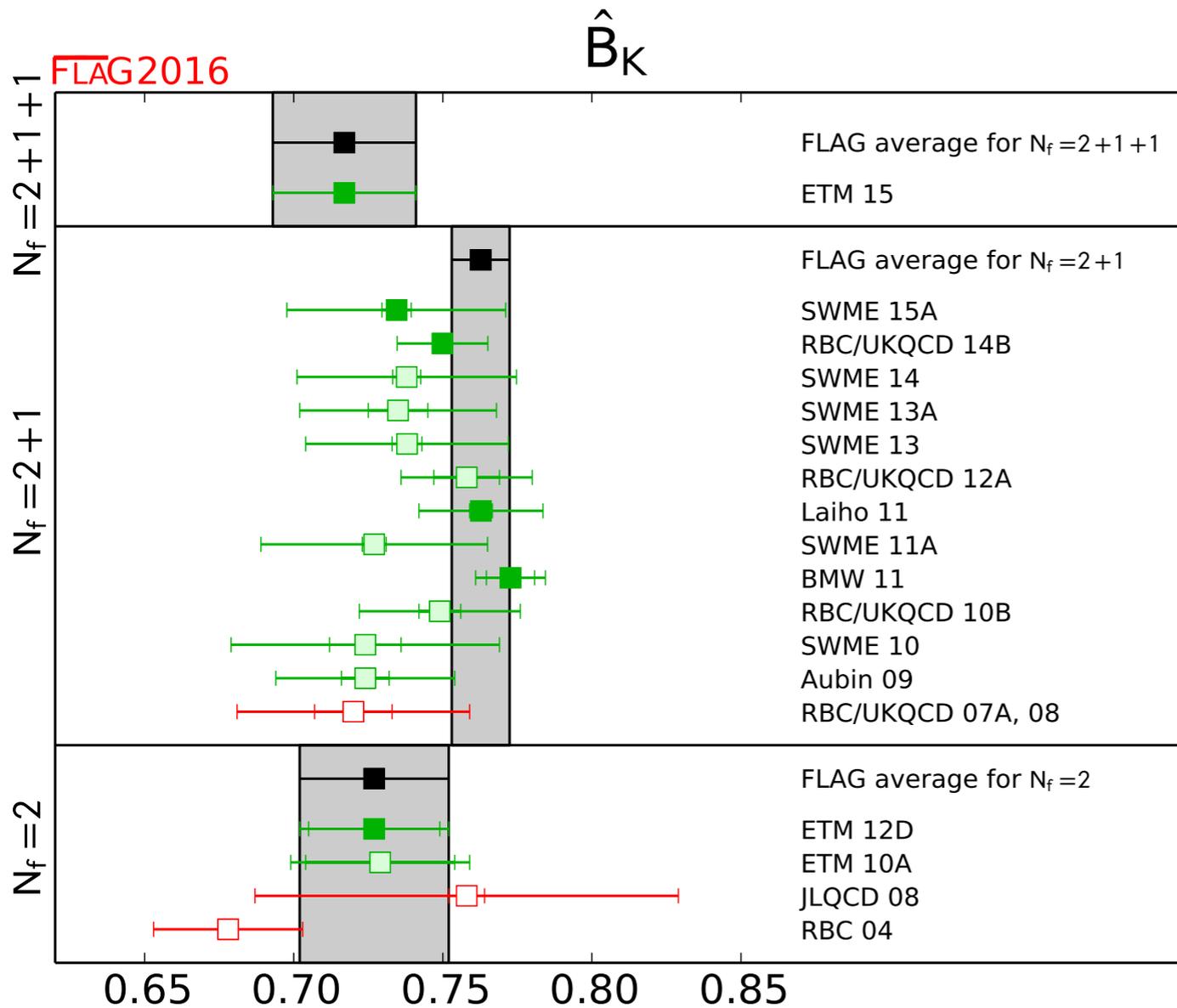
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$$\times \left( \frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[ \frac{\beta_1 \gamma_0 - \beta_0 \gamma_1}{2\beta_0^2} \right] \right\} \langle \bar{K}^0 | Q_R^{\Delta S=2}(\mu) | K^0 \rangle + \text{h.c.}$$

- The product is only nominally  $\mu$ -independent; there is residual  $\mu$ -dependence.
- Alpha-collaboration has a long-term programme of NP renormalisation at scales  $\sim \Lambda_{\text{QCD}}$  and NP-running (continuum) up to scales  $\sim M_W$ ; so far applied at  $N_f=0$

# $B_K$ in the SM



- Different lattice regularisations and renormalisation schemes give compatible, even  $N_f$ -weakly -dependent, results

- Lattice error subdominant in  $\epsilon_K$  (1.6%); dominant error arises from  $V_{cb}$  (40%)

$$\hat{B}_K = 0.717(18)(16)$$

$$\hat{B}_K = 0.7625(97)$$

$$\hat{B}_K = 0.727(22)(12)$$

$$N_f = 2 + 1 + 1$$

$$N_f = 2 + 1$$

$$N_f = 2$$

# $B_K$ beyond the SM

- Analyse New Physics (NP) effects in a model-independent way: assume a generalisation of the effective  $\Delta S = 2$  Hamiltonian which contains operators beyond the SM one; the amplitude is:

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = C_1 \langle \bar{K}^0 | O_1 | K^0 \rangle + \sum_{i=2}^5 C_i \langle \bar{K}^0 | O_i | K^0 \rangle$$

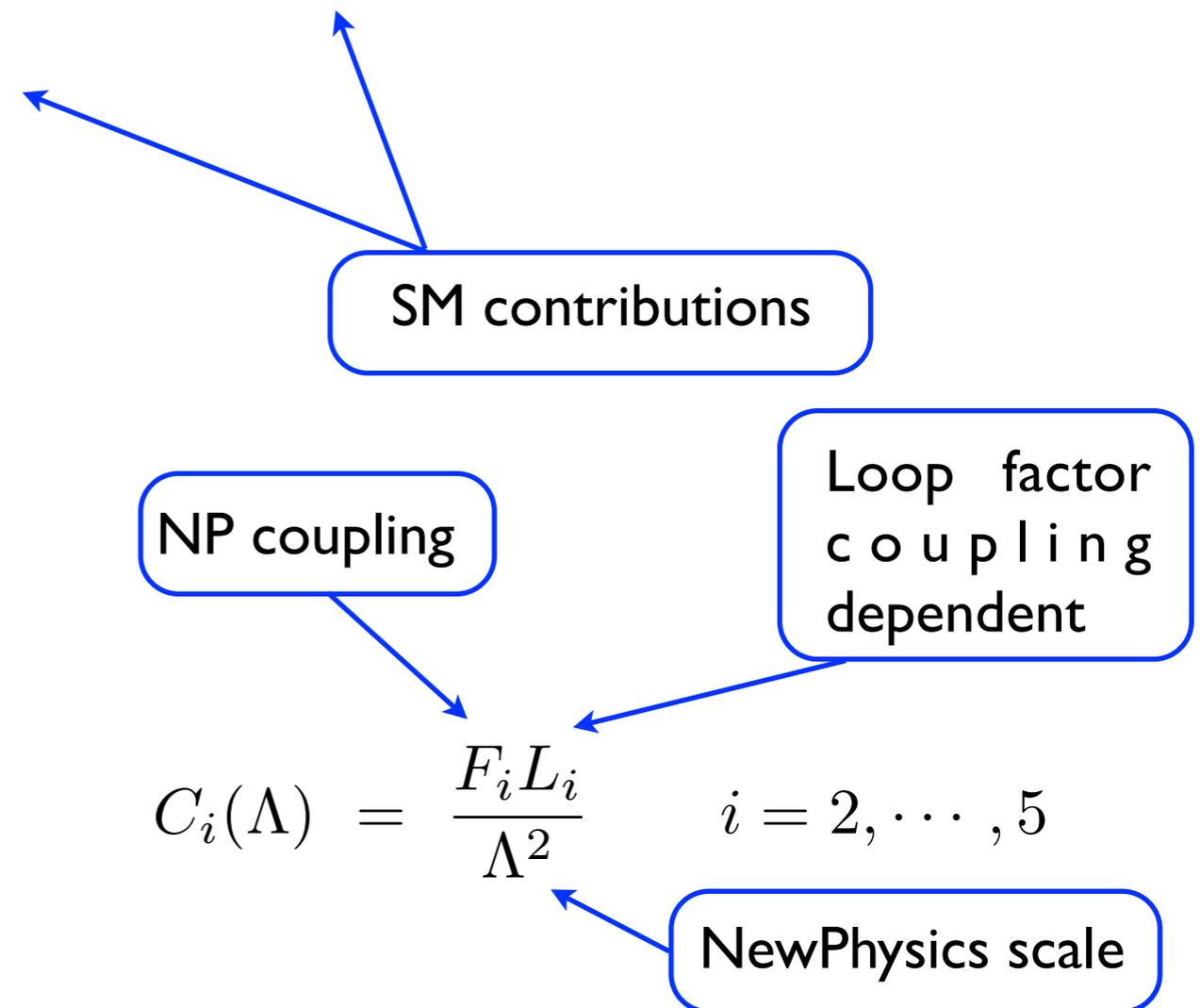
$$O_1 = [\bar{s}^\alpha \gamma_\mu (1 - \gamma_5) d^\alpha] [\bar{s}^\beta \gamma_\mu (1 - \gamma_5) d^\beta]$$

$$O_2 = [\bar{s}^\alpha (1 - \gamma_5) d^\alpha] [\bar{s}^\beta (1 - \gamma_5) d^\beta]$$

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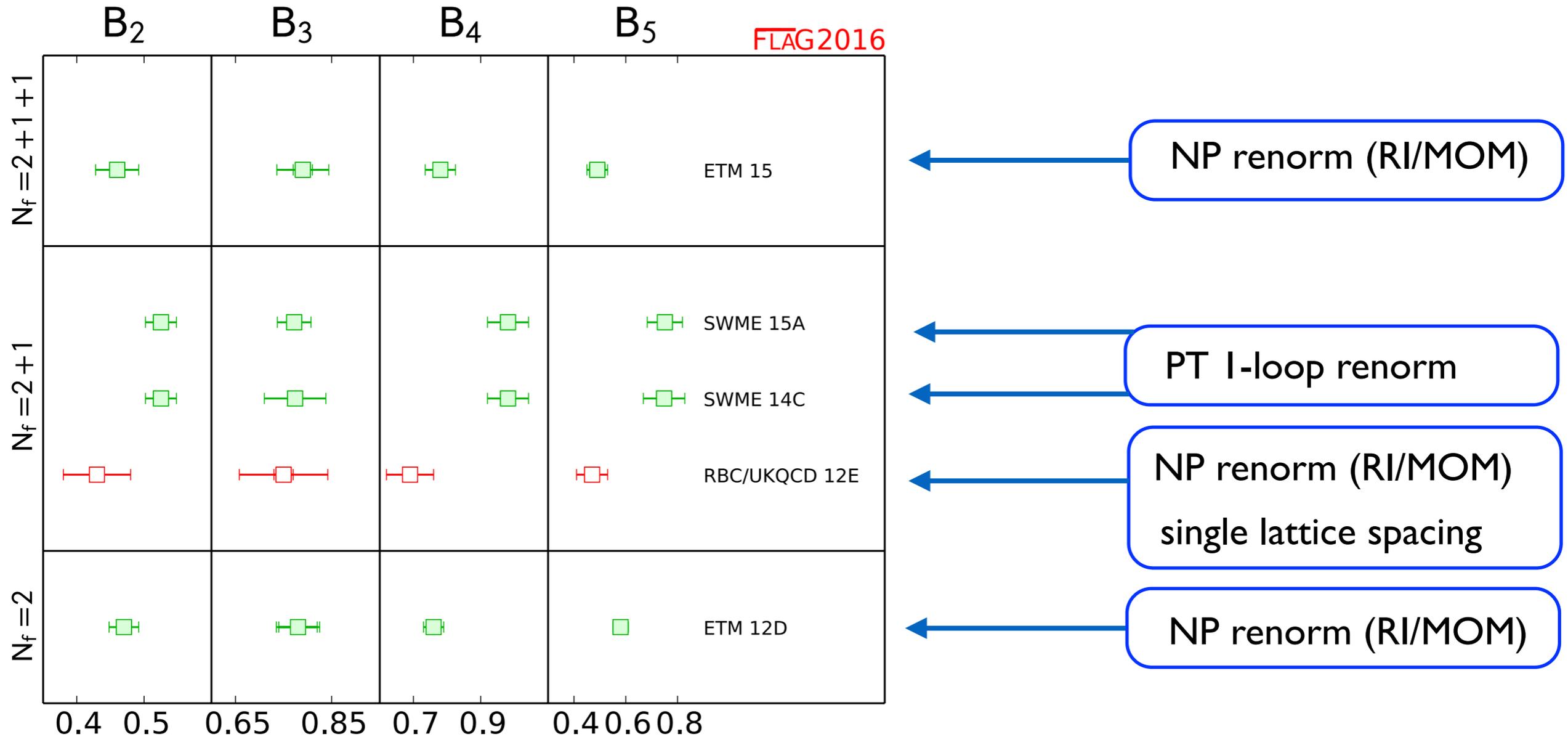
$$O_4 = [\bar{s}^\alpha (1 - \gamma_5) d^\alpha] [\bar{s}^\beta (1 + \gamma_5) d^\beta]$$

$$O_5 = [\bar{s}^\alpha (1 - \gamma_5) d^\beta] [\bar{s}^\beta (1 + \gamma_5) d^\alpha]$$



- $B$ -parameters defined analogously for all operators
- Assuming  $F_i \sim L_i \sim 1$ , generalised UT-fit analysis produces lower bounds for  $\Lambda$ ; these depend very strongly (several orders of magnitude) on this assumption.

# $B_K$ beyond the SM



- RBC/UKQCD 2017: new results with 2 RI/SMOM schemes (at 2 lattice spacings) agree with SWME; authors suggest that RI/MOM may be the culprit for these operators
- ALPHA results (with SF renormalisation scheme) and NP-running are badly needed

# $B_K$ in the SM

- $\Delta S = 2$  transitions are governed by the transition amplitude of the effective Hamiltonian:

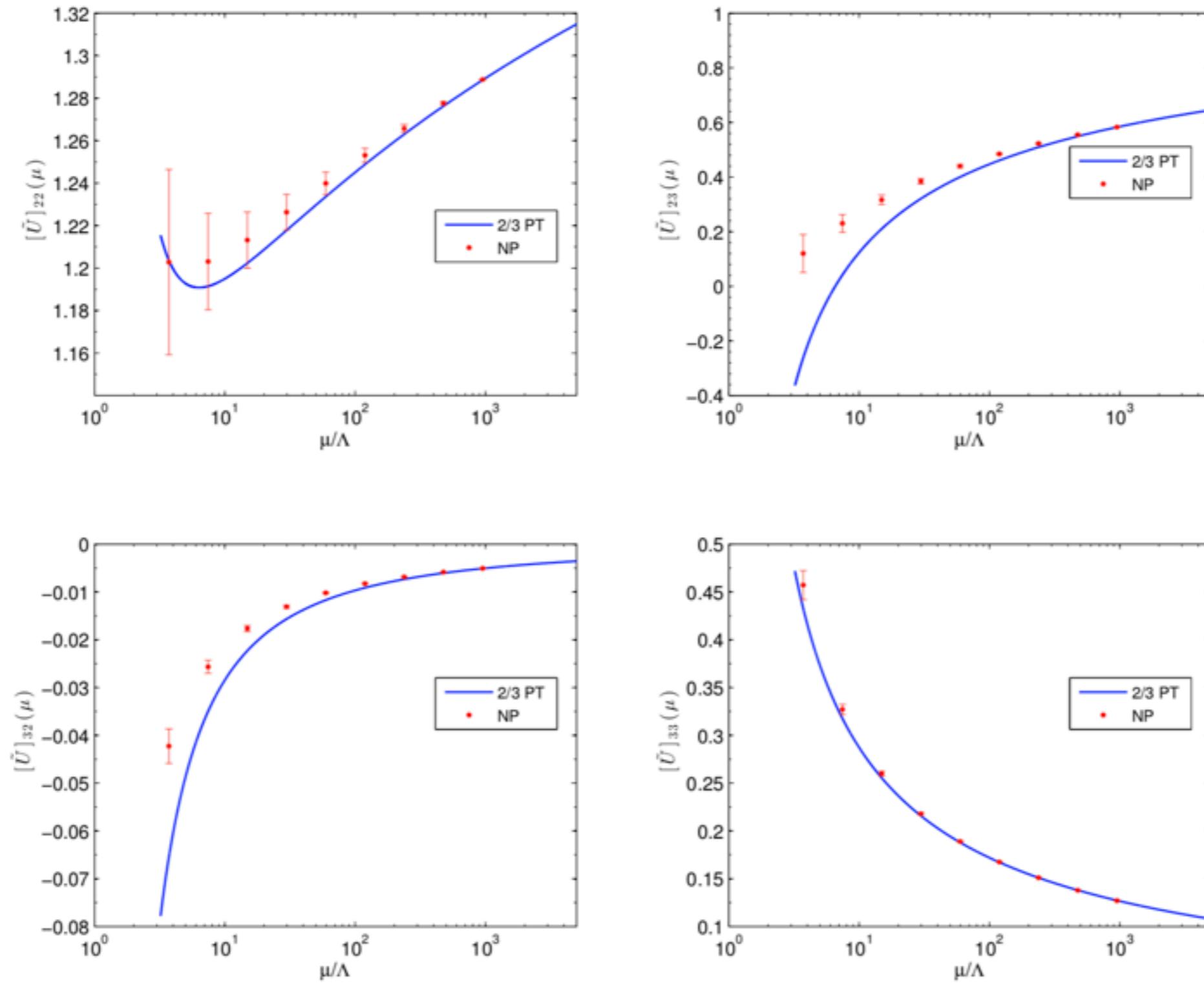
$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = \frac{G_F^2 M_W^2}{16\pi^2} \left[ \lambda_c^2 S_0(x_c) \eta_1 + \lambda_t^2 S_0(x_t) \eta_2 + 2\lambda_c \lambda_t S_0(x_c, x_t) \eta_3 \right]$$

$$\times \left( \frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[ \frac{\beta_1 \gamma_0 - \beta_0 \gamma_1}{2\beta_0^2} \right] \right\} \langle \bar{K}^0 | Q_R^{\Delta S=2}(\mu) | K^0 \rangle + \text{h.c.}$$

- The product is only nominally  $\mu$ -independent; there is residual  $\mu$ -dependence.
- Alpha-collaboration has a long-term programme of NP renormalisation at scales  $\sim \Lambda_{\text{QCD}}$  and NP-running (continuum) up to scales  $\sim M_W$ ; so far applied at  $N_f=0$
- In the late '90s the Schroedinger Functional scheme was introduced and results were obtained for the QCD coupling and quark masses (albeit in the quenched approximation)
- Generalised to BK (quenched) in the early 00's
- Generalised to QCD coupling and quark masses ( $N_f=2$ ) in the early 00's
- Generalised to BK - SM and beyond ( $N_f=2,3$ ) nowadays

# $B_K$ beyond the SM

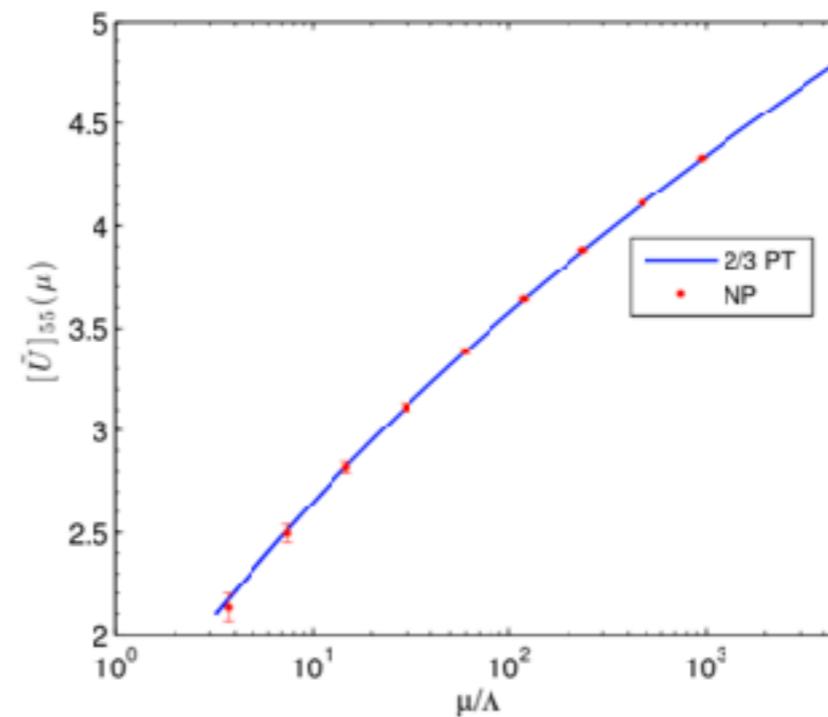
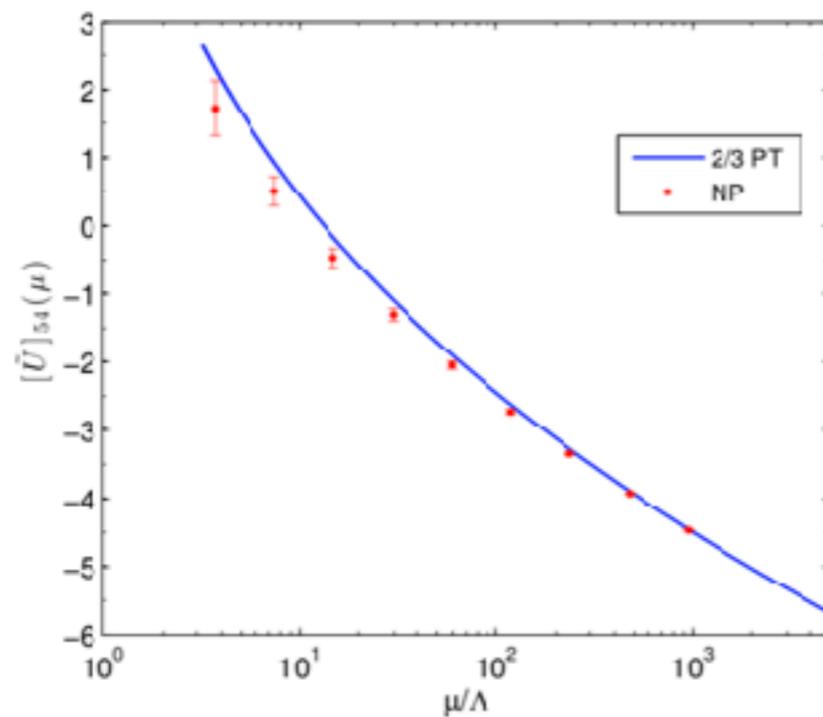
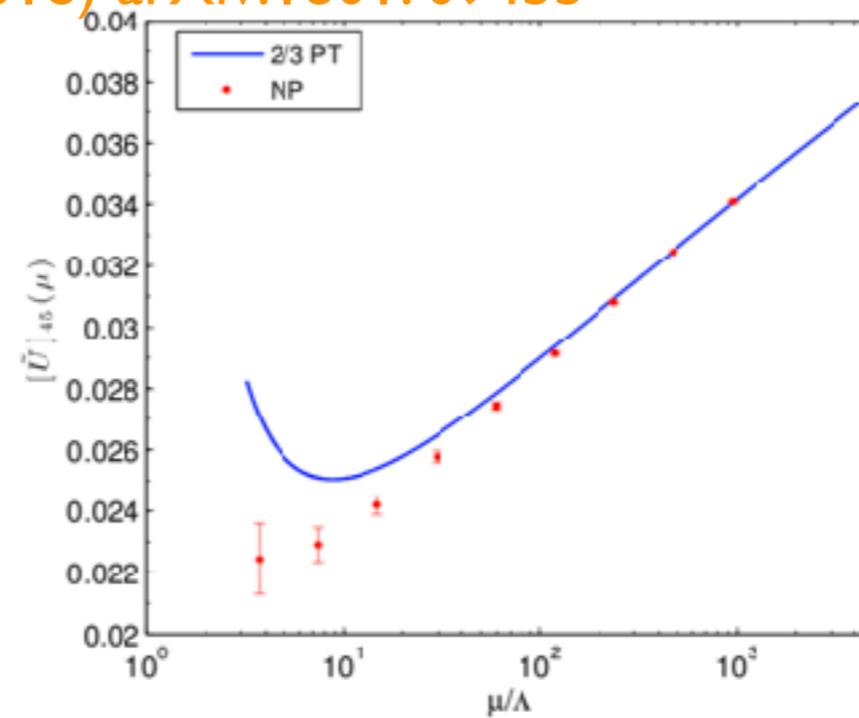
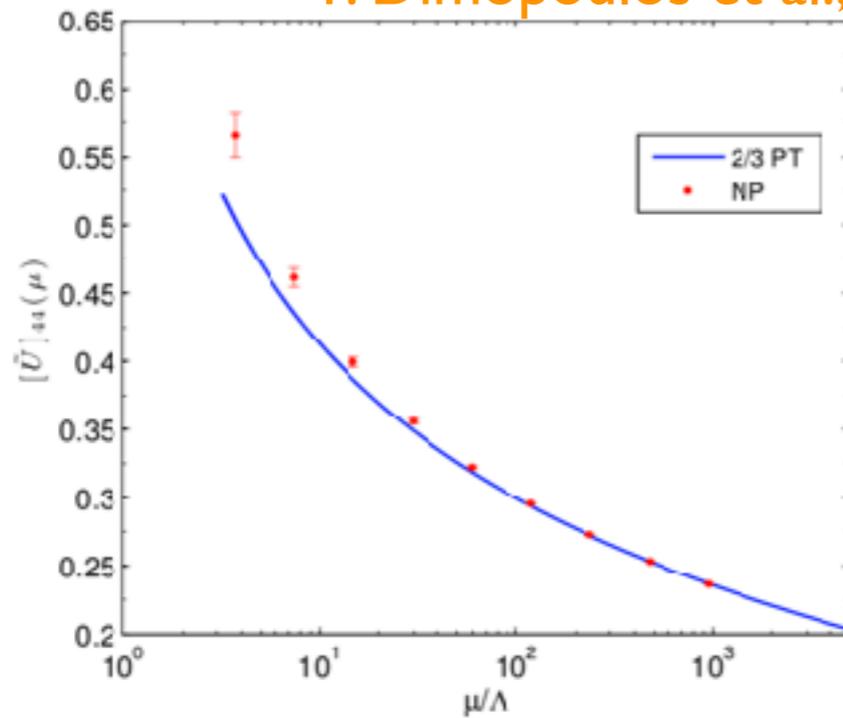
P. Dimopoulos et al., (2018) arXiv:1801.09455



- Alpha 2018 warm-up exercise: RG-running for operators  $O_2$  and  $O_3$  for  $N_f = 2$

# $B_K$ beyond the SM

P. Dimopoulos et al., (2018) arXiv:1801.09455



- Alpha 2018 warm-up exercise: RG-running for operators  $O_4$  and  $O_5$  for  $N_f = 2$

# Conclusions

- Lattice nowadays competes with the accuracy of experiments (in recent years we moved from 5% to 1%-2%).
- SM results are in many cases well under control; activity beyond SM.
- It is the responsibility of the lattice community to provide experimentalists and non-lattice theorists with a review of phenomenologically relevant lattice results with conservative error estimates (FLAG), not only for the SM but also beyond.
- **2011:** G. Colangelo et al., “Review of Lattice Results Concerning Low-Energy Particle Physics”, Eur. Phys. J. C 71 (2011) 1695
- **2014:** S. Aoki et al., “Review of Lattice Results Concerning Low-Energy Particle Physics”, Eur. Phys. J. C 74 (2014) 2890
- **2016:** S. Aoki et al., “Review of Lattice Results Concerning Low-Energy Particle Physics”,

# Backup pages

# $B_K$ beyond the SM

- Analyse New Physics (NP) effects in a model-independent way: assume a generalisation of the effective  $\Delta S = 2$  Hamiltonian which contains operators beyond the SM one; the amplitude is:

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = C_1 \langle \bar{K}^0 | O_1 | K^0 \rangle + \sum_{i=2}^5 C_i \langle \bar{K}^0 | O_i | K^0 \rangle$$

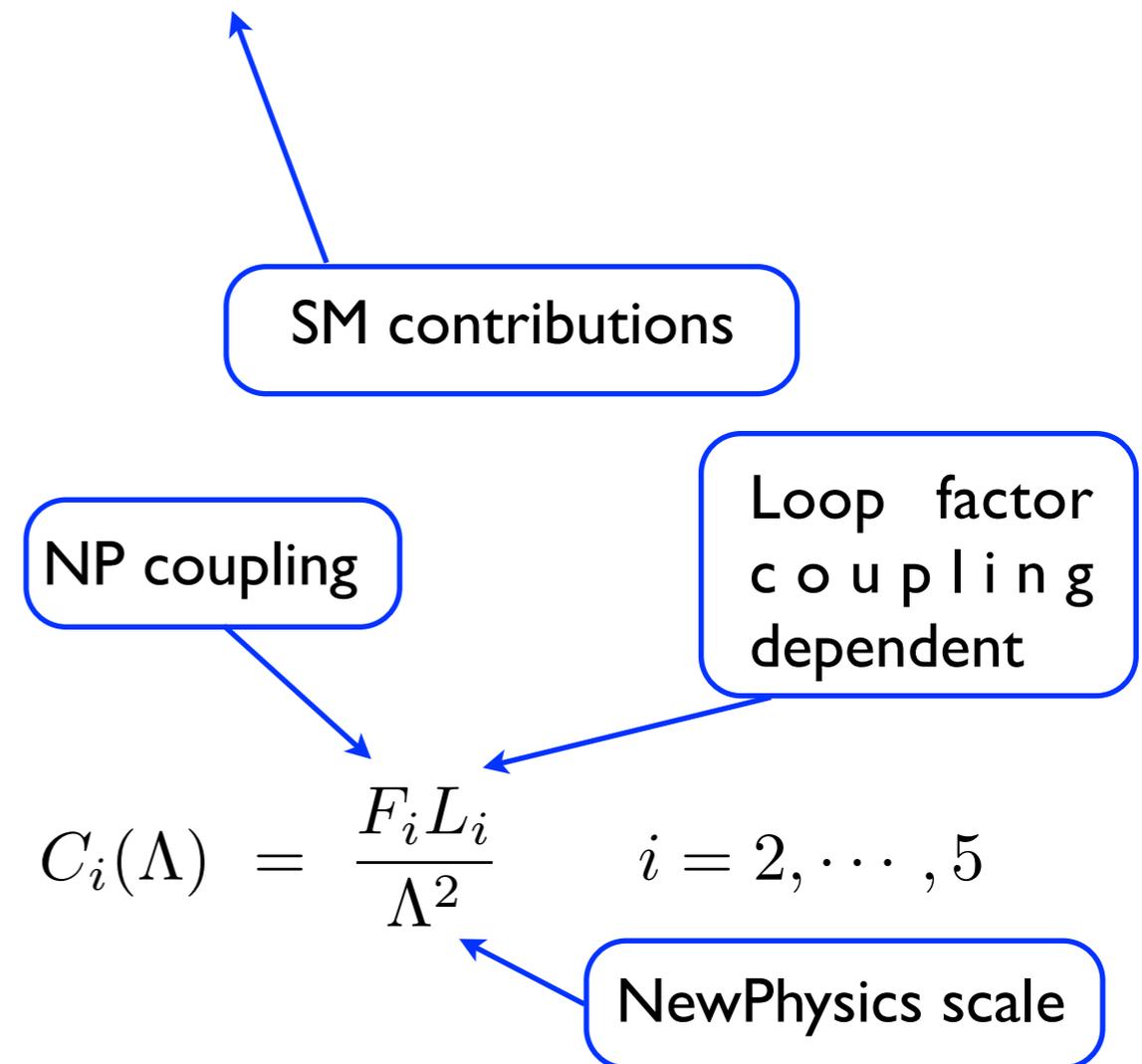
- Minimal Flavour Violation (MFV) models:

$$F_1 = F_{\text{SM}}; F_{2,3,4,5} = 0$$

- Assuming  $L_i \sim 1$ , corresponds to strongly-interacting and/or tree-level coupled New Physics

- gluino exchange in the minimal supersymmetric SM  $L_i \sim \alpha_s^2$

- all models with SM-like loop-mediated weak interactions  $L_i \sim \alpha_W^2$



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NP coupling

Loop factor  
coupling  
dependent

$$C_i(\Lambda) = \frac{F_i L_i}{\Lambda^2} \quad i = 2, \dots, 5$$

NewPhysics scale

In particular, in the Randall-Sundrum (RS) scenario one has

$$L_4 = (g_s^*)^2, \quad F_4 = \frac{2m_d m_s}{Y_*^2 v^2}, \quad \Lambda = M_G, \quad (3.2)$$

where  $M_G$  and  $g_s^* \sim 6$  are the mass and coupling of Kaluza-Klein excitations of the gluon,  $Y_* \sim 3$  is the five-dimensional Yukawa coupling (whose flavour structure is assumed to be anarchic),  $m_d \sim 3 \text{ MeV}$  and  $m_s \sim 50 \text{ MeV}$  are  $\overline{\text{MS}}$  quark masses at the high scale and  $v = 246 \text{ GeV}$  is the Higgs vev. Running from a reference scale of  $5 \text{ TeV}$ , we obtain at 95% probability  $\text{Im } C_4^K \in [-4.7, 10.6] \cdot 10^{-18}$ , from which we get

$$M_G > 43 \text{ TeV}. \quad (3.3)$$

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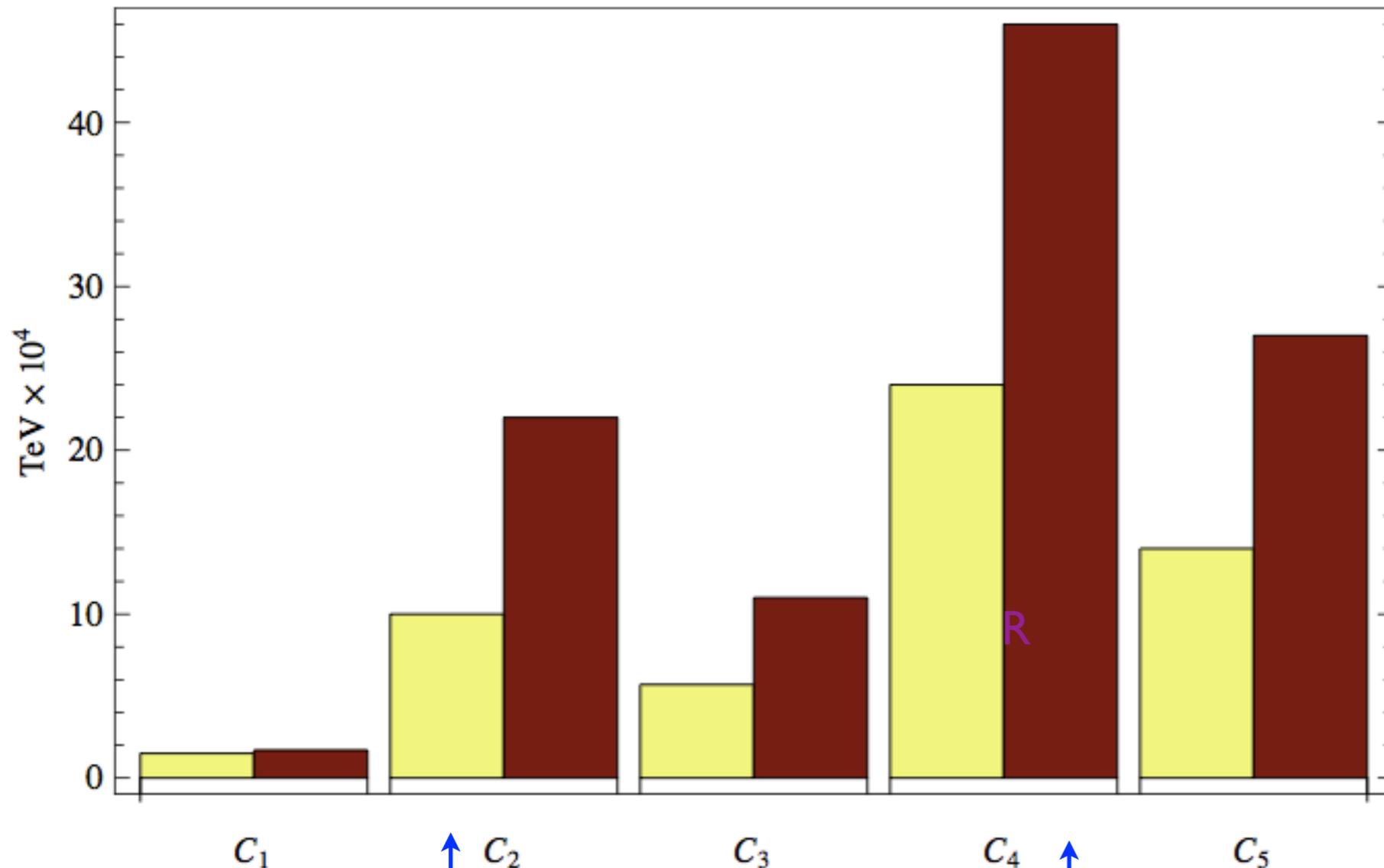
Considering instead gauge-Higgs unification (GHU) models, from ref. [26] we have

$$L_4 = (g_s^*)^2, \quad F_4 \sim \frac{8m_d m_s}{g_*^2 v^2}, \quad \Lambda = M_G, \quad (3.4)$$

where in this case  $g_* \sim 4$  is the five-dimensional gauge coupling in units of the radius of the compact dimension. We obtain the bound

$$M_G > 65 \text{ TeV}. \quad (3.5)$$

# $B_K$ beyond the SM



$$R_i = \frac{\langle \bar{K}^0 | O_i | K^0 \rangle}{\langle \bar{K}^0 | O_1 | K^0 \rangle} \quad i = 2, \dots, 5$$

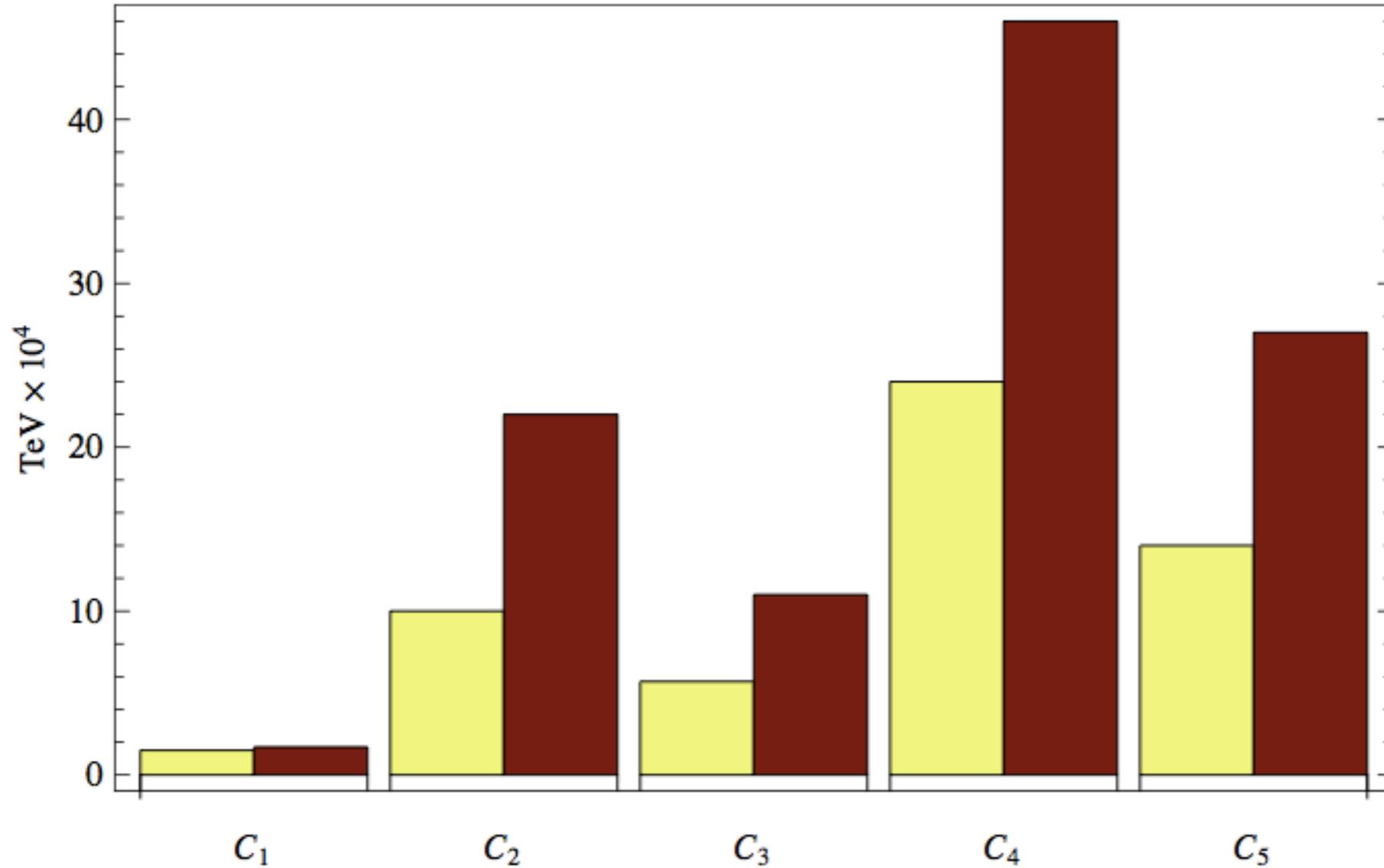
$N_f=0$  data; accuracy of ratios  $R_i \sim 20\%-23\%$

UTfit: M.Bona et al., JHEP03(2008)049

$N_f=2$  data; accuracy of ratios  $R_i \sim 3\%-6\%$

ETM: V.Bertone et al., JHEP03(2013)089

# $B_K$ beyond the SM



- NB: each contribution analysed separately (avoids accidental cancellations).
- NB: SM bound is several orders of magnitude weaker than those arising from BSM operators.